TURNING NORTON'S DOME AGAINST MATERIAL INDUCTION

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John Norton has proposed a position of "material induction" that denies the existence of a universal inductive inference schema behind scientific reasoning. In this vein, Norton has recently presented a "dome scenario" based on Newtonian physics that, in his understanding, is at variance with Bayesianism. The present note points out that a closer analysis of the dome scenario reveals incompatibilities with material inductivism itself.

1: Introduction

In a number of recent papers (2003, 2007, 2010, 2013), John Norton has developed a position of "material induction" that denies the existence of a universal inductive inference schema behind scientific reasoning. Norton argues that schemas which have been developed to date in order to provide a systematic foundation for all forms of inductive inference in science always suffer from either of two fatal problems: they are too general to enforce the necessary restrictions on inductive praxis or they fail to fulfil their claim of universality by being inapplicable to some examples of inductive inference in science. Norton draws the conclusion that the hierarchies should be inverted. Rather than understand the principle of induction as a meta-scientific principle that defines the basic character of all allowed inference patterns in science, he suggests viewing inductive inference schemas in science as being exhaustively determined by scientists' knowledge about the given context of scientific reasoning. All viable instances of inductive inference on that view are licensed by valid scientific theories and/or by other facts regarding the context of scientific research (like the quality or trustworthiness of involved scientists, etc). Each instance of inductive inference may in principle be withdrawn or altered based on the acceptance of a new framework of scientific theories or new information about more general facts. Norton most succinctly formulates his position in (Norton 2003), p 650: *All inductions ultimately derive their licenses from facts pertinent to the matter of the induction.* No room thus remains for a universal principle of induction that can be formulated independently from the scientific status quo. And no principle of induction that does not derive its license from physical background knowledge plays any role in modern science.

Beyond suggesting a new take on the character of inductive reasoning in science, the material theory of induction also affects the understanding of the Humean problem of induction. While Norton concedes that the principle of material induction does not offer an outright solution to the Humean problem, he claims that, at any rate, it makes the latter much more difficult to formulate: if present day reasoning is fully justifiable based on scientific background knowledge, the elements of genuine Humean induction retreat to the archaic construction periods of human world views, where they are notoriously difficult to grasp.

The currently most prominent conception that proposes one universal schema for inductive reasoning and therefore is rejected by the material theory of induction is Bayesianism. Bayesianism asserts that all scientific reasoning is based on attributing a

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probability to a theory's or a statement's truth. That probability measures the degree of belief a scientist has in the given theory or prediction. Experiments on this account alter a theory's or statement's probability according to the following formula:

$$P(H/E) = \frac{P(E/H)}{P(E)} P(H),$$

where P(H) is the prior probability of the hypothesis H before the empirical data E has been considered; P(E) is the probability of the empirical data E regardless of whether H is true; P(H/E) is the probability for H when E has been taken into account; and P(E/H) is the probability for E given that H is true. Probabilities according to this schema can only be assigned based on prior probabilities attributed to theories and empirical data, which themselves must either be posited or assessed based on another Bayesian procedure. No algorithm exists for determining initial probabilities. According to the Bayesian, this does not constitute a serious problem for Bayesianism because repeated empirical tests tend to create 'fixed points' for probabilities that are derived from a wide range of early prior probabilities. Probabilities in mature science are thus taken to be fairly well 'decoupled' from the probabilities attributed at early stages of the scientific development.[2]

In (2007, 2010), Norton argues against Bayesianism by suggesting that specific forms of inductive reasoning in science do not conform to the schema laid out by Bayesianism. He exemplifies his claim in the context of classical Newtonian physics. The latter constitutes a prime example of a scientific theory that is well-confirmed within certain limits. In order to amount to a thought experiment on inductive inference from empirical data, Norton's thought experiment must rely on the assumption that Newtonian physics is consistent with the available data. To put it in Bayesian terms: in Norton's thought experiment, empirical data leads the Bayesian to the attribution of a high probability to the truth of Newtonian physics based on a wide spectrum of priors. On that basis, Newtonian physics should be expected to provide a viable basis for applying Bayesian principles of inductive inference if the latter constitute general characteristics of scientific reasoning. Norton now points out that Newtonian physics allows for various instances of indeterminacy which cannot be grasped based on probabilistic statements and therefore imply inference schemes which are not covered by Bayesianism.

The example he discusses most extensively is a circular symmetric dome with a height

$$h= (2/3g)r^{3/2},$$

where r is the radial distance along the surface, and g the gravitational acceleration.[3] Let t denote the elapsed time between the moment when the ball is put on top of the dome and the last moment before the ball starts moving downwards.[4] If the system is assumed to be frictionless, the Newtonian equations of motion for a point-like massive 'ball' put at the dome's apex turn out to have solutions for any value of t. There is also the solution in which the ball stays on top forever. In other words, Newtonian physics tells us that the ball can slide down but it leaves undetermined whether and when it actually does so. The theory does not even give a probabilistic prediction with respect to the latter question. It states a possibility

[2] Arguments against the universal viability of this understanding have been presented recently by (Belot 2013)

[3] For a full presentation of the dome scenario, see (Norton 2007a).

[4] There is a last point in time when the ball is at rest rather than a first point when it is moving. (Zinkernagel 2010).

without providing any probability for that possibility to be instantiated.[5] Norton argues that a scientist who believes that Newtonian physics is true has to believe in a possibility without assuming a probability for the ball's movement. This, according to Norton, contradicts the Bayesian understanding that "the probability calculus is adequate as the universal logic of induction" (Norton 2010, p769).

A number of recent papers have addressed the question of the viability of the dome and similar scenarios as genuine examples of physical indeterminacy. (Malament 2008, Korolev 2007, Wilson 2009, Zinkernagel 2010, Fletcher 2012, Laraudogoitia 2013). The present note has a different agenda. It assumes that Norton's dome constitutes a good example of indeterminacy and analyses its implications for Norton's view of induction on that basis. It shall be argued that the example actually is at variance with a rigid understanding of material induction:[6] Norton's dome itself can provide a particularly clear example of a case of induction that does not derive its license *from facts pertinent to the matter of the induction.*

2: The Argument against Material Induction

Before entering the discussion of the dome's implications for material induction, we have to make some qualifications. The dome is an idealized model that does not conform to the world as we know it. It is based on pure Newtonian physics, neglecting friction and quantum effects. Moreover, the initial conditions which lead to the indeterminacy phenomenon have measure zero in the space of all possible initial conditions even when only a finite spatial region within which the dome is situated is considered. But the contingent fact that, at the present time, we do not know of scientific models for which the specific form of indeterminism represented by the dome scenario is relevant for the actual analysis of empirical data does not affect the point that our methodology of scientific reasoning about the world does allow for non-probabilistic claims.

In order to understand the problem faced by material induction in the given context, it is important to distinguish two parts of the dome scenario: a theoretical part that consists in everything Newtonian physics has to say about the dome scenario; and an empirical part that is represented by some possible empirical test of the dome system's properties. It is essential to take the empirical part into consideration since only the understanding as to how empirical data can affect a theory's validity makes that theory a genuine element of natural science. In the following, we will thus treat the dome example as a thought experiment that includes both the scientific model and its empirical testing. To that end, we will assume that it is possible for an "idealized experimentalist" to create the initial conditions for indeterminacy.

In (2007, 2010), Norton only addresses the dome scenario's theoretical part. Both his criticism of Bayesianism as well as his assessment of material induction are discussed with respect to the theoretical part of the dome scenario. Norton can rightly point out that material induction nicely accounts for the dome at that level. Rather than suggesting any universal inference schema, the material inductivist asserts that science fully determines the inference schemas. That position is neatly exemplified by the dome scenario. Applied to the dome,

[5] Intuitively, the situation can be grasped more easily by looking at the time-reversed system. The system then has solutions where a ball sliding upwards stops exactly at the apex after a finite period of time.

[6] By "rigid material induction" I mean the understanding that *all* viable instances of inductive inference are licensed by valid scientific theories and/or by other facts regarding the context of scientific research. The weaker idea than *most* such instances are licensed in that way is not affected by the presented discussion of the dome scenario but does not amount to a universal theory of scientific induction.

Newtonian physics determines the ball's possible behaviour but does not offer any probabilities with respect to the time a ball spends on the dome's apex before it starts sliding down. The fact that inductive inference in the given case leads to a statement on potentiality without stating a probability is indeed fully determined by the scientific theory itself.

Let us now go beyond Norton's analysis by turning to the scenario's second part, to a possible empirical testing of the dome-ball system's behaviour. We assume that an idealized experimentalist tests the dome's properties by building a hundred domes, putting balls on top of each of them and measuring the times t when each of these balls start sliding. We have already noted that the scientific status quo does not attribute prior probabilities to any potential outcome of the sliding ball experiment.

In order to understand the material inductivist's position in this case, we have to look closer at the experiment's possible outcomes. At first glance, it may seem that, in view of what has been said in section one, Newtonian mechanics predicts the absence of any stable regularity pattern or definite shape of distribution with respect to t. It is easy to understand, however, that this is not correct. The system's indeterminacy implies that Newtonian physics cannot be refuted by any empirical data on the times when balls start sliding from the dome's apex. Any imaginable distribution pattern of starting times for sliding is compatible with Newtonian mechanics. If all hundred balls in the experiment above start sliding exactly after 16,8 seconds, this does not contradict the theory any more than any other experimental outcome. It is true, of course, that Newtonian mechanics has nothing to say about the outcome of 101^{st} experiment even after one hundred experiments have been carried out with identical results. That, however, is exactly the point where the dome turns against material induction.

Let us imagine that the scientist doing the dome experiment indeed measures 100 times out of 100 experimental runs t=16,8s. The rigid material inductivist must adhere to the following line of reasoning in that case.

- Inductive inference in science is always and entirely licensed by the scientists' knowledge about the world.[7]
- The description of the dome example relies exclusively on Newtonian physics.
- Newtonian physics remains completely silent with respect to regularity patterns of the times t when the balls start sliding from the top of the dome.

=> The empirical data collected on that matter can not be used for inductive reasoning.

=> The experimenter who sees a hundred balls start sliding consistently after 16,8 seconds thus is not licensed to make an inference from that observation to the next ball's behaviour.

A conclusion along these lines would not be logically invalid. However, to refrain from predictions is quite obviously no advice a scientist in the given situation would follow. Since science aims at providing predictions, and a specific kind of prediction must look strikingly obvious for any observer in the given situation and state of knowledge, an experimenter would say something like this: the regularity pattern shown by experiment seems to justify the prediction that the next balls in the same experimental setting will start sliding after 16,8 seconds; our present scientific knowledge is not at variance with this regularity but does not give us the slightest clue as to why it occurs. In other words, it seems that the experimenter would apply a scheme of inductive inference that is not based on her

[7] This includes, as will be discussed below, knowledge about the context of scientific research.

scientific knowledge about the world.[8] In the given case, material induction thus does not provide an adequate characterization of the way scientific reasoning would proceed.

Let me once more emphasise an important distinction here. Since induction is no matter of logical deduction, the problem with the 16,8 seconds scenario is not that adhering to material induction in the given case would be logically inconsistent. But refraining from inductive inference in a case as clear-cut as the 16,8 seconds scenario would be clearly at variance with the way a scientist would have to deal with the situation in order to be acknowledged by the scientific community as behaving in accordance with principles of scientific behaviour.[9]

Norton responds to this thought experiment the following way (email communication):

When a scientist experimenting on a dome finds this result of motion at 16.8 seconds in one hundred of one hundred trials, the scientist would likely conclude that the background Newtonian theory is either incomplete or incorrect. This inductive inference is warranted by a deeper fact presumed in science, that phenomena as regular as this are law-governed. The complete law cannot be existing Newtonian theory since it does not predict this regularity. Alternatively, the extraordinary nature of the result might lead the scientist to speculate that other more prosaic facts might obtain, such as the employing of a dishonest lab assistant. On the warrant of that fact, the scientist might infer that the anomalous result arose through fraud. All this conforms well with the material theory of induction.

In the second part of his response, Norton emphasises that explanations of the regularity pattern in question need not be based on scientific theories but could also rely on external reasoning regarding the experimental process. Resorting to explanations of that kind would be in full agreement with material induction. We fully agree on that point. We want to assume in our example, however, that our scientific observer does not find any plausible explanation of the observed regularity in terms of external influences. We assume that no viable or plausible explanation of the observed regularity pattern, be it external or based on a new scientific theory, is available to the scientist.

Regarding that case, Norton offers the following argument. He suggests that, even if no scientific theory capable of accounting for the observed regularity was available, "meta-inductive" reasoning would lead the scientist to expect that an as yet unknown theory with that capability existed. Scientists have learned to presume that conspicuous and stable observational regularity patterns are law-governed. Norton goes on to argue that a scientist who applies inductive inference in the given case would in fact base that step on assuming the existence of as yet unknown natural laws governing the observed regularity. He then claims that the latter assumption is sufficient for vindicating the principle of material induction.

We share Norton's understanding that scientists would expect the existence of an unknown theoretical scheme if they were confronted with the regularity pattern of our example.[10] However, we disagree with Norton's claim that this expectation is of significance for an understanding of inductive inference in the given case.

The step of inductive inference we are interested in leads from acknowledging the regularity pattern in the collected data towards predicting that the next events measured in the same empirical setting will be in agreement with that regularity pattern. Let us imagine that

[8] The experimenter does, of course, rely on scientific knowledge for framing the question, defining the class of relevant experiments, etc. There is no scientific knowledge, however, that can support or contradict the inductive inference itself.

[9] In terms of rationality, the argument is based on a historicist notion of rationality (see Matheson and Dallman 2014).

[10] One might add that this expectation could lose strength if no adequate theoretical description was found for a long period of time and if other regularities without general natural laws were observed.

the scientist who analyses the data assumes the existence of some unknown theoretical reason for the observed regularity without being able to specify the corresponding theory. The unknown theory she is assuming then might in principle predict any imaginable outcome for future events. The mere assumption that an unspecified theory explains the observed regularity pattern thus offers no basis for any kind of inductive inference. The only basis the scientist has under such circumstances for assuming that the so far unknown viable theory will not contradict raw enumerative induction with respect to future events is the trust in raw enumerative induction itself. Thus, based on the available level of scientific understanding, there remains only one way to carry out inductive inference in the given case: resorting to raw enumerative induction.

Raw enumerative induction amounts to the straightforward continuation of an observed striking regularity pattern. A number of considerations demonstrate the limitations of this strategy:

- As David Hume was the first to point out (Book I, Part III, Section VI), no principle of logic enforces the validity of raw inductive inference even in the most clear-cut cases.
- Raw induction does not *per se* imply a specific probabilistic evaluation of the claims generated on its basis and does not require the knowledge that would be necessary for carrying out any probabilistic evaluation at all.[11]
- Not all regularity patterns allow for a univocal scheme of continuation based on raw induction.
- There is no well-defined sharp limit that designates the point beyond which raw induction may be said to provide univocal predictions.
- Even in cases where raw induction may be said to offer a univocal prediction, endorsing that prediction is not always the favoured way to proceed. Conclusions based on raw induction can be overruled by scientific theories which provide other predictions.

In the light of this list of caveats, this paper is in agreement with Norton in denying to raw induction the role of a universal foundation of inductive reasoning in science. It does assert contra Norton, however, that, in cases like our 16.8 seconds scenario, raw induction does play a role in science and thereby exemplifies a form of induction in science that does not derive its license *from facts pertinent to the matter of the induction.*

Why aren't the data, and the historicist commitment to the method of raw enumerative induction, themselves facts pertinent to the matter of the induction? Norton must use a concept of (theory-based) fact that goes beyond a simple scheme of induction. When scientific theories establish facts pertinent to the matter of induction, those theories must state more than the inductive rule itself. Otherwise, every deployed form of induction could be licensed by the 'fact' that the given rule of induction applies in a given case and Norton's core claim of material induction would be empty.

[11] In our 16.8 seconds Norton's dome example, no measure on the space of alternatives is provided that could support any probabilistic analysis. In cases where a probabilistic evaluation is possible and carried out, such probabilistic analysis reaches out beyond raw induction and is not univocal even in cases where raw induction itself univocally suggests one conclusion. For example, getting 10 times 'up' and no 'down' in a binary experiment generates a prediction based on raw induction: the next result will be 'up'. The scenario also does allow for straightforward frequentist data analysis, which leads to probability 1 for 'up'. However, in order to avoid probability 1 for a specific outcome (and therefore be compatible with Bayesian data analysis) one might deviate from the simple frequentist scheme and rather adhere to Laplace's rule of succession.

If a large number of events shows a straightforward regularity of the kind considered in our 16,8 seconds scenario, there is a clear sense in which a specific form of inductive inference is enforced upon the observer. Confronted with observed regularity patterns, it is the scientist's aim to account for them. If, based on the present state of scientific knowledge, adhering to raw induction is the only strategy open to the scientist confronted with the data, it would thus be at variance with principles of scientific reasoning to ignore predictions that can be made based on a strong case of raw induction. Deployments of raw induction along those lines do occur within the framework of genuine scientific reasoning and can instill a considerable degree of trust in the predictions extracted. (See the beginning of Section 3 for a couple of examples.)

Note that the applicability of raw induction at a certain stage does not imply that future scientific theories won't deviate from its predictions. Raw induction only remains the scientist's best option as long as no scientific theory that supports or overrides raw induction is available.[12] The process of overruling raw induction by new scientific theory building is an important element of scientific progress. To give one example, our raw inductive inference that the sun will always rise the next morning is rejected by astrophysics.

The 16,8 seconds scenario constitutes a clear case of a research context where reliance on raw induction is unavoidable at a given point in time in order to account for the observed regularity patterns. Therefore it demonstrates, contra material induction, that a principle of induction that is not licensed by scientific facts about the matter of inference has its place in science.

3: General Assessment

One can think of many instances in science where observed regularities weren't predicted by the scientific theories known at the time. For example, the law of equal proportions was not understood to be implied by atomism at the time of its discovery. The regular frequencies of pulsars were discovered before the physics behind the phenomenon was understood. Nevertheless, in those cases and others, the observed regularity patterns were acknowledged as part of scientific knowledge and as a legitimate basis for predicting the continuation of the observed regularity in future observations.

Any instance of this kind constitutes an example where the first conceptual step of data analysis isn't motivated by the known scientific theories but seems based on raw induction. Any such instance thus may be taken to support the claim that material inductivism is insufficient for giving a characterisation of all inductive inference in science. In most of those cases, however, reference to a wider framework of scientific background knowledge offers a basis for relating the observed regularities to the general scientific picture and thereby for assessing their plausibility. The viability of some scientific theories may be extendable to a new regime; general scientific principles may be deployed in a new context to explain a certain experimental outcome; or one may have a suspicion what a scientific explanation of the phenomenon might look like even if no satisfactory theory has yet been found. The material inductivist in such cases may still sustain his claim that scientific background knowledge licenses the inductive inferential scheme.

The dome scenario constitutes a more forceful counterexample to material induction. In the dome example, Newtonian physics provides a "well-defined" and universal framework

[12] The presented view is fully consistent with Norton's understanding that the discovery of new theories is itself not an inductive process. On the proposed view, raw induction is deployed in the absence of a more elaborate theoretical understanding of a regularity pattern. It is not deployed for finding new theories.

that implies a clear-cut distinction between the deterministic regime, within which most of the phenomena to which the theory is applicable are described, and the indeterministic regime, where the theory explicitly states it has nothing to say.[13] Thus, the dome provides a "laboratory case" of a situation where the accepted scientific theory itself shields certain of the systems it describes from background knowledge by stating explicitly that any regularity pattern found there would be fully coherent with but entirely inexplicable by the scientific theory itself. All escape routes for the material inductivist are thus blocked.

The occurrence of raw induction in its pure form in the dome example weakens the general case for material induction. If raw induction can be clearly identified in some scientific contexts, one may suspect that it can occur elsewhere in science as well. The dome example thus may be taken to show in its pure and isolated form an elementary kind of inductive inference that constitutes a significant though elusive element in many contexts of scientific reasoning.

Acknowledgements:

I am very grateful to John Norton and a number of undisclosed referees for very helpful comments. Work on the paper was supported by the FWF research grant P19450-N16 and the DFG research grant DA 1633 2-1.

[13] Of course, even the system with a ball at the dome's apex is heavily constrained by physics. We know that the ball will not fly away, not stop on the way down, etc. The indeterminism of the dome scenario is confined to only two parameters: t, which is the last time the ball is at rest, and the radial direction in which the ball falls.

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