The No Miracles Argument without the Base Rate Fallacy

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Abstract

According to an argument by Colin Howson, the no-miracles argument is contingent on committing the base-rate fallacy and is therefore bound to fail. We demonstrate that Howson's argument only applies to one of two versions of the no-miracles argument. The other, more considerate version is not adequately reconstructed in Howson's approach and thus remains unaffected by his line of reasoning. We provide a Bayesian reconstruction of this version of the no-miracles argument and show that it is valid. We then proceed to discuss a number of aspects of NMA on that basis.

1   Introduction

The No Miracles Argument (NMA) arguably is the most influential argument in favour of scientific realism. First formulated under this name in Putnam 1975, the NMA asserts that the predictive success of science would be a miracle if predictively successful scientific theories were not (at least) approximately true. The NMA may be framed as a three step argument. First, it is asserted that the predictive success we witness in science does not have any satisfactory explanation in the absence of a realist interpretation of scientific theory. Predictive success is often understood (see e.g. Musgrave 1988) in terms of the novel predictive success of science. It is then argued that a realist interpretation of scientific theory can provide an explanation of scientific success. Finally, abductive reasoning is deployed to conclude that, given the first two points, scientific realism is probably true.

All three steps of NMA were questioned already shortly after its formulation. It was argued that scientific success needs no explanation beyond what can be given in an empiricist framework (van Fraassen 1980), that scientific

realism cannot provide the kind of explanation of predictive success aimed at by the realist (Laudan 1981) and that the use of abductive reasoning in a philosophical context already presumes a realist point of view (Fine 1986). The debate on all these points continues until this day.

In the year 2000, Colin Howson presented an interesting new line of criticism (Howson 2000) that did not look at one of the three individual steps of the NMA but questioned the overall logical validity of the argument. Howson argued that a Bayesian reconstruction of the NMA revealed a logical flaw in the argument's structure: it commits the base-rate fallacy. Since then, a number of articles have been devoted to the discussion of Howson's point. Howson's line of reasoning was followed and extended by Magnus and Callender (2003). It was criticised on various accounts in Psillos (2009) (which was answered by Callender 2013), Worrall (2007), Menke (2014), Sprenger (2015), and Henderson (forthcoming).

In this article, we present a different formalization of the NMA than what has been provided by Howson. This new formalization will clarify the status and the limits of Howson's argument and put the critical points mentioned in the above-cited literature into a bigger context. We start with a rehearsal of Howson's argument in Section 2. Section 3 then presents the observation, first made in Dawid (2008), that one has to distinguish two kinds of the NMA, which we will call *individual theory-based NMA* and *frequency-based NMA*. Menke (2014) and Henderson (forthcoming) pointed out that Howson's formalization applies to the former but not to the latter kind of NMA. In Section 4, we present a formalization of frequency-based NMA that demonstrates that (i) the individual theory-based NMA is sub-argument of the frequency-based NMA, and that (ii) Howson only reconstructs this sub-argument. In Section 5, we argue that, while there is a strand in scientific realism that reduces NMA to the individual theory-based NMA, Putnam and Boyd, the first main exponents of the argument, did clearly endorse the frequency-based NMA. Sections 6 then provides a more thorough discussion of the differences in perspective between individual theory-based and frequency-based NMA. Section 7 finally formally demonstrates that a core worry with respect to a frequency-based understanding of the NMA can be dispelled.

2 Howson's Argument

In his book Howson (2000), Colin Howson makes the remarkable claim that the NMA commits the base-rate fallacy and therefore is invalid on logical grounds. Howson provides the following Bayesian formalization of this ar-

gument: Let S be a binary propositional variable with values S: Hypothesis H is predictively successful, and ¬S: H is predictively unsuccessful. Let T be the binary propositional variable with values T: H is approximately true, and ¬T: H is not approximately true. Next, the adherent of the NMA makes two assumptions:

A₁: $P(\text{S}|\text{T})$ is quite large.

A₂: $P(\text{S}|¬\text{T}) < k \ll 1$.

A₁ states that approximately true theories are typically predictively successful, and **A₂** states that theories that are not at least approximately true are typically not predictively successful. Note that ¬T is the so-called *catch-all hypothesis*. For the sake of the argument, we consider both assumptions to be uncontroversial, although anti-realists objected to both of them (see, e.g., Laudan (1981) and Stanford (2006)). The adherent of the NMA then infers:

C: $P(\text{T}|\text{S})$ is large.

As Howson correctly points out, the stated argument commits the base-rate fallacy: **C** is only justified if the prior probability $P(\text{T})$ is sufficiently large. This condition, however, is not in the set $\mathcal{A} := \{\mathbf{A_1}, \mathbf{A_2}\}$ of assumptions. If it were, we would beg the question because the truth of a predictively successful theory would then be derived from the assumption that the truth of the theory in question is a priori sufficiently probable, which is exactly what an anti-realist denies.[1]

3   Two Versions of the NMA

Does Howson's formal reconstruction constitute a faithful representation of the NMA? It has been pointed out in Dawid (2008) that two conceptually distinct versions of the NMA have to be distinguished. They differ with respect to the issue as to what exactly has to be explained by the realist conjecture.[2]

The first version is the following: We consider one specific predictively successful scientific theory. The approximate truth of that theory is then deployed for explaining why it is predictively successful. We shall call a NMA

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[1]Howson (2015) indeed calls the statement 'one should endorse the truth of an empirically confirmed theory if one believes $P(T) > P(S|¬T)$' the only valid conclusion from NMA-like reasoning. However, since no reason for believing the stated relation is given, this form of "NMA" cannot seriously be called an argument for scientific realism.

[2]In Dawid (2008), the two forms of the NMA are called "analytic NMA" and "epistemic NMA".

based on individual predictive success an *individual theory-based NMA*. According to the second version, what is to be explained by the realist conjecture is the observation that theories which are developed and held by scientists tend to be predictively successful. In this version, the NMA does not rely on the observation that one specific theory is predictively successful but rather on an observed characteristic of science as a whole or at least of a more narrowly specified segment of science. Theories that are part of that segment, such as theories that are part of mature science or that are part of a specific mature research field, are expected to be predictively successful. We will call a NMA based on the frequency of predictive success *frequency-based NMA*.[3] Menke (2014) and Henderson (forthcoming) have pointed out that Howson's argument only addresses what we call individual theory-based NMA. The frequency-based version of the NMA is not adequately represented by Howson's reconstruction.

In the following section, we develop a formalization of frequency-based NMA and therefore provide the basis for a fully formalized analysis of NMA. This will help us to investigate how and to what extent the frequency-based version of the NMA reaches beyond Howson's reconstruction.

4   Formalizing Frequency-Based NMA

To begin with, we specify a scientific discipline or research field $\mathcal{R}$.[4] We count all n_E theories in $\mathcal{R}$ that have been empirically tested and determine the number n_S of theories that were empirically successful. We can thus state the following observation:

O: n_S out of n_E theories in $\mathcal{R}$ were predictively successful.

Now let us assume that we are confronted with a new and so far empirically untested theory H in $\mathcal{R}$. We want to extract the probability $P(\text{S}|\text{O})$ for

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[3]A related but different distinction between two forms of NMA was made in (Barnes 2003). Barnes calls the argument from a theory's success the "miraculous theory argument" and contrasts it with the "miraculous choice argument" from the scientists' actual development and choice of successful theories. The miraculous theory argument is necessarily an individual theory-based NMA, since it presumes the predictive success of the individual theory under consideration. Therefore, no frequency of predictive success can be specified within the framework of the miraculous theory argument. A miraculous choice argument may be either of the individual theory or of the frequency-based type.

[4]One might argue that a strong statement on predictive success in a research field $\mathcal{R}$ always requires the specification of some conditions $\mathcal{C}$ that separate more promising from less promising theories in the field. However, accounting for this additional step does not affect the basic structure of the argument, which is why, in order to keep things simple, we won't explicitly mention these conditions in our reconstruction.

the predictive success S of H given observation O. In order not to beg the question by assuming the predictive success of H a priori, we assume a prior probability $P(\mathrm{S}) = \epsilon$, where ϵ can be arbitrarily small.

We then assume that each new theory that comes up in $\mathcal{R}$ can be treated as a random pick with respect to predictive success. That is, we assume that there is a certain overall rate of predictively successful theories in $\mathcal{R}$ and that, in the absence of further knowledge, the success chances of a new theory should be estimated according to our best estimate R of that success rate:

$$P(\mathrm{S}|\mathrm{O}) = R \tag{1}$$

R will be based on observation O. The most straightforward assessment of R is to use the frequentist information and to identify R with

$$R_{freq} = \frac{n_S}{n_E} . \tag{2}$$

We will adopt it in the remainder. To proceed with our formal analysis of the frequency based-NMA, we need to make two assumptions similar to $\mathcal{A}$:

$\mathbf{A}_1^O$: $P(\mathrm{S}|\mathrm{T}, \mathrm{O})$ is quite large.

$\mathbf{A}_2^O$: $P(\mathrm{S}|\neg\mathrm{T}, \mathrm{O}) < k \ll 1$

Note that scientific realists assume that the truth of H is the dominating element in explaining the theory's predictive success. If that is so, then S is roughly conditionally independent of O given T and we have $P(\mathrm{S}|\mathrm{T}, \mathrm{O}) \approx P(\mathrm{S}|\mathrm{T})$ and $P(\mathrm{S}|\neg\mathrm{T}, \mathrm{O}) \approx P(\mathrm{S}|\neg\mathrm{T})$. The conditions $\mathbf{A}_1^O$ and $\mathbf{A}_2^O$ can then be roughly equated with the conditions $\mathbf{A}_1$ and $\mathbf{A}_2$. For the sake of generality, we will nevertheless use conditions $\mathbf{A}_1^O$ and $\mathbf{A}_2^O$ in the following analysis.

Let us now come to our crucial point, viz. *to show that accounting for observation O blocks the base-rate fallacy.* The base-rate fallacy in individual theory-based NMA consisted in disregarding the possibility of arbitrarily small priors $P(\mathrm{T})$. In frequency-based NMA, however, the crucial probability is $P(\mathrm{T}|\mathrm{O})$ rather than $P(\mathrm{T})$. Updating the probability of S on observation O has an impact on $P(\mathrm{T}|\mathrm{O})$. To see this, we start with the law of total probability,

$$P(\mathrm{S}|\mathrm{O}) = P(\mathrm{S}|\mathrm{T}, \mathrm{O})P(\mathrm{T}|\mathrm{O}) + P(\mathrm{S}|\neg\mathrm{T}, \mathrm{O})P(\neg\mathrm{T}|\mathrm{O}), \tag{3}$$

and obtain

$$P(\mathrm{T}|\mathrm{O}) = \frac{P(\mathrm{S}|\mathrm{O}) - P(\mathrm{S}|\neg\mathrm{T}, \mathrm{O})}{P(\mathrm{S}|\mathrm{T}, \mathrm{O}) - P(\mathrm{S}|\neg\mathrm{T}, \mathrm{O})} . \tag{4}$$

(The proof is in Appendix A.) Two points can be extracted from eq. (4) and assumptions $\mathbf{A}_1^O$ and $\mathbf{A}_2^O$. First, we conclude that

$$P(\text{S}|\text{T}, \text{O}) \geq P(\text{S}|\text{O}). \tag{5}$$

In particular,

$$P(\text{S}|\text{T}, \text{O}) = P(\text{S}|\text{O}) \quad \text{iff} \quad P(\text{T}|\text{O}) = 1. \tag{6}$$

Second, we obtain that

$$P(\text{T}|\text{O}) > P(\text{S}|\text{O}) - k = R - k. \tag{7}$$

(The proof is in Appendix B.) Hence, $P(\text{T}|\text{O})$ is bounded from below if $k < R$. This will typically be the case as (i) k is small (by assumption $\mathbf{A}_2^O$) and (ii) $R \approx n_S/n_E$ (by eq. (2)) is large (see, however, our discussion in Section 7). Thus, if a supporter of the NMA uses a form of assumption $\mathbf{A}_1^0$ that satisfies $k < R$, then the base-rate fallacy is avoided. Note that the first and crucial inference in frequency-based NMA is made before accounting for the predictive success of H itself and relies on relating $P(\text{S}|\text{O})$ to $P(\text{T}|\text{O})$ based on the law of total probability.

Once H has been empirically tested and found to be empirically successful, we can, just as in the case of individual theory-based NMA, update on S, the predictive success of H.[5] Using eqs. (5) and (6) and the identity

$$P(\text{T}|\text{S}, \text{O}) = \frac{P(\text{S}|\text{T}, \text{O})}{P(\text{S}|\text{O})} \cdot P(\text{T}|\text{O}), \tag{8}$$

it is easy to see that Bayesian updating from $P(\text{T}|\text{O})$ to $P(\text{T}|\text{S}, \text{O})$ further increases the probability of T for predictively successful theories as long as $P(\text{T}|\text{O}) < 1$.

Comparing this formalization of frequency-based NMA with Howson's reconstruction, we see that Howson only reconstructs the second part of frequency-based NMA, which leads from $P(\text{T}|\text{O})$ to $P(\text{T}|\text{S}, \text{O})$. He leaves out the part where $P(\text{T}|\text{O})$ is extracted from the observed success frequency n_S/n_E based on the prior $P(\text{T})$. Howson's base rate fallacy charge crucially relies on the understanding that the specification of the truth probability before updating under the predictive success of the given theory is *not* part of the NMA. Howson's reconstruction thus is insensitive to the observed frequency of predictive success and amounts to a formalization of individual theory-based NMA.

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[5]Note that the predictive success of H will change the value of R. However, this change will be small if n_E is large.

5   Do Realists Use Frequency-Based NMA?

The NMA has a long and chequered history. Who among its exponents endorsed individual theory-based NMA and who endorsed its frequency-based counterpart? Here is the precise wording of Hilary Putnam's famous first formulation of NMA:

> The positive argument for realism is that it is the only philosophy that doesn't make the success of science a miracle. That terms in mature science *typically* refer [. . .], that the theories in a mature science are *typically* true, that the same term can refer to the same thing even when it occurs in different theories – these statements are viewed by the scientific realist not as necessary truths but as the only scientific explanation of the *success of science* and hence as part of any adequate scientific description of science and its relations to its objects. (Putnam 1975, our emphasis)

Note that Putnam speaks of the success of science rather than of the success of an individual scientific theory. He clearly understands the *success of science* as a general and observable phenomenon. Since he obviously would not want to say that each and every scientific theory is always predictively successful, he thereby asserts that we find a high success rate n_S/n_E based on our observations of the history of (mature) science. He then infers from the success of science that mature scientific theories are *typically* approximately true. That is, he infers $P(\text{T}|\text{O})$ from the observed rate n_S/n_E. We conclude that Putnam presented a clear-cut frequency-based version of NMA.

The other early main exponent of the NMA, Richard Boyd (see, e.g., Boyd (1984) and Boyd (1985)), is committed to frequency-based NMA as well. Boyd emphasizes that only what he calls the "predictive reliability of well-confirmed scientific theories" and the "reliability of scientific methodology in identifying predictively reliable theories" provides the basis for the NMA. In other words, an individual case of having predictive success could be explained by good luck and therefore would not license a NMA. Only the reliability of generating predictive success in a field, i.e. a high frequency of predictive success, provides an acceptable basis for the NMA.

Later expositions of NMA at times are not sensitive to the distinction between individual theory-based NMA and frequency-based NMA and therefore are not clearly committed to one or the other version of the argument. In some cases, Putnam's phrase "success of science" is used at one stage of the exposition while the thrust of the exposition seems to endorse an individual theory-based NMA perspective. A classic example of this kind is Musgrave

(1988). Psillos (2009) defends individual theory-based NMA in his attempt to refute Howson's criticism of the NMA.

6 Discussion

Having thus established that the frequency-based NMA may be called the canonical form of the NMA, we now want to deepen our understanding of the distinction between individual theory-based and frequency-based NMA. To that end, it is helpful to look at another aspect of Howson's line of reasoning. More specifically, the following objection to Howson's argument may be made.[6] A subjective Bayesian approach by definition relies on prior probabilities of the hypotheses under scrutiny. In other words, in a Bayesian framework even the most convincing set of empirical data leads to an endorsement of the tested hypothesis only for a given range of priors. Why should this be a lethal problem for (individual theory based) NMA when it isn't for all kinds of scientific analysis that can be reconstructed in Bayesian terms?

In order to answer this objection to Howson's argument, one has to take a look at the reason why, from a Bayesian perspective, science can reach stable and more or less objective conclusions despite the necessity of starting from subjective priors. The reason is that, in a standard scientific context, posteriors converge under repeated empirical testing. Therefore, if one wants to test a hypothesis, any prior probability one may choose (apart from dogmatic denial, which corresponds to a prior probability of zero), can lead to posteriors beyond a probability threshold set for acknowledging conclusive confirmation if a sufficient amount of data is collected. The fact that science is modelled as allowing for an infinite series of empirical testing therefore neutralizes the threat of subjective priors to the reliability of scientific claims.

Howson's claim that the NMA involves a base rate fallacy in this light amounts to the claim that the evidence structure that enters the NMA is not of the kind to be found in scientific testing. It does not project a series of updatings under a sequence of observations like in scientific testing. Rather, it relies on acknowledging exactly one characteristic of a given theory: its novel predictive success. In Howson's reconstruction of the NMA, there is only one updating under the observation of predictive success. This, Howson argues, is no adequate structure for establishing the hypothesis under scrutiny as long as no limits to the priors are assumed. Since no anti-realist would subscribe to such limits, the (individual theory-based) NMA cannot establish scientific realism.

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[6] We thank Brian Skyrms for bringing it up.

Howson makes a crucial point about the individual theory-based NMA. The problem of individual theory-based NMA is not that the sample size happens to be 1. The problem runs deeper: individual theory-based NMA does not provide a framework for the sample testing of a rate of predictive success at all. The logic of the argument starts with selecting a theory that is known to have made correct novel predictions. The argument is not based on an observation about the research process that has led up to developing and endorsing scientific theories but only on an observation about the theory's relation to empirical data. Since the predictively successful theory is being selected ex post, individual theory-based NMA provides no basis for establishing whether the predictive success of the given theory is surprising – is "a miracle", if you want – on any account. To use an analogy, if one selects the winning ticket of a lottery after the draw, the fact that it won is no surprise. Frequency-based NMA, to the contrary, relies on an observation about the research process: within a given field, scientific theories that satisfy some set of conditions happen to be predictively successful with a significantly high probability. This observation is grounded in a series of individual observations of the predictive successes of individual theories. The empirical testing of each theory serves as a new data point. The observation $\mathbf{O}$ of a certain frequency of predictive success therefore is the result of a quasi-scientific testing series. The probability $P(\mathrm{T}|\mathrm{O})$, which is extracted from $\mathbf{O}$ based on the law of total probability, can be decoupled from the prior probability $P(\mathrm{T})$ as it is inferred from a quasi-scientific testing series. The mechanism of objectifying results that work in a scientific framework is also applicable in the case of NMA: anyone who takes the available data to be inconclusive can resort to further testing.

Specifically, if a certain number of data points are available for extracting $\mathbf{O}$ and an anti-realist observer (who has a very low prior $P(\mathrm{T})$ before observing predictive success in the field but, for the sake of the argument, accepts conditions $\mathbf{A_1^O}$ and $\mathbf{A_2^O}$) considers the available amount of data too small for overcoming her very small prior probability $P(\mathrm{T})$, one can just wait for new theories in the research context to get tested. If the additional data also supports a high frequency of predictive success, our anti-realist would at some stage acknowledge that the empirical data has established realism.[7] The deep

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[7]The present analysis implies that the proposal by Menke (2014) to use NMA only with respect to theories that show multiple predictive successes is not satisfactory. Since Menke's suggestion remains within the framework of individual theory-based NMA, it does not solve the structural problem Howson is pointing to. A theory that happens to make two novel predictions and is successful in both cases may be more likely to be true than one with only one case of novel predictive success (albeit one may object to treating instances of novel predictive success statistically as independent picks as it is proposed by

reason why frequency-based NMA avoids the base rate fallacy therefore lies in the fact that it provides a framework in which the convergence behaviour of posteriors under repeated updating can be exploited.

7   Does NMA need a high Frequency of Predictive Success?

References to success frequencies in the context of the NMA have been avoided by a number of philosophers of science for one reason: it seemed imprudent to ground an argument for realism on a claim that seemed quite questionable. Given the many failures of scientific theories, it looked unconvincing to assert a high frequency of predictive success in any scientific field. In this light, it is important to have a clear understanding of the success frequencies that are actually required for having a convincing NMA. In the following, we address this question within our formalized reconstruction.

The strength of the NMA is expressed by $P(\mathrm{T}|\mathrm{S},\mathrm{O})$. In order to make a strong case for scientific realism, we demand that

$$P(\mathrm{T}|\mathrm{S},\mathrm{O}) > K, \tag{9}$$

where K is some reasonably high probability value. $K = 1/2$ may be viewed as a plausible condition for taking the NMA seriously. How does a condition on $P(\mathrm{T}|\mathrm{S},\mathrm{O})$ translate into a condition on $P(\mathrm{S}|\mathrm{O})$ and therefore on the (observed) success frequency R? Obviously, this depends on the posited values of $P(\mathrm{S}|\mathrm{T},\mathrm{O})$ and $P(\mathrm{S}|\neg\mathrm{T},\mathrm{O})$. Eqs. (4) and (8) imply that

$$P(\mathrm{T}|\mathrm{S},\mathrm{O}) = \frac{P(\mathrm{S}|\mathrm{T},\mathrm{O})}{P(\mathrm{S}|\mathrm{O})} \cdot \left(\frac{P(\mathrm{S}|\mathrm{O}) - P(\mathrm{S}|\neg\mathrm{T},\mathrm{O})}{P(\mathrm{S}|\mathrm{T},\mathrm{O}) - P(\mathrm{S}|\neg\mathrm{T},\mathrm{O})} \right). \tag{10}$$

We know already that the NMA works only for $P(\mathrm{S}|\mathrm{O}) < P(\mathrm{S}|\mathrm{T},\mathrm{O})$. From eq. (10) we can further infer (proof omitted) that, for fixed $P(\mathrm{S}|\mathrm{O})$ and fixed $P(\mathrm{S}|\neg\mathrm{T},\mathrm{O}) < P(\mathrm{S}|\mathrm{O})$, $P(\mathrm{T}|\mathrm{S},\mathrm{O})$ decreases with increasing $P(\mathrm{S}|\mathrm{T},\mathrm{O})$. Therefore, it is most difficult for $P(\mathrm{T}|\mathrm{S},\mathrm{O})$ to reach the value K if $P(\mathrm{S}|\mathrm{T},\mathrm{O})$ has the maximal value 1. It thus makes sense to focus on this case, which gives

$$P(\mathrm{T}|\mathrm{S},\mathrm{O}) = \frac{1}{P(\mathrm{S}|\mathrm{O})} \cdot \left(\frac{P(\mathrm{S}|\mathrm{O}) - P(\mathrm{S}|\neg\mathrm{T},\mathrm{O})}{1 - P(\mathrm{S}|\neg\mathrm{T},\mathrm{O})} \right). \tag{11}$$

Menke). But individual theories just give us the predictions they happen to imply. There is no perspective of a series of novel predictions that can be understood in terms of an open series of empirical testing. Howson's core objection thus remains valid.

Condition (9) then turns into

$$\frac{1}{P(\text{S}|\text{O})} \cdot \left(\frac{P(\text{S},\text{O}) - P(\text{S}|\neg\text{T},\text{O})}{1 - P(\text{S}|\neg\text{T},\text{O})}\right) > K, \tag{12}$$

which leads to the following condition for $P(\text{S}|\text{O})$:

$$P(\text{S}|\text{O}) > \frac{P(\text{S}|\neg\text{T},\text{O})}{1 - K + K \cdot P(\text{S}|\neg\text{T},\text{O})} \tag{13}$$

(The proof is in Appendix C.) For small $P(\text{S}|\neg\text{T},\text{O})$, a good approximation of the condition expressed in eq. (13) is

$$P(\text{S}|\text{O}) > \frac{P(\text{S}|\neg\text{T},\text{O})}{1 - K}. \tag{14}$$

For $K = 1/2$, we thus obtain

$$P(\text{S}|\text{O}) > 2\,P(\text{S}|\neg\text{T},\text{O}). \tag{15}$$

In the large n_E-limit, this implies

$$n_S/n_E > 2\,P(\text{S}|\neg\text{T},\text{O}). \tag{16}$$

We thus see that we don't need a high rate of predictive success in a scientific field for having a significant argument in favour of scientific realism. The ratio n_S/n_E may be small as long as it is larger than the assumed value of $P(\text{S}|\neg\text{T},\text{O})$. In a sense, the understanding that the NMA needs a high frequency of predictive success is based on the inverse mistake to the one committed by the endorser of the individual theory-based NMA. While the latter only focuses on the updating under the novel predictive success of an individual theory, the former does not take this updating into account.

The possibility to base a NMA on a small n_S/n_E is significant because many arguments and observations which lower our assessments of n_S/n_E in a scientific field at the same time enter our assessment of $P(\text{S}|\neg\text{T},\text{O})$. Take, for example, the following argument against a high ratio of predictive success. "Scientists develop many theories which they don't find promising themselves. Clearly, those theories have a very low frequency of predictive success." It is equally clear, though, that most of these theories are false and therefore lower $P(\text{S}|\neg\text{T},\text{O})$. Examples of this kind show that lines of reasoning that isolate segments of the theory space where both n_S/n_E and $P(\text{S}|\neg\text{T},\text{O})$ are very small provide a framework in which the claim $n_S/n_E > P(\text{S}|\neg\text{T},\text{O})$ can make sense even if the overall value n_S/n_E in a research field is rather unimpressive.

8 Conclusion

The following picture of the role of the base rate fallacy with respect to the NMA has emerged. Howson's argument decisively destroys the individual theory-based NMA, which has been endorsed by some adherents to NMA and clearly was not understood to be logically flawed by many others. This is an important step towards a clearer understanding of the NMA and the scientific realism debate as a whole. However, the individual theory based NMA is only one part of the NMA as it was presented by Putnam and Boyd. The full argument, which we call the frequency-based NMA, does not commit the base rate fallacy.

A clearer understanding of what the base rate fallacy amounts to in the context of the NMA can be achieved by phrasing it in terms of the convergence behaviour of posteriors. An argument commits the base rate fallacy if it (i) ignores the role of the subjective priors and if it (ii) does not offer a perspective of convergence behaviour under a sequence of updatings under incoming data. The individual theory-based NMA is structurally incapable of providing such a sequence of updatings because it addresses only the spectrum of novel predictions provided by one single theory. The frequency-based NMA, to the contrary, is based on a general observation about the research process (the frequency of predictive success in a research field) that can be tested by collecting a sequence of data points, where each data point corresponds to the observed novel predictive success of an individual theory. Therefore, the process of testing the hypothesis "theories that have novel predictive success are probably true" under the assumptions $\mathbf{A_1^O}$ and $\mathbf{A_2^O}$ is of the same type as scientific testing and does not commit the base rate fallacy.

One worry about the frequency-based NMA is related to the understanding that the high frequencies of predictive success necessary for having a convincing NMA cannot be found in actual science. The formalization of the frequency-based NMA demonstrates that such high frequencies are not necessary for achieving high truth probabilities with respect to theories with novel predictive success.

To end with, let us emphasise one point. The fact that the NMA, if correctly reconstructed, does not commit the base-rate fallacy doesn't imply its soundness. A supporter of the frequency-based NMA must justify assumptions $\mathbf{A_1^O}$ and $\mathbf{A_2^O}$ and must explain on which grounds she takes a sufficiently high frequency of predictive success to be borne out by the data. Whether or not that can be achieved lies beyond the scope of this article.

Acknowledgements

A Derivation of Eq. (4)

We use the law of total probability and obtain:

$$
\begin{aligned}
P(\text{S}|\text{O}) &= P(\text{S}|\text{T},\text{O})P(\text{T}|\text{O}) + P(\text{S}|\neg\text{T},\text{O})P(\neg\text{T}|\text{O}) \\
&= P(\text{S}|\text{T},\text{O})P(\text{T}|\text{O}) + P(\text{S}|\neg\text{T},\text{O}) \cdot [1 - P(\text{T}|\text{O})] \\
&= P(\text{T}|\text{O}) \cdot [P(\text{S}|\text{T},\text{O}) - P(\text{S}|\neg\text{T},\text{O})] + P(\text{S}|\neg\text{T},\text{O})
\end{aligned}
$$

Hence,

$$
P(\text{T}|\text{O}) = \frac{P(\text{S}|\text{O}) - P(\text{S}|\neg\text{T},\text{O})}{P(\text{S}|\text{T},\text{O}) - P(\text{S}|\neg\text{T},\text{O})} \, .
$$

Here we have used that $P(\text{S}|\text{T},\text{O}) > P(\text{S}|\neg\text{T},\text{O})$, which follows from assumptions $\mathbf{A}_1^O$ and $\mathbf{A}_2^O$.

B Derivation of Eq. (7)

We start with eq. (4) and obtain

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\begin{aligned}
P(\text{T}|\text{O}) &= \frac{P(\text{S}|\text{O}) - P(\text{S}|\neg\text{T},\text{O})}{P(\text{S}|\text{T},\text{O}) - P(\text{S}|\neg\text{T},\text{O})} \\
&\geq \frac{P(\text{S}|\text{O}) - P(\text{S}|\neg\text{T},\text{O})}{1 - P(\text{S}|\neg\text{T},\text{O})} \\
&\geq P(\text{S}|\text{O}) - P(\text{S}|\neg\text{T},\text{O}) \qquad (17) \\
&> P(\text{S}|\text{O}) - k. \qquad\qquad\qquad (18)
\end{aligned}
$$

Here eq. (17) follows from assumptions $\mathbf{A}_1^O$ and $\mathbf{A}_2^O$ and eq. (18) follows from assumption $\mathbf{A}_2^O$.

C   Derivation of Eq. (13)

We start with eq. (12) and obtain:

$$\frac{1}{P(\text{S}|\text{O})} \cdot \left(\frac{P(\text{S}|\text{O}) - P(\text{S}|\neg\text{T},\text{O})}{1 - P(\text{S}|\neg\text{T},\text{O})} \right) \;>\; K$$

$$\frac{1}{P(\text{S}|\text{O})} \cdot (P(\text{S}|\text{O}) - P(\text{S}|\neg\text{T},\text{O})) \;>\; K\,(1 - P(\text{S}|\neg\text{T},\text{O}))$$

$$1 - \frac{P(\text{S}|\neg\text{T},\text{O})}{P(\text{S}|\text{O})} \;>\; K\,(1 - P(\text{S}|\neg\text{T},\text{O}))$$

$$1 - K\,(1 - P(\text{S}|\neg\text{T},\text{O})) \;>\; \frac{P(\text{S}|\neg\text{T},\text{O})}{P(\text{S}|\text{O})}$$

$$P(\text{S}|\text{O}) \cdot (1 - K(1 - P(\text{S}|\neg\text{T},\text{O}))) \;>\; P(\text{S}|\neg\text{T},\text{O})$$

Hence,

$$P(\text{S}|\text{O}) > \frac{P(\text{S}|\neg\text{T},\text{O})}{1 - K + K \cdot P(\text{S}|\neg\text{T},\text{O})} \;.$$

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