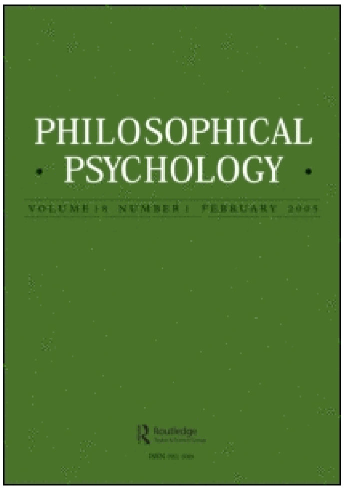

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An Extended Mind Perspective on Natural Number Representation

Helen De Cruz

Experimental studies indicate that nonhuman animals and infants represent numerosities above three or four approximately and that their mental number line is logarithmic rather than linear. In contrast, human children from most cultures gradually acquire the capacity to denote exact cardinal values. To explain this difference, I take an extended mind perspective, arguing that the distinctly human ability to use external representations as a complement for internal cognitive operations enables us to represent natural numbers. Reviewing neuroscientific, developmental, and anthropological evidence, I argue that the use of external media that represent natural numbers (like number words, body parts, tokens or numerals) influences the functional architecture of the brain, which suggests a two-way traffic between the brain and cultural public representations.

Keywords: Cognitive Scaffolding; Counting; Extended Mind; Hebbian Learning; Natural Numbers; Symbolic Number Representation

1. Introduction: The Bounds of Cognition

Human cognition is characterized by an extensive interaction between brain and external environment. We are part of distributed cognitive systems, which include other people's minds, external storage devices such as books, and instruments such as pocket calculators. Delegating cognitive operations to the external world clearly enhances our cognitive capacities: multiple-digit multiplications with pen and paper are far easier than mental arithmetic. Moreover, without external cognitive artifacts many concepts (such as HELIOCENTRISM) or solutions to computational problems

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(such as algebra) are not just harder to obtain, they are unthinkable. Despite this importance of external media in human cognition, relatively little research has been conducted to clarify how the brain and the environment interact to enhance and extend our cognitive capacities.

As a result, there is considerable disagreement on the viability of the extended mind as a philosophical concept. An ongoing debate focuses on whether cognition actually takes place outside the brain. While Clark and Chalmers (1998) assert that there is little difference between an Alzheimer patient who consults his notebook to remember the location and date of an exhibition and a neurologically normal person who consults her memory to recall the same occasion, Adams and Aizawa (2001, p. 57) object that this opens the threat of cognitive bloat: cognition oozing into everything that is somehow causally connected to it. Donald's (1991) classic essay on human cognitive evolution presents another possible view: he claims that the extensive use of writing and other material symbols results in a radical restructuring of the brain during ontogeny. This view, however, seems incompatible with modular, nativist accounts of cognition that many psychologists and philosophers of mind endorse (e.g., Carruthers, 2002). If conceptual information is processed through a set of innately channeled conceptual modules, how could external media influence the way we think about such domains?

The aim of this paper is to come to a more precise formulation of brain-world interactions through an examination of the various ways in which natural numbers are represented. My starting point is a fundamental difference between numerical representations in humans and nonhuman animals: humans are unique in their ability to denote exact cardinal values above three or four. Although number words often play a significant role in the conceptual development of natural numbers, I will argue that natural language is neither necessary nor sufficient for their development. I develop a model of multiple cognitive pathways that lead to successful natural number representation, including tokens (e.g., abacus beads, tallies), body parts, numerical notation systems and gestures. Taking evidence from neuropsychology, anthropology and history of mathematics, I argue that these cognitive pathways influence the structure of the human brain, which supports the view that culture influences cognitive architecture without repudiating domain-specific accounts of human cognition.

2. Domain-Specificity in Numerical Representations

A growing body of experimental and neuropsychological literature suggests that human numerical competence is rooted in cognitive evolution and that we possess some elementary innate numerical skills that we share with other animals. Animals spontaneously use numerical cues in ecologically relevant tasks, such as making foraging decisions. When given the choice between two groups of live prey (two or three flies), red-backed salamanders choose the larger quantity (Uller, Jaeger, Guidry, & Martin, 2003). Lionesses (McComb, Packer, & Pusey, 1994) decide whether or not to attack an intruding group, based on a comparison of the number of unfamiliar individuals they hear roaring and the number of members of their own pride present. Similar capacities have been found in infants, prior to schooling or language acquisition. Newborns can discriminate small collections up to three objects (Antell & Keating, 1983), and five-month-olds can predict the outcome of simple additions or subtractions, such as $1 + 1 = 2$ (Wynn, 1992).

Neuropsychological studies indicate that number processing depends on specialized neural circuitry. The bilateral posterior parietal cortex is reliably activated by numerical tasks such as mental arithmetic (e.g., Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999) and even by elementary tasks that do not involve calculation, such as the presentation of single arabic digits (Eger, Sterzer, Russ, Giraud, & Kleinschmidt, 2003) or collections of dots (Cantlon, Brannon, Carter, & Pelphrey, 2006). Patients with brain lesions in number-sensitive areas, such as the intraparietal sulci (e.g., Lemer, Dehaene, Spelke, & Cohen, 2003) are poor at counting and arithmetic. Studies that measure the activation of single neurons in rhesus monkeys (e.g., Nieder, Diester, & Tudusciuc, 2006) show that individual neurons within the horizontal segment of the intraparietal sulci respond selectively to changes in number in a visual display. These neurons exhibit a peak activity to a specific quantity and gradually decrease in activity as the presented number differs from that of the preferred quantity, e.g., a neuron that responds to two items responds less to three items, and even less to four or more items. This suggests that numerosities are converted into a mental format of approximate magnitudes.

Animals and infants can only enumerate small collections (up to three or four) precisely. Higher numbers are represented approximately; they can only be discriminated if the ratio difference between them is large enough. Six-month-olds, for example, can discriminate between 8 and 16, but not between 8 and 12 (Xu & Spelke, 2000). One way to explain this finding (e.g., Dehaene, 2003) is to posit a logarithmically compressed mental number line on which magnitudes are mapped conforming to the natural logarithm of the number. Each magnitude is represented by a Gaussian tuning curve, which increasingly overlaps with curves distributed along other magnitudes. Accordingly, the perceived distance between larger numbers (e.g., 8 and 10) is smaller than between smaller numbers with the same absolute difference (e.g., 2 and 4), which could explain why numerosities <4 are easier to tell apart. For the purpose of this paper, I shall take this logarithmic number line as a model for intuitive numerical representation.[1]

Remarkably, adult numerical cognition still depends on this approximate, logarithmic scale. This is most evident in number comparison tasks, where adults respond faster for small numerosities (e.g., two vs. three is easier than eight vs. nine) and for numerosities with a large absolute difference (e.g., three vs. eight is easier than seven vs. eight); in fact, their performance is almost identical to that of rhesus monkeys who are trained to perform the same task (Brannon & Terrace, 2002). People from cultures without exact number words (e.g., Pica, Lemer, Izard, & Dehaene, 2004) display an intuitive number representation that is very similar to that of nonhuman animals; they perform relatively accurately in the lower numerosities and exhibit a rapid increase in error rate as numerosities increase.

3. Two Cognitive Routes for Number Words

3.1. Counting

How are humans able to overcome these cognitive limitations and to represent larger cardinal values precisely? Most scholars (e.g., Pica et al. 2004, p. 503) agree that a counting routine is necessary and sufficient for natural-number representation. To count, one establishes a one-to-one correspondence between each countable item and consecutive symbols in a list with a fixed ordinality (e.g., the number words 'one', 'two', etc.). The final item tagged determines the last tag from the counting sequence, which in turn denotes the cardinality of the set. Making correspondences between collections is part of our innate cognitive architecture: both infants and nonhuman animals can recognize whether two sets of stimuli have the same number of elements (see also Decock, this volume). Seven-month-olds, for example, can match the number of voices they hear to the number of speakers they see (Jordan & Brannon, 2006). Rhesus monkeys, chimpanzees and lions show similar capacities: they can compare the number of conspecifics they hear to the number of individuals they see (e.g., McComb et al., 1994). However, humans are not innately furnished with a stably ordered counting list. External media, such as body parts or tallies, may be able to supplement our approximate numerical representation with a list of counting symbols (De Cruz, 2006). They provide the necessary conceptual stability, thus enabling us to discriminate between numerosities that would otherwise be indistinguishable (e.g., five and six).

To be semantically accessible, external symbolic number representations are linked to the logarithmic mental number line. This is most aptly demonstrated by neuroimaging studies (e.g., Eger et al., 2003), that indicate that passively hearing spoken number words or perceiving arabic numerals activates the same neural circuits that are implicated in approximate numerical cognition, namely the bilateral intraparietal sulci. The tendency to understand symbolic numerical stimuli in terms of approximate numerosities seems irresistible, and even occurs when we are confronted with numerical symbols of which we do not know the precise meaning. Masataka, Ohnishi, Imabayashi, Hirakata and Matsuda (2007) presented monolingual Japanese adults with roman numerals (e.g., CMXCIX), which are unknown in Japan. Initially, the subjects' neural activation (as indicated by the blood oxygen level dependent signal in the functional magnetic resonance imaging [fMRI] experiment) showed the typical linguistic pattern for letter processing. Upon learning that these shapes actually represent numerosities, subjects immediately showed strong activation in a network of brain areas involved in numerical processing, such as the bilateral inferior and superior parietal lobule. Although none of the subjects could even remotely identify these numerals, their brains showed numerical activation patterns at the simple presentation of the unknown numerical stimuli.

Given that counting lists provide us with exact cardinal values, it seems remarkable that numerate individuals should continue to rely on their approximate number representation. One reason to associate symbolic representations with the fuzzy and imprecise number sense is that only the latter gives us semantic access to numerosities, enabling us, for example, to judge very quickly and without calculation or counting that 6 is smaller than 9, or that $3 + 5$ cannot be 15. Indeed, while patients with brain damage to the intraparietal area can typically recite calculations they learnt by heart (e.g., multiplication facts), they make puzzling errors in calculations that are not learnt by heart, such as division or subtraction (Lemer et al., 2003). Furthermore, the linkage between numerical symbols and approximate numerosities allows us to reason about numerosities in a purely symbolic format, without actually observing collections of items, which significantly expands our ability to deal with number.

3.2. Approximate Number Words

People from innumerate cultures do not have a conventionalized counting routine, but rather estimate numerosities directly and only subsequently convert this approximate representation into a linguistic format. The Mundurucu (an Amazonian culture from Brazil) only possess number words up to five. However, they do not use these words in counting routines, but make approximate comparisons between numerosities and collections that are easily available. For example, they use the expression *pũg põgbi* ('a hand') not only for collections of 5 items, but also for collections ranging from 4 to 12 items. Thus, the mere presence of number words is not sufficient to promote natural number representation. Rather, number words can emerge from two distinct cognitive routes: they can be representations of exact cardinal values, emerging from the use of counting words, or they can be linguistic expressions of perceived approximate numerosities, as is the case of the Mundurucu and other cultures with few number words. In numerate adults, this second, approximate use of number words is also apparent in expressions like 'a couple of days' or 'about 50 people'. When there is no strong cultural incentive to denote cardinal values (e.g., lack of monetary commerce), this latter system may be sufficient, and counting routines may not develop.

Once the need arises to denote numerosities more precisely, approximate number words can serve as the basis of a counting list, which explains why in so many languages, some number words are etymologically related to the word for 'hand' or 'finger'. For example, while older speakers of Martu Wangka (a Western Australian Aboriginal language) use the words *marakuju* and *marakujarra* in an approximate fashion to denote 'about a hand' and 'about two hands', younger speakers use these terms as the precise numbers 5 and 10 (Harris, 1982, p. 167). This is probably due to an increasing participation in the monetary economy, as is aptly illustrated by the Tiwi, an Aboriginal culture from Melville Island, Australia, whose numerical competence strongly correlates with the degree to which they participate in monetary economic activities (McRoberts, 1990, pp. 35–36).

4. Interactions between External Media and Internal Cognitive Processes

There are differing views on the relationship between external media and internal cognitive processes. One popular position in cognitive science and philosophy of mind (e.g., Fodor, 1975) holds that external representations merely serve as input to the internal mind. According to this approach, numerical cognition only takes place after we convert arabic numerals and number words into some inner code. Some scholars (e.g., Clark, 2006) instead argue that external representations need not be represented internally to be involved in numerical cognition. In this view, much of human cognition is essentially hybrid: it involves a complex interplay between the brain and external resources, and it is often impossible to demarcate sharp boundaries between internal and external cognition. A third position (e.g., Donald, 1991) asserts that external cognitive resources shape the mind in the strong sense that they actually alter our cognitive architecture. Compelling evidence for this claim comes from studies on the effects of literacy and music on the brain. Petersson, Silva, Castro-Caldas, Ingvar, and Reis (2007) compared MRI scans of literate and illiterate subjects from similar socioeconomic background and found illiterates to be more right-lateralized and to possess more white matter. In their comparison of brains of professional musicians, musical amateurs and nonmusicians, Gaser and Schlaug (2003) found that musical competence correlates with an increase in grey matter in motor, auditory and visual-spatial brain regions. This influence of culture on the brain can be explained by the mechanism of Hebbian learning, which assumes that a repeated and persistent excitement of one neuron by another results in metabolic changes in both cells which increases their connectivity, a process known as long-term synaptic potentiation. In the case of number, cultural exposure to symbolic numerical representations during early cognitive development could result in long-term synaptic potentiation between populations of number-sensitive neurons, such as those in the intraparietal sulci, and neurons involved in high-level processing of other domains, such as body-part representation or language. In the following subsections, I discern five ways to represent natural numbers externally that are salient across cultures: number words, body parts, tallies and tokens, numerical notation systems and gestures. For each of these instances, I will attempt a more precise formulation of how internal and external representations interact to yield natural numbers.

4.1. Number Words and Language

The role of language in the development of natural number concepts has been the focus of intense debate. This controversy is fuelled by the fact that both grammatical language and the ability to represent natural numbers accurately seems to be restricted to humans. Even chimpanzees who received intensive training on arabic numerals never generalize to the counting procedure that children master with ease (Biro & Matsuzawa, 2001). Hauser, Chomsky and Fitch (2002) explain these observations by invoking a uniquely human domain-general recursive capacity that underlies both natural language and natural numbers. This ability, which allows us to generate a potentially infinite array of expressions from a limited set of elements, would help children to realize that they can come up with higher and higher numbers. However, there is little support in the neuropsychological literature for the implicit prediction that linguistic and mathematical recursive tasks would recruit similar brain networks. Varley, Klessinger, Romanowski and Siegal (2005) investigated numerical skills in three profoundly agrammatic patients. Despite their inability to produce grammatical language, all patients could accurately solve multiple-digit calculations. They also scored well on questions on infinity, e.g., when asked to produce several numbers smaller than 2 but larger than 1, subjects came up with solutions like 1.9999. Varley et al. (2005) proposed that the mature cognitive system contains autonomous, domain-specific structures for both language and mathematics that deal with recursive structure. Martín-Loeches, Casado, Gonzalo, de Heras and Fernández-Frías (2006) examined the event-related potential (ERP) responses in subjects presented with mathematical reasoning tasks in an order-relevant format, such as bracket expressions in the form of $4 \div (10 - 3)$. Although these problems closely mirror syntactical structures, they did not recruit any brain areas which are related to syntax or linguistic working memory. Moreover, in many cultures that possess few number words, counting is supplemented by tallies (such as marks on the ground) or body-part counting (see, e.g., Wassman & Dasen, 1994, on Yupno counting). Therefore, language may be only one of many cognitive routes that lead to successful natural number representation.

Natural language may play a role in the weaker sense that it is a possible external medium through which we can denote exact cardinal values. The frequent use of number words in everyday discourse helps children to understand that natural numbers are specific, rather than approximate, as their innate number sense would lead them to suggest. Indeed, Sarnecka and Gelman (2004) found that 2.5-year-old children understand that number words are specific, before they can actually count. These children can predict, for example, that a box with six objects to which another object is added, no longer contains six objects.

Since number words play an important role in denoting exact cardinal values, we can expect that linguistic areas are mainly recruited for numerical tasks that involve exact numerical magnitudes. Neuroimaging studies (e.g., Dehaene et al., 1999) show that exact calculation recruits extensive parts of the perisylvian language-related area, whereas approximate calculation yields a stronger activation of the intraparietal sulci. Patients with brain lesions in language-related areas (e.g., Lemer et al., 2003) have impaired exact calculation, while their ability to perform approximate calculations remains relatively intact. As we have seen, in subsection 3.1., number words can be used as external anchors to denote exact cardinal values and are thus more extensively used in exact calculation. This concurs with Locke's (1689/2004, bk. 2, ch. 16) idea that language is not necessary for number representation, but that it can "conduce to our well-reckoning" by providing publicly distributed counting symbols.

4.2. Body Parts

Body-part counting features in many cultures. Several Indo-European number words derive from body-part terms, suggesting an underlying body-part counting system: the word 'four' is related to the Proto-Indo-European word for 'finger'; 'five' in many Indo-European languages is related to the Proto-Indo-European word for 'hand' (Butterworth, 1999, pp. 66–67). Some cultures possess more elaborate systems, such as the Papua New Guinean Okapsmin, who count on their fingers, facial features, shoulders and arms, which are touched and named in a conventionalized stable order (Saxe, 1981). In such systems, individual body parts refer to quantities in much the same way as our number words: in a counting context, the Okapsmin term for 'right shoulder' always denotes 18. Intriguingly, body parts are also often used by people who do not possess formalized counting routines, as was already reported by Locke (1689/2004, bk. 2, ch. 16). The Mundurucu numerical expressions, studied by Pica et al. (2004), include such terms as *pũg põgbi* ('one hand'), *eba* ('your two arms'), and even 'all the fingers of the hands and then some more' (given by one subject in response to 13 dots). A possible reason why body parts feature so prominently in counting and approximate number word representation may be that the neural structure that represents fingers and other body parts—the *body schema*, situated in the left intraparietal lobule—lies anatomically close to number-sensitive neurons in the intraparietal sulci. Establishing a synaptic potentiation is easier between areas that are anatomically close (De Cruz, 2006). Moreover, the body schema is an ideal candidate for a list of symbols with fixed ordinality because it represents body parts in an ordered fashion. Experimental studies (e.g. Le Clec'H et al., 2000) show that the comparison of body parts is prone to a distance effect similar to that in number comparison: subjects are faster at judging that the eyes are higher than the knees than at judging that they are higher than the nose; likewise it is easier to assess that eight is bigger than two, than to see that eight is bigger than seven.

To what extent does the cultural link between magnitudes and body parts influence the structure of the human brain? Studies that measure changes in corticospinal excitability of the hand muscles (e.g., Andres, Seron, & Olivier, 2007) indicate that numerical tasks such as counting and number comparison result in an increased excitability of the hand muscles, while other muscles (such as those of the foot) remain unaffected. Sandrini, Rossini, and Miniussi (2004) found that briefly disrupting the left intraparietal lobule (implicated in finger recognition) through repetitive transcranial magnetic stimulation causes a marked increase in reaction time when subjects complete a number comparison task. Similarly, a temporary disruption of the right angular gyrus results in both finger agnosia (the inability to individualize one's own fingers) and a decline in number processing (Rusconi, Walsh, & Butterworth, 2005). Taken together, these studies suggest that Western adults, who no longer use fingers to solve simple arithmetical problems, nevertheless continue to use the internal cognitive architecture that represents fingers in numerical tasks. Western children, especially preschoolers and first-graders often resort to finger counting when they solve arithmetical problems, perhaps because Western number words are quite irregular (Geary, Bow-Thomas, Lin, & Siegler, 1996). Hebbian learning provides a compelling explanation to account for this continued use of finger representation in adult numerical cognition.

4.3. Tallies and Tokens

Tallies are the oldest archaeologically attested representations for numerosities. Artifacts in bone or antler which show regular incisions or notches are present in the archaeological record from about 70,000 to 50,000 Before Present (Cain, 2006). Some of these objects were probably tally sticks, with engravings that gradually accumulated over time. Other notched artifacts, such as the La Marche antler (d'Errico, 1995) show a clear and intentional morphological differentiation between sets of notches, indicating that several collections of items were being counted.

However, tokens have also played an important role in historical literate cultures. In Western culture up to the introduction of the hindu-arabic positional system, calculations were frequently made by moving counters on a surface known as the *abacus*. Western abacus calculation was positional. Values were assigned on an *ad hoc* basis: the same positions and objects could stand for 10s or 100s, depending on the calculation required. As Netz (2002, p. 9) observed, abaci were not just some aid in the manipulation of numbers, they were the principal medium of calculation. The then available numerical notation systems, such as the Greek alphabetic numerals, proved far too cumbersome for calculation and were therefore rarely used as such. The Greek alphabetic system used 27 distinct signs for the numerical ranges 1–10, 20–90, and 100–900 (Chrisomalis, 2004), which meant one had to learn many multiplication and addition facts by heart, placing heavy demands on working memory and often leading to substantial errors. Consider the addition $80 + 20$. In Greek alphabetic numerals, one needs to retrieve the addition $\pi + \kappa = \rho$ from memory, whereas an abacus user can simply place one counter on the 'fifty' line and three counters on the 'ten' line, where he adds two more counters. Five counters on the 'ten' line means that he can move one counter to the 'fifty' line, which in turn allows him to put one counter on the 'hundred' line. Since no rules further allow him to move counters, the calculation is completed. The only internal cognitive operation he has to perform, is counting up to five. Another example is ancient Chinese arithmetic and algebra, which were based on the manipulation of counting rods, which were arranged in groups of five. By manipulating these rods, Chinese mathematicians of the Han dynasty (206 BC to AD 220) came up with matrix solutions for simultaneous linear equations: they simply arranged the counting rods in rows and columns, where each row corresponded to the coefficient of an unknown and each column represented an equation (De Cruz, 2007). In contrast, European mathematicians only invented matrix solutions in the eighteenth century, probably because the arabic numerals are less suited for this external cognitive operation. However, the use of counting rods in Chinese arithmetic and algebra preserved the concreteness of the calculations, thereby preventing Chinese mathematicians from developing general solutions to higher-degree equations (Chemla, 2003), which Western mathematicians could do by using symbols to denote variables and unknowns. The abacus and the counting rods were integral and irreducible parts of mathematical cognition, allowing hybrid modes of thought in which internal and external resources were tightly interwoven.

Does the enduring use of tokens also influence our cognition in the stronger sense that they cause synaptic reorganization? Support for this view comes from an fMRI study by Tang et al. (2006), which compared brain activation of native Chinese speakers and English speakers who performed comparison and addition tasks. Although intraparietal sulcus activation was common to both groups, English speakers relied more on the language-related left perisylvian area, whereas Chinese speakers showed more activation in the premotor cortex. A plausible explanation for this finding lies in cultural differences in arithmetic teaching, in particular abacus instruction in Chinese education. Chinese abacus users can mentally visualize and manipulate abacus beads while solving mathematical problems. The premotor cortex is normally involved in planning complex movements in response to particular stimuli. The activation of this area during numerical tasks in Chinese speakers could thus be explained by their use of a mental abacus. It is interesting to note that Japanese expert abacus users can perform mental arithmetic involving very large numbers (up to 16 digits) with remarkable accuracy by imagining a *soroban*, the traditional Japanese abacus. When primed with subliminally presented abacus beads in configurations not related to the problem, their performance drops markedly (Negishi et al., 2005). These studies suggest that users of the abacus internalize the operations they perform in the world (the manipulating of abacus beads) when they do mental arithmetic, fostering synaptic potentiation between number-sensitive neurons and neurons in the premotor cortex.

4.4. Numerical Notation Systems

Numerical notation systems are visual and primarily nonphonetic structured systems for representing numbers. Signs such as 9 or IX are part of numerical notation systems, words like 'nine' or 'quatre-vingt' are not. Over the past 5,000 years, more than 100 numerical notation systems were developed worldwide, many of which have now been replaced by the hindu-arabic numerals (Chrisomalis, 2004). Numerical notation systems typically emerge in large-scale societies, where trade, public works or taxation require calculation with large numbers. They enhance our cognitive capacities by representing some aspects of numerical tasks externally, so that they do not need to be represented internally, which would require additional cognitive resources (Zhang & Norman, 1995). For example, positional systems use place value, which externalizes some information on the size of the number: the value of a given numeral sign is partly determined by its position among the signs in the numeral phrase. However, this does not allow us to decide which of two numbers is largest if they have the same highest power value, such as 94 and 49. Whether 4 is smaller or bigger than 9 cannot be derived from the shape of the numerals. Before we can decide whether 49 is bigger or smaller than 94, we have to retrieve the cardinal values of 4 and 9 from memory. In contrast, the Egyptian hieroglyphic system represents 1 as |, 2 as ||, and 3 as |||, thereby representing some information on magnitude externally through the shape of the numerals. Consequently, all calculations with numerical notation systems involve an interplay of internal and external cognitive resources.

For example, a multiple-digit calculation in arabic numerals requires one to retrieve the value of the shapes of the numerals from memory and to remember multiplication and addition facts, while carrying numbers and remembering partial solutions can be performed externally. Several studies (e.g., Zhang & Wang, 2005) support this important role of external representations in numerical tasks, showing that the format in which numbers are presented influences processing speed and accuracy. In this sense, pen-and-paper calculations are hybrid modes of thought, where cognitive performance depends on the complex interplay between internal cognitive operations and external media.

Nevertheless, even though numerical notations are not simply translated into an internal symbolic code, they influence internal cognitive processing. Children as young as five years show a number-specific brain response (as measured by ERP) when they compare arabic numerals, a response virtually indistinguishable from comparing nonsymbolic numerical presentations such as collections of dots (Temple & Posner, 1998). Moreover, as we have seen in subsection 3.1., passively viewing arabic digits (e.g., '3') yields activation in the parietal sulci, whereas viewing letters (e.g., 'A') does not (Eger et al. 2003). This indicates that the brain converts arabic digits into numerosities fast and automatically. Psychological studies on symbolic and nonsymbolic numerical representation (see, e.g., Verguts & Fias, this issue) have yielded conflicting results: some studies show that numerals are converted into an internal mental format very similar to that of nonsymbolic number; others suggest an irreducible role of numerical notation systems in cognitive processing. This problem can be elucidated when we consider it from an extended mind perspective: although numerical notation systems are irreducible to internal cognitive processes, we nevertheless need to convert symbolic numerical representations into mental magnitudes to be able to manipulate them and to gain semantic access to them.

4.5. Gestures

Across the world, gestures are used to denote cardinalities. Contemporary Chinese use hand gestures to denote numerosities up to 20, and well into the twentieth century, French peasants employed an elaborate system of hand gestures to perform multiplications, such as 7×8 (Dantzig, 1947). Young children spontaneously point and gesture when they count. Gesturing lightens cognitive demands by establishing which objects have already been counted. Indeed, under experimental conditions, children have more difficulties in counting when they are prevented from gesturing or pointing (Alibali & DiRusso, 1999). Even in numerate adults, preventing pointing and touching has marked effects on numerical performance. If adult subjects are asked to count a collection of coins without being allowed to touch or point to them, the result is that over half of the subjects give a wrong answer. Once they are allowed to touch or gesture, the error rate falls to nearly 20% (Kirsh, 1995). Interestingly, neural structures that are typically recruited during gesturing, pointing and visual attention lie very close to the number-sensitive neurons in the intraparietal sulci (Simon, Mangin, Cohen, Le Bihan, & Dehaene, 2002). The formation of new synaptic connections may be easier between two anatomically adjacent areas. Culture may key in on this architectural property of the human brain by creating synaptic connections between them.

5. Discussion and Concluding Remarks

This examination of natural number representation shows that external media are a necessary and irreducible part of human numerical cognition. In accordance with Clark (2006), I argue that external media together with the internal cognitive processes involved in number form a hybrid cognitive process. Next to this, I make a relatively strong claim for the interaction between internal and external cognitive resources. The enduring use of external media results in structural changes in the brain: the cognitive scaffolding we use to accurately represent cardinalities (number words, body parts, tokens, numerical notation systems and gestures) is recruited in numerical cognition alongside the number-sensitive neurons. For instance, body-part recognition (finger counting) is recruited for solving numerical tasks involving arabic digits. Natural number representation is only possible when we supplement the internal cognitive architecture involved in numerical processing with external resources. One could object to this view that it is easy to calculate 3×7 mentally and that external media are therefore not necessary for natural number representation. However, such calculations are only possible through an extensive cultural familiarization with arabic numerals or number words. It is interesting in this respect to compare the performance in arithmetical tasks of two small-scale societies, the Munduruсu and the Yupno. Although the Munduruсu have approximate number words up to five, they fail to produce a correct result when subtracting four dots from a total set of six dots (i.e., $6 - 4$), although the result is small enough to be named in their approximate number word system (Pica et al., 2004). The reason for their failure lies in the fact that their number words are not natural numbers but approximate number words. In contrast, the Yupno, an Aboriginal culture from Papua New Guinea, who have a body-part counting system that goes up to 33, can solve tasks such as $12 + 13$ or $19 - 8$, the latter by reversing the problem into an addition, counting up from the smaller to the larger number by naming the different body parts (Wassman & Dasen, 1994, p. 89). They can do this, because in a counting context the body-parts serve as exact number symbols.

It is worthwhile to consider Dartnall's (2005) *internalism* in this discussion. Internalism mirrors externalism in the sense that "the world leaks into the mind." As an illustration, Dartnall considers a person who observes an uncompleted jigsaw, who leaves the room and then realizes—through mental rotation—how one of the remaining pieces fits in the puzzle. This mental rotation of the observed piece is an epistemic action, since it tells him something he did not know before. We could speculate that any kind of extended mind entails some kind of internalism: we use external media that represent numerosities as epistemic tools by manipulating mental representations of them to solve numerical problems that are otherwise intractable to us, as in the case of skilled abacus users who resort to a mental abacus to solve multiple-digit calculation (Negishi et al., 2005).

External symbolic representations of natural numbers are not merely converted into an inner code; they remain an important and irreducible part of our numerical cognition. Natural language is one among several tools that allow us to map exact cardinalities onto our approximate logarithmic mental number line. During cognitive development, the structure of the brain is adapted to the external media that represent natural numbers in the culture where one is raised. In this way, the interaction between internal cognitive resources and external media is not a one-way traffic but an intricate bidirectional process: we do not just endow external media with numerical meaning, without them we would not be able to represent cardinalities exactly.

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Note

[1] There are many alternative theoretical models to explain numerical skills in infants and nonhuman animals, including linear mental number lines with scalar variability, two core systems of number (one for small numerosities up to three or four and another for larger, approximate magnitudes) and object files. However, the choice of the model does not matter for the argument I am developing here.

References

Adams, F., & Aizawa, K. (2001). The bounds of cognition. *Philosophical Psychology*, *14*, 43–64.

Alibali, M., & DiRusso, A. A. (1999). The function of gesture in leaning to count: More than keeping track. *Cognitive Development*, *14*, 37–56.

Andres, M., Seron, X., & Olivier, E. (2007). Contribution of hand motor circuits to counting. *Journal of Cognitive Neuroscience*, *19*, 563–576.

Antell, S. E., & Keating, D. P. (1983). Perception of numerical invariance in neonates. *Child Development*, *54*, 695–701.

Biro, D., & Matsuzawa, T. (2001). Chimpanzee numerical competence: Cardinal and ordinal skills. In T. Matsuzawa (Ed.), *Primate origins of human cognition and behavior* (pp. 199–225). Tokyo: Springer.

Brannon, E. M., & Terrace, H. S. (2002). The evolution and ontogeny of ordinal numerical ability. In M. Beko, C. Allen, & G. M. Burghardt (Eds.), *The cognitive animal: Empirical and theoretical perspectives on animal cognition* (pp. 197–204). Cambridge, MA: MIT Press.

Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.

Cain, C. R. (2006). Implications of the marked artefacts of the Middle Stone Age of Africa. *Current Anthropology*, *47*, 675–681.

Cantlon, J. F., Brannon, E. M., Carter, E. J., & Pelphrey, K. A. (2006). Functional imaging of numerical processing in adults and 4-y-old children. *PloS Biology*, *4*(5), 844–854.

Carruthers, P. (2002). The cognitive functions of language. *Behavioral and Brain Sciences*, *25*, 657–726.

Chemla, K. (2003). Generality above abstraction: The general expressed in terms of the paradigmatic in mathematics in ancient China. *Science in Context*, *16*, 413–458.

Chrisomalis, S. (2004). A cognitive typology for numerical notation. *Cambridge Archaeological Journal*, *14*, 37–52.

Clark, A. (2006). Material symbols. *Philosophical Psychology*, *19*, 291–307.

Clark, A., & Chalmers, D. (1998). The extended mind. *Analysis*, *58*, 7–19.

Dantzig, T. (1947). *Number: The language of science*. London: Allen & Unwin.

Dartnall, T. (2005). Does the world leak into the mind? Active externalism, ''internalism'' and epistemology. *Cognitive Science*, *29*, 135–143.

De Cruz, H. (2006). Why are some numerical concepts more successful than others? An evolutionary perspective on the history of number concepts. *Evolution and Human Behavior*, *27*, 306–323.

De Cruz, H. (2007). How does complex mathematical theory arise? Phylogenetic and cultural origins of algebra. In C. Gershenson, D. Aerts, & B. Edmonds (Eds.), *Worldviews, science and us: Philosophy and complexity* (pp. 338–351). London: World Scientific.

Dehaene, S. (2003). The neural basis of the Weber-Fechner law: A logarithmic mental number line. *Trends in Cognitive Sciences*, *7*, 145–147.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*, *284*(5416), 970–974.

d'Errico, F. (1995). A new model and its implications for the origin of writing: The La Marche antler revisited. *Cambridge Archaeological Journal*, *5*, 163–206.

Donald, M. (1991). *Origins of the modern mind: Three stages in the evolution of culture and cognition*. Cambridge, MA: Harvard University Press.

Eger, E., Sterzer, P., Russ, M. O., Giraud, A.-L., & Kleinschmidt, A. (2003). A supramodal number representation in human intraparietal cortex. *Neuron*, *37*, 1–20.

Fodor, J. A. (1975). *The language of thought*. New York: Crowell.

Gaser, S., & Schlaug, G. (2003). Brain structures differ between musicians and non-musicians. *Journal of Neuroscience*, *23*, 9240–9245.

Geary, D. C., Bow-Thomas, C. C., Lin, F., & Siegler, R. S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development*, *67*, 2022–3044.

Harris, J. W. (1982). Facts and fallacies of Aboriginal number systems. In S. Hargrave (Ed.), *Language and culture* (Work Papers of SIL-AAB, Series B, Vol. 8, pp. 153–181). Darwin: Summer Institute of Linguistics.

Hauser, M. D., Chomsky, N., & Fitch, W. E. (2002). The faculty of language: What is it, who has it, and how did it evolve? *Science*, *298*(5598), 1569–1579.

Jordan, K. E., & Brannon, E. M. (2006). The multisensory representation of number in infancy. *Proceedings of the National Academy of Sciences of the United States of America*, *103*, 3486–3489.

Kirsh, D. (1995). Complementary strategies: Why we use our hands when we think. In J. Moore & J. F. Lehman (Eds.), *Proceedings of the XVII annual conference of the Cognitive Science Society* (pp. 212–217). Mahwah, NJ: Erlbaum.

Le Clec'H, G., Dehaene, S., Cohen, C., Mehler, J., Dupoux, E., Poline, J. B., et al. (2000). Distinct cortical areas for names of numbers and body parts independent of language and input modality. *NeuroImage*, *12*, 381–391.

Lemer, C., Dehaene, S., Spelke, E. S., & Cohen, L. (2003). Approximate quantities and exact number words: Dissociable systems. *Neuropsychologia*, *41*, 1942–1958.

Locke, J. (2004). *An essay concerning human understanding*. Retrieved September 20, 2007, from http://etext.library.adelaide.edu.au/l/locke/john/l81u/index.html (Original work published in 1689.)

Martín-Loeches, M., Casado, P., Gonzalo, R., de Heras, L., & Fernández-Frías, C. (2006). Brain potentials to mathematical syntax problems. *Psychophysiology*, *43*, 579–591.

Masataka, N., Ohnishi, T., Imabayashi, E., Hirakata, M., & Matsuda, H. (2007). Neural correlates for learning to read roman numerals. *Brain and Language*, *100*, 276–282.

McComb, K., Packer, C., & Pusey, A. (1994). Roaring and numerical assessment in contests between groups of female lions, *Panthera leo. Animal Behaviour*, *47*, 379–387.

McRoberts, R. W. (1990). Counting at Pularumpi: A survey of a traditional mathematics and its implications for modern learning. *Aboriginal Child at School*, *18*, 19–43.

Negishi, H., Ueda, K., Kuriyama, M., Kato, M., Kawaguchi, H., & Atsumori, H. (2005). Change of mental representation with the expertise of mental abacus. In B. G. Bara, L. Barsalou, & M. Bucciarelli (Eds.), *Proceedings of the XXVII annual conference of the Cognitive Science Society* (pp. 1606–1611). Mahwah, NJ: Erlbaum.

Netz, R. (2002). Counter culture: Towards a history of Greek numeracy. *History of Science*, *40*, 321–352.

Nieder, A., Diester, I., & Tudusciuc, O. (2006). Temporal and spatial enumeration processes in the primate parietal cortex. *Science*, *313*(5792), 1432–1435.

Petersson, K. M., Silva, C., Castro-Caldas, A., Ingvar, M., & Reis, A. (2007). Literacy: A cultural influence on functional left–right differences in the inferior parietal cortex. *European Journal of Neuroscience*, *26*, 791–799.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, *306*(5695), 499–503.

Rusconi, E., Walsh, V., & Butterworth, B. (2005). Dexterity with numbers: rTMS over left angular gyrus disrupts finger gnosis and number processing. *Neuropsychologia*, *43*, 1609–1624.

Sandrini, M., Rossini, P. M., & Miniussi, C. (2004). The differential involvement of inferior parietal lobule in number comparison: A rTMS study. *Neuropsychologia*, *42*, 1902–1909.

Sarnecka, B. W., & Gelman, S. A. (2004). Six does not just mean a lot: Preschoolers see number words as specific. *Cognition*, *92*, 329–352.

Saxe, G. B. (1981). Body parts as numerals: A developmental analysis of numeration among the Oksapmin in Papua New Guinea. *Child Development*, *52*, 306–316.

Simon, O., Mangin, J.-F., Cohen, L., Le Bihan, D., & Dehaene, S. (2002). Topographical layout of hand, eye, calculation, and language-related areas in the human parietal lobe. *Neuron*, *33*, 475–487.

Tang, Y., Zhang, W., Chen, K., Feng, S., Ji, Y., Shen, J., et al. (2006). Arithmetic processing in the brain shaped by cultures. *Proceedings of the National Academy of Sciences of the United States of America*, *103*, 10775–10780.

Temple, E., & Posner, M. I. (1998). Brain mechanisms of quantity are similar in 5-year-old children and adults. *Proceedings of the National Academy of Sciences of the United States of America*, *95*, 7836–7841.

Uller, C., Jaeger, R., Guidry, G., & Martin, C. (2003). Salamanders (*Plethodon cinereus*) go for more: Rudiments of number in an amphibian. *Animal Cognition*, *6*, 105–112.

Varley, R. A., Klessinger, N. J. C., Romanowski, C. A. J., & Siegal, M. (2005). Agrammatic but numerate. *Proceedings of the National Academy of Sciences of the United States of America*, *102*, 3519–3524.

Wassman, J., & Dasen, P. R. (1994). Yupno number system and counting. *Journal of Cross-Cultural Psychology*, *25*, 78–94.

Wynn, K. (1992). Addition and subtraction by human infants. *Nature*, *358*, 749–750.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*, B1–B11.

Zhang, J., & Norman, D. A. (1995). A representational analysis of numeration systems. *Cognition*, *57*, 271–295.

Zhang, J., & Wang, H. (2005). The effect of external representations on numeric tasks. *Quarterly Journal of Experimental Psychology*, *58*, 817–838.