

THE ONTOLOGICAL FOUNDATION OF
RUSSELL'S THEORY OF MODALITY*

ABSTRACT. Prominent thinkers such as Kripke and Rescher hold that Russell has no modal logic, even that Russell was indisposed toward modal logic. In Part I, I show that Russell had a modal logic which he repeatedly described and that Russell repeatedly endorsed Leibniz's multiplicity of possible worlds. In Part II, I describe Russell's theory as having three ontological levels. In Part III, I describe six Parmenidean theories of being Russell held, including: literal in 1903; universal in 1912; timeless in 1914; transcendental in 1918–1948. The transcendental theory underlies the primary level of Russell's modal logic. In Part IV, I examine Rescher's view that Russell and modal logic did not mix.

It is well known that not only did Russell not have a modal logic, but he ignored modal logic, and was even against modal logic. Call this view *V*. Saul Kripke, for example, says in *Naming and Necessity* that not only did Russell have a theory “plainly incompatible with our direct intuitions of rigidity”,¹ but the one reason for this was that Russell “did not consider modal questions”.² Nicholas Rescher goes further in his article, ‘Russell and Modal Logic’. There he holds that Russell, with his ‘massive influence’³ and “deliberately held negative views toward modal conceptions”,⁴ was almost single-handedly responsible for “the stunted development of modal logic for two generations”.⁵

The scholarly objective of this paper is to combat view *V*. In Part I, I show that Russell repeatedly offered a modal theory and that he repeatedly upheld Leibniz's multiplicity of possible worlds. In Part II, I explain Russell's full theory as having three levels of depth. In Part III, I note six Parmenidean theories of being Russell held from 1903 to 1948. In Part IV, I examine Rescher's advocacy of view *V* to see what may be said for the other side of this issue.

There is much that is true and important in *V*. Many sorts of development in modal logic probably were impeded for many of the reasons Rescher cites. But *V* is not the whole truth. Further study of Russell reveals that not only was Russell concerned with modal logic, but he did have a modal theory, and even developed it throughout his philosophical career. Call this opposing view *V**. I shall, of course, be

arguing in favor of V^* . Indeed, it is arguable that holders of V have been largely responsible for the currently stunted scholarly development of Russell's modal logic. But I shall leave this explanation of the shocking neglect of Russell's theory for the readers of this journal to judge.

1. PROPOSITIONAL FUNCTIONS AND POSSIBLE WORLDS

Is Russell against modal logic? Is he against modality? Consider, for example, what he says in his 1927 *The Analysis of Matter*:

I do not think much can be made of modality, the plausibility of which seems to have come from confusing propositions with propositional functions.⁶

But what follows shows that Russell is not rejecting modality as such, but only the view that necessity is predicated of propositions. Russell goes on to assert what may only be called a theory of logical modality on which the logical modalities are properties of propositional functions. Russell says:

Propositional functions . . . are of three kinds: those which are true for all values of the argument or arguments, those which are false for all values, and those which are true for some arguments and false for others. The first may be called necessary, the second impossible, the third possible.⁷

Kripke and Rescher do not seem to be aware of this theory, which Russell states very openly also in famous works such as 'The Philosophy of Logical Atomism' and *Introduction to Mathematical Philosophy*. Now call this theory of logical modality MDL. Very simply, MDL is the theory that the logical modalities are certain specific properties of propositional functions. To say that MDL is not a modal theory is exactly as absurd as to say that Russell had no theory of existence when he held that existence is a property of propositional functions. In fact Russell had a sophisticated and bold theory of existence roughly like Frege's. And even more might be said for MDL, since nobody at all had offered it before, not even Frege.

Just as levels of Russellian quantification may be in principle nested any finite number of times in a single sentence, as many have seen, so too may levels of modality in MDL, as apparently no one has cared to see. One may nest predications of necessity over different variables of the same level, as well as over variables of different levels. One may

even iterate predications of analyticity (not: necessity) over quoted whole sentences. All three sorts of nesting and iteration are evident in the following sentence:

- S. “‘It is necessary of x that {it is necessary of F that [it is necessary of y that (if Fx , then (if Fy then Fx))]}’ is analytic” is analytic.

S would be true for Russell. Clearly one may predicate necessity over the variables F and y even though the variable x is already modally predicated over. Russell himself did not trouble to point this out. But if this is to tinker with his notation at all, it is to tinker with it very little indeed. So that if a capacity for nesting is a necessary condition of being a modal logic, MDL may meet the criterion. But those who are impressed by strings of repeated symbols would do well to remember that technics are the least part of modality.

Russell continues in the same passage we have been quoting to develop an epistemic modal logic based on MDL. Call it MDL-E. Russell says:

And these terms [necessity, possibility, impossibility] may be transferred to propositions when they are not known to be true on their own account, but what is known as to their truth or falsehood is deduced from knowledge of propositional functions.⁸

Russell, of course, had already expounded both MDL and MDL-E in his 1918 ‘The Philosophy of Logical Atomism’, although there he had perhaps not made explicit that MDL-E applies only to whole propositions. But if MDL-E is a *de dicto* theory of epistemic modality, does that not make MDL a *de re* theory of logical modality? The answer to the question depends on the nature of Russell’s propositional functions. And that in turn concerns the nature of Russell’s propositions. Now unfortunately the notion of a proposition was one of Russell’s more kaleidoscopic notions, as Alan R. White has shown.⁹ There are two main options. Option (1): Sometimes Russell sees propositions as linguistic in nature, in which case propositional functions are probably also linguistic in nature. Now insofar as a propositional function is linguistic it is syncategorematic, i.e., nothing.¹⁰ On option (1) necessity is best viewed as also nothing, just as Russell should view existence on option (1). For nothing can be a property of nothing. Option (2): At other times Russell holds that propositions have nonlinguistic constituents with which we must be acquainted in

order to understand propositions. And insofar as a propositional function is a concept,¹¹ and a concept is "a universal of which we are aware",¹² logical necessity for Russell is best viewed as a universal predicable *de re* of other universals, again just as Russell should view existence on option (2).

Why would Russell offer MDL? There are four reasons he might give. I have been emphasizing how Russell treats the logical modalities the same way he treats existence, or more precisely, everything, something, and nothing. I now suggest that two *reasons* why Russell would do this are also the same. The first one, best expressed in 'The Philosophy of Logical Atomism', is that "There is no sort of point in a predicate which could not be conceivably false".¹³ So that if existence were a property of things, then there would be no point in asserting the existence of anything. But there is a clear sense in which there is great point in saying that certain things do not exist (no need to fear dragons). Hence, existence is not a property of things, and neither are everything and nothing. But similarly, too, for logical possibility. There is much point in saying that certain things are not logically possible (no trying to square the circle). Hence, logical possibility is not a property of things, and neither are logical necessity and logical impossibility. The arguments are so close that existence and logical possibility are both construed as the very same property by Russell, the property of being true for some values and false for others. Even we can accept at least that what is actual is, as such, possible, if not the converse.

The second argument is that universal quantification expresses a property not of things but of propositional functions, and existential quantification is definable in terms of universal quantification and negation. "All *S* is *P*" cannot be about all *S*'s. For we do not know each *S*, but we do understand "All *S* is *P*". But this very argument may be given for logical necessity's being a property of propositional functions. For we can understand "It is logically necessary that (all) things be *F*" even though we do not know each (possible) *F*. Again, the arguments are so close that Russell defines logical necessity in the same way he does the universal quantifier, i.e., as the property of being true for all values of the propositional function in question. And even we can accept at least that what is necessary is, as such, (universally) actual, if not the converse.

Russell's two explicit arguments will be our third and fourth. Third,

then, in his 1912 'On the Notion of a Cause', Russell argues that since a proposition is simply true or false, then if 'necessary' means "what is true under all circumstances", then if it is "worth saying of something" that it is necessary, then that something must be a propositional function, ' Fx ', which "is true for all possible values of x ", "for all values of its argument or arguments".¹⁴ There is a similarity to the first argument in that being worth saying of something is the same as being a predicate which has a point. Note that this third argument speaks of all *possible* values. (And surely Russell means by all circumstances, all *possible* circumstances.) I shall return to this point later.

The question might be raised about the third argument, Are there not logically necessary propositions? And if so, is not logical necessity plainly a property of these propositions? Russell's fourth argument addresses this issue. In his 1919 *Introduction to Mathematical Philosophy*, Russell doubts that there is "any clear account of what [is] added to truth by the conception of necessity".¹⁵ I view this as a requirement that the content of the concept of necessity be different from that of the concept of truth if we are to say significantly of a true proposition that it is necessary. And logical necessity ought to be something different from truth, since some but not all truths are logically necessary. But Russell views truths as timeless and unchanging. What sort of necessity, then, could be intelligibly added to the timeless necessity of the truth of any true proposition? (Could any true proposition be more necessary than it already is? Could logical truths be more necessary than contingent truths?) The difficulty of finding something may be measured by the fact that many philosophers simply explain logically necessary truths as expressing timeless relations between timeless entities. Russell suggests this fourth argument as early as 1903 in *Principles of Mathematics*.¹⁶

The insight of the fourth argument applies when Russell explains not logical necessity but only logical truth as truth in virtue of form, and truth in virtue of form as being a tautology (being analytically true). The question is here, What is added to analyticity by the conception of necessity? Naturally the meaning of 'necessary' can scarcely be identified with the meaning of 'analytically true'. Otherwise the theory that all necessary truths are analytic would not be significant. Once again, we can find nothing to add. This is why Russell does not explain logical necessity in terms of analyticity – unlike the early Wittgenstein.

Those are the four arguments in Russell for MDL. The first two are implicit in his discussions of existence. The second two are explicit. Now let us consider four howlers which might be found in MDL.

First, one howler of which Russell might be easily accused is that he has confounded logical modality with epistemic modality in his account. It does look that way in 'The Philosophy of Logical Atomism'. But I trust that the distinction between MDL and MDL-E, based on the clearer text of *The Analysis of Matter*, lays that howler to rest. Indeed, there it is Russell who can and does claim to be *detecting* this confusion of logic with epistemology.¹⁷

Second, in 'The Philosophy of Logical Atomism' surely poor Russell has 'carelessly'¹⁸ conflated accidental generality with universal necessity by defining them both as the same property, being true for all values. The way out of the second howler is to rely again on the clearer *The Analysis of Matter*. There necessity seems to be the property of being true for all *possible* values.

But what does Russell mean by 'possible value'? – What, indeed, does he mean by 'value'? Does he mean the *argument* which a propositional function may take? For Russell, values are not arguments, except, apparently, in *Principles of Mathematics*¹⁹ and *An Inquiry into Meaning and Truth*.²⁰ Usually propositional functions map arguments onto values for Russell. But Russell's values are not truth values. Nor are they particulars at all. Russell's values are *propositions*. Russell makes this clear in *Principia Mathematica*.²¹ Thus Russell's possible values are possible propositions.

We are thus back to the notion of a proposition with a vengeance. On option (1), on which propositions are spoken or written events, the second howler remains a howler unless we admit merely possible such events. On option (2), on which there are arguably infinitely many actual propositions which have never been spoken or written, the howler might be silenced. For on option (2) the distinction between actual and possible propositions, that is, the distinction between actual and possible values of propositional functions, might arguably be collapsed. (If all constituents of propositions are timeless, these distinctions arguably collapse on either option. But this way out is more plausible on option (2).)

In *Principia* Russell does seem to collapse the distinction between a proposition and a possible proposition when he asserts that the totality of a function's values "comes to the same thing"²² as "the totality of

its possible arguments".²³ He also says that "all possible propositions are obtainable from matrices by the process of turning the arguments to the matrices into apparent variables".²⁴ He speaks repeatedly of possible values (possible propositions)²⁵ as well as of possible arguments.²⁶ But by 'possible argument' Russell might mean here only "actual argument which a given function logically can take".

In his 1940 *An Inquiry Into Meaning and Truth*, Russell advances a theory of syntactically possible sentences.²⁷ On this theory every significant sentence has syntactic possibility, which Russell admits "is perhaps narrower than logical possibility".²⁸ (It is certainly wider, as every logical falsehood is syntactically well formed.) But ignoring this issue, "Jones is a unicorn" would appear to be possible just in case "Jones is a horse" and "Jones has a horn" are possible. More precisely, as long as atomic sentences are possible, their molecular conjunctions are possible.²⁹ This takes much pressure off. But one may still ask if, say, "Spot *S* is red" is possible if red is an atomic quality and nothing is red? Russell's answer seems to be that: (i) if spot *S'* is, say, green, then "Spot *S'* is green" is possible; (ii) green and red belong to "the same category",³⁰ and similarly for spots *S* and *S'*; thus (iii) substituting 'red' for 'green' and '*S*' for '*S'*' results in an equally significant (syntactically possible) statement. But what if nothing has any color? Is "*S* is red" then possible? For Russell, if "*S* is not-red" is significant (syntactically possible), then so is "*S* is red".³¹

Now in this 1940 work propositions are beliefs sentences express.³² So that we might explicate 'possible value' as 'possible proposition', where 'possible proposition' is "belief expressible by a syntactically possible sentence". But Russell speaks of possible values many times in this book.³³ And what he means seems to be not propositions but arguments, i.e., members of the class determined by the function.³⁴ This seems to indicate that Russell's possible arguments are nothing more than those actual arguments which a function logically can take. Thus our proposed explication seems not to be faithful to this particular work. Worse, the explication does not change or refute Russell's conception of a propositional function as possible if it is sometimes true. That is, "Jones is a unicorn" may be a *syntactically* possible sentence expressing a *syntactically* possible proposition. But without any such proposition being true, "*x* is a unicorn" remains a *logically* impossible propositional function for Russell (if he still holds MDL in this work).

This brings us to the third howler. It is that on MDL a propositional function is not possible unless at least one actual argument satisfies it. But consider the propositional function “ x is a unicorn”. Since nothing actually is a unicorn, “ x is a unicorn” is an impossible function for Russell. But surely that should be a possible propositional function, since its values are logically contingent propositions. A closely related question is, Since “Jones is a unicorn” is for Russell synthetic in logical form, does this mean that impossible propositional functions may have synthetic propositions as values?

Russell seems perfectly serious about necessary functions having synthetic propositions as values. He gives two examples in ‘On the Notion of a Cause’:

For example, “if Socrates is a man, Socrates is mortal”, is necessary if Socrates is chosen as argument, but not if *man* or *mortal* is chosen. Again, “if Socrates is a man, Plato is mortal” will be necessary if either Socrates or *man* is chosen as argument, but not if Plato or *mortal* is chosen . . .³⁵

The first example is a material implication whose consequent will be made true by any argument substituted for its logical subject which, when substituted for the logical subject of the antecedent as well, makes the antecedent true. The second example is a material implication whose true consequent is left untouched while the antecedent’s subject or predicate is changed to any other argument. Russell then refines his definition of ‘necessary’:

A proposition is *necessary* with respect to a given constituent if it remains true when that constituent is altered in any way compatible with the proposition remaining significant. [This inspired my sentence S.]³⁶

The obvious implication is that a genuinely necessary proposition is necessary relative to *all* of its constituents (and in this 1912 article, relative to all possible substitute constituents). But on this new definition “Smith is a unicorn” is still impossible, relative to Smith. For “Smith is *not* a unicorn” is *necessary* relative to Smith, in the absence of actual unicorns.

A fourth howler is that on MDL, which propositional functions are possible logically depends on contingent matters such as whether unicorns exist. And this is a most unusual conception of logical possibility.

The third howler seems to be the heart of the matter with respect to the second, third, and fourth howlers alike. So I shall discuss mainly

that one. Let us consider two ways out of the third howler. The first way out would be to show that in some sense only actual things are possible things. Then if there are no unicorns, it really *is* impossible for anything to be a unicorn. This way out is challenging but has ancient roots in Parmenides and Diodorus Cronos which we may cultivate.

The second way out is to show that in some sense only actual propositions (values) are possible propositions (values). Now things themselves may or may not be propositional constituents (or arguments). If not, then propositional constituents are either (a) names or definite descriptions or (b) nonlinguistic universals expressible by definite descriptions. On (a), there is the howler that someday a new definite description may be spoken or written which, when substituted as an argument, makes a supposedly necessary proposition false. The escape would be to show that only actual definite descriptions are possible definite descriptions. But this amounts to the first way out. On (b), if universals exist timelessly, then only actual universals would be possible constituents. But even admitting the universal that would be expressed by the description "the unicorn in the forest" as a possible argument for "*x* is a unicorn", that function will still be impossible for Russell if there are no existent unicorns. Again we are driven back to the first way out.

The notion of a possible argument retains importance. For if the only arguments not to satisfy "*x* is a unicorn" are those which do not even appear to be the right sort of thing, such as apples, then there will be no interesting existence assertions at all. And the only sort of argument that can appear to satisfy "*x* is a unicorn" without succeeding is for Russell a description such as "the unicorn in the forest", or else the universal such a description would express, perhaps accompanied by the presentation of a lone sense-datum (phantom or hallucination). The sense-datum by itself could not appear to do this, for a lone sense-datum as such, unaccompanied by correlated data or even by habitual expectations of such correlated data, is far too unlike what we call an animal.

Thus MDL has its difficulties: three howlers and a challenging way out. But Russell's *intentions* concerning MDL are good. Many facts support this. First, Russell frequently affirms that he is upholding Leibniz's panorama of possible worlds. In *The Problems of Philosophy*, Russell says:

Moreover, we feel some quality of *necessity* about the proposition 'two plus two are four', which is absent from even the best attested empirical generalizations. Such generalizations always remain mere facts: we feel that there might be a world in which they were false, though in the actual world they happen to be true. In any possible world, on the contrary, we feel that two and two would be four: this is not a mere fact, but a necessity to which everything actual and possible must conform.³⁷

In *Our Knowledge of the External World*, Russell says,

Between philosophy and mathematics there is a certain affinity, in the fact that both are general and a priori. Neither of them asserts propositions which, like those of history and geography, depend on the actual concrete facts being just what they are. We may illustrate this characteristic by means of Leibniz's conception of the many *possible* worlds, of which one only is *actual*. In all the many possible worlds, philosophy and mathematics will be the same; the differences will be only in respect of those particular facts which are chronicled by the descriptive sciences.³⁸

In *Introduction to Mathematical Philosophy*, Russell says, "Among 'possible' worlds, in the Leibnizian sense, there will be worlds having one, two, three, . . . individuals".³⁹ In *The Analysis of Mind*, Russell says, "Leibniz's conception of many possible worlds seems to accord much better [than the coherence theory of truth] with modern logic and with the practical empiricism which is now universal".⁴⁰ In *Human Knowledge: Its Scope and Limits*, Russell says, "I hold, rather, to Leibniz's multiplicity of possible worlds".⁴¹

Second, in *Our Knowledge of the External World*, Russell admits possible sense-data⁴² as described ideals providing theoretical continuity for constructions.⁴³

Third, in *The Analysis of Mind* Russell says, "We may identify propositions in general with the contents of actual and possible beliefs".⁴⁴ He admonishes us that "logic is not interested in what people do in fact believe, but only in the conditions which determine the truth or falsehood of possible beliefs".⁴⁵ This definition succeeds in equating actual propositions with possible ones.

Fourth, in *An Inquiry into Meaning and Truth* Russell admits not only actual but also possible object words, and allows us to "suppose an indefinite extension of our perceptive faculties"⁴⁶ to be able to admit a sufficiently large range of possible object words.

Fifth, in *Human Knowledge* Russell holds that it is significant to assert that there are facts we cannot imagine, since the contradictory of that assertion is significant, and by molecular combination, the negation of every significant assertion is also significant.⁴⁷

Sixth, in *Human Knowledge* Russell says we can imagine general facts where we cannot imagine particular facts which would be their instances.⁴⁸

Seventh, in *An Introduction to Mathematical Philosophy* Russell summarizes his discussion of modality not as if he rejected it, but as if he accepted it: "In all such cases, as in regard to modality in general, the propositional function is relevant".⁴⁹ So too in 'The Philosophy of Logical Atomism', where Russell says, "It is important to realize that the whole doctrine of modality only applies to propositional functions, not to propositions".⁵⁰

Can these be the views by which Russell almost single-handedly stunted the development of modal logic for two generations? It seems by far the best course to dig deeper than the explicit MDL theory and delve into Russell's ontology. For it is an ontological question whether only actual things are possible.

2. RUSSELL'S THEORY OF MODALITY

What I call Russell's full modal logic appears in his 1918 'The Philosophy of Logical Atomism'. I have argued elsewhere that in that essay Russell used the expression 'exists' (and its synonyms 'is real', 'has being', and 'is actual') in three senses.⁵¹ I now assert that each of these three senses is also a different level of modal understanding for Russell. That is, just as I have argued that Russell's ontology is rich, having a depth of three levels, I now wish to show against V and on behalf of V^* that Russell's modal logic is equally rich, having a corresponding depth of three levels. I shall call this modal theory MDL {1, 2, 3}. Our old MDL is level {3} in the {1, 2, 3}. To sum up my earlier results:

The primary sense of 'exists' is Parmenidean. It is that to be is not to be nothing. For Russell, there is no such thing as a merely possible thing.⁵² Since everything has being in this sense, it is a non-classificatory, anti-Meinongian sense. Indeed, it is Russell's robust sense of reality.

The secondary sense of 'exists' is Berkeleyan and Humean. It is that to be real is to be correlated with other particulars (sense-data) in appropriate ways.⁵³ Tables, persons, and electrons are real in this sense. Phantoms and hallucinations, as single particulars, are not real in this sense.⁵⁴

The tertiary sense of 'exists' is Fregean. It concerns the logical structure of existence assertions. It is that existence is a property of a propositional function, namely, the property of being satisfied.⁵⁵

These three senses are not rival views, but work together as follows: (1) The relation of the primary sense to the secondary is that particulars (sense-data) which exist in the primary sense are the fundamental building blocks for the constructions or logical fictions which exist in the secondary sense. (2) The relation of the primary sense to the tertiary sense is that conforming to the fundamental principle that all things exist in the primary sense is for Russell a logical requirement of the adequacy of any analysis of the tertiary assertoric sense.⁵⁶ That is, we may *say* that certain things do not exist, but we must not construe our assertions as being *about* nonexistent things. This is just what led to Russell's theory of the elimination of definite descriptions from our assertions. (3) The relation of the secondary sense to the tertiary sense is the most neglected relation among Russell scholars. In the case of persons, tables, and electrons, if not also numbers, the applicability of the secondary sense of 'exists' is the criterion of which arguments satisfy propositional functions for Russell, i.e., of which existence assertions in particular *are true*.⁵⁷ That concludes my summary.

I now wish to distinguish correspondingly between: (i) a primary sense of 'possible' in which all and only existents are possible; (ii) a secondary sense of 'possible' in which only groups of correlated particulars are possible; and (iii) a tertiary sense of 'possible' which concerns the logical structure of possibility assertions. We have already seen that for Russell, possibility in the tertiary sense is predicated of propositional functions. Just as the primary sense of 'exists' is the deepest sense in Russell's theory of being, so the primary sense of 'possible' is the deepest sense in Russell's theory of modality. Paradoxically, neither primary sense is significantly assertable, even though they are the basis of Russell's tertiary theory of assertion.

On the primary level, there are three modal features of interest. I shall mention only particulars, but the features apply to existents generally. (i) The primary existence of a particular is absolutely contingent and can be known only through empirical acquaintance.⁵⁸ (ii) There is no such thing as a merely possible particular.⁵⁹ What Russell calls ideal or possible particulars are not Meinongian nonexistents.⁶⁰ They are logical fictions, and belong to the primary level only out of logical courtesy. (iii) The existence of a particular is transcendently

necessary with respect to thought and language. That is, it is logically necessary, for us to be acquainted with or to be able to name a particular, that it exists in the primary sense. Russell says that a name must always be a name *of* something, and that the meaning of a proper name literally is the particular it names.⁶¹ Call feature (iii) transcendental Parmenideanism.

Concerning modal feature (iii) of particulars, proper names of particulars are obviously rigid designators in Kripke's sense. That is, they denote the same particulars in all possible worlds. That is because (a) to change the particular is to change the meaning of its name. It is also because (b) one fully knows the particulars one is acquainted with,⁶² and one knows *p* only if it is logically impossible for one to be mistaken about *p*'s identity. So that if one knows *p*, then one rigidly designates *p* by '*p*'. It follows that for Russell, proper names rigidly designate the particulars we are acquainted with. D. F. Pears saw reason (a) sixteen years before I did,⁶³ and speculated about reason (b) as well.⁶⁴ Thus, far from being "plainly incompatible with our direct intuitions of rigidity",⁶⁵ as Kripke said, Russell's theory plainly endorses these intuitions on its deepest level.⁶⁶

On the secondary level, there are correspondingly three modal features of ordinary things (groups of correlated particulars). (i) The secondary existence of a group of correlated particulars is not radically contingent, but plainly relatively contingent. That is, it logically depends on the actual primary existence of at least some of the correlated particulars, though others may be ideal or possible for continuity purposes.⁶⁷ (ii) There is a clear sense of structural possibility of secondary existence given the primary existence of some particulars. That is, the secondary existence of an ordinary thing depends on both the possibility and the actuality of these existing particulars' being suitably correlated together. This secondary possibility of secondarily real things is in effect what Michael Loux and William Lycan would call a modal theory of combinatorial actualism.⁶⁸ (iii) The secondary existence of ordinary things is not transcendently necessary. Trees and tables are not secondarily real just because we think about them, are acquainted with certain particulars, or use certain descriptions. For the objects of our thought or perception may be 'hallucinated' or 'phantom',⁶⁹ i.e., single particulars. And descriptions need not be satisfied.

On the tertiary level, there are two possible versions of MDL. On

version (i), we count things as arguments of propositional functions. On version (ii), we count propositional constituents, as opposed to things, as the arguments. Here a propositional function is satisfied if and only if some constituent which is an argument for it corresponds to some thing that exists. Russell's examples of actual arguments are always version (i). But possible sense-data⁷⁰ as arguments could only be version (ii). Russell does hold that we can describe things we are acquainted with.⁷¹ Presumably this would include actual sense-data. So there is no reason why we cannot describe merely possible sense-data. On version (iia) constituents are descriptions, or 'incomplete symbols'.⁷² On version (iib) each constituent is perhaps "composed wholly of particulars and universals with which we are acquainted."⁷³

The three modal features of the tertiary level are: (i) A propositional function is possible if and only if it is sometimes true. (ii) A propositional function is possible, therefore, if and only if it describes something which actually has secondary reality (in the case of logical fictions), or which actually has primary reality (in the case of simple existents). (iii) A tertiary existence assertion may be said to have transcendental necessity in a derivative sense if and only if it is logically deduced by quantifying over a logically proper name (which would, of course, denote a simple existent).

That sums up my presentation of MDL {1, 2, 3}. When we dig beneath the surface, we see that Russell has a rich and sophisticated approach to modality. Perhaps not everything is as fully or as clearly stated as we would like. Only primary level modal features (i)–(iii) and tertiary level modal feature (i) are explicitly stated. But as the rest are obviously logically implicit in Russell's views, it is very reasonable to suppose that MDL {1, 2, 3} articulates Russell's modal logic as well as can be. Thus V^* is confirmed.

The first way out of the third logical howler may now be explained. The third howler was that "x is a unicorn" is impossible on Russell's view, yet most of us would regard the existence of unicorns as logically possible. Now anything that exists in the secondary sense logically must also have existence in the primary sense. That is, anything that is a group of particulars logically must not be nothing. This includes tables, trees, and unicorns. It follows that anything that has secondary existence not only has secondary level modal features (i)–(iii) relative to its secondary existence, but also has primary level modal features (i)–(iii) relative to its primary existence. In particular,

unicorns, if there are any, would have primary level modal feature (ii). That is, on the primary level, the only possible unicorns *are* actual unicorns. And since there are no actual unicorns, unicorns *are* impossible. And this is perfectly compatible with primary level modal feature (i) of radically contingent existence, which any actual unicorns would have.

It is perfectly consistent to assert that actual horses are radically contingent beings on the primary level of existence, and are relatively contingent beings on the secondary level of existence. This concerns modal feature (i) of each of those two levels. With respect to modal feature (ii) on the secondary level of existence, unicorns are combinatorially possible. And this is perfectly consistent with their being impossible according to modal feature (ii) on the primary level of existence. Finally, if some superior being outside space and time could *name* a horse 'H3', we could even assert that H3's primary existence is transcendently necessary relative to the name 'H3', but that H3's secondary existence is not. This concerns modal feature (iii) of each of the two levels.

Let us consider the logical howler as it would appear on the tertiary level of modal assertion. Here we simply specify the version of modal assertion and the modal level, primary or secondary, which is being asserted. For example, consider version (i) with respect to the primary level. Version (i) faithfully reflects the primary level modal features of things, since on it arguments simply *are* things. This includes primary level feature (ii). So that on version (i) with respect to primary modality, "x is a unicorn" is indeed impossible. For if we 'run through' all existents throughout endless time, we simply find no unicorns (no groups of suitably correlated particulars). So to speak, on the primary level there is nothing to *be* a unicorn.

On version (ii), the same explanation is applied *mutatis mutandis* to those things which satisfy the candidate constituents which are arguments for propositional functions. I conclude that Russell's reply to the third howler has been made clear by our triple distinction between modal levels.

3. ITS ONTOLOGICAL FOUNDATION

W. V. Quine notes that Russell "stopped talking of subsistence [being] . . . by 1914".⁷⁴ This is accurate but overlooks the major

changes Russell made. Russell subscribed to six Parmenidean theories of being in his career: (1) being is an entity, 1903; (2) being is the world of universals, 1912; (3) being is general timelessness, 1914; (4) primary being is transcendentally necessary for logical atoms, 1918; (5) primary being is transcendentally necessary for object words, 1940–1948; (6) qualities are substantive (atoms), 1940–1956. The last three theories come from the same period as Russell's theory that analytic statements are tautologies, and must not be confused with that latter theory.

I suggest that in theories (4) and (5), while Russell rejected literal being which is a literally necessary object, he accepted what may be called transcendental being which is transcendentally necessary. It ontologically underlies MDL {1, 2, 3}. Thus an account of transcendental being and its development in theories (4) and (5) is crucial to understanding Russell's motives for MDL {1, 2, 3}. (Theory (6) is but an improved version of theory (4).)

The trend from theory (1) to theory (5) has been from the most literally Parmenidean to the least. (Beginning with his 1901 'Mathematics and the Metaphysicians', Russell has held that Zeno was simply right: there *is* no change.⁷⁵) I shall describe Russell's six Parmenidean ontologies in order.

First, there is the ontology of the 1903 *Principles of Mathematics*. Here Russell distinguishes between being and existence, between entities and existents. Some but not all entities exist. Existents would be a proper subclass of the class of entities if class membership were not restricted by Russell to existents for his mathematical logic to succeed. Perhaps this is to ensure that the null class has no nonexistent members.⁷⁶ In any case entities, or terms, are so many "immutable and indestructible"⁷⁷ Parmenidean beings.

Being is 'being simpliciter'.⁷⁸ The intension of 'is' is radically different from the intension of 'exists'. Russell discusses being in positive terms: "Being is that which belongs to every conceivable term, to every possible object of thought...".⁷⁹ Russell discusses being using two particular sorts of positive terms. First, he uses cognitive terms such as *conceivable* and *thought*. Second, he uses modal terms such as *conceivable* and *possible*. But it does not follow from these discussions that Russell's 1903 being was at all intrinsically conceptual or modal in character. Russell speaks in *Principles of Mathematics* of analysis, definition (by description), characterization,

and criteria.⁸⁰ Now being is being simpliciter, so it is not analysable. And being is presupposed by every term,⁸¹ so it is not definable. That leaves characterization or criteria. (Russell did not analyze, define, characterize, or provide a criterion for these last two terms.) On the face of it, Russell's discussion is a characterization. There is no need of a criterion for being, since everything has being anyway.⁸²

Thus Russell's 1903 being is apparently *intensionally* (intrinsically) nonmodal and nonconceptual, even though Russell seems to be suggesting that being is *co-extensive* with being thinkable and being possible. This fact is interesting because Russell nonetheless held that "every term is immutable and indestructible",⁸³ *term* being Russell's 'widest word'.⁸⁴ For it seems to show that the immutability of terms is not due to any *de re* necessity of being, but simply to the timelessness of being. I see no grounds for thinking Russell ever changed his mind about the nonmodal character of being (Quine notes that Russell retained 'subsistence' (being) from 1903 to 1914,⁸⁵ but does not note the three changes; and there is transcendental being after 1914.) If this is so, then MDL {1, 2, 3} may be described as nonmodal actualism (recall modal feature (i) of the primary level). This is something MDL {1, 2, 3} has in common, interestingly enough, with Rescher's own modal theory.⁸⁶

Clearly Russell's 1903 being is thoroughly Parmenidean. He himself later wrote, "I had been a realist in the scholastic or Platonic sense; I had thought that cardinal integers, for instance, had a timeless being".⁸⁷

Though the being of Russell's 1903 being may have been inferred from the timelessness of truth, Russell did have an explanation of how an actual chair is contingent.⁸⁸ Still, the difficulty of distinguishing necessity from timelessness apparently led him to hold that necessity could pertain only to implications between propositions.⁸⁹

The second Parmenidean theory of being Russell offers is in his 1912 *The Problems of Philosophy*. In this work entities and existents are mutually exclusive classes. No entities are existents, and no existents are entities.

Here universals have timeless being, while particulars come into and pass away from existence. This is more or less Plato's two world view: a Parmenidean world of universals and an Heraclitean world of change.⁹⁰ Thus there is still a strong connection between Russell's being and Parmenides' being: "The world of universals . . . may also be

described as the world of being. The world of being is unchangeable, rigid, exact . . ."⁹¹

The third theory of Parmenidean being is given in the 1914 *Our Knowledge of the External World*. There, after quoting Parmenides' argument, Russell says:

A truer image of the world, I think, is obtained by picturing things as entering into the stream of time from an eternal world outside, than from a view which regards time as the devouring tyrant of all that is.⁹²

Now the whole universe seems to be eternal being. Thus Parmenides' argument now leads not to an entity, *being*, as in 1903, nor to a world of timeless entities, universals, as in 1912, but to general timelessness. This general timelessness is more embracing than the timeless world of universals, since it presumably also embraces particulars.

Anthony Quinton omits the whole third theory from his account of Russell's views on necessity.⁹³ But the 1914 'Mysticism and Logic' shows Russell's great interest in it. Here Russell sees Timelessness as due to Parmenides' denial of plurality: past, present, future would be a plurality.⁹⁴ Russell claims that he does not know whether Parmenidean reality is real. But he argues against the reliability of mystical intuition of such reality.⁹⁵ He mentions that the basis of the doctrine of oneness is Parmenides' logical argument as to the impossibility of nonbeing.⁹⁶ Russell then goes on to consider the unreality of time as a *separate* issue from that of unity and plurality. There the discussion is almost identical with that in *Our Knowledge of the External World*.⁹⁷

Transcendental Parmenideanism was Russell's fourth theory. Russell held that his theory that 'one' and 'being' are predicable of descriptions destroyed the being of Parmenides and Plato,⁹⁸ yet being structurally survives as the foundation of the very theory, transcendently on the primary level.

By 'transcendental' I mean a theory about what is, which is inferred from considerations about thought and language. For language to be thus and such, the world must be so and so. Thought and language are thus and such; therefore the world is so and so. Russell's transcendental theory is: Acquaintances and names must be *of* something at some ultimate level. We have acquaintances and names of certain sorts; thus it must be that certain things exist.

Concerning 'The Philosophy of Logical Atomism', the essay in which MDL {1, 2, 3} is most fully present, Pears gives well the stan-

dard account of why Russell's logical atoms must exist. It is that all analysis of what complex propositions are about must come to an end with simples that exist.⁹⁹ That is, given that there are complexes, there must be existent simples. The transcendental aspect is obvious. Pears clearly separates this logical issue from Russell's empiricism of identifying these simples with sense-data.¹⁰⁰ I disagree with Pears' view that Moore's famous argument against Russell's claim that simple sense-data must exist 'demolished'¹⁰¹ Russell's claim. As Moore put it, you can "always say with truth of a sense-datum, 'This might not have existed'".¹⁰² Pears goes on to say that the most that can be said for Russell is that "if a particular is dubbed 'a'", it is a pragmatic contradiction to continue with the words "... does not exist".¹⁰³ Pears then adds that "if the dubbing is *correct*, the particular must exist".¹⁰⁴ This last remark suggests what is really wrong with Moore's criticism. Moore is right but misses the mark. Russell establishes only the transcendental necessity, not the intrinsic necessity, of simples' existing (not being nothing). Concerning modal features (i) and (iii) of the primary level of modality, Moore shows only that (i) is correct, and does not even see (iii). Pears' contradiction is not merely pragmatic. It is transcendental. (On transcendental argument #21 below, it is also deductively logical.)

In *A History of Western Philosophy*, Russell gives a scholarly statement of the transcendental Parmenideanism which was feature (iii) of the primary modal level. Quoting Parmenides' main argument exactly as in 1914, Russell now says:

The essence of the argument is: When you think, you think *of* something; when you use a name, it must be the name *of* something. Therefore both thought and language require objects outside themselves

This is the first example in philosophy of the argument from thought and language to the world at large. It cannot of course be accepted as valid, but it is worth while to see what element of truth it contains

... [the element of truth is that] it is obvious that, in most cases, we are not speaking of words, but of what the words mean. And this brings us back to the argument of Parmenides, that if a word can be used significantly it must mean *something*, not nothing, and therefore what the word means must in some sense exist¹⁰⁵

It even appears here that Parmenides himself was philosophy's very first transcendental Parmenidean! Russell continues:

What subsequent philosophy, down to quite modern times, accepted from Parmenides, was not the impossibility of all change, which was too violent a paradox, but the indestructibility of *substance*.¹⁰⁶

Russell goes on to reject both substance and the related notion of essence as “a transference to metaphysics of what is only a linguistic convenience”.¹⁰⁷ He says in the later 1948 *Human Knowledge* that “proper names, as ordinarily understood, are ghosts of substances”.¹⁰⁸ But in the earlier 1918 ‘The Philosophy of Logical Atomism’, Russell says:

Particulars have this peculiarity, among the sorts of objects that you have to take account of in an inventory of the world, that each of them stands entirely alone and is completely self-subsistent. It has that sort of self-subsistence that used to belong to substance, except that it usually only persists through a very short time, so far as our experience goes (PLA 201–202). . . . In that respect particulars differ from the old substances but in their logical position they do not.¹⁰⁹

The 1918 work is closer to Parmenides; in it particulars, not proper names, are Russell’s replacements for Parmenidean substances or substantial atoms. So that properly speaking there are two types of transcendental Parmenideanism Russell has held: the 1918 version and the 1948 version, i.e., theories (4) and (5).

I suspect that it is the transcendental character of Russell’s fourth ontology that has largely kept it hidden, besides Russell’s synonymous use of ‘has being’, ‘is real’, and ‘exists’. Like the three earlier Parmenidean ontologies, it might have led to many nonmodal actualist theories of modality. Only such theories can be ontologically reductive (eliminative) of modality, as is Russell’s theory MDL in terms of propositional functions and being sometimes true. Only in this sense is Russell ‘against’ modality. But then all theories which offer significant explanations of what modality is are against modality.

When Rescher says that Russell’s program of logical constructions “provides yet another facet of his rejection of modality”,¹¹⁰ we see how completely Rescher has misunderstood Russell. For atomism is the whole basis of Russell’s MDL {1, 2, 3}! It is just Russell’s logical atoms which are named by names which must be of something, or are given in an acquaintance which must be with something, and which therefore *cannot be nothing*. Without this Parmenidean transcendental necessity that all things have primary existence, Russell’s tertiary theory of existence and modality *would never have been developed*. The only other ground I can think of would have been the intrinsic plausibility of holding with Frege that a merely possible object is no object at all.¹¹¹

It is in the 1940–1948 period that Russell embraces a fully sys-

tematic transcendental Parmenideanism. By my own count he presents, explicitly or implicitly, some 23 transcendental arguments that for psychology, epistemology, and language to be the way they are, there must be things out there which are something, not nothing. From language, Russell gives positive transcendental arguments concerning object language, and negative transcendental arguments concerning secondary language. Primary or object language may be about either primary level or secondary level things. Secondary language concerns tertiary level assertions. Concerning psychology and epistemology, it is important to note that the arguments concern matters which Russell regards not only as prelinguistic, but even as prehuman.¹¹² Russell's arguments are as follows:

(Psychological) Argument #1. There are no illusions of the senses, but only errors of interpretation.¹¹³ Therefore sensations are as they appear to be. Therefore if sensations appear to exist, then they do exist, and are not nothing.

(Psychological) Argument #2. There is no distinction between a sensation and an object of sensation,¹¹⁴ or between a percept and an object of perception.¹¹⁵ Therefore objects of sensation or perception are as they appear to be. And if they appear to be, then they exist and are not nothing.

(Psychological) Argument #3. Perception cannot be pure interpretation, that is, interpretation that is *of* nothing. Therefore there must be something perceived which is not nothing.¹¹⁶

(Epistemological) Argument #4. There must be a pure datum. For data cannot be conceptualization alone. Conceptualization must be *of* something.¹¹⁷

(Epistemological) Argument #5. For fear of vicious infinite regress of classification, there must be sensations which are pre-classificatory.¹¹⁸

(Epistemological) Argument #6. Each datum has its own evidential weight which is not nothing. Otherwise knowledge will be impossible due to a vicious epistemic regress.¹¹⁹

In the following Primary and Secondary Language Arguments, Russell treats object words as nonexpectational, i.e., as meaning nothing beyond present perception, even though most object words, such as 'cat', are expectational.

(Primary Language) Argument #7. If there are no expectations going beyond present experience, then an object word simply,

directly, and simultaneously means and affirms the existence of some present sensation. An object word always asserts or indicates the presence of what it names.¹²⁰ Therefore it has a simple affirmative nature and cannot involve denial. To deny the existence of what the object word affirms would *ipso facto* rob the object word of its very meaning, so that it would not be an object word at all.

(*Primary Language*) *Argument #8*. Since meaning in the primary language (object language) is causal, then with the right causal relation obtaining, it is tautological that there cannot be reference failure. That is, if an object word is caused to be uttered by the object it means or named, then that object cannot be nothing, but must exist. Similarly, the truth of a judgment of perception is logically guaranteed by the causal relation it has to the fact it asserts.¹²¹

(*Primary Language*) *Argument #9*. Spontaneous language cannot lie.¹²² It is tautological that spontaneous speech expresses the speaker's state of mind.¹²³ Many object words are spontaneous. Therefore there are many sensations which are not nothing.

(*Primary Language*) *Argument #10*. In some object language assertions the belief expressed and the fact indicated are the same state of mind. In such assertions the possibility of falsehood 'does not arise'.¹²⁴

(*Primary Language*) *Argument #11*. Language cannot be learned without an essential presupposition of correctness of meaning in teaching and learning situations.¹²⁵ Indeed, the only possible error in primary language is social error, which is insignificant for our purposes.¹²⁶

(*Primary Language*) *Argument #12*. Language must begin with words that can be learned independently of other words. On pain of vicious infinite regress of verbal definitions, some words must be learned ostensively. Therefore there must be words whose meaning is external to language.¹²⁷

(*Primary Language*) *Argument #13*. In the object language, truth is best defined as the assertion's being made about what it indicates. But then in the object language, no true object assertion is about nothing.¹²⁸

(*Primary Language*) *Argument #14*. All object words are names. And the use of expressions as names has an implication much like that intended by the ontological argument. That is, a name must be a name of something.¹²⁹

(*Secondary Language*) *Argument #15*. Lying, falsehood, and deliberate misleadingness require a consciousness of signs as signs. Therefore such linguistic acts are at most made *about* primary language in secondary language. Thus object words cannot be *deliberately* about nothing.¹³⁰

(*Secondary Language*) *Argument #16*. A distinction between expressed significance (or belief) and fact indicated is needed to explain the possibility of falsehood. Such a distinction is present only in secondary language. Therefore all false existence assertions are in the secondary language, and there are no such assertions in primary language.¹³¹

(*Secondary Language*) *Argument #17*. All logical connectives, including negation, belong to the secondary language.¹³² Thus all denials of existence belong to the secondary language. Therefore denials of existence cannot be expressed in primary language.

(*Secondary Language*) *Argument #18*. Negation is not literally part of the world of observed particulars.¹³³ There is no such thing as a 'not' going around which we can sense or experience. And the world can be described without employing negation.¹³⁴ Therefore no denial of existence can be expressed in primary language.

(*Secondary Language*) *Argument #19*. Negation itself is on the most primitive level a *positive* inhibition of a *positive* response.¹³⁵ Thus there is no negation in the world for an object word to name.

(*Secondary Language*) *Argument #20*. Not only are logical connectives part of secondary language, but so is generalization. Therefore existence assertions are secondary language.¹³⁶ Once again, denials of existence cannot even be expressed in primary language.

(*Secondary Language*) *Argument #21*. Existence assertions are tautologically implied by primary language assertions.¹³⁷ That is because existence assertions are actually secondary language abstractions from the meaning of primary language, removing the sensational content of 'a' and leaving only the empty syntactical form 'x' of the expression 'a'. Thus it is tautological that object words assertions are always about existents.

(*Secondary Language*) *Argument #22*. Just as the world can be described without negation or the other logical connectives, so too the world can be described without generalization, that is, without *some* and *all*.¹³⁸ Existence and universality and not things going about which we can sense or experience, and are not nameable in object

language. So we cannot use 'All cats' in the object language to be construed as a denial of 'Some noncats'.

Secondary Language Argument #23. On the most primitive level existential and universal generalization are respectively like asserted disjunctions and conjunctions.¹³⁹ And *and* and *or* are not going about in the world. *And* obviously adds nothing to its conjuncts,¹⁴⁰ and *or* has its nonlinguistic psychological root in experienced hesitation between alternatives.¹⁴¹ Therefore there is no indefinite conjunction in the world that is even *like* universality for an object word to be about. Again there will be no object language 'All cats'.

This battery of arguments clearly demonstrates that the 1940–1948 Russell is an extremely thoroughgoing transcendental Parmenidean, even though some of these arguments (#20–#23) actively reject the sort of being Parmenides himself advocated. It is important to gather these arguments in one place to show for once the full extent of Russell's commitment to the transcendental impossibility of nonbeing at the primary level of existence. For Russell himself did not do or show this. And it is too easy to be left with the impression that Russell's commitment to transcendental Parmenideanism is limited to the simple claim that a name must be the name of something.

Russell's sixth Parmenidean ontology is his 1940–1956 continuation of the substance tradition, which, we saw, he regards as Parmenidean. It is important to us only as offering some new nameables, qualities, for theories (4) and (5) to concern. A quality "is a particular, not a universal",¹⁴² (probably) because it has no instances; ordinary things are bundles of qualities.¹⁴³ Qualities are "syntactically [categorially] more akin to substances"¹⁴⁴ than to universals.

4. THE CASE FOR V

I should like to examine certain claims made by Rescher on behalf of V, as it is only fair to consider the other side of the $V-V^*$ dispute – even though Rescher, for his part, did not consider our side at all.

First, Rescher sees Russell as "unwilling to recognize the merely possible (i.e. the *contingently* possible) as a distinct category".¹⁴⁵ – Well, we have absolute contingency on the primary level and relative contingency on the secondary. And Russell himself said he advocated the tertiary theory precisely because it made necessity, possibility, and impossibility distinct.¹⁴⁶

Second, Rescher suggests that the "philosophical roots of the early Russell's discontent with mere factual truth"¹⁴⁷ are found in "Spinozistic necessitarianism",¹⁴⁸ adding that Russell is a "more or less classical"¹⁴⁹ determinist. – We have seen logical fatalism, i.e., the timelessness of truth, even in Russell's first theory of being. But we have not mentioned determinism at any point. Russell's modal roots are in Parmenides.

Third, Rescher says of Russell that "Like his hero, Spinoza, he was prepared to maintain that there will, in the final analysis, be a *collapse* of modality: that the actual is itself is more or less necessary, so that the possible vanishes as a distinct category".¹⁵⁰ – We have seen that the *fear* of such a collapse is Russell's very motive for making necessity a relation between propositions on his first theory of being,¹⁵¹ and a property of propositional functions on his fourth theory.¹⁵² Rescher also ignores feature (i) of the primary level of modality, that particulars are absolutely contingent.¹⁵³ So that in 1918 the actual is not necessary at all. (Even with his Parmenidean being of 1903, Russell made room for contingency.¹⁵⁴)

Fourth, Rescher notes that "...in mathematics it is altogether otiose to differentiate between the actual and the necessary, and there is no room at all for the contingently possible".¹⁵⁵ And if mathematics and logic are at bottom identical, then "...a modal *logic* becomes almost a contradiction in terms".¹⁵⁶ Thus Russell's almost exclusive concern with mathematical logic is a strong reason why Russell was predisposed against modal logic.¹⁵⁷ – Now Russell's own account of the matter is somewhat different. Russell clearly states in the Preface to *Principles of Mathematics* that mathematics is concerned not with actual objects but with 'hypothetical objects',¹⁵⁸ and makes hypothetical assertions.¹⁵⁹ Russell says:

Thus in every proposition of pure mathematics...any conceivable entity may be substituted for any one of our variables without impairing the truth of our proposition.¹⁶⁰

Thus Russell cannot even explain what mathematics is, without introducing the very distinction Rescher accused him of finding no room for!

Fifth, Rescher cites three points advanced by Hugh MacColl which are 'foundational' for modal logic:

- a. that there is a crucial difference between propositions that

- obtain merely de facto and those that obtain of necessity
- b. that there is a crucial difference between a *material* implication and genuine implication
 - c. that a satisfactory logic of modality must distinguish between actually existing individuals and merely possible ones¹⁶¹

Rescher then comments that “Russell, of course, would have none of this”.¹⁶² But Russell honors all three points. (a) Among propositions he has a synthetic-analytic distinction. Among propositional functions he distinguishes between the necessary, the possible (the contingent), and the impossible. (b) Russell makes clear the difference between material implication and logically necessary hypothetical implication in ‘On the Notion of a Cause’, where material implications are revealed to be necessary relative only to their antecedent’s constituents being replaced by other arguments.¹⁶³ (c) The whole point of Russell’s theory of descriptions is to account for this distinction without *ontologically* admitting nonexistent objects. With this theory, Russell has everything from possible sense-data¹⁶⁴ to possible spaces.¹⁶⁵

Sixth, Rescher says that “one can search Russell’s pages in vain for any recognition of”¹⁶⁶ C. I. Lewis, Jan Łukasiewicz, and L. E. J. Brouwer. My view is that Russell largely ignored the work of others largely because he was abandoning the whole field of mathematical logic due to exhaustion.¹⁶⁷ In 1943 Russell wrote that he would not reply to Gödel’s ‘Russell’s Mathematical Logic’ because he (Russell) had not worked on such issues in “about eighteen years”,¹⁶⁸ i.e., since about 1925. (Russell worked on the second edition to *Principia* in 1925.¹⁶⁹) This is not to mention Russell’s many nonphilosophical activities after 1918. Let us now “search Russell’s pages” for the three figures Rescher cites. First, as Rescher himself points out, Łukasiewicz became well known only “after the early 1920s”.¹⁷⁰ Second, Rescher singles out Lewis’s 1918 *Survey of Symbolic Logic* as a book about which Russell “preserves total silence”,¹⁷¹ notably in the second edition of *Principia*. Russell and Whitehead recommend Lewis’s 1918 book to the public in the second edition of *Principia Mathematica*.¹⁷² Russell probably missed Lewis’s book in 1918 because he was in jail half that year, working on his own final book in mathematical logic,

Introduction to Mathematical Philosophy. Even then Russell does discuss Lewis in that final book. He discusses a 1912 paper of Lewis. And in *The Analysis of Matter*, Russell explicitly accepts Lewis's notion of strict implication and uses it to clarify the notion of a possible world. Russell says:

On grounds of logic, I hold that nothing existent can imply any other existent except a part of itself, if implication is taken in the sense of what Professor C. I. Lewis calls 'strict implication', which is the relevant sense for our present discussion. If this is true, it follows that any selection of things in the world might be absent, so far as self-contradiction is concerned. Given a world consisting of particulars x, y, z, \dots inter-related in various ways, the world which follows from the obliteration of x must be logically possible.¹⁷³

In these last two works Lewis is cited, under 'Lewis', in the index.

Third, Rescher says that "Russell ignores totally the development of mathematical intuitionism, especially the writings of L. E. J. Brouwer".¹⁷⁴ Russell mentions Brouwer in at least four major works. Russell and Whitehead recommend Brouwer's work to the public on the same page of the second edition of *Principia* on which they recommend Lewis's *Survey*. Brouwer may also be found briefly in *Human Knowledge* and in *My Philosophical Development*. Russell's fullest discussion of Brouwer is Chapters XX–XXI of the *Inquiry*.¹⁷⁵ One may search Rescher's essay in vain for mention of these last three works. Rescher does not even seem to have looked in the index for Brouwer in the last three, though Brouwer is plainly listed under 'Brouwer'.

Might this be how Rescher searched "Russell's pages in vain for any recognition" of modal logic? Modality is plainly listed, under 'modality', in the indices in *Introduction to Mathematical Philosophy* and *The Analysis of Matter*. MDL is described on the very page cited in the former book, and is described on the page after the page cited in the latter book.

Seventh, Rescher says that Russell's constructivism and "dismissal of all inferred entities and processes points towards a demise of potentialities, powers, and causal efficacy that pulls the rug out from the main motivation for recognizing possibility and contingency".¹⁷⁶ Rescher thus ignores the 'chief purpose'¹⁷⁷ of *Human Knowledge*, in which Russell argues that inference as opposed to construction is necessary to science,¹⁷⁸ that scientific construction involves concealed inferences,¹⁷⁹ and that we must admit at least five postulates of

nondeductive epistemic inference to have any scientific knowledge at all.¹⁸⁰ And *The Problems of Philosophy* took an *exclusively* inductive approach toward the external world! Even in the citadel of Russellian constructivism, *Our Knowledge of the External World*, the whole last Lecture is on causal inference. Indeed Russell there *defines* 'causal law' in terms of inference.¹⁸¹ On the whole it is hard to think of anyone who has discussed more hypothetical possibilities as to the physical and spatiotemporal structure of the world than Russell has.¹⁸² Can this be the Russell whom Rescher denounces as dismissing "all inferred entities"?

Eighth, Rescher says, "The logical construction of something real will, quite evidently, be a construction from elements that are themselves altogether actual (real)".¹⁸³ This ignores that Russell's constructivism is not the death of modality, but is the very basis of the combinatorial approach to modality. Saul Kripke, for one, would agree with the combinatorial approach broadly construed. See Kripke's combinatorial dice throwing analogy to possible worlds as being only ways the actual world might have been.¹⁸⁴ Indeed, Russell's constructionism is explicitly modeled on Leibniz's theory of monads¹⁸⁵ in a way that leads easily to his endorsement of Leibniz's full range of possible worlds.¹⁸⁶ Russell's hypothetical construction of the physical world is avowedly Leibnizian; it is constructed from both actual and ideal private worlds of monadic consciousnesses.¹⁸⁷ What could be a more stirring endorsement of possibility short of Meinong's jungle of nonexistent?

Ninth, Rescher says that "Russell was deeply caught up in the ideology of two-valued truth-functionality".¹⁸⁸ Rescher then adds that since the "critical fact"¹⁸⁹ about modal concepts is that they are not truth-functional, Russell was bound to reject modality as illegitimate.¹⁹⁰ Now Russell did not reject, but relocated modality with propositional functions precisely because propositions are only true or false.¹⁹¹ As I said earlier, to say that Russell rejected modality when he made it a property of propositional functions is just as absurd as saying that he rejected existence when he made existence a property of propositional functions. In both cases Russell rejected only a certain naive theory about the subject in question.

Tenth, Rescher says that Russell's distaste for modal logic was due to its being '*philosophically* uncongenial'.¹⁹² My translation is this: Russell had philosophical reasons for advancing the modal logic he

did. – What is wrong with that? Why else would one advance one? What, indeed, is wrong with his reasons? Indeed, it is not clear that Rescher himself has a very congenial or broad attitude about what to count as a modal theory.

In a very kind letter to me dated 30 July 1988, Rescher distinguishes theory of modality from modal logic and suggests that while Russell held views on modality, Russell was against modal logic. My reply is: 1. One might equally distinguish theory of existence from quantificational logic. Nonetheless Russell's theory that existence is a property of propositional functions simply is his quantificational logic. Now quantificational logic is logic. And Russell explicitly identifies MDL with his quantificational logic. It follows that MDL is indeed logic. 2. My Part IV critique rejects Rescher's premisses as false. Thus Rescher's arguments are unsound regardless of what Rescher is concluding – that Russell is against modality or that Russell is against modal logic. This also means that Rescher has given us no positive reason to believe that Russell is in fact against modal logic. 3. Even though Russell's theory of truth admits only two values, Russell does find three valued truth logic both 'possible' and interesting.¹⁹³ Why then would Russell not likewise find modal logic both possible and interesting even if his theory of modality were everything Rescher thinks it is in my Part IV's first, second, third, seventh, and eighth sections?

In this essay I have had only an opportunity to describe Russell's modal theory, its history, its motivations, and its philosophical precedents. I have not evaluated Russell's theory. I have not discussed his views on logical form, analyticity, or tautology. Indeed, I have not even mentioned some of his most suggestive ideas on necessity. There is his refutation of Leibniz's theory of necessary propositions as analytic in *The Philosophy of Leibniz*. There is his refutation of Kant's transcendental theory of necessity, and his claim that there are degrees of implicative necessity, in *Principles of Mathematics*.¹⁹⁴ There are his hints that our feeling of causal necessity comes from the ancient belief in fate in *A History of Western Philosophy*. There is his location of natural kinds not among simple sense-data, which are named "without the medium of any descriptions",¹⁹⁵ but among the most remote constructions of physics in *Human Knowledge: Its Scope and Limits*.¹⁹⁶ There is his presentation of a part-whole theory of logical dependence.¹⁹⁷ Crowning all, there is his celebration in *On Our*

Knowledge of the External World of the liberating influence of the new logic, which allows so many metaphysical possibilities that Occam's razor must be epistemologically wielded right and left.¹⁹⁸

My conclusion is that view *V** has been completely substantiated. Rescher holds that by reading Russell, two generations of logicians were kept from developing modal logic. We have seen that it was, if anything, by *not* reading Russell that this occurred. Russell gave so many statements of MDL and in favor of possible worlds, in so many major works, that ignoring them is incredible. Russell cannot be blamed for several generations of such scholarship.

As a last note, Rescher denies that Russell has a modal theory in 'Russell and Modal Logic', yet inconsistently claims in another article, without giving any argument, that 'Nominalism' is Russell's theory of the "ontology of hardcore possibilities".¹⁹⁹ But even this view of Rescher's, that Russell's theory of *possibilia* is linguistic, is correct at most on the tertiary level of MDL {1, 2, 3}. For the sense-data of the primary level are not only prelinguistic but prehuman, and so are the secondarily real things of the common sense world.²⁰⁰

NOTES

* The United States Naval Academy Research Council kindly provided a summer 1988 research grant for work on this essay. This essay was presented at the Bertrand Russell Society Meeting during the December 1988 Eastern Division Meeting of the American Philosophical Association in Washington, D.C.

¹ Saul Kripke: 1980, *Naming and Necessity*, Harvard University Press, Cambridge, MA, p. 14.

² *Ibid.*

³ Nicholas Rescher: 1979, 'Russell and Modal Logic', in George W. Roberts (ed.), *Bertrand Russell Memorial Volume*, George Allen and Unwin, London, p. 146.

⁴ *Op. cit.*, p. 147.

⁵ *Op. cit.*, p. 148.

⁶ Bertrand Russell: 1954, *The Analysis of Matter*, Dover Publications, Inc., New York, p. 169.

⁷ *Op. cit.*, p. 170.

⁸ *Ibid.*

⁹ See Alan R. White: 'Propositions and Sentences', in *Bertrand Russell Memorial Volume*.

¹⁰ Russell: 1971, 'The Philosophy of Logical Atomism', in Robert C. Marsh (ed.), *Logic and Knowledge*, The Macmillan Company, New York, p. 230; see p. 253.

¹¹ Russell: 1919, *Introduction to Mathematical Philosophy*, Allen and Unwin, London,

- pp. 157–158; see Russell: 1964, *Principles of Mathematics*, W. W. Norton and Company, New York, p. 44.
- ¹² Russell: 1974, *The Problems of Philosophy*, Oxford University Press, London, p. 52.
- ¹³ 'The Philosophy of Logical Atomism', p. 241.
- ¹⁴ Russell: 1951, 'On the Notion of a Cause', in Russell, *Mysticism and Logic*, Doubleday Anchor Books, New York, p. 176.
- ¹⁵ *Introduction to Mathematical Philosophy*, p. 165.
- ¹⁶ *Principles of Mathematics*, p. 454.
- ¹⁷ *The Analysis of Matter*, p. 173.
- ¹⁸ A. J. Ayer: 1971, *Russell and Moore*, Harvard University Press, Cambridge, MA, p. 92.
- ¹⁹ *Principles of Mathematics*, pp. 20–21, 29.
- ²⁰ *Introduction to Mathematical Philosophy*, pp. 203, 260.
- ²¹ Russell and Alfred North Whitehead: 1950, *Principia Mathematica*, Vol. I, second edition Cambridge University Press, London, pp. xx, 38. In *My Philosophical Development*, George Allen and Unwin Ltd, London, 1959, Russell seems to distinguish between "values of the function" and "values of the variable", p. 92. The former seem to be sentences, and the latter, things. Concerning the latter, Russell seems to equate "all possible values of x " with "everything in the universe", p. 52, though probably restricting 'everything' to everything of the proper logical type in his extensional hierarchy of types, p. 53. See pp. 62–63 on totalities. Russell's seeming equation of assigning a value to a variable x with substituting "a proper name for ' x '", p. 92, shows how the second way out of the howlers may come into play, if ordinary names (or better, definite descriptions), as well as logically proper names, are meant.
- ²² Op. cit., p. 54.
- ²³ Ibid.
- ²⁴ Op. cit., p. 51.
- ²⁵ Op. cit., pp. 40, 41, 50.
- ²⁶ Op. cit., pp. 39, 40, 41, 54.
- ²⁷ Russell, 1985, *An Inquiry into Meaning and Truth*, George Allen and Unwin, London, pp. 37, 170, 182.
- ²⁸ Op. cit., p. 170.
- ²⁹ See op. cit., p. 195.
- ³⁰ See op. cit., p. 182.
- ³¹ Op. cit., 172, 195.
- ³² Op. cit., p. 171.
- ³³ Op. cit., pp. 82, 197, 201, 260, 264.
- ³⁴ *An Introduction to Mathematical Philosophy*, pp. 203, 260.
- ³⁵ 'On the Notion of a Cause', p. 177.
- ³⁶ Ibid.; recall variables x , y , and F in statement S .
- ³⁷ *The Problems of Philosophy*, p. 78.
- ³⁸ Russell: 1960, *Our Knowledge of the External World* Mentor Books, New York, p. 145.
- ³⁹ *Introduction to Mathematical Philosophy*, p. 203.
- ⁴⁰ Russell: 1933, *The Analysis of Mind*, George Allen and Unwin Ltd, London, p. 268.
- ⁴¹ Russell: 1976, *Human Knowledge: Its Scope and Limits*, Simon and Schuster, New

York, p. 157. See also Russell's 'The Study of Mathematics', in *Mysticism and Logic*, p. 65, on possible worlds.

⁴² *Our Knowledge of the External World*, p. 68.

⁴³ Op. cit., p. 89.

⁴⁴ *The Analysis of Mind*, p. 241.

⁴⁵ Op. cit., pp. 241–242.

⁴⁶ *An Inquiry into Meaning and Truth*, p. 66.

⁴⁷ *Human Knowledge*, p. 153. See also Reference 31.

⁴⁸ Ibid.

⁴⁹ *An Introduction to Mathematical Philosophy*, p. 166.

⁵⁰ 'The Philosophy of Logical Atomism', p. 231.

⁵¹ Jan Dejnozka: 1988, 'Russell's Robust Sense of Reality: A Reply to Butchvarov', *Grazer Philosophische Studien* 32.

⁵² 'The Philosophy of Logical Atomism', pp. 189, 220, 248, 252; see Russell, 'On the Nature of Acquaintance', in *Logic and Knowledge*, p. 149, and Russell, 'On Denoting', in *Logic and Knowledge*, p. 55.

⁵³ 'The Philosophy of Logical Atomism', pp. 257–258, 274–275. In 1921 Russell replaces sense-data with a distinction between sensations and data.

⁵⁴ I have suggested that in this secondary sense of 'exists', Russell is a neglected major early proponent of Quine's famous "no entity without identity" thesis. This is because the very particulars which allow informative identities also constitute the secondary reality of an ordinary thing. See 'Russell's Robust Sense of Reality: A Reply to Butchvarov'.

⁵⁵ 'The Philosophy of Logical Atomism', p. 232.

⁵⁶ 'On the Nature of Acquaintance', p. 153; *An Introduction to Mathematical Philosophy*, p. 170; see 'The Philosophy of Logical Atomism', pp. 248, 252.

⁵⁷ See 'The Philosophy of Logical Atomism', pp. 271–277. This is normally so.

⁵⁸ Op. cit., pp. 199, 202–203.

⁵⁹ Op. cit., pp. 259–274; compare p. 220.

⁶⁰ *Our Knowledge of the External World*, pp. 68, 89.

⁶¹ See 'The Philosophy of Logical Atomism', pp. 194–195, 200–201, 243.

⁶² Op. cit., pp. 202–203.

⁶³ D. F. Pears: 1972, 'Russell's Logical Atomism', in D. F. Pears (ed.), *Bertrand Russell: A Collection of Critical Essays*, Doubleday and Company, Inc., Garden City, pp. 37–38; see p. 41.

⁶⁴ Op. cit., p. 42, lines 1–5.

⁶⁵ *Naming and Necessity*, p. 14.

⁶⁶ It must have taken Kripke astonishing concentration to ignore Russell's names and establish against Russell only that Russell's descriptions are not rigid designators, as if that meant Russell had no rigid designators at the *primary* level. – And to do this in a book entitled *Naming and Necessity*! Kripke ignores not only Russell's logically proper names, but also that Russell views 'this = this' as necessarily true. See *Naming and Necessity*, pp. 3–10; p. 27, Note 4. See 'The Philosophy of Logical Atomism', p. 201 on 'this', and pp. 245–247 on 'Scott = Scott' as tautologous if 'Scott' is a name.

⁶⁷ 'On the Nature of Acquaintance', pp. 68, 91.

- ⁶⁸ Michael J. Loux (ed.): 1979, *The Possible and the Actual: Readings in the Metaphysics of Modality*, Cornell University Press, Ithaca, p. 48; p. 60, Note 48.
- ⁶⁹ 'The Philosophy of Logical Atomism', p. 275.
- ⁷⁰ *Our Knowledge of the External World*, p. 68.
- ⁷¹ *The Problems of Philosophy*, p. 53.
- ⁷² 'The Philosophy of Logical Atomism', p. 253.
- ⁷³ *The Problems of Philosophy*, p. 59.
- ⁷⁴ W. V. Quine: 1971, 'Russell's Ontological Development', in Elmer D. Klemke (ed.), *Essays on Bertrand Russell*, University of Illinois Press, Urbana, p. 10.
- ⁷⁵ Russell, 'Mathematics and the Metaphysicians', in *Mysticism and Logic*, p. 78. Compare *Principles of Mathematics*, pp. 347, 369–473.
- ⁷⁶ Compare *Principles of Mathematics*, pp. 347, 369–473.
- ⁷⁶ *Principles of Mathematics*, pp. 74–76, 81.
- ⁷⁷ Op. cit., p. 44.
- ⁷⁸ Op. cit., p. 449.
- ⁷⁹ Ibid.
- ⁸⁰ See op. cit., pp. 27, 111–112, 137, 161, 168, 185–186, 200, 249, 530.
- ⁸¹ Op. cit., pp. 43, 451.
- ⁸² Op. cit., pp. 43, 449. But some phrases have no meaning, p. 176. Thus a criterion of being may be in order. There are general marks of terms, p. 44.
- ⁸³ Op. cit., p. 44.
- ⁸⁴ Op. cit., p. 43. (See p. 55, Note *, on *object*.)
- ⁸⁵ 'Russell's Ontological Development', p. 10.
- ⁸⁶ *The Possible and the Actual*, pp. 56, 58; see Rescher, 'The Ontology of the Possible', in *The Possible and the Actual*.
- ⁸⁷ *Principles of Mathematics*, p. 13.
- ⁸⁸ Op. cit., pp. 47–48; Klemke, 'Logic and Ontology in Russell's Philosophy', in *Essays on Russell*, p. 422.
- ⁸⁹ *Principles of Mathematics*, p. 454.
- ⁹⁰ *The Problems of Philosophy*, p. 100.
- ⁹¹ Ibid.; see Anthony Quinton, 'Russell's Philosophy of Mind', in *Bertrand Russell: A Collection of Critical Essays*, pp. 103–104.
- ⁹² *Our Knowledge of the External World*, pp. 130–131.
- ⁹³ 'Russell's Philosophy of Mind', pp. 103–104.
- ⁹⁴ Russell, 'Mysticism and Logic', in *Mysticism and Logic*, p. 10.
- ⁹⁵ Op. cit., pp. 11–17.
- ⁹⁶ Op. cit., p. 18.
- ⁹⁷ Op. cit., pp. 19–20.
- ⁹⁸ Russell, 'Logical Positivism', in *Logic and Knowledge*, p. 370.
- ⁹⁹ 'Russell's Logical Atomism', pp. 28–29.
- ¹⁰⁰ Op. cit., pp. 37–38. Russell allows that there may be entities *simpler* than those which have been atomic in our language, thought, and acquaintance so far. This is just why sense-data must be Russell's named particulars. But he says only sometimes that there must be *simples* if there are complexes, as in 'Logical Atomism', in *Logic and Knowledge*, p. 337. Other times he says that analysis may have no end, – as in 'The Philosophy of Logical Atomism', p. 202. Such simple *possibilia* are ostensibly primary

level, but they have no names. Therefore as descriptions, or logical fictions of a noncorrelative sort, they are secondary level.

¹⁰¹ Op. cit., p. 38.

¹⁰² G. E. Moore: 1966, 'Is Existence a Predicate?', in G. E. Moore, *Philosophical Papers* Collier Books, New York, p. 125.

¹⁰³ 'Russell's Logical Atomism', p. 39.

¹⁰⁴ Ibid.

¹⁰⁵ Russell: 1945, *A History of Western Philosophy*, Simon and Schuster, New York, p. 49.

¹⁰⁶ Op. cit., p. 52.

¹⁰⁷ Op. cit., p. 201.

¹⁰⁸ *Human Knowledge*, p. 73. The *Inquiry*, p. 32, looks like an anticipation.

¹⁰⁹ 'The Philosophy of Logical Atomism', p. 204.

¹¹⁰ 'Russell and Modal Logic', p. 144.

¹¹¹ Gottlob Frege: 1970, *Grundgesetze der Arithmetik*, Vol. ii, in Peter Geach and Max Black (eds.), *Translations from the Philosophical Writings of Gottlob Frege*, Basil Blackwell, Oxford, p. 222.

¹¹² *An Inquiry into Meaning and Truth*, pp. 13, 81; *Human Knowledge*, pp. 94–95.

¹¹³ *Human Knowledge*, p. 167.

¹¹⁴ *An Inquiry into Meaning and Truth*, pp. 114, 116.

¹¹⁵ *Human Knowledge*, p. 205.

¹¹⁶ *An Inquiry into Meaning and Truth*, pp. 124–125.

¹¹⁷ *Human Knowledge*, p. 171.

¹¹⁸ Op. cit., p. 423.

¹¹⁹ *An Inquiry into Meaning and Truth*, pp. 124–125; see *Human Knowledge*, p. 190.

¹²⁰ *An Inquiry into Meaning and Truth*, p. 29.

¹²¹ Op. cit., p. 160. Most philosophers seem totally unaware that Russell offered a sophisticated causal theory of names (object words) in 1940.

¹²² Op. cit., pp. 204–205.

¹²³ Op. cit., p. 212; see p. 215.

¹²⁴ Op. cit., p. 206.

¹²⁵ Op. cit., p. 25.

¹²⁶ *Human Knowledge*, p. 117.

¹²⁷ *An Inquiry into Meaning and Truth*, pp. 25, 65, 128.

¹²⁸ Op. cit., pp. 215, 232.

¹²⁹ Op. cit., p. 32. Compare *Principles of Mathematics*, pp. 36–41, on material implication and formal implication. See also the quotation referenced by Reference 138 on Russell's acceptance of C. I. Lewis's strict implication.

¹³⁰ Op. cit., pp. 73–74, 210, 212.

¹³¹ Op. cit., p. 183.

¹³² Op. cit., pp. 40, 64–65, 73–74, 212.

¹³³ *Human Knowledge*, p. 125.

¹³⁴ Op. cit., p. 500.

¹³⁵ Op. cit., pp. 120–121, 122, 124–125.

¹³⁶ *An Inquiry into Meaning and Truth*, p. 65.

¹³⁷ Op. cit., pp. 242, 245 ff.; *Human Knowledge*, pp. 141, 142.

- ¹³⁸ *Human Knowledge*, p. 501.
- ¹³⁹ Op. cit., p. 134.
- ¹⁴⁰ Ibid.
- ¹⁴¹ *An Inquiry into Meaning and Truth*, p. 83ff.; *Human Knowledge*, p. 126.
- ¹⁴² Russell: 1944, 'Reply to Criticisms', in Paul A. Schilpp (ed.), *The Philosophy of Bertrand Russell*, Northwestern University Press, Evanston, p. 685. See also p. 714.
- ¹⁴³ See *An Inquiry into Meaning and Truth*, pp. 98, 102, 128, 230.
- ¹⁴⁴ *My Philosophical Development*, p. 127. But qualities are not traditional substances, p. 126. Also, Russell does retain *generic* universals, p. 127.
- ¹⁴⁵ 'Russell and Modal Logic', p. 139.
- ¹⁴⁶ *An Introduction to Mathematical Philosophy*, p. 165.
- ¹⁴⁷ 'Russell and Modal Logic', p. 140.
- ¹⁴⁸ Ibid.
- ¹⁴⁹ Ibid.
- ¹⁵⁰ Ibid.
- ¹⁵¹ *Principles of Mathematics*, p. 454.
- ¹⁵² *An Introduction to Mathematical Philosophy*, p. 165; *The Analysis of Matter*, p. 170.
- ¹⁵³ 'The Philosophy of Logical Atomism', p. 202.
- ¹⁵⁴ *Principles of Mathematics*, pp. 47–48; see 'Logic and Ontology in Russell's Philosophy', p. 422.
- ¹⁵⁵ 'Russell and Modal Logic', p. 140.
- ¹⁵⁶ Op. cit., p. 141.
- ¹⁵⁷ Ibid.
- ¹⁵⁸ *Principles of Mathematics*, p. xvii.
- ¹⁵⁹ Ibid., p. 6.
- ¹⁶⁰ Ibid., p. 7.
- ¹⁶¹ 'Russell and Modal Logic', pp. 141–142.
- ¹⁶² Op. cit., p. 142.
- ¹⁶³ 'On the Notion of a Cause', p. 177. See also *Principles of Mathematics*, pp. 475–476 on causal relations and choices, and Reference 138 on Lewis.
- ¹⁶⁴ *Our Knowledge of the External World*, p. 68.
- ¹⁶⁵ 'On the Nature of Acquaintance', p. 146.
- ¹⁶⁶ 'Russell and Modal Logic', p. 143.
- ¹⁶⁷ G. Kreisel, 'Bertrand Russell's Logic', in *Bertrand Russell: A Collection of Critical Essays*, pp. 173–174. Compare Hao Wang: 1986, *Beyond Analytic Philosophy: Doing Justice to What We Know*, The MIT Press, Cambridge, MA, p. 241, on Russell's busy parenthood.
- ¹⁶⁸ 'Reply to Criticisms', p. 741.
- ¹⁶⁹ Russell: 1968, *Autobiography of Bertrand Russell*, Vol. II, Allen and Unwin, London, p. 152.
- ¹⁷⁰ 'Russell and Modal Logic', p. 143.
- ¹⁷¹ Ibid.
- ¹⁷² *Principia Mathematica*, p. xlvi.
- ¹⁷³ *The Analysis of Matter*, pp. 199–200.
- ¹⁷⁴ 'Russell and Modal Logic', p. 143.
- ¹⁷⁵ *My Philosophical Development*, p. 83, calls attention to this discussion.

- ¹⁷⁶ 'Russell and Modal Logic', p. 144.
- ¹⁷⁷ Russell, 'Addendum to my "Reply to Criticisms"', in *The Philosophy of Bertrand Russell*, p. xvii.
- ¹⁷⁸ *Human Knowledge*, p. xiv.
- ¹⁷⁹ *Op. cit.*, p. 139.
- ¹⁸⁰ *Op. cit.*, p. 487.
- ¹⁸¹ *Our Knowledge of the External World*, p. 164.
- ¹⁸² See *op. cit.*, pp. 10–11, 53. Indeed, Russell's constructions themselves are only metaphysically conservative hypothetical possibilities. See *op. cit.*, pp. 15, 79, 89, 116, and 'Logical Atomism', pp. 341–343.
- ¹⁸³ 'Russell and Modal Logic', p. 144.
- ¹⁸⁴ *Naming and Necessity*, pp. 15–20.
- ¹⁸⁵ *Our Knowledge of the External World*, pp. 72–73, 89.
- ¹⁸⁶ *Op. cit.*, pp. 144–145.
- ¹⁸⁷ *Op. cit.*, pp. 72–73, 89.
- ¹⁸⁸ 'Russell and Modal Logic', p. 144.
- ¹⁸⁹ *Op. cit.*, p. 145.
- ¹⁹⁰ *Op. cit.*, p. 144.
- ¹⁹¹ 'On the Notion of a Cause', p. 176; *The Analysis of Matter*, p. 170.
- ¹⁹² 'Russell and Modal Logic', p. 147.
- ¹⁹³ 'Reply to Criticisms', pp. 681–682.
- ¹⁹⁴ *Principles of Mathematics*, p. 454. But transcendental argument #22 has a slight Kantian flavor. Russell's 1943 'Reply to Criticisms', p. 710, "I think I agree when he [W. T. Stace] says that generality is peculiar to thought, ..." has even more. In view of the analogies between generality and modality shown in Parts I and II, may MDL {1, 2, 3} be considered roughly Kantian in orientation? Compare p. 689.
- ¹⁹⁵ 'Russell's Logical Atomism', p. 41.
- ¹⁹⁶ *Human Knowledge*, pp. 318, 444.
- ¹⁹⁷ *Our Knowledge of the External World*, p. 63; *An Inquiry into Meaning and Truth*, p. 242.
- ¹⁹⁸ *Our Knowledge of the External World*, pp. 10–11, 53. See also *Principles of Mathematics*, p. 374, and Russell: 1974, *An Outline of Philosophy*, Meridian, New York, pp. 307–308.
- ¹⁹⁹ 'The Ontology of the Possible', p. 180.
- ²⁰⁰ *An Inquiry into Meaning and Truth*, pp. 13, 81; *Human Knowledge*, pp. 94–95.

Manuscript submitted 27 April 1988

Department of the Navy
 United States Naval Academy
 Sampson Hall
 Annapolis, MD 21402-5000
 U.S.A.