Logic and Logical Philosophy
Volume 9 (2001), 25–33

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SOME RESULTS ON JAŚKOWSKI'S DISCURSIVE LOGIC

Abstract. Jaśkowski [3] presented a new propositional calculus labeled "discussive propositional calculus", to serve as an underlying basis for inconsistent but non-trivial theories. This system was later extended to lower and higher order predicate calculus ([1], [2]). Jaśkowski's system of discussive or discursive propositional calculus can actually be extended to predicate calculus in at least two ways. We have the intention using this calculus of building later as a basis for a discussive theory of sets. One way is that studied by Da Costa and Dubikajtis. Another one is developed in this paper as a solution to a problem formulated by Da Costa. In this work we study a first order discussive predicate calculus J^{**}.

The paper consists of three parts. In the first part we introduce the calculus J^{**} and, following Prof. D. Makinson's suggestion, we show that it is not identical with the predicate calculus [2] of Da Costa and Dubikajtis. An axiomatization of J^{**} is presented. In the second one, we introduce new discussive connectives and study some of the properties. We observe that the usual Kripke semantics can be adapted to the calculus J^{**}.

1. Introduction

Jaśkowski [3] presented a new propositional calculus called "discursive propositional calculus" which serve as an underlying basis for inconsistent but non-trivial theories. This system was later extended to lower and higher order predicate calculus (see [1] and [2]).

Jaśkowski's system of discursive propositional calculus can actually be in at least two ways. One way is that studied by Da Costa and Dubikajtis.

Another one is developed in this paper as a solution to a problem formulated by Da Costa. In this work we study a first order discursive predicate calculus J^{**}, using this calculus as a basis for a theory of discursive sets.

The paper consists of three parts. In the first part we introduce the calculus J^{**} and, following Prof. D. Makinson's suggestion, we show that it is not identical with the predicate calculus [2] of Da Costa and Dubikajtis. An axiomatization of J^{**} is presented. In the second one, we introduce new discursive connectives and study some of their properties. We observe that the Kripke usual semantics can be adapted to the calculus J^{**}. Finally, we present an axiomatization of J^{**} based on the discussive operatives (see [4] and [6]).

The calculus present here can, of course, be extended to higher orders, in particular building a theory of discursive sets.

2. The discursive predicate calculus J^{*}

We characterize the discursive predicate calculus J^{*} [5], an extension of the discursive propositional calculus J, through the following definition.

Definition 2.1. Let α be a formula of $\mathrm{S5}^{*}$. α is a thesis of J^{*} iff $\Diamond\alpha$ is a thesis of $\mathrm{S5}^{*}$. In symbols:

$$\vdash_{\mathrm{J}^{*}} \alpha \iff \vdash_{\mathrm{S5}^{*}} \Diamond\alpha$$

We can, however, introduce another discursive predicate calculus associated with $\mathrm{S5}^{*}$, and this will be the object of study in the next section.

3. The discursive functional calculus J^{**}

Definition 3.1. Let α be a formula of $\mathrm{S5}^{*}$ containing free occurrences of the variables x_1, x_2, …, x_n only. We designate the formula

$$\forall x_1 \forall x_2 \dots \forall x_n\, \alpha$$

the universal closure of α. The universal closure of α will be denoted by $\forall\alpha$.

Definition 3.2. Let α be a formula of $\mathrm{S5}^{*}$. α is a thesis of J^{**} iff $\Diamond\forall\alpha$ is a thesis of $\mathrm{S5}^{*}$. In symbols:

$$\vdash_{\mathrm{J}^{**}} \alpha \iff \vdash_{\mathrm{S5}^{*}} \Diamond\forall\alpha\,.$$

Observation. Of course if in α there is no occurrence of free variables then α will be a thesis of J^{*} iff α is a thesis of J^{**}.

A system of axioms for J**

1. The alphabet of J** is the same as S5*.
2. The formulae of J** are the same as S5*.
3. The rules of formation of J** as well as the usual definitions are the same as S5*.
4. Schemes of Axioms and rules of inference

In the following α, β and γ are formulae. Axiomatics C** for J**:

AJ**1. $\Box\forall(\alpha \supset (\beta \supset \alpha))$

AJ**2. $\Box\forall((\alpha \supset (\beta \supset \gamma)) \supset ((\alpha \supset \beta) \supset (\alpha \supset \gamma)))$

AJ**3. $\Box\forall((\neg\alpha \supset \neg\beta) \supset (\beta \supset \alpha))$

AJ**4. $\Box\forall(\Box(\alpha \supset \beta)) \supset (\Box\alpha \supset \Box\beta))$

AJ**5. $\Box\forall(\Box\alpha \supset \alpha)$

AJ**6. $\Box\forall(\Box\alpha \supset \Box\Box\alpha)$

AJ**7. $\Box\forall(\Box\alpha \supset \Box\Diamond\alpha)$

AJ**8. $\Box\forall(\forall x\alpha(x) \supset \alpha(t))$

RJ**1. $\dfrac{\Box\forall\alpha, \quad \Box\forall(\alpha \supset \beta)}{\Box\forall\beta}$

RJ**2. $\dfrac{\Box\alpha}{\alpha}$

RJ**3. $\dfrac{\Box\forall\alpha}{\Box\forall\Box\alpha}$

RJ**4. $\dfrac{\Diamond\forall\alpha}{\alpha}$

RJ**5. $\dfrac{\Box\forall(\beta \supset \alpha(x))}{\Box\forall(\beta \supset \forall x\alpha(x))}$

RJ**6. $\dfrac{\forall\alpha}{\alpha}$

RJ**7. $\dfrac{\Box\forall\Box\alpha}{\Box\forall\alpha}$

We must prove that the axiomatics C** characterize J**, i.e.:

$$\vdash_{\mathrm{C}^{**}} \alpha \iff \vdash_{\mathrm{S5}^*} \Diamond\forall\alpha \iff \vdash_{\mathrm{J}^{**}} \alpha$$

LEMMA 3.3. *If* $\vdash_{\mathrm{C}^{**}} \alpha$ *then* $\vdash_{\mathrm{S5}^*} \Diamond\forall\alpha$.

PROOF. The proof is made by induction on the length c of α, if in C**.

First case. Let $c = 1$. In this case α is one of the axioms C**, and α is the form of $\Box\forall\beta$, where β is axiom of S5*. Then

$$\vdash_{\mathrm{S5}^*} \beta \implies \vdash_{\mathrm{S5}^*} \forall\beta \implies \vdash_{\mathrm{S5}^*} \Box\forall\beta \implies$$
$$\left.\begin{array}{r}\vdash_{\mathrm{S5}^*} \forall\Box\forall\beta \\ \vdash_{\mathrm{S5}^*} \forall\Box\forall\beta \supset \Diamond\forall\Box\forall\beta\end{array}\right\} \implies \vdash_{\mathrm{S5}^*} \Diamond\forall\underbrace{\Box\forall\beta}_{\alpha}$$

Second case. For the induction hypothesis, suppose that the Lemma is valid for proofs in C** of length $c \leq k$. We must show that it is valid for proofs in C** of length $k+1$. We have the following cases to consider:

a) $k = 1$. This is the case considered above.

b) α is obtained in C** by RJ**1 from $\Box\forall\beta$ and $\Box\forall(\beta \supset \gamma)$, therefore the form of α is $\Box\forall\gamma$.

Then, the proofs of $\Box\forall\beta$ and $\Box\forall(\beta \supset \gamma)$ in C** have lengths $\leq k$. By the induction hypothesis: $\vdash_{\mathrm{C}^{**}} \Box\forall\beta$ and $\vdash_{\mathrm{C}^{**}} \Box\forall(\beta \supset \gamma)$, i.e., $\vdash_{\mathrm{S5}^*} \Diamond\forall\Box\forall\beta$ and $\vdash_{\mathrm{S5}^*} \Diamond\forall\Box\forall(\beta \supset \gamma)$.

We must prove $\vdash_{\mathrm{S5}^*} \Diamond\forall\underbrace{\Box\forall\gamma}_{\alpha} \therefore \vdash_{\mathrm{C}^{**}} \alpha$

But

$$(1) \qquad \vdash_{\mathrm{S5}^*} \Diamond\forall\Box\forall\beta \implies \vdash_{\mathrm{S5}^*} \Box\forall\beta$$

$$(2) \qquad \vdash_{\mathrm{S5}^*} \Diamond\forall\Box\forall(\beta \supset \gamma) \implies \vdash_{\mathrm{S5}^*} \Box\forall(\beta \supset \gamma) \implies$$
$$\vdash_{\mathrm{S5}^*} \Box(\forall(\beta \supset \gamma) \supset \Box(\forall\beta \supset \forall\gamma) \implies \vdash_{\mathrm{S5}^*} \Box(\forall\beta \supset \forall\gamma) \supset \Box(\Box\forall\beta \supset \Box\forall\gamma)$$
$$\implies \vdash_{\mathrm{S5}^*} \Box(\Box\forall\beta \supset \Box\forall\gamma) \supset (\Box\forall\beta \supset \Box\forall\gamma) \implies \vdash_{\mathrm{S5}^*} (\Box\forall\beta \supset \Box\forall\gamma)$$

From (1), (2) and *modus ponens* in S5* we have:

$$\vdash_{\mathrm{S5}^*} \Box\forall\gamma \implies \vdash_{\mathrm{S5}^*} \forall\Box\forall\gamma \implies \vdash_{\mathrm{S5}^*} \Diamond\forall\underbrace{\Box\forall\gamma}_{\alpha} \implies \vdash_{\mathrm{C}^{**}} \alpha\,.$$

c) In C** α is obtained by RJ**2, from $\Box\alpha$. In this case the proof of $\Box\alpha$ in C** have length $\leq k$. By the induction hypothesis,

$$(3) \qquad \vdash_{\mathrm{S5}^*} \Diamond\forall\Box\alpha \implies \vdash_{\mathrm{S5}^*} \Diamond\Box\forall\alpha \implies \vdash_{\mathrm{S5}^*} \Box\forall\alpha\,,$$

$$(4) \qquad \vdash_{\mathrm{S5}^*} \Box\forall\alpha \supset \Diamond\forall\alpha\,.$$

From (3), (4) and *modus ponens* in S5* we have:

$$\vdash_{S5^*} \Diamond\forall\alpha \;\therefore \vdash_{C^{**}} \alpha\,.$$

d) In C** α in obtained por RJ**3, from $\Box\forall\beta$ and has the form $\Box\forall\Box\beta$. By the induction hypothesis, as the length of the proof of $\forall\Box\beta$ in C** has length $\leq k$, we have:

$$\vdash_{S5^*} \Diamond\forall\Box\forall\beta \implies \vdash_{S5^*} \Diamond\forall\Box\Box\forall\beta \implies \vdash_{S5^*} \Diamond\forall\underbrace{\Box\forall\Box\beta}_{\alpha} \implies \vdash_{C^{**}} \alpha\,.$$

e) In C** α is obtained by RJ**4 from $\Diamond\forall\alpha$. In this case the proof of $\Diamond\forall\alpha$ in C** has length $\leq k$. By induction hypothesis

$$\vdash_{S5^*} \Diamond\forall\Diamond\forall\alpha\,. \tag{5}$$

We have:

$$\vdash_{S5^*} \Diamond\forall\Diamond\forall\alpha \supset \forall\Diamond\Diamond\forall\alpha\,. \tag{6}$$

From (5), (6) and *modus ponens* in S5* we have:

$$\vdash_{S5^*} \forall\Diamond\Diamond\forall\alpha \implies \vdash_{S5^*} \forall\Diamond\forall\alpha \implies \vdash_{S5^*} \Diamond\forall\alpha \implies \vdash_{C^{**}} \alpha$$

f) In C** α is obtained by RJ**5 from $\Box\forall(\beta \supset \gamma(x))$ and has the form $\Box\forall(\beta \supset \forall x\gamma(x))$. In this case, as the length of the proof of $\Box\forall(\beta \supset \forall x\alpha(x))$ in C** has length $\leq k$. We use the induction hypothesis to write: $\vdash_{S5^*} \Diamond\forall\Box\forall(\beta \supset \gamma(x))$, i.e.,

$$\vdash_{S5^*} \Diamond\forall\underbrace{\Box\forall(\beta \supset \forall x\gamma(x))}_{\alpha} \implies \vdash_{C^{**}} \alpha\,.$$

g) In C** α is obtained by RJ**6 from $\forall\alpha$ where α contains no free variables. In this case as the length of the proof of $\forall\alpha$ in C** is $\leq k$, the induction hypothesis yields: $\vdash_{S5^*} \Diamond\forall\forall\alpha$. But,

$$\vdash_{S5^*} \Diamond\forall\forall\alpha \implies \vdash_{S5^*} \Diamond\forall\alpha \implies \vdash_{C^{**}} \alpha\,.$$

h) The case RJ**7 is analogous. □

Lemma 3.4. *If* $\vdash_{S5^*} \alpha$ *then* $\vdash_{C^{**}} \Box\forall\alpha$.

Proof. The proof is by induction on the length c of the proof of α in S5*. We have the following cases to consider.

First Case. Let $c = 1$. In this case α is an axiom of S5*. In other words $\vdash_{S5^*} \alpha$. By the definition of C** we have that $\Box\forall\alpha$ is an axiom of C**, i.e., $\vdash_{C^{**}} \Box\forall\alpha$.

Second Case. From the induction hypothesis, suppose that the lemma is valid for all proofs in S5* with length $c \leq k$. Let us suppose that the proof of α in S5* has length $k+1$. We have the following cases to consider:

a) α is obtained from β and $\beta \supset \alpha$. In this case the proofs in S5* of β and $\beta \supset \alpha$ have length $\leq k$. Then, by the induction hypothesis

$$\left.\begin{array}{r}\vdash_{C^{**}} \Box\forall\beta \\ \vdash_{C^{**}} \Box\forall(\beta \supset \alpha)\end{array}\right\} \underset{RJ^{**}3}{\Longrightarrow} \begin{array}{rr}\vdash_{C^{**}} \Box\forall\Box\beta & (1) \\ \vdash_{C^{**}} \Box\forall\Box(\beta \supset \alpha) & (2)\end{array}$$

By AJ**4 we have:

$$(3) \qquad \vdash_{C^{**}} \Box\forall(\Box(\beta \supset \alpha) \supset (\Box\beta \supset \Box\alpha)\,.$$

From (2), (3) and RJ**1 we have:

$$(4) \qquad \vdash_{C^{**}} \Box\forall(\Box\beta \supset \Box\alpha)\,.$$

From (1), (4) and RJ**1 we have:

$$\vdash_{C^{**}} \Box\forall\Box\alpha \underset{RJ^{**}2}{\Longrightarrow} \vdash_{C^{**}} \forall\Box\alpha \underset{RJ^{**}6}{\Longrightarrow} \vdash_{C^{**}} \Box\alpha\,.$$

b) α has the form of $\Box\beta$ and it is obtained from β. As the proof of β in S5* has length $\leq k$, the induction hypothesis yields:

$$\vdash_{C^{**}} \Box\forall\beta \underset{RJ^{**}3}{\Longrightarrow} \vdash_{C^{**}} \Box\forall \underbrace{\Box\beta}_{\alpha}\,.$$

c) α has the form $\beta \supset \forall x\gamma(x)$ and it is obtained, from $\beta \supset \gamma(x)$. As the length of the proof of $\beta \supset \gamma(x)$ in S5* has length $\leq k$, we can write:

$$\vdash_{C^{**}} \forall\Box(\beta \supset \gamma(x)) \underset{RJ^{**}5}{\Longrightarrow} \vdash_{C^{**}} \Box\forall \underbrace{(\beta \supset \forall x\gamma(x))}_{\alpha}\,. \qquad \Box$$

THEOREM 3.5. C** *characterizes* J**. *In other words,*

$$\vdash_{C^{**}} \alpha \iff \vdash_{J^{**}} \alpha\,.$$

PROOF. "$\Rightarrow$" $\vdash_{C^{**}} \alpha \Longrightarrow$ (by Lemma 4.1) $\vdash_{S5^*} \Diamond\forall\alpha \Longrightarrow$ (by Def. J**) $\vdash_{J^{**}} \alpha$.

"$\Leftarrow$"

$$\vdash_{J^{**}} \alpha \underset{\text{Def. } J^{**}}{\Longrightarrow} \vdash_{S5^*} \Diamond\forall\alpha \underset{\text{Lemma 4.2}}{\Longrightarrow} \vdash_{C^{**}} \Box\forall\Diamond\forall\alpha \underset{RJ^{**}3}{\Longrightarrow} \vdash_{C^{**}} \Box\forall\Box\Diamond\forall\alpha \quad (1)\,.$$

By AJ**5 we have:

$$(2) \qquad \vdash_{C^{**}} \Box\forall(\Box\Diamond\forall\alpha \supset \Diamond\forall\alpha)\,.$$

From (1), (2) and RJ**1 we have:

$$\vdash_{C^{**}} \Box\forall\Diamond\forall\alpha \underset{RJ^{**}2}{\Longrightarrow} \vdash_{C^{**}} \forall\Diamond\forall\alpha \underset{RJ^{**}6}{\Longrightarrow} \vdash_{C^{**}} \Diamond\forall\alpha \underset{RJ^{**}4}{\Longrightarrow} \vdash_{C^{**}} \alpha\,. \qquad \Box$$

4. The relationship between J* and J**

In this section we set up a relationship between J* and J**, which will be expressed by Theorem 5.1. First, however, we developed the following lemmas:

LEMMA 4.1. *If* $\vdash_{J^{**}} \alpha$ *then* $\vdash_{J^*} \alpha$.

PROOF. $\vdash_{J^{**}} \alpha \Longrightarrow \vdash_{S5^*} \Diamond\forall\alpha \Longrightarrow \vdash_{S5^*} \forall\Diamond\alpha \Longrightarrow \vdash_{S5^*} \Diamond\alpha \Longrightarrow \vdash_{J^*} \alpha$. □

LEMMA 4.2. $\vdash_{J^*} \alpha$ *does not imply* $\vdash_{J^{**}} \alpha$.

PROOF. The proof consists in the presentation of a formula α such that $\vdash_{J^*} \alpha$ (i.e. $\vdash_{S5^*} \Diamond\alpha$) and $\nvdash_{J^{**}} \alpha$ (i.e. $\nvdash_{S5^*} \Diamond\forall\alpha$).

Let α be the formula $(\Diamond Px \supset Px)$. We have: $\vdash_{S5^*} (\Diamond Px \supset Px) \Longrightarrow \vdash_{S5^*} \Diamond(\Diamond Px \supset Px) \Longrightarrow \vdash_{J^*} \Diamond(\Diamond Px \supset Px)$

But $\nvdash_{S5^*} \Diamond\forall x(\Diamond Px \supset Px)$ (i.e. $\nvdash_{J^{**}} (\Diamond Px \supset Px)$). The verification of this assertion consists in obtaining a Kripke's model that validates the expression: $\neg\Diamond\forall x(\Diamond Px \supset Px)$ (i.e. $\Box\exists x(\Diamond Px \wedge \neg Px)$).

We consider the Kripke's model $\langle W, R, D, V\rangle$, where $W = \{w_1, w_2\}$, $D = \{a, b\}$, R and V are the generally used for S5*, $v(P, w_1) = \{a\}$ and $v(P, w_2) = \{b\}$.

Suppose, by contradiction, that $v(\Diamond\forall x(\Diamond Px \supset Px), w_1 = V$ then either (i) $v(\forall x(\Diamond Px \supset Px), w_1) = V$ or (ii) $v(\forall x(\Diamond Px \supset Px), w_2) = V$.

Case (i): Let v' be a valuation like v except that $v'(x) = b$. We have $v'(\Diamond Px \supset Px), w_1) = V$. But $v'(Px, w_2) = V$. So $v'(x) = b$ and $v'(Px, w_2) = \{b\}$ and $b \in \{b\}$. Then $v'(Px, w_1) = V$. Therefore $v'(Px, w_1) = V$, i.e., $v'(x) \in v'(Px, w_1) = \{a\}$, which is a contradiction.

Case (ii): Let v'' be a valuation exactly like v except that $v''(x) = a$. We have $v''(\Diamond Px \supset Px), w_2) = V$. But $v''(Px, w_1) = V$, since $v''(x) = a$, $v''(P, w_1) = v(P_1, w_1) = \{a\}$ and $a \in \{a\}$. Then $v''(\Diamond Px, w_2) = V$. Therefore $v''(Px, w_2) = V$, i.e., $v(x) \in v(Px, w_2)$, and we have a new contradiction.

Hence $\vdash_{S5^*} \neg\Diamond\forall x(\Diamond Px \supset Px)$, i.e., $\nvdash_{S5^*} \Diamond\forall x(\Diamond Px \supset Px)$ and $\nvdash_{J^{**}} (\Diamond Px \supset Px)$. □

THEOREM 4.3. J* *is strictly stronger than* J**.

PROOF. Immediate consequence of lemmas 5.1. and 5.2. □

5. Some results concerning J**

DEFINITION 5.1. We define $\neg_d$, $\rightarrow_d$, $\vee_d$ and $\wedge_d$, the discursive operation negation, implicated, disjunction and conjunction, as follows:

$$\neg_{\mathrm{d}}\, \alpha \overset{\mathrm{df}}{=} \neg\Diamond\forall\alpha$$

$$\alpha \to_{\mathrm{d}} \beta \overset{\mathrm{df}}{=} \Diamond\forall\alpha \supset \beta$$

$$\alpha \vee_{\mathrm{d}} \beta \overset{\mathrm{df}}{=} \Diamond\forall\alpha \vee \beta$$

$$\alpha \wedge_{\mathrm{d}} \beta \overset{\mathrm{df}}{=} \Diamond\forall\alpha \wedge \beta$$

The following schemes of theorems and rules are valid in J**:

$$\alpha \to_{\mathrm{d}} (\beta \to_{\mathrm{d}} \alpha)$$
$$(\alpha \to_{\mathrm{d}} \beta) \to_{\mathrm{d}} ((\alpha \to_{\mathrm{d}} (\beta \to_{\mathrm{d}} \gamma)) \to_{\mathrm{d}} (\alpha \to_{\mathrm{d}} \gamma)$$
$$((\alpha \to_{\mathrm{d}} \beta) \to_{\mathrm{d}} \alpha) \to_{\mathrm{d}} \alpha$$
$$(\alpha \wedge_{\mathrm{d}} \beta) \to_{\mathrm{d}} \alpha$$
$$(\alpha \wedge_{\mathrm{d}} \beta) \to_{\mathrm{d}} \beta$$
$$\alpha \to_{\mathrm{d}} (\beta \to_{\mathrm{d}} (\alpha \to_{\mathrm{d}} \alpha \wedge_{\mathrm{d}} \beta))$$
$$\alpha \to_{\mathrm{d}} (\alpha \vee_{\mathrm{d}} \beta)$$
$$\beta \to_{\mathrm{d}} (\alpha \vee_{\mathrm{d}} \beta)$$
$$(\alpha \to_{\mathrm{d}} \gamma) \to_{\mathrm{d}} ((\beta \to_{\mathrm{d}} \gamma)) \to_{\mathrm{d}} (\alpha \vee_{\mathrm{d}} \beta) \to_{\mathrm{d}} \gamma))$$
$$\alpha \to_{\mathrm{d}} \neg_{\mathrm{d}} \neg_{\mathrm{d}} \alpha$$
$$\neg_{\mathrm{d}} \neg_{\mathrm{d}} \alpha \to_{\mathrm{d}} \alpha$$
$$(\alpha \to_{\mathrm{d}} \beta) \to_{\mathrm{d}} ((\alpha \to_{\mathrm{d}} \neg_{\mathrm{d}} \beta) \to_{\mathrm{d}} \neg_{\mathrm{d}} \alpha)$$
$$\alpha \vee_{\mathrm{d}} \neg\alpha$$
$$\frac{\alpha, \quad (\alpha \to_{\mathrm{d}} \beta)}{\beta}$$

Da Costa and Dubikajtis [2] gave a new axiomatization J based upon the connectives: $\to_{\mathrm{d}}$, $\wedge_{\mathrm{d}}$, $\vee$ and $\neg$. Lopes dos Santos [6] using a similar procedure, gave an axiomatization for J*.

Based on that occur with J* we could think about an axiomatization for J** based on the connectives: $\to_{\mathrm{d}}$, $\wedge_{\mathrm{d}}$, $\vee_{\mathrm{d}}$, $\neg$, and $\forall$. We can verify easily that in J** the following schemes are valid:

$$\alpha \to_{\mathrm{d}} \neg\neg\alpha$$
$$\neg\neg\alpha \to_{\mathrm{d}} \alpha$$
$$\neg(\alpha \vee_{\mathrm{d}} \neg\alpha) \to_{\mathrm{d}} \beta$$

However, we have that

$$\nvdash_{\mathrm{J}^{**}} \neg(\alpha \vee_{\mathrm{d}} \beta) \to_{\mathrm{d}} (\beta \vee_{\mathrm{d}} \alpha)$$

We cannot develop in J^{**} an axiomatization base upon the last set of connectives above. It is evident that to obtain a "natural" axiomatization for J^{**} is very difficult.

The previous considerations make clear that the fundamental problem of discursive logic in the present state of development is the problem of giving an axiomatic system that is easily applied to the predicate calculus.

What we have presented here makes evident that J^{**} can be axiomatized. It shows that this system is not easy to work with. It is important to try at least, to obtain a new axiomatization for J^{**}, parallel to the axiomatization in [6] of J^{*}.

Acknowledgements. The second author is supported by FAPESP grant 97/02328-9.

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