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A NOTE ON ZENO B3

Εἰ πολλά ἐστιν, ἀνάγκη τοσαῦτα εἶναι ὅσα ἐστί καί
οὔτε πλείονα αὐτῶν οὔτε ἐλάττονα. Εἰ δέ τοσαῦτά
ἐστιν ὅσα ἐστί, πεπερασμένα ἄν εἴη.

If there are many things, it is necessary that
they are just as many as they are, neither more
nor less. And if they are just as many they are,
they would be finite in number.

Let V be the set of things that there are. Suppose that V contains infinitely many members. We will now introduce the premiss:

(1)   V contains just as many members as V, neither more nor less.

This is the only premiss to be deployed in my reconstruction that represents something explicit in Zeno's Greek. But it has at least the advantage of representing Zeno's words quite literally. Contrast the interpretation whereby Zeno contends in these crucial words that 'any plurality of things must consist of a *definite number* of things and so be finite in number'[1]. The only sense in which it follows from V's infinity that it contains no definite number of members would be that for no natural number n does V contain only n members. But this is not a thought happily framed as a denial that the members of V 'are as many as they are'; and in any case to infer this thought is merely to restate V's infinity, not to draw from it an evidently absurd conclusion. A similar contrast may be drawn also with the paraphrase of

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[1] H.D.P.Lee, **Zeno of Elea** (Cambridge, 1936) p.31. Lee's interpretation seems to be shared by practically all other commentators.

Barnes, whereby Zeno reasoned thus: 'If there are many *As*, then there is some true proposition of the form: "There are as many *As* as *Bs*"'[2]. If Zeno's thought was that any plurality must contain just as many members as *another* plurality, then he was quite inept in framing it as a thought that any plurality contains just as many members as *itself*. Furthermore, so to reconstruct Zeno's argument is to leave him with what is, as Barnes himself puts it, 'an uninstructive sophism'. My more literal interpretation of these words will in the end also produce a sophism: for how could a 'proof' of falsehood be otherwise? Nevertheless I trust that it will not be entirely uninstructive.

Our premiss (1) is not of itself enough to reduce to absurdity our supposition that V contains infinitely many members. My reconstruction must therefore, like others, supply further premisses to give Zeno the semblance of a sound argument. Let me therefore supply:

(2)   An infinite set is one that contains the same number of members as some proper subset of itself.

(2) may seem to have a suspiciously anachronistic air. After all, it first achieved a fully clear articulation and proof in the nineteenth century. Nevertheless, intimations of it can be found closer to Zeno's time. Some Stoics held that 'The man is not composed of more parts then the finger, nor the universe than the man; for division produces bodies to infinity, and of infinities none is greater or lesser'[3]. And it is not altogether implausible to see such ideas as present in Zeno himself. For, as the second horn of the B3 dilemma indicates, it was points on a line that above all Zeno has in mind here. And, however hard it may be to grasp (2) as a general definition of infinities, its particular application to infinities of points on a line is evident enough. For it takes but little reflection to see that if a line can be divided at an infinite number of points then the same holds of any part into which it is divided.

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[2] Jonathan Barnes, **The Presocratic Philosophers** (London, 1979) Vol.1, p.252.

[3] Plutarch **De communibus notitiis adversus Stoicos** 1079a (=SVF 2.484).

There is a third premiss we will use in our reconstruction.

(3)   Each set contains more members than any proper subset of itself.

(3) is undeniably true if we restrict its application to sets of finite size, and it takes some sophistication to see that it is false when applied to infinities. If I am to spend every day from now for ever onwards in the Isles of the Blessed, and you are to spend only every other day there, then you have evident cause to envy me; and it would be only too natural for you to give the reason that I am to have more days in the Isles of the Blessed than you are. Again, it is only too natural to suppose that the line ABC can be divided at more points than can its segment AB; for the line ABC can be divided at all those points at which the segment AB can, and also at those further points at which the segment BC can be divided. The sophistication required to see the error here was in all likelihood not possessed by Zeno's contemporaries. For it seems to have been lacked by Aristotle, who in his argument against the view that infinity is a substance treats as evidently absurd its alleged consequence that the infinite has a part which is itself infinite[4]. Nor indeed is it entirely stupid to suppose that (3) is in general true. For the chief reason that we have to doubt (3) is simply that (1) and (2) are ·true, that V is infinite, and that (3) in conjunction with (1) and (2) entails that all sets are finite in size.

How then does it entail this? How did Zeno's argument from (1), (2) and (3) proceed? V, we are supposing, contains infinitely many members. Hence there is, by (2), a proper subset of V, containing just as many members as V itself. Call such a subset S. The number of members of V is now the same as the number of members of S. But by (3) the number of members of S is *less* than the number of members of V. So the number of members of V is less than itself. Similarly, the number of members of V is, by (3), *greater* than the number of members of S. But this is, by (2), the same as the number of members of V. So the number of members of V is greater than itself. But all this contradicts (1). We have thus reduced to absurdity our assumption that V contains infinitely many members; and if there are many things they are in consequence only finite in number.

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[4] **Physics** 204a20ff (= **Metaphysics** 1066b11ff.).