Quantum Superpositions and the Representation of Physical Reality Beyond Measurement Outcomes and Mathematical Structures

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Abstract

In this paper we intend to discuss the importance of providing a physical representation of quantum superpositions which goes beyond the mere reference to mathematical structures and measurement outcomes. This proposal goes in the opposite direction of the orthodox project which attempts to "bridge the gap" between the quantum formalism and common sense "classical reality" —precluding, right from the start, the possibility of interpreting quantum superpositions through non-classical notions. We will argue that in order to restate the problem of interpretation of quantum mechanics in truly ontological terms we require a radical revision of the problems and definitions addressed within the orthodox literature. On the one hand, we will discuss the need of providing a formal redefinition of superpositions which captures their contextual character. On the other hand, we attempt to replace the focus on the measurement problem, which concentrates on the justification of measurement outcomes from "weird" superposed states, and introduce the superposition problem which concentrates instead on the conceptual representation of superpositions themselves. In this respect, after presenting three *necessary conditions* for objective physical representation, we will provide arguments which show why the classical (actualist) representation of physics faces severe difficulties to solve the superposition problem. Finally, we will also argue that, if we are willing to abandon the (metaphysical) presupposition according to which 'Actuality = Reality', then there is plenty of room to construct a conceptual representation for quantum superpositions.

Keywords: quantum superposition, physical reality, measurement problem.

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Introduction

Quantum superpositions are being used today in laboratories all around the world in order to create the most outstanding technological and experimental developments of the last centuries. Indeed, quantum computation, quantum teleportation, quantum cryptography and the like technologies are opening up an amazing range of possibilities for the near future. This new quantum technological era is founded on one of the main principles of Quantum Mechanics (QM), the so called *superposition principle* —which in turn gives rise to *quantum superpositions* and *entanglement*. However, while many experimentalists are showing that Schrödinger's cats are growing fat [7], and it becomes increasingly clear that quantum superpositions are telling us something about quantum physical reality even at the macroscopic scale [44, 42], the philosophy of science community has not been capable of providing a coherent physical representation of them. In the orthodox literature —apart from expensive metaphysical solution provided by many worlds interpretations which claim we live in a "superposition of many worlds" or "multiverse"[1]— there is no conceptual explanation of the physical meaning of quantum superpositions which goes beyond the formal reference to a mathematical structure or the empirical reference to measurement outcomes.

In this paper, we will argue that the reason for the lack of conceptual analysis regarding quantum superpositions is directly related to the exclusive emphasis that has been given in the orthodox literature to the infamous measurement problem. Taking as a standpoint a representational realist stance, we will argue for the need to replace the measurement problem, which only focuses on the justification of observable measurement outcomes, by the superposition problem, which concentrates instead on the conceptual account of the mathematical expression itself. In order to discuss the possibility of interpreting superpositions we will present three *necessary conditions* which attempt to constrain any objective conceptual representation of an empirically adequate mathematical formalism. Our analysis will require a formal redefinition of the notion of quantum superposition which considers not only the meaningful physical statements that can be derived from them, but also their contextual character as basis-dependent formal elements of the theory. Taking into account our proposed three *necessary conditions* for the objective representation of physical reality, we will provide arguments which show the severe difficulties faced by the classical (actualist) representation of physics to produce a conceptual interpretation of quantum superpositions. In the final part of the paper, we will also argue that, if we are willing to discuss the possibility that 'Quantum Physical Reality $\neq$ Actuality', then there is plenty of room to represent quantum superpositions in terms of (non-actual) physical reality.

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[1] See for example [54]. We leave the analysis of quantum superpositions in many worlds interpretation for a future work.

1 The Representational Realist Stance

Within philosophy of physics, realism has been characterized as a stance which assumes the existence of a reality independent of the actions of any human subject or conscious being. In short, it is claimed that realism is committed to the belief of an objective (subject independent) reality. However, we will argue that this account remains insufficient when attempting to grasp the *praxis* of realist physicists themselves. Representational realism attempts to capture exactly this aspect; i.e., the specific way through which realist physicists *produce* such representational (meta-physical) account of reality. In this respect, the main presupposition of representational realism is that physical theories relate to reality, not only through their mathematical formalisms, but also through a network or structure of physical concepts.

The coherent interrelation between mathematical structures and physical notions allows physical theories to represent (in various ways) the world and reality; it allows the physicist to imagine different physical phenomena. For example, in Newtonian mechanics the notions of space, time, inertia, particle, force, mass, etc., are related to infinitesimal calculus in such a way that we physicists are capable to imagine and predict very different phenomena which range from the motion of planets to the free fall of an apple from a tree. But infinitesimal calculus does not wear Newton's physical notions on its sleeves. There is no notion of 'inertia', of 'particle' or 'absolute space' which can be read off the formalism of the theory. Mathematical equations do not posses, by themselves, physical concepts. Space', 'time', 'particle', etc. are not —according to our stance— "self evident" givens of experience; they are concepts that have been developed by physicists and philosophers through many centuries.[2]

Every theory possesses its own specific set of physical notions and mathematical equations. As we all know, Maxwell's electromagnetic theory has a mathematical structure and set of concepts different to that of classical mechanics and Einstein's relativity theory. The fact that theories share the same names for distinct physical concepts should not confuse us, their meaning in many cases differ radically. As remarked by Heisenberg [33, pp. 97-98]: "New phenomena that had been observed could only be understood by new concepts which were adapted to the new phenomena. [...] These new concepts again could be connected in a closed system. [...] This problem arose at once when the theory of special relativity had been discovered. The concepts of space and time belonged to both Newtonian mechanics and to the theory of relativity. But space and time in Newtonian mechanics were independent; in the theory of relativity they were connected." From this acknowledgment, the representational realist argues that the task of a realist physicist or philosopher of physics must be the creation of a closed theory which coherently interrelates the specific mathematical formalism, the particular physical concepts and the phenomena described by that theory.

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[2] See for example, in this same respect, the detailed analysis of the concept of space in the history of physics provided by Max Jammer in his excellent book: *The Concepts of Space. The History of Theories of Space in Physics* [37].

According to our representational realist stance, which will be presupposed through the rest of the paper, 'experimental data' and 'observability' should be regarded only in terms of a confirmation procedure about the empirical adequacy (or not) of a given theory. Our realist stance —which relates to Heisenberg's closed theory approach[3] and some of the main elements of Einstein's philosophy of physics— assumes that physical observation is both theoretically and metaphysically laden. In this respect, the basic principles of classical logic and metaphysics, namely, the Principle of Existence (PE), the Principle of Identity (PI) and the Principle of Non-Contradiction (PNC) determine, right from the start, the possibilities and limits of classical observation itself. Classical observation is both defined and constrained by these ontological and logical principles. Identities or non-contradictory properties are not something we find "outside in the world", they are instead the basic conditions of possibility for discussing about any object of experience. Exactly the same point was made already by Hume himself regarding the physical notion of causality. As we all know, causation is not something grounded empirically, it is never found in the observable world. Rather, as Hume clearly exposed, it is a metaphysical presupposition which allows us to make sense of physical phenomena.

PE, PNC and PI have been, not only the fundamental cornerstones of both Aristotle's metaphysics and classical logic itself, but also the main principles behind the physical picture imposed by Newton's classical mechanics (see [52]). The idea of "classical reality", which encompasses the whole of classical physics (including relativity), can be condensed in the notion of *Actual State of Affairs* (ASA).[4] This particular (metaphysical) representation was developed by Newton and can be formulated in terms of *systems constituted by a set of actual (definite valued) preexistent properties.* Objects and its properties are intrinsically defined through PE, PNC and PI as existents in the actual mode of being. In the 18th century these same principles constituted the basis for the metaphysical definition of the notion of actual entity in classical physics. Newton conceived a Universe constituted by bodies with always definite valued (actual) properties. This allowed Laplace to imagine a demon which, given the complete and exact knowledge of all particles in the Universe (i.e., the set of all actual properties), would have access through the equations of motion to the future and the past of the whole Universe.

Physical representation allows us to think about experience and predict phenomena without the need of performing any actual measurement. It allows us to imagine physical reality beyond the here and now. This is of course the opposite standpoint from empiricists who argue instead that the fundament of physics is 'actual experimental data'. As remarked by van Fraassen [51, pp. 202-203]: "To develop an empiricist account of science is to depict it as involving a search for truth only about the empirical world, about what is actual and observable." In this case, "actual" has a very different meaning to the one just mentioned above. It is understood as making reference to *hic et nunc* observations (by subjects)

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[3] For a detailed discussion of the closed theory approach see [8, 16].

[4] See [18] for discussion and definition of this notion in the context of classical physics.

and not to the Aristotelian metaphysical definition of a mode of existence (independent of subjects). While the empiricist considers that the construction of theories always begins from observable data, the representational realist recognizes that theory construction is an entangled process of production of concepts, mathematical structures and fields of phenomena.

Contrary to the positivist-empiricist stance according to which observables in physics must be accepted as *givens* of experience, the representational realist considers observability as both theoretically and metaphysically laden. For her, the conceptual scheme is a *necessary condition* to produce not only the physical representation of a closed theory but also the condition of possibility to provide a categorical description of the (closed) physical phenomena discussed by that theory. The task of both physicists and philosophers is to jointly construct new mathematical formalisms and networks of physical concepts which allows us to imagine new physical phenomena.

According to our stance, the subject must play no role at all in the description of physical reality. As Einstein [21, p. 175] made the point: "[...] it is the purpose of theoretical physics to achieve understanding of physical reality which exists independently of the observer, and for which the distinction between 'direct observable' and 'not directly observable' has no ontological significance" even though "the only decisive factor for the question whether or not to accept a particular physical theory is its empirical success." For the representational realist, empirical adequacy is part of a *verification procedure*, not that which "needs to be saved". Following Einstein's dictum which allowed Heisenberg to derive his indeterminacy relations "It is only the theory which decides what we can observe" [34, p. 63]. The theory allows us to describe what is observable. It is not, as argued by empiricists, the other way around.

This understanding of observability as being theoretically and metaphysically dependent goes not only against empiricism and instrumentalism but also against scientific realism itself. As Musgrave [10, p. 1221] explains: "In traditional discussions of scientific realism, common sense realism regarding tables and chairs (or the moon) is accepted as unproblematic by both sides. Attention is focused on the difficulties of scientific realism regarding 'unobservables' like electrons." The main danger of all these philosophical positions —namely, scientific realism, empiricism and instrumentalism— is that they close the door to the development of new physical representations, since they assume that we already know what reality *is* in terms of the (naive) observation of tables and chairs —also known as "classical reality". 'Tables' and 'chairs' are naively considered as "common sense" givens of experience and represented by mathematical models. Within these philosophical positions, contrary to representational realism, there is no reference whatsoever to metaphysical considerations regarding the meaning of 'tables' and 'chairs'. The orthodox project is then focused in trying "to bridge the gap" between the strange quantum formalism and "classical reality" [26]. They attempt to do so, forgetting that our present "common sense" understanding of the world was also part of a creative process —and not the final conditions of human understanding.

Empirically adequate mathematical structures are not enough to produce a meaningful physical representation of reality. Physics cannot be exclusively reduced to mathematical models which predict measurement outcomes simply because neither mathematical models nor empirical facts contain in themselves the physical concepts the theory talks about. The experience of looking to a 'chair' or a 'table' does not produce by itself PI or PNC. There are no physical notions to be found within mathematical models either. Every physical theory is intrinsically constituted through specific physical concepts which are defined in metaphysical terms —through specific principles. It is only through these representation that the experience described by a physical theory is made possible. As Heisenberg [35, p. 264] makes the point: "The history of physics is not only a sequence of experimental discoveries and observations, followed by their mathematical description; *it is also a history of concepts.* For an understanding of the phenomena the first condition is the introduction of adequate concepts. *Only with the help of correct concepts can we really know what has been observed.*"

Representational Realism: *A representational realist account of a physical theory must be capable of providing a physical representation of reality in terms of a network of physical concepts which coherently relates to the mathematical formalism of the theory and allows to articulate and make predictions of definite phenomena. Observability in physics is always theoretically and metaphysically laden, and thus, it must be regarded as a consequence of each particular physical representation.*

2 Representing Quantum Physical Reality

For the representational realist, the task of both physicists and philosophers of physics is to produce conceptual representations which allow us to grasp the features of the Universe beyond mathematical schemes and measurement outcomes. In order to provide such representation we must necessarily complement mathematical formalisms with networks of physical concepts. It is simply not enough to claim that "according to QM the structure of the world is like Hilbert space", or that "reality, according to QM is described through the quantum wave function". That is just mixing the formal and conceptual levels of discourse (see for discussion [18, Section 4]). That is not doing the job of providing a conceptual physical representation in the sense discussed above. Mathematics is simply incapable of producing physical concepts.

If we accept the representational realist challenge and we are willing to discuss the conceptual meaning of QM, there seems to be two main very general lines of research to consider. The first one is to investigate the possibility that QM makes reference to the same physical representation provided by classical physics; i.e. that it talks about an ASA —or in other words, about "classical reality". This is the main idea behind, for example, the hidden variables program which, as noticed by Bacciagaluppi [6, p. 74], attempts to "restore a classical way of thinking about *what there is*." This attempt has been faced with severe difficulties, mainly due to the empirical test of Bell inequalities, Kochen-Specker theorem, the infamous measurement problem, and the list continues. Due to the impossibility to account for a non-contextual classical representation, this line of research has abandoned the orthodox formalism and proposed instead new variants which attempt to rescue "classical reality". The second line is to consider the possibility that QM might describe physical reality in a different —maybe even incommensurable— way to that of classical physics. It is this very different, second line of research, that interests us in this paper.

In order to discuss and analyze physical interpretations we need to agree on the definition of Meaningful Operational Statements (MOS) within a theory.

DEFINITION 2.1 MEANINGFUL OPERATIONAL STATEMENTS: *Every operational statement within a theory capable of predicting the outcomes of possible measurements must be considered as meaningful with respect to the representation of physical reality provided by that theory.*

From a realist perspective MOS must be related, in the final stage of a theory construction, to the representation of reality provided by the theory. The intuition behind this requirement is remarked by Griffiths [30, p. 361]: "If a theory makes a certain amount of sense and gives predictions which agree reasonably well with experimental or observational results, scientists are inclined to believe that its logical and mathematical structure reflects the structure of the real world in some way, even if philosophers will remain permanently skeptical." As we argued above, the representation must be established not only in formal mathematical terms but also in conceptual terms through the introduction of appropriate physical notions. MOS are predictive statements about specific physical situations. It is the task of both physicists and philosophers of physics to complement MOS with adequate physical notions, allowing us to construct a coherent conceptual and categorical representation which allows us to explain what is exactly going on within the addressed phenomenon.

The problem introduced by QM is that, even though we possess MOS such as, for example, 'the spin of the quantum particle is + with probability 0.5' or 'the atom has a probability of decaying of 0.7', we do not possess an adequate set of concepts that would allow us to grasp the physical content of these statements —not in the same way as we do in Newton's mechanics or Maxwell's theory. Let us be clear about this point, in QM we do not understand conceptually what is the meaning of 'quantum particle', 'spin', 'atom' or 'probability'. Unlike classical mechanics, until today we cannot picture what is gong on according to QM. When asked about such notions we can only provide a mathematical explanation or recall the predictive empirical success of the theory. But prediction is not explanation. Neither is mathematical formalization. Prediction and formalization do not allow us to imagine what these "quantum notions" refer to —as we do in classical physical theories.

In order to come up with a conceptual representation which explains the MOS of a theory, the produced physical discourse must allow for Counterfactual Reasoning (CR). This is a necessary condition for without counterfactual discourse there is no true possibility of a subject independent representation of reality. CR is used and analyzed by different disciplines. In the case of logicians and philosophers, CR is studied in terms of Kripke semantics, or possible worlds semantics. Even though this logical approach to counterfactuals has become popular in philosophy of QM (e.g. [30]), it has never been popular among physicists in general. In fact, physicists have always used counterfactuals in a rather (undefined) intuitive way in order to discuss physical experience as related to an objective description of reality. Let us provide thus a general definition of counterfactual reasoning which attempts to consider the actual *praxis* of physicists themselves.

DEFINITION 2.2 COUNTERFACTUAL REASONING: *If the theory is empirically adequate then the MOS it provides must be related to physical reality through a conceptual scheme. The possibility to make MOS related to an objective physical representation requires necessarily a counterfactual discourse. MOS are not necessarily statements about future events, they can be also statements about past and present events. CR about MOS comprises all actual and non-actual physical experience. CR is the objectivity condition for the possibility of physical discourse.*

CR is an indispensable element for the physical discourse of a theory which attempts to discuss an objective representation of physical reality. Many of the most important debates in the history of physics are thought experiments which make explicit use of counterfactual discourse. In the 18th century, Newton and Leibniz imagined different physical situations in order to draw conclusions about classical mechanics. At the beginning of the 20th century, Einstein's famous *Gedankenexperiments* in relativity theory made clear that the notion of *simultaneity* in Newtoninan mechanics had to be reconsidered, producing a revolution in our understanding of space and time. During the 1920s Solvay meetings, Bohr and Einstein discussed in depth many *Gedankenexperiments* related to QM. Some years later, Schrödinger imagined a strange situation in which a (quantum) cat was 'dead' and 'alive' at the same time. More than 50 years had to pass by in order to empirically test the existence of quantum superpositions allowing technicians and experimentalists to explore amazing possibilities for quantum information processing. Also Einstein, Podolsky, Rosen and Bell had to wait till the 80s for Aspect and his group in order to be certain that the hidden variable project —with which they wanted to replace QM by a "complete theory"— was not going to work out without giving up either *reality* or *locality.* These few examples show the crucial role played by conceptual representation and CR within the *praxis* of physicists.

If we assume a representational realist stance, the conceptual representation must be capable of conceptually explaining the MOS produced by the theory, it must also produce a discourse which respects CR. Without CR in physical discourse one cannot imagine objective reality nor experience beyond the here and now. Of course, this might not be regarded as a problem for an empiricist. CR allows us to state that "if I performed this (or that) experiment" then —if it is, of course, a MOS— the physical theory would tell me that "the outcome will be x (or y)", and I do not need to actually perform the experiment! I know what the result will be independently of performing the experiment or not. And that is the whole point of being a realist about physics, that I trust the theory to be talking about a physical representation of reality of which I can make sense without making any reference to the *hic et nunc* observation of a subject.

Physicists are accustomed to play with the counterfactual statements produced by a theory. CR in physical discourse has nothing to do with time, evolution nor dynamics, it has to do with the possibility of representing objective physical reality and experience. A physical theory allows me to make counterfactual statements about the future, the present or the past, just in the same way physical invariance in classical mechanics connects the multiple frames of reference without anyone actually being in any of them. CR is the discursive condition with respect to the objective physical description of phenomena. Indeed, in classical mechanics (or relativity theory), we do not need to actually *be* in a specific frame of reference to *know* what will happen in that specific frame, or a different one. We can imagine and calculate what will happen in each frame, we can physically represent them to ourselves and translate what will happen in each of them through the Galilean (or Lorentz) transformations. And this makes explicit use of CR. For a realist, the possibility to imagine reality is the magic of physics. Once we believe to have an empirically adequate theory, we realists —contrary to empiricists— still have a lot of work to do, we still need to produce a *representation* of what the theory is talking about. And exactly that, is what is lacking in the case of QM.

We physicist can imagine how a distant star will collapse and transform into a white dwarf many, many years from now; we can also understand what would happen to space and time when traveling on a ray of light; or the tremendous consequences of what could happen to us inside a back hole; we can even discuss what already happened 13.800 million years ago during the first minutes of the Universe after the Big Bang, long before any conscious being existed. It is the trust in the physical representations provided by different theories which allows physicists to imagine situations which escape not only the spacial region in which they live, but also the technical possibilities of their time. And that is the reason why, as Einstein remarked, imagination is more important than knowledge.

From the previous analysis, we propose the following three *necessary conditions* for producing an objective physical representation of a theory:

Necessary Condition 1 (NC_1). *Every physical theory must be capable of producing MOS which can be empirically tested.*

Necessary Condition 2 (NC_2). *Every MOS the theory produces must be directly related to the representation of physical reality, provided through a specific conceptual scheme which adequately explains the phenomenon.*

Necessary Condition 3 (NC_3). *The conceptual representation of a physical theory must be capable of producing a coherent counterfactual physical* *discourse which includes all MOS of the theory.*

In QM, the complementarity scheme produced by Bohr violates explicitly CR making impossible to provide an objective representation. As we have discussed elsewhere [16], the orthodox Bohrian scheme faces severe difficulties in this respect. The demand to provide a realist conceptual representation of QM implies not only a different perspective, it also presupposes the consideration of new problems. That will be discussed in the following section.

3 The Superposition Problem: Representation Beyond Actual Outcomes

The orthodox line of research deals with a specific set of problems which analyze QM from a classical perspective. This means that all problems assume as a standpoint "classical reality" and only reflect about the formalism in "negative terms"; that is, in terms of the failure of QM to account for the classical representation of reality and the use of its concepts: separability, space, time, locality, individuality, identity, actuality, etc. The "negative" problems are thus: *non*-separability, *non*-locality, *non*-individuality, *non*-identity, etc.[5] These "no-problems" begin their analysis considering the notions of classical physics, assuming implicitly as a standpoint the strong metaphysical presupposition that QM should be able to represent physical reality in terms of such classical notions. The most famous of all interpretational problems of QM, is the so called "measurement problem".

Measurement Problem: *Given a specific basis (or context), QM describes mathematically a quantum state in terms of a superposition of, in general, multiple states. Since the evolution described by QM allows us to predict that the quantum system will get entangled with the apparatus and thus its pointer positions will also become a superposition,*[6] *the question is why do we observe a single outcome instead of a superposition of them?*

The measurement problem is also a way of discussing the quantum formalism in "negative terms". In this case, the problem concentrates in the justification of observable measurement outcomes. It should be remarked that the measurement problem presupposes that the basis (or context) —directly related to a measurement set up— has been already determined (or fixed). Thus it should be clear that there is no question regarding the contextual character

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[5] I am grateful to Bob Coecke for this linguistic insight. Cagliari, July 2014.

[6] Given a quantum system represented by a superposition of more than one term, $\sum c_i|\alpha_i\rangle$, when in contact with an apparatus ready to measure, $|R_0\rangle$, QM predicts that system and apparatus will become "entangled" in such a way that the final 'system + apparatus' will be described by $\sum c_i|\alpha_i\rangle|R_i\rangle$. Thus, as a consequence of the quantum evolution, the pointers have also become —like the original quantum system— a superposition of pointers $\sum c_i|R_i\rangle$. This is why the measurement problem can be stated as a problem only in the case the original quantum state is described by a superposition of more than one term.

of the theory within this specific problem. As we have argued extensively in [17], the measurement problem has nothing to do with contextuality. The problem raises when, within a definite basis, the actualization process is considered. There is then a mix of subjective and objective elements when the recording of the experiment takes place —as Wigner and his friend clearly explained. The problem here is coherency between the physical representation provided when the measurement was not yet performed, and the system is described in terms of a quantum superposition; and when we claim that "we have observed a single measurement outcome", which is not described by the theory. Since there is no physical representation of "the collapse", the subject (or his friend) seems to define it explicitly. The mixture of objective and subjective is due to an incomplete description of the state of affairs within the quantum measurement process (or "collapse").

The focus of the measurement problem is the *hic et nunc* actual realm of experience. In this sense, the measurement problem is an empiricist problem which presupposes the controversial idea that actual observation is perfectly well defined in QM. But from a representational realist stance things must be analyzed in a radically different perspective. Indeed, here it is only the theory which can tell you what can be observed. This means that if we are willing to truly investigate the physical representation of quantum superpositions then we need to "invert" the measurement problem and focus on the formal-conceptual level —instead of trying to justify what we observe in "common sense" classical terms. This means that the attention should be focused on the conceptual representation of the mathematical expression itself instead of attempting to somehow "save" the measurement outcomes justifying through *ad hoc* rules the "collapse" of the quantum superposition to one of its terms.

The important developments we are witnessing today in quantum information processing demands us, philosophers of QM, to pay special attention to the novel requirements of this new technological era. In this respect, we believe that a task of outmost importance is to produce a coherent physical representation of quantum superpositions.

Superposition Problem: *Given a situation in which there is a quantum superposition of more than one term,* $\sum c_i \; |\alpha_i\rangle$*, and given the fact that each one of the terms relates trough the Born rule to a MOS, the question is how do we conceptually represent this mathematical expression? Which is the physical notion that relates to each one of the terms in a quantum superposition?*

The superposition problem opens the possibility to truly discuss a physical representation of reality which goes beyond the classical representation of physics in terms of a Newtonian ASA, or its mere reference to measurement outcomes and mathematical structures. We are convinced that without a replacement of the measurement problem by the superposition problem, there is no true possibility of discussing an interpretation of QM which provides an objective non-classical physical representation of reality. We know of no reasons to believe that this is not doable.

4 Contextual Formal Redefinition of Quantum Superpositions

Both the measurement and the superposition problems imply the necessary requirement that the formal definition of a quantum superposition is contextually specified. The fact that a quantum state Ψ can be mathematically represented in multiple bases must be explicitly considered within such definition. This obvious remark might be regarded as controversial due to the fact the contextual character of quantum superpositions has been completely overseen within the orthodox literature. As we have discussed in detail in [12], the semantic interpretation used in order to interpret the syntactical level of the quantum formalism presupposes implicitly PE, PNC and PI. This "common sense" classical interpretation has been uncritically accepted without the necessary consideration of the coherency between the addressed semantical and syntactical levels of the theory.

Another consequence of this classical-type semantical interpretation is the complete omission of the obvious distinction between two different levels of mathematical representation present within the orthodox formalism. Indeed, the use of notions of 'system', 'state' and 'property' has camouflaged the important distinction between, on the one hand, an abstract vector, Ψ, and on the other, its different basis-dependent representations, $\sum c_i \; |\alpha_i\rangle$. Let us remark again that this distinction is already implicit within the measurement problem itself. The measurement problem requires the necessary specification of the particular basis in which the state is represented as a linear combination of terms. Thus, in order to make explicit these two different formal levels, we state now the following contextual definition of quantum superpositions:

DEFINITION 4.1 QUANTUM SUPERPOSITION: *Given a quantum state, Ψ, each i-basis defines a particular mathematical representation of Ψ, $\sum c_i \; |\alpha_i\rangle$, which we call a quantum superposition. Each one of these different basis dependent quantum superpositions defines a specific set of MOS. These MOS are related to each one of the terms of the particular quantum superposition through the Born rule. The notion of quantum superposition is contextual for it is always defined in terms of a particular experimental context (or basis).*

The contextual character of quantum superpositions is an aspect of outmost importance when attempting to conceptually represent them. Let us discuss an explicit example in order to clarify these ideas. Consider a typical Stern-Gerlach type experiment where we have produced a quantum state Φ mathematically represented in the x-basis by the ket $|\uparrow_x>$. This can be easily done by filtering off the states $|\downarrow_x>$ of a Stern-Gerlach arranged in the x-direction. It is a mathematical fact that the state can be represented in different bases which diagonalyze a complete set of commuting observables. Each basis-dependent representation of the sate Φ is obviously different when considering its physical content. Indeed, it is common to the *praxis* of physicists to relate each different basis with a specific measurement context.

The notion of basis, and thus also of superposition as we defined it above, possesses a physical content which relates a specific set of epistemic inquiries regarding the abstract state Φ to a set of MOS which provide an answer to each specific question. In our example, we know that if we measure Φ in the context given by the x-basis we will observe with certainty 'spin up'. Thus, the MOS related to the $| \uparrow_x >$ is of course: "if the SG is in the x-direction then the result will be '+' with certainty (probability = 1)".

Physicists are taught that if they want to learn what are the possible outcomes in a different context, for example if they turn the SG to the y-direction, then they just need to write the state $| \uparrow_x >$ in the y-basis. Through this change of basis, and according to our previous definition, physicists obtain a different quantum superposition: $c_{y1} | \uparrow_y > + \, c_{y2} | \downarrow_y >$. Writing the state in the y-basis produces a new superposition which relates to the following two MOS. The first one is that "if the SG is in the y-direction then the result will be '+' with probability $|c_{y1}|^2$". The second MOS is: "if the SG is in the y-direction then the result will be '-' with probability $|c_{y2}|^2$". The same will happen with any i-context of inquiry (determined by a particular i-basis), each one of them will be related to a particular quantum superposition $c_{i1} | \uparrow_i > + \, c_{i2} | \downarrow_i >$ and a specific set of MOS arising from it. Following NC_1, all these different context-dependent MOS can be tested and provide empirical content to each one of the different superpositions. Thus, according to our NC_2 and NC_3, each one of these particular MOS must be related to physical reality through a conceptual physical representation which respects counterfactual discourse and reasoning. Following our previous analysis we will show that if we assume that there is only one world —and not many, as Oxford Everettians believe—, quantum superpositions cannot be represented in terms of the actual mode of existence.

5 Quantum Superpositions and the Actual Realm

According to the three *necessary conditions* imposed by representational realism, each MOS, like the ones described in our Stern-Gerlach example of the previous section, must be related to a representation of physical reality. However, if we attempt to do so within the realm of actuality the conflict becomes evident. On the one hand, the realm of actuality —determined through PE, PNC and PI— cannot allow a physical description in terms of *contradictory properties*. On the other hand, QM produces, at least within some specific contexts, contradictory statements such as, for example, 'the atom has the property of being decayed' and 'the atom has the property of being not-decayed'. But obviously, within the actual realm of existence, an atom cannot posses both properties simultaneously, for that would flagrantly violate PNC. According to our classical understanding, the atom must be necessarily *either* 'decayed' or 'not-decayed'.[7]

The simplest way to escape this paradox would be to argue that the probabilities that accompany the states are *epistemic probabilities*. That would allow

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[7] See for a detailed discussion of the paraconsistent character of quantum superpositions: [11]. Also, for a debate regarding this interpretation: [4, 5, 15].

us to argue that the atom *is* in fact, *either* 'decayed' or 'not-decayed'; the problem would then be that we do not *know* which is the actual state of the atom —implying that the atom has in fact a non-contradictory actual state. Unfortunatelly, orthodox QM precludes such epistemic interpretation of quantum probability (see for example [47]). As it is well known, the quantum probability arising from the orthodox formal structure via Gleason's theorem is non-Kolmogorovian and does not accept an epistemic interpretation. Quantum probability simply cannot be understood as making reference to our ignorance about an ASA.

Regardless of the formal constraints, quantum Bayesianism (QBism for short) [29] does interpret quantum probability in epistemic terms, but this is done at the very high cost —at least for a realist— of having to claim that "QM does not talk about physical reality", that it is "an algorithm" which accounts for observations (of subjects) [28, p. 71]. Following QBism, D'Ariano and Perinotti have explained in a recent paper [14, pp. 280-281] that: "What happens in the Schrödinger cat thought experiment is that the nonlocal test has no intuitive physical interpretation, since it is incompatible with all possible local observations. [...] But if one reasons operationally, it is evident that there is no logical paradox, and the described experiment is only highly counterintuitive." In fact, when reasoning in operational terms, the Schrödinger cat paradox cannot be posed. If one does not accept the premise that QM should be related to a representation of physical reality, then the realist question regarding the physical meaning of quantum superpositions —exposed by the superposition problem— cannot be discussed. This is not solving the problem, it is rather assuming a philosophical standpoint which invalidates the question regarding the conceptual meaning of quantum superpositions.

An analogous interpretation would be to argue that quantum probabilities need to be interpreted in terms of future events (see e.g. [50]). This response also escapes the question at stake. The problem of interpretation of quantum superpositions is not that of epistemic prediction, it is that of ontological representation. We are not discussing here whether QM predicts the correct measurement outcomes in an experiment —we already know it does—, we want to understand how this occurs in terms of a conceptual physical representation. And this is why the physical explanation we seek requires necessarily a conceptual level.

One of the first to see the difficulties of interpreting quantum superpositions was Paul Dirac who wrote in his famous book [24]: "The nature of the relationships which the superposition principle requires to exist between the states of any system is of a kind that cannot be explained in terms of familiar physical concepts. One cannot in the classical sense picture a system being partly in each of two states and see the equivalence of this to the system being completely in some other state." Also Schrödinger made very clear, in his famous 1935 paper [49, p. 153], the deep difficulties one is immersed in when attempting to represent quantum superpositions in terms of our classical (actualist) representation of physical reality: "The classical concept of *state* becomes lost, in that at most a well-chosen *half* of a complete set of variables can be assigned definite numerical values." He also remarked that the problem cannot be solved by making reference to epistemic uncertainty or the measurement process. "One should note that there was no question of any time-dependent changes. It would be of no help to permit the [quantum mechanical] model to vary quite 'unclassically,' perhaps to 'jump.' Already for the single instant things go wrong." As he made the point: "[...] if I wish to ascribe to the model at each moment a definite (merely not exactly known to me) state, or (which is the same) to *all* determining parts definite (merely not exactly known to me) numerical values, then there is no supposition as to these numerical values *to be imagined* that would not conflict with some portion of quantum theoretical assertions."[8]

The necessity of considering the multiple terms of a quantum superposition as physically real is supplemented by the fact that the terms 'evolve' and 'interact' according to the Schrödinger equation producing specific predictions which can be empirically tested and are in accordance to such 'evolution' and 'interaction'. According to our representational realist stance, such empirical findings must be necessarily related to a physical representation of reality. Some modal interpretations have attempted to escape this problem by arguing that one can relate these MOS to *possibilities* rather than *actualities* [9, 22]. The problem is that in classical physics, possibilities never interact! Only actualities are allowed to evolve and interact with other actualities. The "interaction of possibilities" is an idea that simply makes no sense within the classical actualist Newtonian representation of physical reality. In classical physics, possibilities are always epistemic possibilities. As Dieks makes the point:

> "In classical physics the most fundamental description of a physical system (a point in phase space) reflects only the actual, and nothing that is merely possible. It is true that sometimes states involving probabilities occur in classical physics: think of the probability distributions ρ in statistical mechanics. But the occurrence of possibilities in such cases merely reflects *our ignorance* about what is actual. The statistical states do not correspond to features of the actual system (unlike the case of the quantum mechanical superpositions), but quantify our lack of knowledge of those actual features. This relates to the essential point of difference between quantum mechanics and classical mechanics that we have already noted: in quantum mechanics the possibilities contained in the superposition state may interfere with each other. There is nothing comparable in classical physics. In statistical mechanics the possibilities contained in ρ evolve separately from each other and do not have any mutual influence. Only one of these possibilities corresponds to the actual situation. The above (putative) argument for the reality of modalities can therefore not be repeated for the case of classical physics." [23, pp. 124-125]

The fact that quantum possibilities do interact is, according to us, the main mystery introduced by the theory of quanta. The Humean interpretation proposed by Dieks in the same paper [*Op. cit.*] —which only considers the events

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[8] [*Op. cit.*, p. 156].

predicted by QM— seems to us an empiricist escape which does not address the representational realist superposition problem defined above.

All the just mentioned interpretations shift the debate from the conceptual meaning of a mathematical element of the theory to the possibility of prediction given that same formal element. But as we argued above, for a realist, prediction is not explanation. Fortunately, there is also a vast literature composed by two main lines of research which investigate the possibility to consider quantum superpositions from realist perspectives. The first line is related to the idea that each term in a superposition relates to the existence of a "world", "branch", "mind" or "history". Examples of these interpretations are Everett's many worlds interpretation [48, 54] the many minds variant proposed by Albert and Lower [3], the consistent histories interpretation proposed by Griffiths [30] and the decoherent variant by Gell-Mann, Hartle [32] and Omnès [45]. Even though this family of interpretations might seem to provide an answer to the measurement problem arguing that actual reality is multiple, they still have two serious problems to confront. The first is the basis problem, which attempts to justify the subjective choice of a particular "preferred" basis between the many incompatible ones. The proposed solution to this problem in terms of the process of decoherence has found serious criticisms [13, 40]. The second problem is the interpretation of probability, which according to the orthodox formalism is incompatible with an epistemic interpretation. Following Everett's epistemic viewpoint regarding measurement, Deutsch and Wallace [19, 53] have proposed to join QBism and interpret quantum probability as a subjective epistemic belief of "rational agents" or "users" —like Mermin prefers to call them [43]. This proposal also confronts very serious difficulties [38]. Others like Griffiths, understand the probability related to the terms in a superposition simply as a "tool" to calculate outcomes [31]. The main problem surrounding the epistemic and instrumentalists interpretations of probability within supposedly realist interpretations of QM is that they provide no conceptual understanding of the weird interaction of probable states described by the theory. Once again, using a formal scheme that "works" and provides the correct measurement outcomes in probabilistic terms, is clearly very different from understanding and representing what is really going on according to the theory. The ontological question about *what there is* (independently of subjects) according to a theory obviously cannot be solved from an epistemic viewpoint which assumes the opposite standpoint —according to which theories only make reference to the *prediction of observations* (by subjects).

The second realist line of research investigates the idea of the existence of *indefinite properties* described in terms of propensities, dispositions, potentialities, possibilities or latencies. There are many different examples of this large family of interpretations. Let us mention at least some of them. Heisenberg's potentiality interpretation developed in terms of operational quantum logic by Piron and Aerts [1, 46], Popper's propensity interpretation and Margenau's latency interpretation presently developed by Suarez, Dorato and Esfeld [26], the modal interpretations of Dieks and Bub [21, 9], and the more recent transactional interpretation of Kastner [39]. However, all these interpretations share a common difficulty. As remarked by Dorato himself with respect to dispositions:

> "[...] dispositions express, directly or indirectly, those regularities of the world around us that enable us to predict the future. Such a predictive function of dispositions should be attentively kept in mind when we will discuss the 'dispositional nature' of microsystems before measurement, in particular when their states is not an eigenstate of the relevant observable. In a word, the use of the language of 'dispositions' does not by itself point to a clear ontology underlying the observable phenomena, but, especially when the disposition is irreducible, refers to the predictive regularity that phenomena manifest. *Consequently, attributing physical systems irreducible dispositions, even if one were realist about them, may just result in more or less covert instrumentalism.*" [25, p. 4] (emphasis added)

This deep criticism to dispositions can be easily extended to the description of *indefinite properties* in terms of propensities, possibilities and potentialities. The reason is that the just mentioned interpretations end up defining propensities, possibilities and potentialities exactly in the same way as it is done by dispossitionalists, namely, in terms of the future actualization of measurement outcomes. And for this reason, they all fall pray of Dorato's criticism. The lack of a (conceptual) categorical definition of these notions does not allow to imagine what these strange *indefinite properties* amount to beyond their predictive capacity regarding future observations. But let us recall, it is the conceptual level which the representational realist considers as a necessary element for the production of a closed theory. Without conceptual representation there is no explanation of what physical reality amounts to, and thus the main question remains also unanswered.

Both of these general lines of research have concentrated their efforts in trying to solve the measurement problem. None of these general schemes break with the actualist understanding of physical reality. While the former line of research attempts to restore classicality by multiplying our world into a multiverse of many (un-observable) branching worlds, minds or histories; the latter introduces potential, propensity or dispositional type properties explicitly defined in terms of their future actualization. As remarked by Dorato this solution seems to result in nothing else than "more or less covert instrumentalism". Both lines have deep problems in order to meet the requirements of a representational realist project according to which the understanding of QM requires an explicit conceptual and categorical description of *what physical reality is* according to the theory beyond the mere reference to measurement outcomes or abstract mathematical structures.

A detailed analysis of these interpretations exceeds the space of this paper which we leave for a future work. In the present paper we attempt to consider a radically different path. That is, to address the question of how to extend the notion of reality in order to produce an objective description of physical reality in accordance with the orthodox formalism of QM. The price we are willing to pay is the abandonment of a metaphysical equation which has become a silent dogma within philosophy of physics, the idea that ‘Reality = Actuality’.

6 Representing Quantum Superpositions Beyond the Actual Realm

Today, experimentalists in laboratories are playing with superpositions and entanglement all around the world. Physicists are developing a new era of quantum technology far away from the measurement problem. It is in fact this *praxis* of physicists which should call our attention as philosophers of QM. How are quantum superpositions being treated by physicists in the lab? We believe this is an important question we should definitely consider. After more than one century of not being able to interpret QM in terms of the actualist representation of “classical reality”, it might be time we admitted that QM confronts us with the fact that the classical representation of physics might not be the end of the road. We might be in need of abandoning physical representation exclusiveley in terms of “classical reality” (see for discussion [16]).

Quantum superpositions ‘evolve’ and ‘interact’ according to the Schrödinger equation of motion. Their MOS can be empirically tested in the lab through the Born rule. But when in physics a mathematical element of a theory ‘evolves’, ‘interacts’ and ‘can be predicted’ according to a mathematical formalism, then —always from a representational realist perspective— the elements of such mathematical expression need to be related (in some way) to physical reality through specific physical notions which are capable of producing a coherent counterfactual discourse and representation of physical reality (NC_2 and NC_3). Newton was capable of explaining the movement of the planets in the sky and the bodies on Earth through the creation of specific concepts such as ‘inertia’, ‘absolute space’, ‘absolute time’, ‘mass’, etc. After centuries, these concepts became —to great extent— part of our “common sense” representation of the world. Also Maxwell was capable of explaining many different experiments and equations through the introduction of the notions of ‘electromagnetic filed’ and ‘charge’. In the case of QM, we know that each quantum superposition is related to a specific set of MOS within a particular measurement context. However, we do not understand what they *represent* in conceptual terms. We do not know what do they refer to.

Superpositions impose a difficult crossroad when attempting to interpret the orthodox formalism of QM. So it seems, either the formalism should be changed in order to restore “classical reality”, or we should create a new understanding of physical reality itself beyond the constraints imposed by the classical actualist representation of physics —which boil down to the metaphysical scheme imposed through PE, PNC and PI. The latter path implies taking seriously the logical possibility that ‘Quantum Physical Reality $\neq$ Actuality’. A first step that we propose in this direction is to reconsider the meaning of *element of physical reality* beyond actuality (certainty).

Indeed, our representational realist stance seems to force us, given the predictions provided by QM, to extend the realm of what is considered to be real. Since both *certain* (probability equal to unity) and *statistical* (probability between zero and unity) predictions about physical quantities provide empirical knowledge, we believe there is no reason —apart from dogmatism regarding actualist metaphysics— not to relate both predictions to physical reality. This means we need to be creative enough to produce a new understanding of probability in terms of objective knowledge, abandoning its classical understanding in terms of ignorance about an ASA.

If we accept the challenge of representational realism and admit that quantum superpositions must be related to a conceptual level of description, then there are two main mathematical elements we need to conceptually represent in terms of objective physical concepts. Firstly, we need to provide a clear representation of the kets that constitute each quantum superposition —orthodoxly interpreted through their one-to-one relation to projection operators as properties of a quantum system. Secondly, we need to explain the physical meaning of the numbers that accompany the kets —orthodoxly interpreted as related to the probability of finding the respective property. If we were able to extend the limits of what can be considered as physically real, we might be also able to open the door to a new understanding of QM beyond the orthodox classical reference to 'systems', 'states' and 'properties'.

But how to extend physical reality beyond the limits of the actual realm? We believe that a good starting point would be the generalization of Einstein's realist definition of an *element of physical reality* in [27]. The famous definition by Einstein reads as follows:

Einstein's (Actual) Element of Physical Reality: *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.*

As remarked by Aerts and Sassoli [2, p. 20]: "the notion of 'element of reality' is exactly what was meant by Einstein, Podolsky and Rosen, in their famous 1935 article. An element of reality is a state of prediction: a property of an entity that we know is actual, in the sense that, should we decide to observe it (i.e., to test its actuality), the outcome of the observation would be certainly successful." Indeed, certainty, taken as the condition of possibility to make reference to the actual realm, has been up to the present the restrictive constraint of what can be considered as part of physical reality. Our redefinition must keep the relation imposed between predictive statements and physical reality, but leaving aside both the actualist constraint imposed by certainty —restricting existence only to probability equal to unity— and the focus in the process of measurement —which should be only regarded as confirming or disconfirming a specific prediction of the theory. Taking into account these general remarks we propose the following generalization:

Generalized Element of Physical Reality: *If we can predict in any way (i.e., both probabilistically or with certainty) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.*

The problem is now clearly framed: we need to find a physical concept that is capable of being statistically defined in objective terms. That means to find a notion that is not defined in terms of *yes-no experiments* (as it is the case of classical properties), but is defined instead in terms of a *probabilistic measure*. Of course, this first step must be accompanied by developing a network of physical notions that account for what QM is talking about —beyond measurement and mathematical structures. In the end, our new non-classical physical scheme will also have to be capable of taking into account the main features brought in by the orthodox formalism.

According to our representational realist stance, mathematical structures are not capable of providing by themselves a physical representation of a theory. As we argued above, physical theories are also *necessarily* related to a network of specific physical concepts. Mathematical structures provide a quantitative understanding about phenomena. But this mathematical description does not provide the qualitative understanding produced by physical notions. Thus, until we do not find a conceptual scheme which coherently relates to the orthodox formalism we will no be able to say we have understood QM.

We believe it is possible to come up with a physical network of concepts that take into account the non-classical features of QM. The price to pay may-be the abandonment of the attempt to explain the theory of quanta in terms of "common sense" or the the classical Newtonian metaphysics of actuality. This abandonment might allow us to construct a new non-classical metaphysical scheme with physical concepts specifically designed in order to account for the orthodox formalism of QM. We know of no reason to believe this is not doable.

7 Conclusion

In this paper we criticized the dogmatic constrains of the orthodox literature in order to discuss and analyze the quantum formalism in general, and more specifically, quantum superpositions. From a representational realist stance, we argued in favor of the necessity to consider a conceptual representational level defined through a metaphysical architectonic which describes quantum superpositions beyond the reference to mathematical structures and measurement outcomes. We have also provided a formal redefinition of quantum superpositions which takes into account their contextual nature. We presented three necessary conditions for any objective physical representation and presented the superposition problem which, contrary to the measurement problem, focuses on the conceptual interpretation of superpositions themselves. We provided arguments which point in the direction of considering quantum superpositions as real physical existents. In the final part of the paper we discussed, firstly, why the actualist interpretations fail to provide such a representation, and secondly, the possibility of producing a physical representation beyond the constraints imposed by the classical metaphysics of actuality.

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