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MATHEMATICS AND THE PHYSICAL WORLD: A RECONSIDERATION

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THE PRINCIPAL purpose of this article is to suggest two things: first, that a widely held interpretation of the fathers of the scientific revolution is dubious; and second, that this same interpretation can best be replaced by another which harmonizes more with what these and other physicists do in fact say.

The interpretation in question arises when asking why a mathematical approach to nature is not only successful, but even possible, and it consists in maintaining that according to the majority opinion of the physicists themselves, nature is "essentially" or "fundamentally" mathematical. Thus E.A. Burtt, in his now classic *Metaphysical Foundations of Modern Science*, summarizes what he believes to be the position of post-Renaissance scientists with these words:

> We have observed that the heart of the new scientific metaphysics is to be found in the ascription of ultimate reality and causal efficacy to the world of mathematics, which world is identified with the realm of material bodies moving in space and time. Expressed somewhat more fully... the real world in which man lives is no longer regarded as a world of substances possessed of as many ultimate qualities as can be experienced in them, but has become a world of atoms (now electrons), equipped with none but mathematical characteristics, and moving according to laws fully statable in mathematical form.[1]

Alexander Koyré follows suit when he writes, "Elle (modern science at its inception) choisit la précision comme principe; elle affirme que le réel est géométrique par essence et soumis, par conséquent, à la détermination et à la mesure rigoureuses..."[2] And A.C. Crombie maintains, "The momentous change that Galileo, along with other platonizing mathematicians like Kepler, introduced into scientific ontology was to identify the substance of the real world with the mathematical entities contained in the theories used to describe the 'appearances'".[3]

1. London, Routledge & Kegan Paul Ltd., 1964; p. 300.
2. "Une Expérience de mesure", in *Études d'histoire de la pensée scientifique*, Paris, Presses Universitaires de France, 1966, p. 259; cf. also pp. 69, 150-151 ff.
3. *Robert Grosseteste and the Origins of Experimental Science*, Oxford, Clarendon Press, 1953, p. 310.

Now if this position supposedly propounded by the physicists is true, then the successes of a purely mathematical physics are easily explicable; yet certain further consequences accrue that are quite disconcerting. For one, any investigation of nature employing non-mathematical tools or concepts is simply barking up a wrong tree. The biological sciences, psychology, etc., are all superfluous enterprises to the degree that they eschew considerations of the mathematical aspects of their subjects; and the little they can contribute to learning is restricted to discoveries concerning certain épiphenomena, the knowledge of which is hardly worth the effort. Secondly, and allied to the first, the natural thing that is man becomes largely inexplicable. The thoughts and aspirations that characterize him are seemingly barred from mathematical analysis, and hence ipso facto subjective, or unreal. Ironically enough, the scope of mathematical physics would preclude consideration of the very being responsible for that science to begin with.

These consequences, being grave, force us to re-examine the conclusions of the aforementioned historians. Do all of the renowned physicists, or even a majority, maintain that nature is in essence mathematical? Is this the most faithful manner of understanding the "heart of the new scientific metaphysics"? The central thesis of what follows is that it is not. The statements of Kepler, Galileo and others need not be read as Burtt has insisted, for there exists another interpretation far more in keeping with what they actually do say. This second interpretation, we hope to show, is more characteristic of scientists than the first, as witness a close examination of the key passages of their works. Moreover, certain modifications of what we consider to be the actual position of these seminal thinkers will be presented, as made by several physicists of this century. These modifications, we suggest, render the position very much akin to an outlook proposed by a number of physicists as far back as Aristotle, which outlook indeed provides a basis for a mathematical approach to nature, yet without the adverse consequences above enumerated.

These being our claims, four things are required to substantiate them. First, a brief criticism of Burtt's interpretation must be adduced,[4] followed by a summary of our own; thirdly, representative texts from 20th century scientists will be taken in relation to the views of their predecessors, manifesting to what extent they agree and disagree with what we consider Newton, Galileo, Kepler, etc., to be actually saying; lastly, we will attempt to link the position of the current scientists to a tradition beginning with Aristotle — a tie that would emphasize all the more strongly that we need not adhere to any mathematization of nature to explain the possibility and successes of a mathematical physics.

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To begin with, it is not Burtt's allegation that all physicists considered nature to be in essence mathematical. He is careful to expose the "non-mathematical

4. According to Koyré himself, Burtt best exposed the metaphysical pre-suppositions of modern science (cf. *op. cit.*, p. 172, n. 2), and for this reason, in addition to Burtt's analysis being more exhaustive than any of Koyré's or Crombie's, we will restrict ourselves to a critique of the former's *Metaphysical Foundations, etc.*

current", represented by thinkers such as Gilbert and, to a certain extent, such men as Boyle and Harvey.[5] These, though, were not responsible for the "scientific revolution", nor for the supposed world view consequent upon that upheaval. Others are held accountable for this, most notably Copernicus, Kepler, Galileo, Descartes, Hobbes and Newton. According to Burtt, the mathematical or quantified view of nature was clearly affirmed by these thinkers, hence to his treatment of them we must now turn.

In the case of Copernicus, our author would draw this conclusion: "He had become convinced that the whole universe was made of numbers, hence whatever was mathematically true, was really or astronomically true."[6] It is to be noted that no citations from Copernicus' writings are offered in support of this; instead, two arguments are advanced that in Burtt's mind necessarily lead one to identifying Copernicus' thought with that of Pythagoras'. The first stems from Copernicus' reason for professing his revolutionary heliocentric hypothesis, viz., that such a conception "threw the facts of astrónomy into a simpler and more harmonious mathematical order."[7] Now for Burtt, the *only* possible justification for this premise is the statement "the universe as a whole, including our earth," is "fundamentally mathematical in its structure".[8] What we have then, is a conditional argument constructed by Burtt, in which Copernicus affirms the consequence, while he (Burtt) goes on to affirm the antecedent, to thereby conclude that this very antecedent must be Copernicus' position. Yet to so reason is to fall into the fallacy of the consequent. Our author has not shown that if, and *only* if nature is essentially mathematical, then one must always strive for the more simple mathematical hypothesis. In fact, there definitely exists an alternative antecedent premise to this argument, one to which Ptolemy ascribed, as did Aristotle before him. This arose from the ancients' conception of the middle science of astronomy, wherein the aim was "to save the appearances" in terms of the simplest, mathematical hypothesis.[9] In justifying such a science, it was not necessary to equate nature and mathematics, but merely assume — or better yet recognize — the close proximity between the trajectories of the heavenly bodies and certain geometrical configurations. This assumption was Aristotle's,[10] Ptolemy's,[11] and more likely than not, Copernicus',[12] for in making it, one is at perfect liberty to heliocentrize the universe if this

5. *Metaphysical Foundations*, see especially pp. 156–160.

6. *Ibid.*, p. 44.

7. BURTT, *ibid.*, p. 26.

8. *Ibid.*, p. 40.

9. Cf. P. DUHEM, *To Save the Phenomenon*, Chicago, University of Chicago Press, 1969.

10. Cf. *De Caelo*, I, ch. 2 and 3, as found in *The Basic Works of Aristotle*, ed. R. McKeon, New York, Random House, 1941, pp. 399–403; cf. also *De Caelo* II, ch. 4 & ff.; *Physics* II, ch. 2; *Metaphysics* XII, ch. 8.

11. Cf. "The Almagest", Bk 1, Preface, in *The Great Books of the Western World*, ed. R.M. Hutchins; Chicago, Encyclopaedia Britannica, Inc., 1952; Vol. 16, pp. 5-6.

12. "Revolution of the Heavenly Spheres", Preface and Dedication to Pope Paul III, in *ibid.*, pp.506-7; Bk 1, Introductory Paragraphs, pp. 510-11; Kuhn's opinion of Copernicus in this respect would seem to concur with our own — see his *Copernican Revolution* (N.Y., Vintage Books, 1959) pp. 136, 144, 148, 182–84. See also DREYER'S *A History of Astronomy from Thales to Kepler*, N.Y., Dover Publications Inc., 1953, pp. 312-13, 319–321.

accounts for the phenomena in a simpler, more efficient manner. And although the theory of Copernicus runs counter to the teachings of the aforementioned ancients on points of astronomy, it is not thereby necessarily opposed to their position on the relation of mathematics to nature.

The second argument is even weaker. Apparently Copernicus spent several years studying with the Pythagorean Dominicus Maria de Novara, whose influence was a determining factor in the former's world view.[13] Undoubtedly it was, but is that sufficient to make the point? Novara may very well have bolstered Copernicus' aspirations to discover the simplest mathematical hypothesis, but there is no textual evidence in Copernicus indicating a wholesale adoption of Novara's Pythagoreanism.

The next thinker to have supposedly accepted the mathematical view of nature was Kepler. In his case, several texts are cited which ostensibly indicate this. Thus, to support his claim that for Kepler, "The real world is a world of quantitative characteristics only; its differences are differences of number alone," Burtt cites the passage: "Wherever there are qualities, there are likewise quantities, but not vice versa". To justify his having Kepler say, "quantity is the fundamental feature of things", Burtt quotes his assertion that quantity is "primarium accidens substantiae".[14] But neither of these statements are equivalent to Burtt's interpretation of them; on the contrary, both are mere repetitions of wellknown Aristotelian doctrines — something of which assuredly Kepler was aware. Thus we have every reason to think that he conceded their literal import: to wit, quantity is an accident of physical things, as distinguished from their substance, yet the first and most important accident, being the subject of other accidents such as figure, color, etc. Quantity, then, is not the substance or essence of things and, if anything, Burtt's alluding to these passages discloses a fundamental concord between Kepler and Aristotle on this issue, rather than any disagreement.[15]

Nor do other doctrines of Kepler — referred to in Burtt's analysis — attesting that "God was a mathematician", or "perfect knowledge is always mathematical" necessarily suggest an identity between quantity and physical essences. We will have occasion below to exhibit the possibility of upholding these, yet all the while denying the nature-mathematics equivalence.

The third physicist of outstanding repute cited by Burtt as a proponent of the "new metaphysical outlook" is Galileo. Here potent textual evidence exists seemingly supporting an identification of mathematics and the nature of the physical world. The famous passages from the "Assayer" are quoted in full, including the "Language of Nature" excerpt, in addition to the one wherein the reality of the proper sensibles, or "secondary qualities", are denied.[16] Moreover, there is refe-

13. BURTT, *op. cit.*, pp. 42–44.

14. *Ibid.*, pp. 56-57.

15. Even Koyré acknowledges Kepler's "failure" to have mathematized the universe as his successor Galileo evidently succeeded in doing. See KOYRÉ, *op. cit.*, p. 46.

16. BURTT, *op. cit.*, pp. 68, 74–76, 78–80; the same passages in Galileo can be found in *Discoveries and Opinions of Galileo*, Stillman Drake, ed., N.Y., Double Day Anchor Books, 1957, pp. 237-8, 274–7.

rence to the Florentine's atomism — i.e., the existence of infinitely small, indivisible particles, which in Burtt's view, "possess *none* but mathematical qualities".[17] Let us examine each of these doctrines, to see if our author's reading of them is the correct one.

"The Book of nature is written in the Mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it."[18] It should be noted at the outset that Galileo's assertion is not an outright equating of the mathematical and the substance of the natural; it is rather a figurative manner of emphasizing the role which mathematical entities play in nature. The figure employed is a metaphor, and to discern the Florentine's meaning here, the sense of the metaphor must be elaborated. Now without doubt a grasp of the language is imperative for understanding the book, but is the language all there is, or even what is most essential? Is not the thought, or content of the book, even more reflective of its essence than the language in which it is composed, and could not Galileo have meant this in using the particular metaphor which he did? In this interpretation, mathematics (i.e. quantity) is fundamental to nature, but on no account do the whatnesses of physical things consist in their quantifiable aspects.[19]

The negation of the so called secondary qualities does not oppose our reading Galileo in this way. At one point in the *Two New Sciences* Salviati states:

> I desire, before passing to any other subject, to call your attention to the fact that these forces, resistances, moments, figures, etc., may be considered either in the abstract, disassociated from matter, or in the concrete, associated with matter. Hence the properties that belong to figures that are merely geometrical and non-material must be modified when we fill these figures with matter and give them weight.[20]

For Galileo, then, what distinguishes the mathematical from the physical, and thus what is proper to the latter, is "matter" and "weight" — two "concrete" characteristics in some way sensible. The secondary qualities may not be as real as the primary ones, yet they must be trusted at least to the extent that these two properly physical qualities can be recognized. In this respect, Galileo's teaching on secondary qualities anticipates Eddington's single, colored-blind eye; granted its reportings are imprecise and subjective — nevertheless they cannot be done away with.[21] In short, this text both qualifies the Florentine's extreme doctrine in the "Assayer", as well as lends credence to our interpretation outlined above.

Yet what of Galileo's atomism? Burtt's assertion is that they possess none but

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17. BURTT, *op. cit.*, p. 76.

18. DRAKE, *op. cit.*, p. 238.

19. Werner Heisenberg, when discussing whether or not elementary particles are "all there is", uses the same metaphor as Galileo with the same interpretation of it as our own. See his *Philosophical Problems of Nuclear Physics*, N.Y., Fawcett World Library 1966, pp. 121-122.

20. *Two New Sciences*, New York, Dover Publications, Inc. 1954; "Second Day", p. 112.

21. Cf. A.S. EDDINGTON, *New Pathways in Science*, Ann Arbor, The University of Michigan Press, 1959, ch. 1, pp. 12-13.

mathematical properties. Galileo himself does not say this, although even if it were true, one need not then conclude that the Florentine ascribed to the further position in question. There is reference to sensible matter being proper to physical things not only in the passage just cited, but elsewhere in his works; furthermore, there is the neglected connotation of the metaphor explained above that points to something more critical in nature than mathematical entities alone; finally, there is Galileo's deliberate restriction of his method to inorganic phenomena, a restriction that conduces to our interpreting his famous statements as applying to certain parts of the physical world, but not to nature in her totality.[22]

Following Burtt's analysis of Galileo come those of Descartes and Hobbes, with which we readily concur: the substance or essence of things is clearly extension for Descartes, and Hobbes unabashedly follows suit.[23] Yet it is consequential to add that neither of these thinkers were physicists, and it is with the physicists that we are concerned. Our principal intent, again, is to reveal a basic unanimity amongst the more important scientists as to the relation between mathematics and nature — a unanimity severed by philosophers of Descartes' persuasion. It is certainly to Burtt's and Koyré's credit to have disclosed the affinities which join Descartes to the scientists discussed thus far; however the identification of their respective positions is what we are forced to deny.

Prior to examining Newton, Burtt briefly discusses the "non-mathematical current", as well as several less renowned thinkers such as Barrow and Boyle, both of whom, he thinks, follow Descartes' assimilation of the real to the mathematical.[24] Barrow was a mathematician and deeply imbued with the very customs that led to Descartes' reduction of matter to extension. Does he, though, acquiesce to the Cartesian view? Certainly Barrow considers geometry the paradigm of the sciences, and physics to be simply applied mathematics; he even asserts that "quantitative continuity is the object of all science".[25] However, such allegations are not equivalent to saying that only the quantitative *exists*, or that it alone constitutes the essence of physical phenomena. Failure to make this distinction will of course lead to Burtt's interpretation.

With Boyle, our author writes that he "... expresses his complete agreement with the mathematical metaphysics of Galileo and Descartes, the whole world seems to be fundamentally mathematical in structure; 'nature does play the mechanician'; mathematical and mechanical principles are the 'alphabet in which God wrote the world' ".[26] Once more we have Galileo's metaphor; should it be under-

22. There is also Galileo's "positivism", documented by Burtt on pp. 93-94; if the Florentine was unwilling to grant that the essences of such things as gravity were readily accessible, would he not also, *a fortiori*, maintain a similar reluctance in regard to the nature of physical reality? This would further account for his particular choice of metaphor in the "Assayer" text cited above.

23. For Descartes, cf. *ibid.*, pp. 96–107, and Hobbes, pp. 118–127.

24. For Barrow, cf. *ibid.*, pp. 145-146; for Boyle, pp. 165-166.

25. Barrow, in *ibid.*, p. 145.

26. *Ibid.*, pp. 165-166.

stood in the same way? We would not be so willing to conclude as Burtt does in the case of Boyle; the former admits that Descartes and Hobbes were viewed by the latter as "extremists"[27], and that in Boyle's mind, the secondary qualities were just as real as the primary ones. His mechanistic tendencies, though, apparently suggest the contrary. Given the absence of consistency in Boyle's writings, perhaps the more prudent course would be to suspend judgement.[28]

This leaves us with Newton. Burtt concludes of him:

> Just as Boyle, though not a skilled mathematician himself, had accepted without serious question the main structure of the universe as portrayed in Galileo, Descartes and Hobbes, so Newton... took over without criticism the general view of the physical world and of man's place in it which had developed at the hands of his illustrious predecessors. For Newton too the world of matter was a world possessing mathematical characteristics fundamentally.[29]

The reasons for so concluding, as best as we can discern them in Burtt's analysis, are derived from Newton's atomism, a view entailing the reduction of all physical changes to interactions of particles equipped with the "primary qualities"[30], as well as his method — an important aspect of which was the successful use of mathematics.

Concerning the first, Newton is careful to indicate that the qualities proper to the atoms are the very qualities sensed in our common experience. Burtt acknowledges this, but would nonetheless insist that for Newton "the atoms are predominately mathematical". But what is the sense of "mathematical" here? Certainly not the sense that Descartes employed, nor the one supposedly used by Galileo. Extension is only one attribute of the particles, and nowhere does Newton say it is the predominant one; all the other attributes may be viewed in quantifiable relations to one another, but they themselves are qualities — e.g., hardness and impenetrability. Or perhaps he is suggesting that in Newton's eyes the quantifiable relations are of the essence, such that all of the atomic changes — and thus all of the changes in nature — are reducible to mathematical laws. Yet this is not what Newton says; the entire *Principia* may be viewed as an investigation of the mathematical principles of natural philosophy,, instead of the non-mathematical ones, the two being quite different. Ironically enough, Burtt says as much when he writes:

> For Newton... the world is what it is; so far as exact mathematical laws can be discovered in it, well and good; so far as not, we must seek to expand our mathematics or resign ourselves to some other less certain method. This is obviously the spirit of the paragraph from the preface of the *Principia* already quoted...[31]

27. *Ibid.*, p. 170.

28. Nevertheless, there are two further points against Burtt; firstly, his own admission that Boyle leaned towards positivism (pp. 178–182), and secondly, Koyré's pitting Boyle against the "pan-mathematism" of Galileo and Descartes. Cf. *Newtonian Studies*, Chicago, the University of Chicago Press, 1965, pp. 11-12.

29. BURTT, *op. cit.*, p. 228.

30. *Ibid.*, p. 228, ff.

31. *Ibid.*, p. 208.

Unless, then, Burtt wishes to equivocate on "mathematics", we do not see the justification for his uniting Newton's conceptions with Descartes'; a reduction of this sort is to go further than the texts would allow us.[32]

In regard to the second argument, Burtt's very words run counter to his general conclusion. This last, again, would seem to be an embracing (on Newton's part) of a quantified outlook on nature because of the tremendous success rendered by an exclusively mathematical approach.[33] When we turn to the development of the argument, though, where Burtt discusses the effects of the method consequent upon Newton's concept of mass, we read,

> What about the metaphysical bearings of the Newtonian concept of mass? Did Newton conceive of physical bodies as merely masses, that is, possessing none but geometrical qualities and *vis inertiae*? Probably not. And yet the effect of his work was decidedly to encourage others so to convince them.[34]

This is precisely the point that need be emphasized: Newton himself did not subscribe to any identification of mathematics with the substance of nature, but the efficacy of his method and the customs consequent upon it may and did turn the minds of many in this direction. We will have occasion to elaborate on this below; at this juncture it suffices to say that Burtt's train of thought here is indeed well taken, yet antithetical to his general conclusion regarding Newtonian metaphysics.

Further evidence in our favor is Burtt's own assessment of Newton's empiricism. So strong was it that "... if it were possible to wholly separate the two aspects of his method, it would have to be said that Newton's ultimate criterion was more empirical than mathematical".[35] Thus "... it is for Newton, in marked contrast with Galileo and Descartes, there is a distinct difference between mathematical truths and physical truths."[36] Such being the case, we would argue to the intrinsic unlikeliness of Newton ever assenting to an interpretation of his works in which the natural is seen to be essentially mathematical.

To sum up our findings thus far: the achievement that is Burtt's in his *Metaphysical Foundations of Modern Science*, is a notable one in many respects; yet it does not successfully prove the position of the more influential scientists to be in fact the particular metaphysical foundation which is alleged. We are not suggesting that there were no profound disagreements between the conceptions of these physi-

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32. The reason why Burtt alludes to Newton's atomism is simply that this doctrine is a clear sign for him that the author of the *Principia* had adopted the panmathematism of his predecessors. If this is so, however, how can Burtt maintain that "For Newton there was absolutely no a priori certainty, such as Kepler, Galileo and pre-eminently Descartes believed in, that the world is through and through mathematical, still less that its secrets can be fully unlocked by the mathematical methods already perfected."? p. 208.

33. Cf. pp. 226-227.

34. *Ibid.*, pp. 240-241; for an even clearer enunciation of the same point, cf. pp. 242-243. Needless to say, this statement wreaks havoc with Burtt's earlier assertion that Newton's atoms are "predominantly mathematical".

35. *Ibid.*, p. 209.

36. *Ibid.*, p. 208.

cists and those of the ancients, nor do we wish to deny the reality of what was indeed a "revolution in science". Differences there were, but an identification of physical essences with quantity was not one of them.

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Now we must make good our claim to replace Burtt's interpretation with something more faithful. If these scientists were not teaching what our author had supposed on this question, what were they teaching? An interpretation that would fully recount what the scientists did say, in addition to accounting for what they only suggested, is as follows: mathematical properties, or quantities and quantitative relations, constitute the most desireable objects of the intelligence, for only these provide the objectivity, precision, clarity and certitude that the mind always seeks. In brief, the knowledge of such things is the only proper or true knowledge. Obviously quantities and their relations exist in nature, hence it will be to these that the physicist must turn, as only here — and not in the realm of the qualitative or "occult forms" — will his thirst for such knowledge be assuaged. In consequence, it is not surprising that mathematics was incorporated into physics with such vigour, and with such success; one would indeed expect rapid strides forward given the extent to which quantity prevails in nature and the zeal with which this aspect of the world was studied. Thus we interpret the statements of the post-Renaissance physicists in terms of their quest for a certain kind of knowledge, and not in terms of any presupposition regarding the nature of the physical world; in their eyes the mathematical side of reality was paramount, not because it comprised the most important part of things, but rather because it was the most important thing to be known.

This outlook of the early physicists (the existence of which is to be defended shortly) is contrasted with a more severe one, that historians often label the "19th century world view". What characterized this latter was precisely an identification between quantity as the most knowable, and quantity as the *only* knowable thing. The road from the first position to the second was a gradual one, and the earliest traces of it are found in d'Alembert's Preface to the *Encyclopédie*:

> L'usage des connaissances mathématiques n'est pas moins grand dans l'examen des corps terrestres qui nous environnent. Toutes les propriétés que nous observons dans ces corps ont entr'elles des rapports plus ou moins sensibles pour nous: la connaissance ou la découverte de ces rapports est presque toujours le seul objet auquel il nous soit permis d'atteindre, et le seul par conséquent que nous devions nous proposer.[37]

With time, the now famous doctrine of Lord Kelvin grew to express the attitude proper to many thinkers of his day:

37. D'ALEMBERT, *Encyclopédie, ou Dictionnaire Raisonné des sciences des arts et des métiers* (Nouvelle impression en fac-similé de la première édition de 1751–1780 vol. 1, Stuttgart-Bad Connstatt, 1966) Discours Préliminaire p. vi.

> I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be.[38]

This knowable-measurable equivalence was not the only tenet of the world picture espoused by these scientists. Historians like Dampier[39], and Singer[40], not to mention a host of contemporary physicists, generalize the attitude of the last century with the phrase "mechanistic-deterministic", suggesting perhaps that Burtt's interpretation of the early scientists is more apropos of these later ones.[41]

The development in attitude between Newton's day and Kelvin's is not difficult to explain. Physicists and philosophers alike, inured to the methods of science and intoxicated by its awesome explanatory-predictive power, grew to construe the object of science as the exclusive object of the mind. This was most of all the case with those thinkers whose formation was almost exclusively mathematical and who had little real contact with nature. Notice, then, that the resulting 19th century outlook — one still very much alive today — reflected precisely the effects of custom, and not the actual position of the more prestigious physicists. The next order of business is to return to these same thinkers and briefly justify our aforesaid interpretation of them.

Copernicus, we recall, was "highly influenced" by Pythagoreans such as Nicholas of Cusa, Giordano Bruno, and the above-mentioned Novara. We also recall that no textual evidence is found in Copernicus indicating an adoption of authentic Pythagoreanism. Nonetheless, there undoubtedly was an influence exerted by these men, as Burtt insisted, which justifies our asking in what it consisted. Our author is careful to emphasize that for these thinkers "knowledge is always measurement", and more generally, "all certain knowledge that is possible for man must be mathematical knowledge".[42] Sentiments of this kind are definitely implied in Copernicus' writings, suggesting that the influence exerted was an induced enthrallment with the cogency, simplicity and beauty of mathematical explanations.[43] On this hypothesis, his adamant defence of heliocentricity for mathematical reasons is perfectly explicable, and in no way discordant with the traditional understanding of astronomy.

The case of Kepler is even clearer. The texts cited by Burtt to manifest his supposed equating of quantity with the "fundamental structure of the universe"

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38. LORD KELVIN, *Popular Lectures and Addresses, 1891–1894*, London and New York, The Macmillan Co.
39. W.C. DAMPIER, *A History of Science*, Cambridge, at the University Press, 1930, pp. 213–216, 219, 320, 328-9, 370–72.
40. C. SINGER, *A Short History of Scientific Ideas to 1900*, Oxford, at the Clarendon Press, 1959, pp. 418–422; cf. also A. D'ABRO, *The Rise of the New Physics*, N.Y., Dover Publications, 1951, vol. 1, p. 61.
41. Though it is worthwhile remarking that even Kelvin's position is not equivalent to Burtt's interpretation.
42. BURTT, *op. cit.*, p. 42.
43. Cf. "De Revolutionibus", in *op. cit.*, Preface to Pope Paul III, and Bk 1, ch. 1.

can be far better understood in light of our interpretation. Kepler's remarks stressing that quantity is closer to substance than quality plainly reveal his view that the first of these is more stable, more permanent than the second. This in turn supports his further inference, "... that nothing can be known completely except quantities or by quantities, and so it happens that the conclusions of mathematics are more certain and indubitable." Even more clearly, "Just as the eye was made to see colors, and the ear to hear sounds, so the human mind was made to understand, not whatever you please, but quantity."[44] Thus Burtt states with justification, "Quantitative features (for Kepler) are the sole features of things as far as the world of our knowledge is concerned."[45] But, as we have repeatedly argued, Burtt's proceeding from this to the following conclusion is unwarranted: "Therefore, quantity is the fundamental feature of things".[46] The first is definitely present in Kepler's words, while the second is not.

With Galileo, the sundry positions found in his writings (and referred to above) can all be accounted for in terms of his having held that mathematical knowledge is the most perfect knowledge, and thus the most important. Obviously, one adhering to such a position will strive whenever possible to explain and reduce the workings of nature to quantitative analyses, in the form of geometrical demonstrations and mechanical laws — something which in fact Galileo did. That he should turn to a mathematical atomism, and a rejection of the "secondary qualities" is not surprising. Thus mathematics is indeed the key to understanding nature's secrets, with "key" meaning: that in virtue of which the mind can have the most perfect knowledge of the physical world — though not in its entirety. The Florentine, let us repeat once more, is not asserting that nature is in essence mathematical; on the contrary, in his scheme (identical to Kepler's and Newton's) a mathematical approach enables us to know *certain aspects* of nature (i.e., her geometrical "letters") in an exact, certain way. In short, the importance of mathematics is the certitude it offers concerning a part of the physical world, and not a grasp of that world in its essence nor in its totality.

This very distinction is suggested in a passage from his *Dialogue* on the two chief world systems:

> Extensively, that is, with regard to the multitude of intelligibles, which are infinite, the human understanding is as nothing even if it understands a thousand propositions... But taking man's understanding intensively, in so far as this term denotes understanding some propositions perfectly, I say that the human intellect does understand some of them perfectly and thus in these it has as much absolute certainty as nature itself has. Of such are the mathematical sciences alone; that is, geometry and arithmetic, in which the divine intellect indeed knows infinitely more propositions since it knows all. But with

44. BURTT, *ibid.*, cf. p. 57 for all of Kepler's statements cited in this paragraph.

45. *Ibid.*, Though the adjective "sole" is unjustified.

46. *Ibid.*

> regard to those few which the human intellect does understand, I believe that its knowledge equals the divine in objective certainty, for here it succeeds in understanding necessity, beyond which there can be no greater sureness.[47]

The strength of this passage and others[48] indicates our position: Galileo seeks mathematical explanations not because they reveal nature in her essence, but rather because they enable him to participate in the certitude of the Divinity Himself.

The same may equally be said of Barrow and, to a lesser extent, of Boyle.[49] Nonetheless the latter admits that mathematical principles, given their certitude and clarity, "must be ultimate truths superior to God himself." Furthermore, they are (along with metaphysical principles) the "universal foundations and instruments of all the knowledge we mortals can acquire". Thus Boyle's "mathematical view of nature" can very well be construed as a simple concession that the quantitative aspects of things provide us with the most perfect kind of knowledge.

Lastly, and most importantly, we turn to Newton: did he, in fact, maintain this position and not the one attributed to him by Burtt? Again, on the basis of his texts alone we would have to say yes. In the *Principia* he writes:

> Our purpose is only to trace out the quantity and properties of this force (attraction) from the phenomena and to apply what we discover in some simple cases, as principles, by which, in a mathematical way, we may estimate the effects thereof in more involved cases... We said 'in a mathematical way', to avoid all questions about the nature or quality of this force, which we would not be understood to determine by any hypothesis.[50]

Here he explicitly acknowledges the restricted character of his enterprise, in as much as the "natures", "qualities" or ultimate causes lie simply beyond its bounds.[51] What the use of mathematics does give is a clear, rigorous and certain explanation, as witness its ability to predict and control phenomena. As in the case of his predecessors, mathematics for Newton is desireable for the knowledge it gives us — not because it is intimately entwined with the essence of nature. In fact, this last assertion would be tantamount to the particular kind of hypothesis he was always careful to avoid.[52]

Burtt's emphasis on Newton's restricting himself to the "how" of things further supports this. In one pertinent passage, he makes the very point which we intend:

> For Newton, then, science was composed of laws stating the mathematical behavior of nature solely — laws clearly deducible from phenomena and exactly verifiable in phenomena — everything further is to be swept out of science,

47. *Dialogue Concerning the Two Chief World Systems*, translated by Stillman Drake, Berkeley and Los Angeles, The University of California Press, 1967, p. 103.

48. Cf. for example, Koyré's reference in his *Études, etc.*, p. 172.

49. Cf. again in Burtt's analysis, pp. 146-7, and for Boyle, p. 166.

50. Bk III, "The System of the World", as quoted in *ibid.*, pp. 218-19.

51. Cf. also "General Scholium" in "Principia" Bk III, *Great Books*, vol. 34, p. 371.

52. For a discussion of Newton's eschewing of hypotheses, cf. KOYRÉ, *Newtonian Studies*, p. 16 and ch. II.

> which thus becomes a body of absolutely certain truth about the doings of the physical world.[53]

The further inferences, we repeat, to the effect that this "mathematical behavior" is the only behavior, or the only knowable one, are not to be found in Newton, nor must they be assumed in order to explain what he does say. On the contrary, the summation given by Burtt here is all one need say; it stresses the key role of mathematics in our knowledge, and not its role in things.

To summarize thus far, the "heart of the new scientific metaphysics" is not what Burtt and others would have us believe. A new outlook there certainly was, but it did not consist in the reduction of nature to mathematics. As we have laboured to show with the above analysis, it seems best to understand the attitude of post-Renaissance science to consist in a renunciation of the study of natural *qualities* — the approach of the ancient physicists, by and large, — and an embracing of all investigations into natural *quantities*. The reason being that the knowledge of quantity is the most perfect knowledge accessible to inquiring man, and hence the most desireable. In consequence, it was only a matter of time before physics became mathematical, that is, the exact science of physical quantities; in Kelvin's terminology, physics was heretofore the science of the measureable.

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The question now to be faced is in regard to the views of relativity and quantum physicists. Would they subscribe to the doctrine of their "classical" precursors as we have interpreted it? Are the mathematical features of nature the most important, indeed, the sole objects of knowledge (to follow the 19th century outlook) for thinkers such as Einstein, Bohr, Planck, Heisenberg and the like? Our claim is that there is partial agreement, but equally substantial disagreement. Let us begin with the similarities.

For sure, the quantitative, or the measureable, is the proper subject of physics equally for current scientists as for classical. It is also true for modern science that the measureable is not all that exists, nor is it most essential to the physical world. Eddington makes the first point when writing "I should like to make it clear that the limitation of the scope of physics to pointer readings and the like is not a philosophical craze of my own but is essentially the current scientific doctrine."[54] And Heisenberg attests:

> This general tendency of the new science also foreshadows a characteristic feature that has often been discussed, namely the emphasis on the quantitative. The demand for precise experimental conditions, accurate measurements, an

53. BURTT, *op. cit.*, p. 223.

54. *The Nature of the Physical World*, London, Everyman Library, 1927, p. 247. Cf. also pp. 245, 249, 266, 324.

> exact unambiguous terminology and a mathematical presentation of the idealized phenomena has determined the aspect of this science of nature, and brought it the name of "exact science".[55]

Whereas classical physicists lent themselves to mis-interpretation concerning the measureable or quantitative not being equivalent to the essence of the real, scientists of this century are outspokenly unambiguous. To cite Eddington once more:

> Scientific investigation does not lead to knowledge of the intrinsic nature of things. Whenever we state the properties of a body in terms of physical quantities we are imparting knowledge as to the response of various metrical indicators to its presence, and nothing more.[56]

Heisenberg also insists on this in a passage where he comments on the work of Heinrich Hertz:

> "... here it emerges clearly how physics began to remember once more that a natural science is one whose propositions on limited domains of nature can have only a correspondingly limited validity; and that science is not a philosophy developing a world view of nature as a whole or about the essence of things.[57]

In the same work the author concedes that "this modesty was largely lost during the nineteenth century",[58] when, as we indicated earlier, many grew to consider physics as having a greater power and scope than it actually did have. Heisenberg insists, though, that a proclivity of this kind is antithetical to science: "The philosophic content of a science is only preserved if science is conscious of its limits".[59]

The third point of agreement between the two is the acknowledgment that the mathematical approach is the most objective, precise and clear — at least for those initiated to its language. Indeed, this is precisely what sustains physics in its role of paradigm, or model, for the other sciences.[60]

At this point the unanimity ceases, and significant differences arise. We noticed with the classical physicists a tendency to view the non-quantifiable as insignificant, and even unknowable for thinkers of Kelvin's persuasion. Accordingly, sciences such as biology were hardly worth the effort, or their subjects viewed as unknowable, unless, of course, they could employ mathematics and reduce the living to the measureable. Now such efforts of reduction were attempted in the last century, and

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55. *Across the Frontiers*, N.Y., Harper and Row Publishers, 1975, p. 216-17. Cf. also *Physics and Philosophy*, N.Y., Harper & Row, 1958, pp. 101-2.

56. EDDINGTON, *op. cit.*, p. 292; cf. also p. 251, 260–63, 280-1; N. BOHR, *Atomic Physics and Human Knowledge*, N.Y., John Wiley and Sons, vol. 1, p. 7, vol. 2, p. 10; H. POINCARE, *Science and Hypothesis*, N.Y., Dover Publications, 1952, ch. XII, p. 211.

57. *Physicist's Conception of Nature*, Westport, Conn., Greenwood Press, 1970, pp. 152-3.

58. *Ibid.*, pp. 180-81.

59. *Ibid.*

60. Cf. M. PLANCK, *The Philosophy of Physics*, N.Y., W.W. Norton & Company, Inc., 1936, p. 10; L. DE BROGLIE, *Matter and Light*, N.Y., Dover Publications, Inc., pp. 38-39; HEISENBERG, *Across the Frontiers*, p. 216.

had failed. Significantly enough, Bohr, Heisenberg, de Broglie and Einstein often acknowledged the failure, but for that reason did *not* shirk off the biological sciences as impossible or unfruitful, nor did they ban these disciplines from natural science.[61] Granted the objectivity and precision — the "clarity" as Einstein would say — are not present with organic phenomena to the same degree as the inorganic, this does not and should not prevent scientists from pursuing the study of life. Knowledge of non-mathematical kind abounds in this realm, and it is both legitimate and fruitful; for according to Bohr and Heisenberg, it is complementary with the knowledge acquired in mathematical physics. Bohr, for example, writes:

> Owing to this essential feature of complementarity, the concept of purpose, which is foreign to mechanical analysis, finds a certain field of application in biology. Indeed, in this sense, teleological argumentation may be regarded as a legitimate feature of physiological description which takes due regard to the characteristics of life in a way analogous to the recognition of the quantum of action in the correspondance argument of atomic physics.[62]

Thus for these physicists[63] the knowable is not restricted to the measureable; in fact, investigations of the non-measureable are both legitimate and fruitful.

They are also objective and certain — a further claim which classical physicists would undoubtedly deny. Here again, it is in the realm of the living that this is most clearly disclosed. Bohr considers the existence of life an elementary fact, which is as important to biology as the quantum of action is to atomic physics.[64] Secondly, Planck and Eddington bear witness to one's internal experience of freedom and responsibility. This experience, they maintain, is certain, immediate and objective, but obviously not measureable, nor subject to mathematical laws.[65]

In discussing the points of disagreement between classical and modern physicists, we have heretofore exhibited the latter's inclination to uphold the worthwhileness of studying the non-measureable. We must now ask whether the same physicists would grant the legitimacy of an approach that investigated the measureable *without* recourse to measurement. In others words, do twentieth century scientists sever themselves from their predecessors even more strongly by allowing for a successful non-mathematical approach to inorganic phenomena — one that would concentrate on physical qualities rather than quantities alone?

To make such an allowance would fly in the face of Newton and Galileo; nevertheless, several scientists of this era have done so. To see how, it behooves us

61. Cf. HEISENBERG, *Physics and Philosophy*, pp. 102-5, 154-5; DE BROGLIE, *Physics and Microphysics*, N.Y., Harper and Brothers, 1960, pp. 139-40; R.W. CLARK, *Einstein, the Life and Times*, N.Y., World Publishing Co., 1971, p. 35.

62. *Atomic Physics and Human Knowledge*, vol. 1, p. 10; cf. the entire chapters "Light and Life," "Biology and Atomic Physics"; see also HEISENBERG, *Physics and Philosophy*, pp. 154-5; *Philosophical Problems of Nuclear Science*, p. 102.

63. And others in addition; cf. Max BORN, *Physics in My Generation*, N.Y., Springer-Verlag Inc., 1969, p. v and following.

64. BOHR, *op. cit.*, pp. 9, 21.

65. For Planck, cf. *op. cit.*, pp. 79-80, 102–05; EDDINGTON, *New Pathways in Science*, pp. 90-91.

to set forth Heisenberg's remarks on the researches of Goethe in the sphere of color. The poet's scientific method is described in these words:

> For Goethe, all observation and understanding of nature began with the immediate sensory impression; not, therefore, with an isolated phenomenon, filtered out with instruments and so to speak wrung from nature, but with the free natural happening, directly accessible to our senses.[66]

Heisenberg goes on to add that in other respects, Goethe's method (and conclusions) closely resembled that of Newton's; thus the former assuredly leaned on a kind of experimentation, and developed theories to the same extent as the latter.[67] But the distinguishing feature, to repeat, was the absence of measurement and mathematics. Does Heisenberg concede any value to Goethe-type efforts? In response, here are his very words:

> Dividing reality in this way into different aspects immediately resolves the contradiction between Goethe's and Newton's theories of color. In the great structure of science, the two theories take up different positions. It is certain that an acceptance of modern physics cannot prevent the scientist from following Goethe's way of contemplating nature too.[68]

If we follow correctly, only a combination of the mathematical and non-mathematical avenues will produce an understanding of the physical world in its entirety, which is to say that the quantitative and the qualitative aspects of things must *both* be regarded in the total study of nature.

This same point is implicitly made in Heisenberg's (and others) insistence on the physicists constant recourse to "ordinary language".[69] This language, argues Heisenberg, signifies concepts which are derived from an immediate connection with reality. They consequently represent the real more faithfully than the mathematical idealizations of physics, and thus "seem to be more stable in the expansion of knowledge than the precise terms of scientific language".[70] If science hopes to secure stability, as well as remain in contact with the concrete physical world it attempts to explain, then the kind of concepts signified by plain language must never be abandoned. This reveals again the basic insufficiency of a physics exclusively tied to mathematics, and emphasizes the necessity of complementing the mathematical approach with that of the qualitative, or non-mathematical.

At this point it is opportune to make known the similarities which tie together the positions of Heisenberg and Bohr in this regard with those of a multitude of scientists stemming as far back as antiquity. The poet-physicist Goethe is one

66. *Across the Frontiers*, p. 123; cf. 124–135.

67. *Ibid.*, p. 124, 141.

68. *Philosophical Problems of Nuclear Science*, pp. 84-85.

69. Cf. especially *Physics and Philosophy*, pp. 55-56, 75, 200–202; *Across the Frontiers*, p. 120; BOHR, *op. cit.*, vol. 1, p. 67; vol. 2, p. 59; A.N. WHITEHEAD, *The Aims of Education*, N.Y., New American Library, 1952, p. 110.

70. *Physics and Philosophy*, p. 200.

thinker among many in a long tradition of physicists who directed their pursuits more towards the qualitative aspects of things than the quantitative. In this century and the last, the biological sciences have been the principal realm wherein this tradition thrived. The evolutionists such as Lemark, Darwin, Spencer and Huxley, together with the naturalists Fabre, Von Frisch and the ethologists Lorenz, Tinbergen are all cases in point. In the years before Goethe, several non-mathematical researches were conducted in physics, most notably by Gilbert and Boyle.[71] All of these scientists were similar in approach to a large number of medieval scholastics, including Roger Bacon, Albertus Magnus, and his student Thomas Aquinas.

The bulk of the scholastic physicists were Aristotelians, however, and it is properly with Aristotle that the tradition begins. In his *Physics* the Stagirite clearly distinguishes natural philosophy from mathematics [72], yet all the while admits the preponderance of quantity in things and hence the possibility of successfully applying mathematics to nature. Of equal importance is his emphasis on the role of common experience. Here he anticipates Heisenberg's doctrine of "ordinary language" in pointing out that reflection on common experience, as represented by common concepts and signified by common terms, is the necessary starting point of natural science.

Thus the modifications of the 17th century "scientific metaphysics" made by physicists of this century definitely disclose a unanimity of thought amongst scientists of all ages regarding the possibility of a mathematical physics. That which the non-mathematical current has in common with the mathematical explains in a general way why the natural world can be studied mathematically with such great success, and why, correspondingly, an exclusively mathematical physics is deficient. This common ground was ignored with the advent of the "scientific revolution", and certainly denied in the outlook of the last century; nonetheless, contrary to Burtt's interpretation, the prestigious scientists never replaced it with a mathematization of nature.

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By way of summary, two general conclusions arise from our analysis. First, according to the physicists themselves, it is not necessary to explain the possibility and successes of a mathematical physics through any kind of reduction of nature to mathematics. All that needs be said is that nature *has* quantity, but is not quantity in essence. Now to so argue is advantageous, for we not only justify the efficacious application of mathematics to nature, but in addition, avoid the adverse consequences of an essentially quantified world enumerated at the outset.

The above conclusion, while being the chief point intended in this article, is nonetheless followed by another of even greater import. If it is true to say that

71. Cf. BURTT, *op. cit.*, pp. 156–160, and above pp. 56-57.

72. *Physics* II, ch. 2.

quantity is not "all there is" to the physical world, nor even what constitutes its essence, then a qualitative study of nature becomes just as important as a mathematical or quantitative one. Such an approach was conducted by Aristotle in his *Physics*, Gilbert in his *De Magnate* and Goethe in his *Zur Farbenlehre*, and as Heisenberg puts it, mathematical physics is obligated to make room for these kinds of endeavours, for only in virtue of them will physical science ever achieve a knowledge of nature in her totality.