What the Humean cannot say about entanglement

Neil Dewar

January 31, 2016

1 Introduction

There has recently been debate in the literature over whether the metaphysical doctrine popularly known as *Humean supervenience* can be reconciled—in whole or in part—with certain empirical facts about quantum entanglement. In this paper, I undertake a critical analysis of Humean efforts to effect such a reconciliation. I begin with a discussion of the relationship between Humeanism and quantum mechanics; I suggest that there are some difficulties even when considering single-particle quantum mechanics, but agree that the real problems come when considering many-particle quantum mechanics. I then review the available strategies for overcoming this problem, and find them wanting.

2 Single-particle quantum mechanics

Let's begin by reviewing the general quantum formalism. At least at the level of textbook formalism, any quantum system is represented by a *Hilbert space* $\mathcal{H}$: a (possibly infinite-dimensional) complex vector space. A point (a vector, or *ket*) $|\psi\rangle$ in $\mathcal{H}$ represents possible instantaneous states of the system. Taking time to be represented by a one-dimensional space $T \cong \mathbb{R}$, a history of such a system is represented by a function $|\psi(t)\rangle : T \to \mathcal{H}$, associating to each time the state of the system at that time.

The facts about what particular *kind* of system we have are coded up in the structure of the system's *observables*: a collection of self-adjoint operators on the Hilbert space,

$\mathcal{H}$. A given observable $\hat{O}$ is canonically associated with a collection $\{|\chi_i\rangle\}_{i\in I}$ (where I is some index set) of *eigenkets*: points in $\mathcal{H}$ on which $\hat{O}$ acts as

$$\hat{O}\,|i\rangle = o_i\,|\chi_i\rangle \tag{1}$$

where o_i is referred to as the *eigenvalue* of $|\chi_i\rangle$ (with respect to $\hat{O}$). Each observable is associated to a certain measurable quantity: if the quantity associated to $\hat{O}$ is measured, then each eigenvalue o_i is a possible value for the result to take. If this measurement is performed on a system in state $|\psi\rangle$, then the probability of obtaining o_i as the outcome of the measurement is

$$|\,\langle\chi_i|\psi\rangle\,|^2 \tag{2}$$

(Note that in particular, the probability of obtaining outcome o_i upon measuring $\hat{O}$ on $|\chi_i\rangle$ is 1.)

The most important such observable is probably the *Hamiltonian*, $\hat{H}$, whose associated measurable quantity is that of the total energy. The reason for its importance lies in its connection to the dynamics: a history $|\psi(t)\rangle$ of the system is dynamically allowed iff it satisfies Schrödinger's equation,

$$i\hbar\frac{d\,|\psi(t)\rangle}{dt} = \hat{H}\,|\psi(t)\rangle \tag{3}$$

Thus, the Hamiltonian of a system is what determines its evolution over time.

The other useful thing about observables is that the spectrum of eigenkets $\{|\chi_i\rangle\}_{i\in I}$ often forms a *basis* of Hilbert space: a linearly independent set of vectors which collectively span the space. This means that we can represent $|\psi\rangle$ by an indexed set of complex numbers $\{\psi^i\}_{i\in I}$, with the relationship to $|\psi\rangle$ being given by

$$|\psi\rangle = \sum_{i\in I}\psi^i\,|\chi^i\rangle \tag{4}$$

An indexed set of numbers $\{\psi^i\}_{i\in I}$ is essentially the same thing as a function $\psi(i) : I \to \mathbb{C}$, by the correspondence $\psi(i) = \psi^i$. Finally, since each point of $\mathcal{H}$ can be represented by such a function, then if we have a function $|\psi(t)\rangle : T \to \mathcal{H}$, we can represent that by a function $\psi(t,i) : T \times I \to \mathbb{C}$.

Let us turn to the particular case of a single (spinless) particle. This particle's Hilbert space carries a *position operator* $\hat{X}$: that is, an operator whose associated measurable quantity is the particle's position in space. Let $\{|\delta^i\rangle\}_{i\in I}$ be the eigenkets of $\hat{X}$; the possible eigenvalues form a space $X \cong \mathbb{R}^3$. Now it turns out that in this case, there is a one-to-one correspondence between I and X; that is, there is at most one eigenket with a given eigenvalue $x \in X$. For this reason, we standardly use X itself as the index set, writing the eigenkets as $\{|\delta^x\rangle\}_{x\in X}$. All the information about the vector $|\psi\rangle$ can therefore be represented using a function $\psi(x) : X \to \mathbb{C}$, with the property

$$|\psi\rangle = \sum_{x\in X} \psi(x)\, |\delta^x\rangle \tag{5}$$

We can then represent a trajectory $|\psi(t)\rangle$ through $\mathcal{H}$ by a function $\psi(t, x) : T \times X \to \mathbb{C}$. This function is generally known as the *wavefunction* of the particle.

Thus speaketh the quantum textbooks. However, the question of whether the above is adequate as a "description of a single particle" is rather philosophically vexed. Plausibly, knowing the above is enough to engage in experiments with such a particle; but if we are to read off metaphysical lessons from a description, then we want to be confident that that description goes beyond mere instrumentalist instructions. In particular, the description above was rather weaselly on the connection between the dynamics and experiment: in what sense are measurable quantities "associated to" observables, and why do measurements of such quantities yield outcomes with probabilities given by (2)?

Fully answering such questions would require that we confront the notorious *measurement problem* of quantum mechanics.[1] But it is not only controversial how one ought best to solve the measurement problem; it is even controversial what solving it requires. On *primitive-ontology* approaches,[2] the textbook formalism requires supplementation. We must specify an ontology for quantum mechanics in terms of which experimental outcomes can be couched; solving the measurement problem is a matter of demonstrating that quantum theory will predict that the primitive ontology be arranged in a manner that matches our observations. On other approaches to the measurement problem, the challenge is how to "read off" an ontology from the textbook

[1]It is somewhat contested how best to think of the measurement problem. [Albert, 1994] characterises the problem as being how to reconcile the application of the collapse postulate in measurement contexts with the application of the standard unitary dynamics in non-measurement contexts; [Wallace, 2012a] characterises it as the fact that standard application of quantum mechanics does not seem to consistently think of the state space of quantum mechanics as representing either a space of physical states, nor as representing a space of probability distributions—but, rather, shifting between the two as we move from "microscopic" to "macroscopic" contexts.

[2][Goldstein, 1998]; [Maudlin, 2007c]; [Allori et al., 2008]

dynamics (Everett),[3] or perhaps some supplemented dynamics (bare GRW):[4] that is, rather than taking an ontology as understood and showing how it meshes with the dynamics, we seek to extract an understanding of the dynamics in ontological terms, and show how *that* ontology comes to be arranged in a manner consonant with our observations.

Clearly, such issues are unlikely to be settled anytime soon. Nor, however, can we simply ignore them for the purposes of this paper: as just discussed, the position one takes on these issues bears on how one should go about answering questions about quantum mechanics' metaphysical structure (such as whether it is consistent with Humean supervenience). So instead, I simply stipulate that I will suppose we are considering a primitive-ontology solution to the measurement problem: more specifically, that we have signed up to Bohmian mechanics.[5] The reason I do so is not because Bohmian mechanics is my preferred solution to the measurement problem (it's not, as it happens). Rather, I do so because several of the proposals for reconciling quantum mechanics with Humean supervenience make use of some form of primitive-ontology framework; and Bohmian mechanics is not only one of the best-developed versions of such a framework, it is also one which (as we shall see) looks to be the most hospitable to the advocate of Humean supervenience. In other words, given that my aim in this essay is to argue *against* the compatibility of quantum mechanics with Humean supervenience, it seems that going with a Bohmian framework is the most dialectically generous way to proceed. As a result, this essay will be mostly about the metaphysics of entanglement *in Bohmian mechanics*, rather than in general—although at various points, I'll contrast the Bohmian picture with alternatives.

So: what is this framework? The postulated primitive ontology, for Bohmians, consists of pointlike entities; I'll refer to the entities themselves as "corpuscles", reserving "particle" to refer to a combined corpuscle-plus-wavefunction system. (This is just meant as terminological stipulation—in particular, no lessons about the reality or metaphysical nature of the wavefunction ought to be inferred.) The crucial thing about such corpuscles is that at all moments of time, they have quite definite positions and velocities. So for a single (spinless) particle, we supplement the formalism above with an extra X-valued variable Q, representing the position of the corpuscle. A history of the corpuscle is represented by a function $Q(t) : T \to X$. Dynamically allowed

[3] [Wallace, 2012b]; [Saunders et al., 2010]

[4] [Ghirardi et al., 1986]

[5] For an introduction to Bohmian mechanics (especially the form of it appealed to here), see [Dürr et al., 1992].

histories of the corpuscle are those which satisfy the *guidance equation*

$$\frac{dQ}{dt} = \frac{\hbar}{m}\mathrm{Im}\left(\frac{\nabla\psi}{\psi}(Q)\right) \tag{6}$$

where m is the mass of the particle and ψ is the wavefunction (as defined by (5)). The guidance equation (6), together with the Schrödinger equation (3), constitute the Bohmian dynamics.[6]

3 Humean supervenience

How does all this square with the picture of the world advanced by Humean supervenience? That picture is described by David Lewis as follows:

> Humean Supervenience [...] says that in a world like ours, the fundamental relations are exactly the spatiotemporal relations: distance relations, both spacelike and timelike, and perhaps also occupancy relations between point-sized things and spacetime points. And it says that in a world like ours, the fundamental properties are local qualities: perfectly natural intrinsic properties of points, or of point-sized occupants of points. Therefore it says that all else supervenes on the spatiotemporal arrangement of local qualities throughout all of history, past and present and future.[7]

So prima facie, at least, if we want to show that a Bohmian particle is in accordance with Humean supervenience, we need to show that the above formal apparatus can be understood as describing a distribution of perfectly natural instrinsic properties over spacetime points, or over the point-sized occupants of spacetime points. [Maudlin, 2007b] calls this the doctrine of *Separability*: the claim that "the complete physical state of the world is determined by (supervenes on) the intrinsic physical state of each spacetime point (or each pointlike object) and the spatio-temporal relations between those points."[8] One option is to take this talk of the "quantum state" seriously, as the state *of* the Bohmian corpuscle—i.e., as representing a certain kind

[6]This is all working in the case where we are dealing with a single particle in isolation. The manner in which Bohmian dynamics for a single particle "emerge" from the Bohmian dynamics for an aggregate of particles—more generally, the manner in which Bohmian dynamics for a subsystem emerge from the dynamics for the subsystem together with its environment—is a topic we shall get to later in this essay.

[7][Lewis, 1994, p. 474]

[8][Maudlin, 2007b, p. 51]

of property for that corpuscle. This suggests interpreting the points of $\mathcal{H}$ as being the determinates of a determinable property, with the corpuscle as the bearer of this property. This looks pretty good: the corpuscles are indeed "point-sized occupants of spacetime points"; so this construal of things looks to be Humeanistically hygienic.[9]

It is worth noting the work that the commitment to Bohmianism is doing here. Had we not decided to go with Bohmianism, then this way of being a quantum Humean would be rather more strained; attaining Separability by interpreting the quantum state as the state *of* something will only be possible for those who have an appropriately point-like something with which to carry this out. Note that this even rules out other forms of primitive ontology, such as GRWm or GRWf. If the particle is associated, at each time, with a mass density or a distribution of flashes (rather than a Bohmian corpuscle), then it is not a point-sized occupant of one spatial point at a time: it is either a region-sized occupant, or else a point-sized occupant of multiple spatial points at each time (and in the case of flashes, sometimes an occupant of no point). If there is no primitive ontology specified, on the other hand, then the only thing around to be the bearer of the quantum state is the quantum particle itself (conceived of thinly, as whatever it is that bears the quantum state). Presumably, the occupancy-facts for such a thing are just given by the position-basis representation of its quantum state. After all, each position eigenket admits of a natural interpretation as representing a state of definite occupancy: a state, that is, in which the particle bears the occupation relation to exactly one spacetime point. A superposition of such eigenkets may therefore be interpreted as representing a state in which the particle bears the occupancy relation to multiple spacetime points; and, moreover, one in which the occupancy relation is a determinable relation, whose determinates have the structure of the complex numbers.[10] (This seems more appropriate than describing it as one in which the particle occupies different spacetime points to "different degrees": that would seem to be apt only if the determinates had the structure of the positive real numbers.) So again, the picture obtained seems distinctly unHumean, involving an object which is multiply (and complexly, in both senses) located.

[9] There are some subtleties about whether the properties of a Bohmian corpuscle really can be thought of as localised to whatever point the corpuscle occupies [Brown et al., 1996]; in the interests of charity, I put these concerns to one side.

[10] Actually, things will be a bit more complicated than that, since some of the structure of the complex numbers is surplus (as a result of the phase-equivalence of kets). So the real structure they exhibit will be like that of the complex numbers, but without data concerning absolute (as opposed to relative) phase.

Nevertheless: if one is happy with Bohmianism, then interpreting the quantum state as representing a (perfectly natural and intrinsic) property of the corpuscle is one way of making single-particle quantum mechanics consistent with Separability. The other option is to interpret the wavefunction $\psi(t, x)$ as describing (perfectly natural and intrinsic) properties of spacetime points. Indeed, this is arguably the received view on how Humeans should understand the quantum mechanics of particles, as expressed in the following forceful statement of this position:

> The sorts of physical objects that wave functions *are*, on this way of thinking, are (plainly) *fields*—which is to say that they are the sorts of objects whose states one specifies by specifying the values of some set of numbers at every point in the space where they live, the sorts of objects whose states one specifies (in *this*) case by specifying the values of *two* numbers (one of which is usually referred to as an *amplitude*, and the other as a *phase*) at every point [...].[11]
>
> The values of the amplitude and the phase are thought of (as with all fields) as intrinsic properties of the points in the configuration space with which they are associated.[12]

If a Humean does decide to take this path, then they must regard the wavefunction as privileged over other representations of the quantum state—indeed, as privileged over the quantum state itself. The quantum state can be represented as a function $\phi(p)$ of momentum-space, for example, but the advocate of interpreting the wavefunction as a field will not (I presume) want to interpret the momentum function as a field: for that would mean committing to the physical existence of a "space of momenta", of which $\phi(p)$ predicates intrinsic properties. So on this view, there is an asymmetry in how perspicuously the position and momentum wavefunctions represent the metaphysical situation.

However, this doesn't seem like an especial cost to the Humean that we are envisaging: since she is also (we have supposed) a Bohmian, the privileging of position seems perfectly natural. On a technical level, the position representation of the quantum state plays a privileged role in the guidance equation; and on a conceptual level, position-space is privileged over (say) momentum-space by virtue of being the space

[11] This sentence finishes with "in the universe's so-called *configuration* space". This is because Albert is here describing the case for the whole universe, rather than a single particle; we will return to this below.

[12] [Albert, 1996, p. 278]

in which the primitive ontology lives. So once again, Bohmianism seems to be a natural partner for the quantum Humean. That said, it seems that this partnership would work equally well for any primitive ontology: provided that a primitive ontology is required to be "a description of matter in space and time",[13] then the commitment to primitive ontology will already mean privileging the position wavefunction—and so interpreting the wavefunction as a field (property of spacetime points) will not represent a further commitment. However, natural as this commitment may be for primitive ontologists, it bears emphasising that adopting it puts the philosopher of quantum mechanics at odds with standard practice in physics, in which the more general and flexible formalism of kets is primary, and wavefunctions are a means of representing kets. We'll see some consequences of this below.

4 Entanglement

Now that we have a decent grip on what is going on for single quantum particles, let us turn our attention to the issues posed by multiple particles—when those particles are allowed to interact, and thereby become *entangled*. So: suppose (to begin with) that we have two quantum systems, labelled 1 and 2 respectively, and we want to represent the available states for the *pair* of systems together—given the available states for system 1, and the available states for system 2. Now, we know that the states for system 1 make a Hilbert space $\mathcal{H}_1$, and that the states for system 2 make a Hilbert space $\mathcal{H}_2$; and we should expect that the states for the joint system form a Hilbert space as well, which we can denote $\mathcal{H}_{12}$. It is well-known how $\mathcal{H}_{12}$ is related to $\mathcal{H}_1$ and $\mathcal{H}_2$: it's the *tensor product*,

$$\mathcal{H}_{12} = \mathcal{H}_1 \otimes \mathcal{H}_2 \tag{7}$$

Defining the tensor product $U \otimes V$ of a pair of vector spaces U and V is a little tricky, but for our purposes all we need to know about $U \otimes V$ is the following: that it is a vector space, equipped with operations of addition and scalar multiplication; that every pair of vectors $(u, v) \in U \times V$ corresponds to some element of $u \otimes v \in U \otimes V$ (though not vice versa, and not uniquely); and that the vector-space operations on U

[13][Allori et al., 2008, p. 370]

and V separately interact with those of $U \otimes V$ according to

$$(u + u') \otimes (v + v') = (u \otimes v) + (u' \otimes v) + (u \otimes v') + (u' \otimes v') \tag{8a}$$

$$(\alpha u) \otimes v = \alpha(u \otimes v) = u \otimes (\alpha v) \tag{8b}$$

The fact that we can build a vector space which manifests these properties is non-trivial, but not something we need worry about; so long as the reader is willing to take it on faith that this can be done, we can proceed. Note that the rules (8) entail the imperfect correspondence between $U \times V$ and $U \otimes V$. The addition rule means that $(u \otimes v) + (u' \otimes v')$ will not typically correspond to any individual pair in $U \times V$; and the scalar multiplication rule means that $u \otimes v$ corresponds to the same element of $U \otimes V$ as $(\lambda u) \otimes (\frac{1}{\lambda} v)$ (for any scalar λ).

The phenomenon of *entanglement* arises from precisely the fact that not every element of a tensor product space corresponds to some element of the Cartesian product: that is, there are elements of $\mathcal{H}_1 \otimes \mathcal{H}_2$ which do *not* correspond to any pair $(|\psi\rangle_1, |\phi\rangle_2) \in \mathcal{H}_1 \times \mathcal{H}_2$. Consider, for example,

$$\frac{1}{\sqrt{2}} \left(|\psi\rangle_1 \otimes |\phi\rangle_2 + |\psi'\rangle_1 \otimes |\phi'\rangle_2\right) \tag{9}$$

where $\langle\psi|\psi'\rangle_1 = 0 = \langle\phi|\phi'\rangle_2$. The sum (9) cannot be factorised into a single product, and so we cannot treat it as simply arising from some assignment of states to systems 1 and 2 individually.

That said, there are mathematical objects which we could characterise as (at least partially) representing the states of systems 1 and 2. In particular, the *density operators* associated to systems 1 and 2 remain well-defined even under entanglement; the density operator of either system suffices to predict the (probabilities of) outcomes of measurements on that system. However, that pair of density operators do not suffice to uniquely fix the density operator for the joint system (i.e., there are distinct density operators for the joint system which give rise to the same pair of density operators for the subsystems). As a result, the subsystem density operators do not contain enough information to predict the outcomes of *joint* measurements, i.e., do not predict the correlations between the outcomes of measurements on the first system with the outcomes of measurements on the second. Moreover, one cannot give a dynamics for how the pair of density operators will evolve.

What does this entail for the representations of the quantum state? Let us suppose that both systems are particles; then, to find a basis of $\mathcal{H}_{12}$, we can use the operator $\hat{X} \otimes \hat{X}$, just as we used $\hat{X}$ for the individual Hilbert spaces.[14] What we find is that eigenvectors of $\hat{X} \otimes \hat{X}$ in $\mathcal{H}_{12}$ are all of the form

$$|\delta_1^{x_1}\rangle \otimes |\delta_2^{x_2}\rangle \tag{10}$$

where $\{|\delta_i^{x_i}\rangle\}_{x_i \in X}$ are the eigenvectors of $\hat{X}$ in $\mathcal{H}_i$, for $i = 1, 2$. Hence, any element $|\Psi\rangle_{12}$ of $\mathcal{H}_{12}$ may be represented in the form

$$|\Psi\rangle_{12} = \sum_{(x_1,x_2) \in X \times X} \Psi(x_1, x_2)\, |\delta_1^{x_1}\rangle \otimes |\delta_2^{x_2}\rangle \tag{11}$$

where the *joint wavefunction* $\Psi(x_1, x_2)$ is a map $X^2 \to \mathbb{C}$, where $X^2 := X \times X$. As above, a trajectory of joint states can be represented by a function $\Psi(t, x_1, x_2) : T \times X^2 \to \mathbb{C}$. In other words, the natural index set to use for the position basis of a pair of particles is not X itself, but X^2—what is often called the *2-particle configuration space*.[15] If $|\Psi\rangle_{12}$ is a product $|\psi\rangle_1 \otimes |\phi\rangle_2$, then the joint wavefunction $\Psi(x_1, x_2) = \psi(x_1)\phi(x_2)$; if we are dealing with an entangled state, however, then this will not be the case.

Of course, entanglement between pairs of systems is only the beginning—in general, any number N of quantum particles could be entangled with one another. The states of such an N-fold entangled system would be represented as elements of the Hilbert space

$$\boldsymbol{\mathcal{H}} := \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N \tag{12}$$

where $\mathcal{H}_i$ is the Hilbert space of the ith particle; and most elements of this Hilbert space correspond to no N-tuple of individual states in $\mathcal{H}_1 \times \cdots \mathcal{H}_N$. And if an element of this Hilbert space is represented in the position basis, it gets represented by

[14]The tensor product $A \otimes B$ of two linear operators $\hat{A} : \mathcal{H}_1 \to \mathcal{H}_1$ and $\hat{B} : \mathcal{H}_2 \to \mathcal{H}_2$ is the (unique) linear operator which acts on any product ket $|\psi\rangle_1 \otimes |\phi\rangle_2$ as

$$A \otimes B(|\psi\rangle_1 \otimes |\phi\rangle_2) = A\,|\psi\rangle_1 \otimes B\,|\phi\rangle_2$$

[15]There is a bit of a tradition, in the literature on the metaphysics of quantum mechanics, of handwringing about whether the textbook account can make sense of configuration space (given that there is nothing, in the formalism, for points in configuration space to be the configurations *of*). I find this concern almost totally unpersuasive: for the textbook account, points of (2-particle) configuration space are understood as corresponding to pairs of points in (normal) space; this conception is entirely clear, and entirely available to the textbook account. Sure, the *name* configuration space isn't ideal—but who cares? The fact that Star Destroyers can't destroy stars hardly makes the concept unintelligible.

a wavefunction $\mathbf{\Psi} : X^N \to \mathbb{C}$, where X^N is the N-particle configuration space:

$$X^N := \underbrace{X \times \cdots X}_{N \text{ times}} \tag{13}$$

It will be worthwhile making a few brief remarks about the Bohmian perspective on entanglement. If we have a collective of N particles, then in addition to the N-particle joint wavefunction $\mathbf{\Psi}(\mathbf{x})$, we have the locations of all N Bohmian corpuscles: these locations are represented by an N-tuple $(Q_1, \ldots, Q_N)$ of points in X, or (equivalently) by a single point $\mathbf{Q}$ in X^N. The evolution of $\mathbf{Q}$ is given by the guidance equation

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \left(\frac{\nabla_i \mathbf{\Psi}}{\mathbf{\Psi}}(\mathbf{Q}) \right) \tag{14}$$

where m_i is the mass of the ith particle, and ∇_i denotes the position operator associated with the ith coordinate of X^N.

In Bohmian mechanics, there is an another way of representing the state of a subsystem: the *conditional* wavefunction. If we select (say) the first $M < N$ particles as a subsystem, then the *conditional wavefunction* of that subsystem (relative to the remaining $N - M$ particles) is given by

$$\mathbf{\Psi}_{1 \ldots M}(x_1, \ldots, x_M) := \mathbf{\Psi}(x_1, \ldots, x_M, Q_{M+1}, \ldots, Q_N) \tag{15}$$

That is, the conditional wavefunction of the subsystem is obtained by "saturating" the joint wavefunction with the actual locations of the remaining particles.

The importance of the conditional wavefunction is as follows: from the N-particle guidance equation (14), one can immediately see that the Bohmian configuration of the subsystem

$$\mathbf{Q}_{1 \ldots M} := (Q_1, \ldots, Q_M) \tag{16}$$

obeys (14) with respect to the conditional wavefunction—for the simple reason that $\mathbf{\Psi}(\mathbf{Q}) = \mathbf{\Psi}_{1 \ldots M}(\mathbf{Q}_{1 \ldots M})$. If the subsystem is sufficiently decoupled from its environment, then the conditional wavefunction will also abide by Schrödinger's equation; if there is interaction, however, then the conditional wavefunction will not evolve in this unitary fashion. It is in this sense that the Bohmian picture incorporates "effective collapse": although the wavefunction $\mathbf{\Psi}$ of the entire system (in the limiting case, the entire universe) evolves unitarily, the conditional wavefunctions associated to subsystems will not (and will, indeed, collapse in the manner prescribed by quantum measurement theory).

It is, however, important to note that although the conditional wavefunctions of the subsystems can be recovered from the "universal wavefunction" $\mathbf{\Psi}$, the reverse is not true: the conditional wavefunctions associated to subsystems underdetermine the joint wavefunction. For example, in the two-particle case, one can easily have a distinct pair of joint wavefunctions $\Psi(x_1, x_2)$ and $\Phi(x_1, x_2)$ such that $\Psi(x_1, Q_2) = \Phi(x_1, Q_2)$ and $\Psi(Q_1, x_2) = \Phi(Q_1, x_2)$: that agreement only requires that they coincide on certain surfaces within configuration space. Moreover, in the case where the subsystem and the environment are coupled to one another, it is not just that the conditional wavefunction does not evolve according to the Schrödinger dynamics—in general, there will not be any autonomous dynamics for the conditional wavefunction at all. (Note the parallels with the density operators.)

Let us now briefly rehearse how this phenomenon makes trouble for our quantum Humean, who wishes to give a metaphysical account of N quantum particles. First, suppose that they have followed the first strategy for making Humean sense of the quantum mechanics of a single particle: that of interpreting the quantum state as expressing properties of the corpuscle. The basic problem is then that as the above has indicated, there is (in general) no way of assigning states to each of the N particles separately and individually, in such a way that their collective joint state can be recovered. So it seems that the only way to interpret the joint state is as expressing a property that belongs, essentially and irreducibly, to the *collection* of corpuscles. But a collection of point-like things is not itself point-like—so, contra Separability, not all properties are localised to points. Alternatively, suppose that they have interpreted the wavefunction as a field. Then the problem is that (in general) there is no way of reducing the joint wavefunction (a function of X^N) to a pair of single-particle wavefunctions (functions on X). (It is in this sense that the wavefunction of a multi-particle system "lives on configuration space".) So, at any time, the wavefunction can only be thought of as expressing a property of N-tuples of spatial points.[16] But an N-tuple of spatial points is not itself a spatial point. So again, Separability is violated.

5 Modifying Humean supervenience

I now turn to how the Humean could seek to respond to the above argument. We have seen that the problem arises if Humean supervenience (as stated by Lewis) is

[16] cf [Belot, 2011].

combined with an interpretation of quantum mechanics in which the quantum state represents a component of the supervenience basis. In this section, I look at responses which deviate from the exact Lewisian template for articulating the thesis of Humean supervenience; in the next, I consider those which seek instead to expunge the quantum state from the supervenience basis.

The first response to discuss is arguably the best-known: that of taking the space with respect to which separability is measured to be $3N$-dimensional configuration space, rather than 3-dimensional configuration space. Here is one influential statement of that position, due to Barry Loewer (although the position is also strongly associated with David Albert):

> More recently, Lewis has accepted that HS needs to be reformulated to accommodate quantum nonlocality. Here is a suggestion for how to do so in the context of Bohm's theory. The quantum state of an n-particle system is a field in $3n$ dimension [*sic*] configuration space where the value of the field at a point in configuration space is the amplitude of the quantum state at that point. These field values can be thought of as intrinsic properties of points. The ontology of Bohm's theory also includes a "world particle" whose location and motion in configuration space determines the locations and motions of ordinary material particles in three-dimensional space, and the locations and motions of these particles determine the manifest world.[17]

This proposal is cast in terms of taking the wavefunction as a field, but it is clear enough how to modify Loewer's proposal for one who wishes to take the quantum state to be a property: we take as our ontology a single corpuscle ("world particle") moving in a $3N$-dimensional space, and interpret the quantum state (element of $\boldsymbol{\mathcal{H}} := \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N$) as representing a property of this corpuscle. By doing so, we get the desired result, that all the properties knocking about are properties of pointlike entities.

Before critiquing this proposal, I want to argue that this idea—at least in the form just canvassed—is functionally very similar to a seemingly quite different proposal: Darby's suggestion that we should expand the Humean supervenience basis, by allowing in some non-spatiotemporal relations.[18] Darby has his own proposal for how

[17][Loewer, 1996, p. 104]
[18][Darby, 2012]

to do this so as to see off the threat to separability; here is an alternative.[19] First, observe that a relation's holding amongst certain individuals is, for all intents and purposes, the same as a property holding of the ordered sequence of those individuals; or, more carefully, some bit of theoretical gubbins can be interpreted as representing a relation's holding amongst some individuals if and only if it may be interpreted as representing a property holding of the ordered sequence of those individuals. But the elements of $\boldsymbol{\mathcal{H}}$ may be interpreted as properties of an ordered sequence of N corpuscles; hence, they may equally well be interpreted as N-ary relations holding amongst those corpuscles. The additional relations that need to be added to the supervenience base, then, are simply those which are not reducible to properties of the individual corpuscles. These are simply those relations represented by elements of $\boldsymbol{\mathcal{H}}$ which correspond to no element of $\mathcal{H}_1 \times \cdots \times \mathcal{H}_N$.

Having said this much, we can see why the Darby and (modified) Loewer/Albert proposals amount to a very similar move. Both recognise that the Humean base, at least as classically understood, is inadequate as a supervenience base for quantum states. Both propose a way to alter or enrich that base so as to make it adequate; and both do so by allowing that the base be represented by the assignment of an element of $\boldsymbol{\mathcal{H}}$ to the collective of N corpuscles. The only difference is in how they *interpret* this assignment: as representing the possession of a property by the world-corpuscle, which is (in some sense) in correspondence with the collective on N corpuscles; or as representing the possession of a relation by the N corpuscles collectively. Note that we could also modify Darby's proposal to make it akin to the unmodified Loewer/Albert proposal, by taking the wavefunction as privileged: it would then be glossed as representing a (determinable) relation that holds of N-tuples of points of space collectively, rather than as representing a (determinable) property of points of configuration space. But again, the differences between these glosses would not be worth losing much sleep over.

Thus, we have a set of closely related proposals—the primary difference among them being that canvassed in section 3, between views which interpret the quantum state as a property of a quantum system, and views which interpret the wavefunction as a field. All of them propose that we relax our commitment to Separability, by let-

[19] The proposal given here is somewhat simpler than Darby's, as a result of the fact that I have been flat-footedly taking quantum states to correspond to properties. Darby has to engage in some slightly more careful manoeuvring, since he takes the relationship between quantum states and properties to be more delicate: properties, for Darby, are taken to correspond to eigenvectors of projectors on Hilbert space, rather than arbitrary elements of the space.

ting the supervenience basis contain metaphysical structure that cannot be localised to individual spacetime points.[20] But all of these proposals then face the question of whether this modification moves too far away from the purposes that Humean supervenience is supposed to serve.

To see why they might be thought to do so, note that both of these strategies are troublingly easy to generalise. Prima facie, the kind of world that violates Separability is one in which there are necessary connections between distinct existents: that is, in which there are fundamental and irreducible *relations* between pointlike things (irreducible in the sense that they do not supervene on any properties of the parts). But any such relations—of whatever arity—can be redescribed as properties of configurations of the pointlike entities, and thence as properties of entities which are pointlike with respect to the configuration space for those entities. Similarly, but more directly, if we are permitted to add relations into the supervenience basis, then clearly an irreducibly relational world of this kind can be made consistent with Humean supervenience. So what kind of metaphysics is still being ruled out by such liberalised versions of the doctrine?

The natural response is that Humeanism still has a nontrivial content, since it maintains that the modal facts of the world supervene upon the non-modal facts—even if we allow those non-modal facts to be relational in character, rather than pointwise intrinsic. I take this to be guiding thought behind Darby's discussion of a "spirit" of Humean supervenience:

> So, the letter of Lewis's Humean supervenience can be violated along various dimensions, and to varying degrees. It also seems, though this is less obvious, that some dimensions are more significant than others. [...] This line of thought is what suggests a spirit of Humean supervenience. Corresponding to the 'broadly Humean doctrine' of [Lewis, 1980]—which 'holds that all the facts there are about the world are particular facts, or combinations thereof' (and so differs from [Lewis, 1986, pp. ix-x] in saying nothing further about the nature of the particular facts)—this would allow variation along some dimensions (for example, differences in the nature of

[20] A complication: the Loewer/Albert kind of proposal is often presented as one in which we *maintain* Separability—glossed as the claim that metaphysical structure can be localised to individual points of the "fundamental physical space"—but change our mind about what the fundamental physical space is (by taking that to be configuration space rather than ordinary space). I think it is easier to keep track of what is going on if we take Separability as the requirement of localisability to ordinary space, so that the meaning of the doctrine remains constant.

spacetime and the external relations that unify it), but would be violated by moving in others (the addition of unHumean chancemakers or lawmaking relations between universals).[21]

This seems quite correct. At this point, however, we have rather lost track of the dialectic. After all, the discussions of whether Humean supervenience is consistent with quantum mechanics *were only ever about* the locality requirements bound up in Lewis' assertions regarding "the nature of the particular facts", i.e., the doctrine of Separabilty. Indeed, as remarked earlier, the possibility of factorising Humeanism into two components is present in Maudlin's discussion:

> Although he does not remark it, Lewis's Humeanism comprises two logically independent doctrines. The first, which we may call Separability, claims that all fundamental properties are local properties and that spatiotemporal relations are the only fundamental external physical relations. To be precise:
>
> > Doctrine 1 (Separability): The complete physical state of the world is determined by (supervenes on) the intrinsic physical state of each spacetime point (or each pointlike object) and the spatiotemporal relations between those points.
>
> [. . .] The doctrine of Separability concerns only how the total physical state of the universe depends on the physical state of localized bits of the universe. The second component of Lewis's Humeanism takes care of everything else:
>
> > Doctrine 2 (Physical Statism): All facts about a world, including modal and nomological facts, are determined by its total physical state.[22]

Thus, insofar as we were worried about a tension between Humeanism and quantum mechanics, that tension *only ever* pertained to Separability,[23] not to Physical Statism. So to "respond" to this tension by relinquishing Separability is simply to concede the ground that was being challenged.

[21] [Darby, 2012, p. 783]
[22] [Maudlin, 2007b, p. 51]
[23] That is, Separability with respect to ordinary space (see footnote 20).

6 Best-system quantum states

So, are there more substantial responses available—responses which do not just relinquish Separability? There are indeed: in particular, let us consider the responses offered by [Miller, 2014] and [Bhogal and Perry, 2015]. To reiterate, the problem is that Separability is in tension with the non-localised nature of the quantum state. So if we want it to be the case that everything in the Humean supervenience basis abides by Separability, we will have to reject the claim that the quantum state is part of that supervenience basis. Note that this strategy is only available if some form of primitive ontology is knocking around, in order that there are some fundamental physical facts available other than the facts about the quantum state. So the proposal is to take these other facts—in this particular case, the facts about the Bohmian trajectories—to comprise the Humean supervenience basis.

The natural next question, then, is what the status of the quantum state is on this picture. The answer that both Miller and Bhogal & Perry provide is that the quantum state—like everything else—supervenes upon the supervenience basis, i.e., upon the motions of the Bohmian corpuscles. We need to be careful, however, about the exact sense in which this supervenience takes place. One might have thought that the supervenience thesis had the following form: given the trajectories of the Bohmian corpuscles, there is a unique wavefunction which could have brought about those trajectories in a dynamically acceptable way. That is, let $\mathbf{\Psi} : T \times X^N \to \mathbb{C}$ be an N-particle wavefunction, and $\mathbf{Q}^N : T \to X^N$ be a trajectory through N-particle configuration space, such that $\mathbf{\Psi}$ and $\mathbf{Q}$ between them solve the Schrödinger equation (for some specific Hamiltonian H) and the Bohmian guidance equation. Then (the claim goes) there is no distinct wavefunction $\mathbf{\Psi}' : T \times X^N \to \mathbb{C}$ such that $\mathbf{\Psi}'$ and $\mathbf{Q}$ jointly solve the Schrödinger equation (with the same Hamiltonian) and the Bohmian guidance equation.

This kind of supervenience is not to be had, at least in general: there are distinct solutions of the Schrödinger equation which generate the same motions for Bohmian particles.[24] Consider a Bohmian particle in a box: that is, a particle with one positional degree of freedom, which is confined to the unit interval $[0, 1]$ (but is otherwise free). Then the energy eigenfunctions of the system are of the form

$$\phi_n(x) = \sin(n\pi x) \tag{17}$$

[24] The below is taken from [Belot, 2011].

for $n = 1, 2, 3, \ldots$. As an eigenfunction, ϕ_n evolves under the Schrödinger equation only into states which are equivalent to ϕ_n (up to phase). But by the guidance equation, $dQ/dt = 0$ if the wavefunction is ϕ_n, or if it is any wavefunction equivalent to ϕ_n . So any pair of such eigenfunctions are associated to the same Bohmian trajectory: namely, that of the particle remaining at rest. The best that can be hoped for is that cases such as this are exceptional; this is plausible, but it is not clear how to go about proving it.

However, for our purposes here the question is moot. For even if a positive answer to this technical question could be found, it is not one which would be desperately useful to the Humean: for what it would show is that in non-exceptional cases, the Bohmian trajectories *together with the laws of Bohmian mechanics* uniquely determine the wavefunction. And of course, the Humean denies that the laws are to be taken as part of the supervenience basis. However, this also suggests a natural thing the Humean might seek to say instead: that the Bohmian trajectories determine *both* the quantum dynamics *and* the wavefunction. This means that the Humean can finesse the technical question above, by arguing that the wavefunction is determined by the same "best-system" method used to generate the laws. That is, the claim need not be that the Bohmian trajectories uniquely fix the quantum dynamics *and* the specific wavefunction involved in those dynamics: or at least, not in the sense of there being just one dynamics-plus-wavefunction package which would deliver those trajectories. Instead, the idea is that of the candidate packages, precisely one will maximise simplicity and strength (under some appropriate weighting); and *this* package is the one which the Humean takes to be the correct characterisation of what's going on.

This strategy is that advocated by both [Miller, 2014] and [Bhogal and Perry, 2015]. One small clarification, before we proceed. It is clearly intended that the Bohmian trajectories are part of the supervenience basis; but do those trajectories *exhaust* the supervenience basis? Or is the distribution over particles of properties such as mass, or charge, or (total) spin[25] also a part of the basis? On the one hand, both [Miller, 2014] and [Bhogal and Perry, 2015] sympathetically discuss [Hall, 2009]'s proposal that the Humean should introduce mass and charge as part of the best systematisation, rather than treating them as a component of the supervenience basis. On the other, the more austere the supervenience basis is, the less plausible it is that the full quantum dynamics really will supervene upon it (of which more below). So in the interests of charity, let us suppose that mass, charge etc. are indeed included in the basis.[26]

[25]That is, the spin quantum number of the particles (not the projection of the spin along some axis).

[26]That said, as mentioned above, the imputation of appropriately localised mass, charge etc. to Bohmian particles is not entirely straightforward: again, see [Brown et al., 1996].

So, the picture is as follows. We take as given a collection of Bohmian particles, each bearing a certain mass, charge and spin, and each with a certain trajectory through space over time. In order to specify the best system, we need to then introduce a new piece of theoretical vocabulary: that of the quantum state. [Miller, 2014] and [Bhogal and Perry, 2015] both use the wavefunction $\mathbf{\Psi}$ as the appropriate representative for this state. For the reasons discussed already, the Humean could equally well use a ket $|\mathbf{\Psi}\rangle$ in a Hilbert space equipped with a position operator; to facilitate the dialectic, however, I will also suppose that we are using the wavefunction. The Humean should then claim that the best system for codifying those trajectories is one which asserts the following:[27]

- That $\mathbf{\Psi}$ is a complex-valued field on $T \times X^N$
- That $\mathbf{\Psi}(0, \mathbf{x})$ has such-and-such a value at $\mathbf{x}$, for each $\mathbf{x} \in X^N$
- That $\mathbf{\Psi}$ satisfies the Schrödinger equation
- That the location of each corpuscle, together with $\mathbf{\Psi}$, satisfies the Bohmian guidance equation

Miller does not dwell for long on the fact that we are utilising a best system whose vocabulary is not confined to terms referring to individuals and properties in the supervenience basis,[28] remarking only that "what makes it the case that there is a pilot wave is that the best system description of the physical world speaks in terms of it, and this description speaks in terms of it because it is part of an efficient and effective summary of what is fundamental: the positions of particles in space over time."[29] Bhogal and Perry, on the other hand, recognise this liberalisation as a significant departure from more standard presentations of Humeanism:[30]

[27][Bhogal and Perry, 2015] use a best system which postulates a space Q (with the structure of X^N) and a particle ω moving around within Q (whose location at any time is exactly correlated with the configuration of the N particles); the wavefunction is then postulated as a function assigning a complex number to each point of Q, which then acts on ω via the guidance equation. If Q here is intended to simply be defined as X^N (i.e., as the space consisting of N-tuples of points of X), then I take these systems to be essentially the same. If not—that is, if the idea is to stipulate Q's structure separately and then put it into appropriate correspondence with N-tuples of points of X—then it seems to me that the system outlined here will be considerably simpler, at no cost in strength.

[28]If the wavefunction $\mathbf{\Psi}$ did refer to anything in the basis, then we would be back to something like the Albert/Loewer/Darby strategy rejected above.

[29][Miller, 2014, p. 580]

[30]Albeit one which—as they observe—is prefigured by Hall's discussion of mass and charge, and Lewis' discussion of chance.

> The way we do this is by expanding the language that candidate systems can be formulated in. As before [i.e., in standard Humeanism], systems can use vocabulary that refers to perfectly natural properties (the properties that make up the mosaic)—what we've called the "base language." But in addition to this they can introduce and use any other vocabulary so long as it comes in uninterpreted.[31]

So, we now have a concrete proposal on the table: both the Bohmian dynamics and the wavefunction are to be recovered as components of a best systematisation of some (presumably highly structured) collection of Bohmian trajectories. The question that we should now ask is why we ought to believe that they can be so recovered. The most explicit way to show this would be to first determine some scalar field over the space of all sets of differential equations (which measures the "simplicity" and "strength" of each set of equations); fix on a particular solution Σ to the guidance equation, relative to an arbitrary wavefunction, and consider all those sets of differential equations of which Σ is a solution; and then see whether the guidance equation and Schrödinger equation jointly occupy a maximum of that field, relative to some wavefunction (not necessarily the one with which we began—the Humean is committed only to recovering the guidance equation, not the specific form of the wavefunction). The game would then be to find a solution which does indeed have this property, and which (together with its associated wavefunction) is a plausible candidate to represent the actual evolution of the world.

This is an *insanely* difficult problem. First, we need to overcome the formidable hurdles of finding an appropriate scalar field over sets of differential equations. Second, even given such a measure, it would be extraordinary if the project of finding some solution for which the Bohmian equations are indeed the best system proved to be even remotely mathematically tractable. Third, it is rather opaque what would be involved in showing that that a given wavefunction and Bohmian distribution is "a plausible candidate to represent the actual evolution of the world"; but given that at least a necessary condition would be that the distribution contain an unbelievably large number of particles, the prospects for doing so do not look good.[32]

Now, the quantum Humean might respond that these problems are barriers to any form of Humeanism about laws of nature. We might wonder why this response, even

[31] [Bhogal and Perry, 2015, p. 5]

[32] This is even supposing that we spot the Humean the fiction that non-relativistic quantum mechanics is empirically adequate for representing the actual evolution of the world.

if it were granted, should carry any dialectic force: a mathematically hopeless program is not rescued by being only one aspect of a larger mathematically hopeless program. But as a matter of fact, there are barriers to quantum Humeanism over and above those faced by general Humeanism, brought about by the very liberalisation of the Humean project that was thought to be key: the possibility of introducing new theoretical vocabulary when constructing the best system. This means that the available systems of equations to consider are not limited to just those equations employing only a fixed stock of variables and parameters (i.e., those ranging over the supervenience basis); rather, we must consider any equations whose variables and parameters include that fixed stock. So the hopelessness has, at a minimum, substantially increased in magnitude.

Moreover, there are features specific to the quantum case which undermine a classic Humean response to the above concern: namely, that we need not solve this problem, because the actual practice of science provides us with sufficient evidence that the actual theory preferred by scientists is the best system. Bhogal and Perry suggest such a response:

> This worry, that mere positional facts wouldn't be complicated enough to distinguish something like Bohmian Mechanics as the best system of that world, strikes us as far too pessimistic. One of the key motivating thoughts behind the best system account is that whatever an ideal scientist, if she was fully rational and knew everything about the state of the mosaic, would take to be the best overall theory given the evidence is the best system of that world.
>
> Actual scientists are not ideal reasoners and they do not have access to the entirety of the facts about the mosaic. Of the elements of the mosaic, actual scientists only have direct access to facts about positions. [...]
>
> If we look to actual scientific practice, we see that physicists, even with access to only a tiny slice of the position facts, have a great deal of confidence that the world is quantum mechanical (and consider this position very well confirmed). If this, in the grand scheme of things, meager set of position facts is enough to satisfy non-ideal working scientists, then we see very little reason to be skeptical that the ideal scientist, with access to all the position facts at our Bohmian world, would settle on a Bohmian Mechanical

physical theory.[33]

In other words: non-ideal actual scientists, with access to only a small portion of the facts in the Humean supervenience basis, have in fact come up with Bohmian mechanics as the best systematisation of their data; this provides at least some reason to think that an ideal scientist, who had access to all the facts in the supervenience basis, would come up with Bohmian mechanics as the best systematisation of the full set of facts. However, the application of this idea here is highly dubious.

First, note that the above argument elides the distinction between quantum mechanics and Bohmian mechanics: just as a matter of sociological fact, it is *false* that the theory at which actual scientists have arrived as their best theory is Bohmian mechanics. Nor is this explicable as mere interpretational preference: much of the best experimental evidence for quantum mechanics falls outwith the purview of Bohmian analyses! For one thing, it remains the case that no Bohmian version of quantum field theory has been developed, which cuts off support from the predictive success of high energy physics. More generally, the analysis of radiation in Bohmian mechanics is not straightforward: so even the two-slit experiment, conducted with light, is not readily explicable within Bohmian mechanics. Finally, even with regards to quantum systems that are in principle analysable in Bohmian terms, there are plenty of examples where doing so is highly unnatural. The standard means of analysing a quantum system means characterising its dynamics in terms of whatever degrees of freedom are most apt for the problem at hand; but calculating what the corpuscles are up to requires always working in the position basis. So even if a Bohmian analysis is in principle available, it may well fall beyond any practical capacity of working physicists.

In other words, the antecedent of the argument above—that scientists have come up with Bohmian mechanics as the best systematisation of their data—is not true; what they have come up with is quantum mechanics. Moreover, given that Bohmian mechanics is significantly more limited in its explanatory scope than quantum mechanics, it seems that the best explanation of this sociological fact is that Bohmian mechanics is *not* the best systematisation of the data to hand; or at least, that it is not regarded as such by the scientific community. Of course, this isn't to rule out the possibility that some extension of Bohmian mechanics could close that explanatory gap, and thereby come to be accepted by the scientific community as the best system. The point is just that this remains a *possibility*, not actuality: so the Bohmian Humean cannot appeal to actual scientific practice as evidence that their proposed best system is

[33][Bhogal and Perry, 2015, p. 18]

indeed best.

Second, suppose that we *do* confine our attention to just the kinds of situations that Bohmian mechanics is able to give an account of (e.g. the two-slit experiment with electrons). The claim at hand is that Bohmian mechanics ought to be regarded as the best systematisation of the experimental data for such situations. In support of that claim, one could observe that the Born rule can be derived from Bohmian mechanics (together with the quantum equilibrium hypothesis);[34] this at least makes it plausible that the probability distributions for such experiments are best systematised by Bohmian mechanics. However, this argument also highlights a difficulty with inferring, from this claim, that Bohmian mechanics is the best systematisation of the *actual Bohmian trajectories*. For the experimental data in question consists, as just mentioned, of probability distributions over such trajectories, rather than (an incomplete set of) the trajectories themselves: we never gain direct access to exactly what trajectories are being followed. So even the argument for these cases contains a further assumption, that the most compact description of the actual trajectories is as a probability distribution. The Humean shouldn't be alarmed by the general idea that this might be the case.[35] The problem, rather, is that they can't point to anything concrete to explain why we ought to think that it is the case: in particular, given the empirical inaccessibility of the actual trajectories, scientific practice cannot be invoked to justify the claim that probability distributions best systematise the trajectories (even granting, as we are, the claim that Bohmian mechanics best systematises the probability distributions).

Note that the reason these problems arise is that the quantum Humean is, in one crucial respect, worse off than her classical cousin: the latter could, at least, identify the kind of structure in the supervenience basis (i.e., intrinsic properties of points or pointlike things) with the experimental data that (idealised) science collects, and hence argue that the vast parallel-processor of the scientific enterprise has in fact systematised that data into an optimally simple and strong codification. By doing so, the classical Humean can relieve some of the pressure to make precise the nature of the best systematisation they envisage, or to show that such a thing is even possible, since science itself could be taken as demonstrating a proof of principle. The experimental basis for quantum mechanics, on the other hand, is a poor fit with the supervenience basis of the Bohmian Humean. On the one hand, it is too big: it covers many more situations than those to which Bohmianism is readily applied. On the other, it is too

[34][Dürr et al., 1992]; see [Maudlin, 2007a] or [Callender, 2007] for analysis of the notion of "typicality" used in that derivation. I thank an anonymous referee for pressing this point.

[35]Indeed, this is just the standard Humean account of objective chances (see [Lewis, 1994]).

small: the proposed supervenience basis (even over some local region) goes far beyond what could be gathered by empirical investigation (even in principle). Without this tight fit between the supervenience basis and the empirical basis, I don't see how empirical practice can be a source of optimism that Bohmian mechanics is, indeed, the best systematisation of the supervenience basis.

Before wrapping up, I do want to note a somewhat different route that the Miller-Bhogal-Perry Humean could try to take. Throughout this section, I've been working on the assumption that the Humean wants to expunge the *entirety* of the quantum state from their supervenience basis (since that's the proposal most widely defended in the literature). However, it's not clear to me that the Humean need be quite so puritanical. What if the Humean admitted a supervenience basis of Bohmian corpuscles, *plus* some appropriately local "bits" of the quantum state: for example, one-particle density operators or conditional wavefunctions? These seem like Humeanistically acceptable objects: the density operators could be interpreted as representing properties of the corresponding corpuscles, whilst the wavefunctions could be interpreted as representing fields (on good old regular 3-space). As discussed in the section on entanglement, these objects do not uniquely determine the full joint state, nor are they (in general) subject to an autonomous dynamics; but the same goes for the Bohmian trajectories. It's also worth noting that the density operator proposal, at least, doesn't presume Bohmian mechanics or even any primitive ontology at all. So it could offer a way of being Humean to those who aren't signed-up members of the primitive-ontology program.

Furthermore, by employing a richer supervenience basis, this kind of Humean makes it more plausible that full quantum mechanics will turn out to be the best systematisation of the behaviour of that basis: rather than the full quantum state together with the laws, they just need to recover the correlative aspects of the quantum state (i.e. the degrees of freedom in the quantum state not fixed by the individual density operators or conditional wavefunctions), together with the laws. Moreover, by having objects in their supervenience basis which more closely resemble quantum states, the worries I raised above might be ameliorated: it's less clear that one need be tied to working in the position basis, and there is a more obvious link between the supervenience basis and the empirical basis. So there seems more scope to say that the practice of science gives us reason to think that quantum theory is the best systematisation of the supervenience basis. I'm still somewhat pessimistic about the prospects of making this all work out, and—as discussed in the next section—I'm not sure that the motivation for doing so really stands up. But it does seem to me that this proposal (which to my knowledge, has not received much discussion in the literature) is the best way for the Humean to go.

7 Conclusion

Overall, then, I conclude that fitting quantum entanglement into the frame of Humean supervenience remains an unfinished business. It seems appropriate, therefore, to briefly pause and try to take stock of what the benefit of successfully performing this Procrustean feat might be. What would be the gains of having a separable—or more generally, a Humean—picture of the world to hand? Classically, a significant component of the motivation has been taken to be *epistemic*: since what we have direct epistemic access to (the thought goes) are facts about intrinsic properties of individual spacetime points or pointlike entities, we should seek a metaphysics founded upon those facts.

Now, one can certainly criticise this move, from a premise about what is epistemically available to a conclusion about what is metaphysically acceptable.[36] However, it is also worth observing that even the antecedent of this argument seems to be false. We've already seen how our evidence for quantum mechanics is not easily identified with the components of a plausible, separable supervenience basis for quantum mechanics. More generally, though, there is something extremely puzzling about the idea that separability is a precondition for direct epistemic access: for we can perform entanglement experiments, in which we (at least on the account given by quantum mechanics) do indeed observe non-separable phenomena! So what is going on?

The answer, I contend, is that although individual observations are indeed (somewhat) localised, it just does not follow that those observations cannot provide information about or evidence for irreducibly global goings-on. Prima facie, at least, the way in which one does so is about the simplest imaginable: we simply make multiple local observations, and then *aggregate* those observations. So suppose, for example, that mass was not locally conserved, but was conserved on some larger scale—let's say, on the scale of the Earth. It is straightforwardly possible to accumulate evidence for this hypothesis, by making continuous observations at different points of space, and then comparing the results. Of course, it is *harder* to do so than it would be for a purely local phenomenon; and if the scale of the mass conservation were larger still,

[36] See [Maudlin, 2007b] for a particularly biting critique.

then it might well move beyond our capacities to verify it. But the simplistic picture of scientific evidence that seems to motivate the doctrine of separability is long due retirement; and with it, the insistence that our best scientific theories be made to fit that doctrine, at whatever price.

8 Acknowledgments

I'm very grateful to Harjit Bhogal, Elizabeth Miller, Zee Perry, and David Wallace for extensive discussion of previous drafts; to an anonymous referee for their comments; and to the participants in Princeton's Ad Hoc Seminar for their questions.

References

[Albert, 1994] Albert, D. Z. (1994). *Quantum Mechanics and Experience*. Harvard University Press.

[Albert, 1996] Albert, D. Z. (1996). Elementary Quantum Metaphysics. In *Bohmian Mechanics and Quantum Theory: An Appraisal*, volume 184 of *Boston Studies in the Philosophy of Science*, pages 277–284. Kluwer Academic Publishers, Dordrecht.

[Allori et al., 2008] Allori, V., Goldstein, S., Tumulka, R., and Zanghì, N. (2008). On the Common Structure of Bohmian Mechanics and the Ghirardi–Rimini–Weber Theory. *The British Journal for the Philosophy of Science*, 59(3):353–389.

[Belot, 2011] Belot, G. (2011). Quantum states for primitive ontologists. *European Journal for Philosophy of Science*, 2(1):67–83.

[Bhogal and Perry, 2015] Bhogal, H. and Perry, Z. (2015). What the Humean Should Say About Entanglement. *Noûs*.

[Brown et al., 1996] Brown, H. R., Elby, A., and Weingard, R. (1996). Cause and Effect in the Pilot-Wave Interpretation of Quantum Mechanics. In *Bohmian Mechanics and Quantum Theory: An Appraisal*, volume 184 of *Boston Studies in the Philosophy of Science*, pages 309–319. Kluwer Academic Publishers, Dordrecht.

[Callender, 2007] Callender, C. (2007). The emergence and interpretation of probability in Bohmian mechanics. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 38(2):351–370.

[Darby, 2012] Darby, G. (2012). Relational Holism and Humean Supervenience. *The British Journal for the Philosophy of Science*, 63(4):773–788.

[Dürr et al., 1992] Dürr, D., Goldstein, S., and Zanghí, N. (1992). Quantum equilibrium and the origin of absolute uncertainty. *Journal of Statistical Physics*, 67(5-6):843–907.

[Ghirardi et al., 1986] Ghirardi, G. C., Rimini, A., and Weber, T. (1986). Unified dynamics for microscopic and macroscopic systems. *Physical Review D*, 34(2):470.

[Goldstein, 1998] Goldstein, S. (1998). Quantum theory without observers. *Physics Today*, 51(3):42–47.

[Hall, 2009] Hall, N. (2009). Humean reductionism about laws of nature. Unpublished MS. Available at http://philpapers.org/rec/HALHRA.

[Lewis, 1980] Lewis, D. (1980). A Subjectivist's Guide to Objective Chance. In Harper, W. L., Stalnaker, R., and Pearce, G., editors, *IFS*, number 15 in The University of Western Ontario Series in Philosophy of Science, pages 267–297. Springer Netherlands.

[Lewis, 1986] Lewis, D. (1986). *Philosophical papers, volume II*. Oxford University Press, Oxford.

[Lewis, 1994] Lewis, D. (1994). Humean Supervenience Debugged. *Mind*, 103(412):473–490.

[Loewer, 1996] Loewer, B. (1996). Humean Supervenience. *Philosophical Topics*, 24(1):101–127.

[Maudlin, 2007a] Maudlin, T. (2007a). What could be objective about probabilities? *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 38(2):275–291.

[Maudlin, 2007b] Maudlin, T. (2007b). Why Be Humean? In *The Metaphysics Within Physics*. Oxford University Press, Oxford.

[Maudlin, 2007c] Maudlin, T. W. E. (2007c). Completeness, supervenience and ontology. *Journal of Physics A: Mathematical and Theoretical*, 40(12):3151.

[Miller, 2014] Miller, E. (2014). Quantum Entanglement, Bohmian Mechanics, and Humean Supervenience. *Australasian Journal of Philosophy*, 92(3):567–583.

[Saunders et al., 2010] Saunders, S., Barrett, J., Kent, A., and Wallace, D. (2010). *Many Worlds?: Everett, Quantum Theory, & Reality*. OUP Oxford.

[Wallace, 2012a] Wallace, D. (2012a). Decoherence and its role in the modern measurement problem. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 370(1975):4576–4593.

[Wallace, 2012b] Wallace, D. (2012b). *The Emergent Multiverse: Quantum Theory According to the Everett Interpretation*. OUP Oxford.