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An extended grey relational analysis based multiple attribute decision making in interval neutrosophic uncertain linguistic setting

Partha Pratim Dey[1], Surapati Pramanik[2*], and Bibhas C. Giri[3]

[1] Department of Mathematics, Jadavpur University, Kolkata – 700032, India. E-mail: parsur.fuzz@gmail.com
[2*] Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O. –Narayanpur, District – North 24 Parganas, West Bengal, PIN – 743126, India. E-mail: sura_pati@yahoo.co.in
[3] Department of Mathematics, Jadavpur University, Kolkata – 700032, India. E-mail: bcgiri.jumath@gmail.com
*Corresponding author's email [2]:sura_pati@yahoo.co.in

Abstract. This paper investigates an extended grey relational analysis method for multiple attribute decision making problems under interval neutrosophic uncertain linguistic environment. Interval neutrosophic uncertain linguistic variables are hybridization of uncertain linguistic variables and interval neutrosophic sets and they can easily express the imprecise, indeterminate and inconsistent information which normally exist in real life situations. The rating of performance values of the alternatives with respect to the attributes is provided by the decision maker in terms of interval neutrosophic uncertain linguistic variables in the decision making situation. The weights of the attributes have been assumed to be incompletely known or completely unknown to the decision maker and the weights have been calculated by employing different optimization models. Then, an extended grey relational analysis method has been proposed to determine the ranking order of all alternatives and select the best one. Finally, a numerical example has been solved to check the validity and applicability of the proposed method and compared with other existing methods in the literature.

Keywords: Multiple attribute decision making, Interval neutrosophic set, Interval neutrosophic uncertain linguistic variables, Grey relational analysis.

1 Introduction

Multiple attribute decision making (MADM) is a procedure for a decision maker (DM) to get the most desirable alternative from a set of feasible alternatives with respect to some predefined attributes. MADM, an important decision making apparatus have been applied in many kinds of practical fields such as engineering technology, economics, operations research, management science, military, urban planning, etc. However, in real decision making, due to time pressure, complexity of knowledge or data, ambiguity of people's thinking, the performance values of the alternatives regarding the attributes cannot always be represented by crisp values and it is reasonable to describe them by fuzzy information. Zadeh [1] proposed the notion of fuzzy set theory by incorporating the degree of membership to deal with impreciseness. Atanassov [2] extended the concept of Zadeh [1] and defined intuitionistic fuzzy set by introducing the degree of non-membership in dealing with vagueness and uncertainty. However, in many real world decisions making, we often encounter with indeterminate and inconsistent information about alternatives with respect to attributes. In order to handle indeterminate and inconsistent information, the theory of neutrosophic set was incorporated by Smarandache [3-6] by introducing the degree of indeterminacy or neutrality as an independent component. After the ground-breaking work of Smarandache [3-6], Wang et al. [7] proposed single valued neutrosophic set (SVNS) from real scientific and engineering point of view. Wang et al. [8] introduced interval neutrosophic set (INS) which is more realistic and flexible than neutrosophic set and it is characterized by the degree of membership, degree of non-membership and a degree of indeterminacy, and they are intervals rather than real numbers.

In interval neutrosophic decision making environment, Chi and Liu [9] proposed extended technique for order preference by similarity to ideal solution (TOPSIS) method for solving MADM problems in which the attribute weights are unknown and attribute values are expressed in terms of INSs. Ye [10] defined Hamming and Euclidean distances between INSs and proposed a multi-criteria decision making (MCDM) method based on the distance based similarity measures. Broumi and Smarandache [11] defined a new cosine similarity between two INSs based on Bhattacharya's distance [12] and applied the concept to a pattern recognition problem. Zhang et al. [13] developed two interval neutrosophic number aggregation operators for solving MCDM problems. Liu and Shi [14] defined some aggregation operators for interval neutroshic hesitant fuzzy information and developed a decision making method for MADM problems. Zhang et al. [15] further proposed several outranking relations on interval neutrosophic numbers (INNs) based on ELETRE IV and established an outranking approach for MCDM problems using INNs. Ye [16] investigated an improved cross entropy measures for SVNSs and extended it to INSs. Then, the proposed cross entropy measures of SVNSs and INSs are employed to MCDM problems. Şahin and Liu [17] developed a maximizing deviation method for MADM problems with interval-valued neutrosophioc informations. Tian et al. [18] explored a novel and comprehensive approach for MCDM problems based on a cross entropy with INSs. Mondal and Pramanik [19] developed cosine, Dice and Jaccard similarity measures based on interval rough neutrosophic sets and developed MADM methods based on the proposed similarity measures. Ye [20] defined a credibily-induced interval neutrosophic weighted arithmetic averaging operator and a credibily-induced interval neutrosophic weighted geometic averaging operator and established their properties. In the same study, Ye [20] also presented the projection measure between INNs the projection measure based ranking method for solving MADM problems with interval neutrosophic information and credibility information.

Deng [21] initiated grey relational analysis (GRA) method which has been applied widely for solving many MADM problems [22-34] in diverse decision making environments. GRA has been identified as an important decision making device for dealing with the problems with complex interrelationship between various aspects and variables [25-27]. Biswas et al. [28] first studied GRA technique to MADM problems with single valued neutrosophic assessments in which weights of the attributes are completely unknown. Biswas et al. [29] further proposed an improved GRA method for MADM problems under neutrosophic environment. They formulated a deviation based optimization model to find incompletely known attribute weights. They also established an optimization model by using Lagrange functions to compute completely unknown attribute weights. Mondal and Pramanik [30] studied rough neutrosophic MADM through GRA method. Pramanik and Mondal [32] proposed a GRA method for interval neutrosophic MADM problems where the unknown attribute weights are obtained by using information entropy method. Recently, Dey et al. [34] developed an extended GRA based interval neutrosophic MADM for weaver selection in Khadi institution.

Ye [35] introduced interval neutrosophic linguistic variables by combining linguistic variables and the idea of INSs. In the same study Ye [35] proposed aggregation operatos for interval neutrosophic linguistic information and presented a decision making method for MADM problems. Broumi et al. [36] studied an extended TOPSIS method for MADM problems where the attribute values are described in terms of interval neutrosophic uncertain linguistic information and attribute weights are unknown. However, literature review reveals that there has been no work on extending GRA with interval neutrosophic uncertain linguistic information. In this study, we have developed a new GRA method for MADM problems under interval neutrosophic uncertain linguistic assessments where the information about attribute weights are partially known or completely unknown to the DM.

Rest of the paper is designed as follows; In Section 2, we have summarized some basic concepts which are essential for the presentation of the paper. Section 3 has been devoted to develop an extended GRA method for solving MADM problems under interval neutrosophic uncertain linguistic information where the information about attribute weights is partially known or completely unknown. In Section 4, an algorithm of the proposed method has been presented. In Section 5, we have solved a MADM problem to validate the developed method and compared the results with the results of other accessible methods in the literature. Finally, the last Section 6 concludes the paper with future scope of research.

2 Preliminaries

In the Section, we present several concepts regarding neutrosophic sets, single-valued neutrosophic sets, interval neutrosophic sets, uncertain linguistic variable, interval linguistic neutrosophic set, and interval neutrosophic uncertain linguistic set.

2.1 Neutrosophic set

Definition 2.1 [3-6]: Let U be a space of objects, then a neutrosophic set N is defined as follows:

$$N = \{x, \langle \mathrm{T}_N(x), \mathrm{I}_N(x), \mathrm{F}_N(x) \rangle \mid x \in U\} \quad (1)$$

where, $\mathrm{T}_N(x): U \to]^-0, 1^+[$; $\mathrm{I}_N(x): U \to]^-0, 1^+[$; $\mathrm{F}_N(x): U \to]^-0, 1^+[$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively with the condition

$^-0 \le \sup \mathrm{T}_N(x) + \sup \mathrm{I}_N(x) + \sup \mathrm{F}_N(x) \le 3^+$.

2.2 Single – valued neutrosophic set

Definition 2.2 [7]: Assume *U* be a universal space of objects with generic element in *U* represented by *x*, then a SVNS $S \subset U$ is defined as follows:

$$S = \{x, \langle T_S(x), I_S(x), F_S(x) \rangle \mid x \in U\} \quad (2)$$

where, $T_S(x)$; $I_S(x)$; $F_S(x)$: $U \to [0, 1]$ are the degree of truth-membership, the degree of indeterminacy-membership, and the degree of falsity-membership respectively of the element $x \in U$ to the set *S* with the condition $0 \le T_S(x) + I_S(x) + F_S(x) \le 3$.

2.3 Interval neutrosophic set

Definition 2.3 [8]: Assume that *U* be a universal space of points with generic element in *U* denoted by *x*. Then an INS *A* is defined as follows:

$$A = \{x, \langle T_A(x), I_A(x), F_A(x) \rangle \mid x \in U\} \quad (3)$$

where, $T_A(x)$, $I_A(x)$, $F_A(x)$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively with $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq [0, 1]$ for each point $x \in U$ and $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$. For convenience, an INN is represented by $\tilde{a} = ([T^-, T^+], [I^-, I^+], [F^-, F^+])$.

2.4 Uncertain linguistic variable

A linguistic set P = (p_0, p_1, p_2, ..., p_{u-1}) is a finite and completely ordered discrete term set, where u is odd. For example, when u = 7, the linguistic term set P can be defined as given below [36].

P = {p_0 (extremely low); p_1 (very low); p_2 (low); p_3 (medium); p_4 (high); p_5 (very high); p_6 (extremely high)}.

Definition 2.4 [36]: Let $\tilde{p} = [p_\alpha, p_\beta]$, where $p_\alpha, p_\beta \in \tilde{P}$ with $\alpha \le \beta$ be respectively the lower and upper limits of P, then, $\tilde{p}$ is said to be an uncertain linguistic variable.

Definition 2.5 [36]: Consider $\tilde{p}_1 = [p_{\alpha_1}, p_{\beta_1}]$ and $\tilde{p}_2 = [p_{\alpha_2}, p_{\beta_2}]$ be two uncertain linguistic variables, then the distance between $\tilde{p}_1$ and $\tilde{p}_2$ is defined as given below.

$$D(\tilde{p}_1, \tilde{p}_2) = \frac{1}{2(u-1)}(|\alpha_2 - \alpha_1| + |\beta_2 - \beta_1|) \quad (4)$$

2.5 Interval neutrosophic linguistic set

Ye [35] proposed interval neutrosophic linguistic set based on interval neutrosophic set and linguistic variables.

Definition 2.6 [35]: An interval neutrosophic linguistic set *L* in *U* is defined as follows:

$$L = \{x, p_{\varphi(x)}, \langle T_L(x), I_L(x), F_L(x) \rangle \mid x \in U\} \quad (5)$$

where $T_L(x) = [T_L^-(x), T_L^+(x)] \subseteq [0, 1]$, $I_L(x) = [I_L^-(x), I_L^+(x)] \subseteq [0, 1]$, $F_L(x) = [F_L^-(x), F_L^+(x)] \subseteq [0, 1]$ denote respectively, truth-membership degree, indeterminacy-membership degree, and falsity-membership degree of the element *x* in *U* to the linguistic variable $p_{\varphi(x)} \in \hat{p}$ with the condition

$0 \le T_L^+(x) + I_L^+(x) + F_L^+(x) \le 3$.

2.6 Interval neutrosophic uncertain linguistic set

Broumi et al. [36] extended the concept of interval neutrosophic linguistic set [35] and proposed interval neutrosophic uncertain linguistic set based on interval neutrosophic set and uncertain linguistic variables.

Definition 2.7 [36]: An interval neutrosophic uncertain linguistic set *C* in *U* is defined as follows:

$$C = \{x, [p_{\varphi(x)}, p_{\psi(x)}], \langle T_C(x), I_C(x), F_C(x) \rangle \mid x \in U\} \quad (6)$$

where $T_C(x) = [T_C^-(x), T_C^+(x)] \subseteq [0, 1]$, $I_C(x) = [I_C^-(x), I_C^+(x)] \subseteq [0, 1]$, $F_C(x) = [F_C^-(x), F_C^+(x)] \subseteq [0, 1]$ represent respectively, truth-membership degree, indeterminacy-membership degree, and falsity-membership degree of the element *x* in *U* to the uncertain linguistic variable $[p_{\varphi(x)}, p_{\psi(x)}]$ with the condition $0 \le T_C^+(x) + I_C^+(x) + F_C^+(x) \le 3$.

Definition 2.8 [36]: Consider $\tilde{a}_1 = \langle [p_{\varphi(\tilde{a}_1)}, p_{\psi(\tilde{a}_1)}], ([T^-(\tilde{a}_1), T^+(\tilde{a}_1)], [I^-(\tilde{a}_1), I^+(\tilde{a}_1)], [F^-(\tilde{a}_1), F^+(\tilde{a}_1)]) \rangle$ and $\tilde{a}_2 = \langle [p_{\varphi(\tilde{a}_2)}, p_{\psi(\tilde{a}_2)}], ([T^-(\tilde{a}_2), T^+(\tilde{a}_2)], [I^-(\tilde{a}_2), I^+(\tilde{a}_2)], [F^-(\tilde{a}_2), F^+(\tilde{a}_2)]) \rangle$ be two interval neutrosophic uncertain linguistic variables (INULVs) and $\mu \ge 0$, then the operational laws of INULVs are defined as given below.

1. $\tilde{a}_1 \oplus \tilde{a}_2 = \langle [p_{\varphi(\tilde{a}_1)+\varphi(\tilde{a}_2)}, p_{\psi(\tilde{a}_1)+\psi(\tilde{a}_2)}], ([T^-(\tilde{a}_1) + T^-(\tilde{a}_2) - T^-(\tilde{a}_1) . T^-(\tilde{a}_2), T^+(\tilde{a}_1) + T^+(\tilde{a}_2) - T^+(\tilde{a}_1) . T^+(\tilde{a}_2)], [I^-(\tilde{a}_1) . I^-(\tilde{a}_2), I^+(\tilde{a}_1) . I^+(\tilde{a}_2)], [F^-(\tilde{a}_1) . F^-(\tilde{a}_2), F^+(\tilde{a}_1) . F^+(\tilde{a}_2)]) \rangle$
2. $\tilde{a}_1 \otimes \tilde{a}_2 = \langle [p_{\varphi(\tilde{a}_1)\times\varphi(\tilde{a}_2)}, p_{\psi(\tilde{a}_1)\times\psi(\tilde{a}_2)}], ([T^-(\tilde{a}_1) . T^-(\tilde{a}_2), T^+(\tilde{a}_1) . T^+(\tilde{a}_2)], [I^-(\tilde{a}_1) + I^-(\tilde{a}_2) - I^-(\tilde{a}_1) . I^-(\tilde{a}_2), I^+(\tilde{a}_1) + I^+(\tilde{a}_2) - I^+(\tilde{a}_1) . I^+(\tilde{a}_2)], [F^-(\tilde{a}_1) + F^-(\tilde{a}_2) - F^-(\tilde{a}_1) . F^-(\tilde{a}_2), F^+(\tilde{a}_1) + F^+(\tilde{a}_2) - F^+(\tilde{a}_1) . F^+(\tilde{a}_2)]) \rangle$
3. $\mu . \tilde{a}_1 = \langle [p_{\mu\varphi(\tilde{a}_1)}, p_{\mu\psi(\tilde{a}_1)}], ([1 - (1 - T^-(\tilde{a}_1))^\mu, 1 - (1 - T^+(\tilde{a}_1))^\mu], [(I^-(\tilde{a}_1))^\mu, (I^+(\tilde{a}_1))^\mu], [(F^-(\tilde{a}_1))^\mu, (F^+(\tilde{a}_1))^\mu]) \rangle$

4. $\tilde{a}_1^{\mu} = < [p_{\varphi^{\mu}(\tilde{a}_1)}, p_{\psi^{\mu}(\tilde{a}_1)}]$, $([(T^{-}(\tilde{a}_1))^{\mu}, (T^{+}(\tilde{a}_1))^{\mu}]$, $[1 - (1 - I^{-}(\tilde{a}_1))^{\mu}, 1 - (1 - I^{+}(\tilde{a}_1))^{\mu}]$, $[1 - (1 - F^{-}(\tilde{a}_1))^{\mu}, 1 - (1 - F^{+}(\tilde{a}_1))^{\mu}]) >$.

Definition 2.9 [36]: Consider $\tilde{p}_1 = < [p_{\alpha_1}, p_{\beta_1}]$, $([T_A^-, T_A^+]$, $[I_A^-, I_A^+]$, $[F_A^-, F_A^+]) >$ and $\tilde{p}_2 = < [p_{\alpha_2}, p_{\beta_2}]$, $([T_B^-, T_B^+]$, $[I_B^-, I_B^+]$, $[F_B^-, F_B^+]) >$ be two INULVs, then the Hamming distance between them is defined as follows:

$$D_{Ham}(\tilde{p}_1, \tilde{p}_2) = \frac{1}{12(u-1)}(|\alpha_1 \times T_A^- - \alpha_2 \times T_B^-| + |\alpha_1 \times T_A^+ - \alpha_2 \times T_B^+| + |\alpha_1 \times I_A^- - \alpha_2 \times I_B^-| + |\alpha_1 \times I_A^+ - \alpha_2 \times I_B^+| + |\alpha_1 \times F_A^- - \alpha_2 \times F_B^-| + |\alpha_1 \times F_A^+ - \alpha_2 \times F_B^+| + |\beta_1 \times T_A^- - \beta_2 \times T_B^-| + |\beta_1 \times T_A^+ - \beta_2 \times T_B^+| + |\beta_1 \times I_A^- - \beta_2 \times I_B^-| + |\beta_1 \times I_A^+ - \beta_2 \times I_B^+| + |\beta_1 \times F_A^- - \beta_2 \times F_B^-| + |\beta_1 \times F_A^+ - \beta_2 \times F_B^+|) \quad (7)$$

Definition 2.10: Let $\tilde{p}_1 = < [p_{\alpha_1}, p_{\beta_1}]$, $([T_A^-, T_A^+]$, $[I_A^-, I_A^+]$, $[F_A^-, F_A^+]) >$ and $\tilde{p}_2 = < [p_{\alpha_2}, p_{\beta_2}]$, $([T_B^-, T_B^+]$, $[I_B^-, I_B^+]$, $[F_B^-, F_B^+]) >$ be two INULVs, then we define the Euclidean distance between them as follows:

$$D_{Euc}(\tilde{p}_1, \tilde{p}_2) = \frac{1}{12(u-1)}[(\alpha_1 \times T_A^- - \alpha_2 \times T_B^-)^2 + (\alpha_1 \times T_A^+ - \alpha_2 \times T_B^+)^2 + (\alpha_1 \times I_A^- - \alpha_2 \times I_B^-)^2 + (\alpha_1 \times I_A^+ - \alpha_2 \times I_B^+)^2 + (\alpha_1 \times F_A^- - \alpha_2 \times F_B^-)^2 + (\alpha_1 \times F_A^+ - \alpha_2 \times F_B^+)^2 + (\beta_1 \times T_A^- - \beta_2 \times T_B^-)^2 + (\beta_1 \times T_A^+ - \beta_2 \times T_B^+)^2 + (\beta_1 \times I_A^- - \beta_2 \times I_B^-)^2 + (\beta_1 \times I_A^+ - \beta_2 \times I_B^+)2 + (\beta_1 \times F_A^- - \beta_2 \times F_B^-)^2 + (\beta_1 \times F_A^+ - \beta_2 \times F_B^+)^2]^{1/2} \quad (8)$$

3 Extended GRA for MADM problems with interval neutrosophic uncertain linguistic information

Let $G = \{G_1, G_2, \ldots, G_m\}$, $(m \geq 2)$ be a discrete set of alternatives and $H = \{H_1, H_2, \ldots, H_n\}$, $(n \geq 2)$ be the set of attributes in a MADM problem with interval neutrosophic uncertain linguistic information. Also consider $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ be the weighting vector of the attributes with $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^{n} \omega_j = 1$. Suppose the performance values of alternatives with respect to the attributes are represented by INULV $v_{ij} = < [x_{ij}^-, x_{ij}^+]$, $([T_{ij}^-, T_{ij}^+]$, $[I_{ij}^-, I_{ij}^+]$, $[F_{ij}^-, F_{ij}^+]) >$; (i = 1, 2, …, m; j = 1, 2, …, n). Here, $[x_{ij}^-, x_{ij}^+]$ represents uncertain linguistic variable and $x_{ij}^-, x_{ij}^+ \in P = (p_0, p_1, p_2, \ldots, p_{u-1})$, $T_{ij}^-, T_{ij}^+, I_{ij}^-, I_{ij}^+, F_{ij}^-, F_{ij}^+ \in [0, 1]$ with the condition $0 \leq T_{ij}^+(x) + I_{ij}^+(x) + F_{ij}^+(x) \leq 3$. Now, the steps for ranking the alternatives based on extentended GRA method are described as follows:

Step 1. Normalize the decision matrix

Benefit type and cost type attributes are two types of attributes which exist in real world decision making problems. In order to eradicate the impact of the attribute types, we normalize [36] the decision matrix. Suppose $Q = (q_{ij})$ be the normalized decision matrix, where $q_{ij} = < [q_{ij}^-, q_{ij}^+]$, $([\dot{T}_{ij}^-, \dot{T}_{ij}^+]$, $[\dot{I}_{ij}^-, \dot{I}_{ij}^+]$, $[\dot{F}_{ij}^-, \dot{F}_{ij}^+]) >$; (i = 1, 2, …, m; j = 1, 2, …, n), then

For benifit type attribute

$q_{ij}^- = x_{ij}^-$, $q_{ij}^+ = x_{ij}^+$ for (i = 1, 2, …, m; j = 1, 2, …, n)

$$\dot{T}_{ij}^- = T_{ij}^-, \dot{T}_{ij}^+ = T_{ij}^+, \dot{I}_{ij}^- = I_{ij}^-, \dot{I}_{ij}^+ = I_{ij}^+, \dot{F}_{ij}^- = F_{ij}^-, \dot{F}_{ij}^+ = F_{ij}^+ \quad (9)$$

For cost type attribute

$q_{ij}^- = neg(x_{ij}^+)$, $q_{ij}^+ = neg(x_{ij}^-)$ for (i = 1, 2, …, m; j = 1, 2, …, n)

$$\dot{T}_{ij}^- = T_{ij}^-, \dot{T}_{ij}^+ = T_{ij}^+, \dot{I}_{ij}^- = I_{ij}^-, \dot{I}_{ij}^+ = I_{ij}^+, \dot{F}_{ij}^- = F_{ij}^-, \dot{F}_{ij}^+ = F_{ij}^+ \quad (10)$$

Step 2. Identify the positive ideal solution (PIS) $Q^B = (q_1^B, q_2^B, \ldots, q_n^B)$ and negative ideal solution $Q^W = (q_1^W, q_2^W, \ldots, q_n^W)$

Broumi et al. [36] defined PIS (Q^B) and NIS (Q^W) in interval neutrosophic uncertain linguistic environment as follows:

$$Q^B = (q_1^B, q_2^B, \ldots, q_n^B) = [< [q_1^{B-}, q_1^{B+}], ([\dot{T}_1^{B-}, \dot{T}_1^{B+}], [\dot{I}_1^{B-}, \dot{I}_1^{B+}], [\dot{F}_1^{B-}, \dot{F}_1^{B+}]) >; < [q_2^{B-}, q_2^{B+}], ([\dot{T}_2^{B-}, \dot{T}_2^{B+}], [\dot{I}_2^{B-}, \dot{I}_2^{B+}], [\dot{F}_2^{B-}, \dot{F}_2^{B+}]) >; \ldots; < [q_n^{B-}, q_n^{B+}], ([\dot{T}_n^{B-}, \dot{T}_n^{B+}], [\dot{I}_n^{B-}, \dot{I}_n^{B+}], [\dot{F}_n^{B-}, \dot{F}_n^{B+}]) >] \quad (11)$$

$$Q^W = (q_1^W, q_2^W, \ldots, q_n^W) = [< [q_1^{W-}, q_1^{W+}], ([\dot{T}_1^{W-}, \dot{T}_1^{W+}], [\dot{I}_1^{W-}, \dot{I}_1^{W+}], [\dot{F}_1^{W-}, \dot{F}_1^{W+}]) >; < [q_2^{W-}, q_2^{W+}], ([\dot{T}_2^{W-}, \dot{T}_2^{W+}], [\dot{I}_2^{W-}, \dot{I}_2^{W+}], [\dot{F}_2^{W-}, \dot{F}_2^{W+}]) >; \ldots; < [q_n^{W-}, q_n^{W+}], ([\dot{T}_n^{W-}, \dot{T}_n^{W+}], [\dot{I}_n^{W-}, \dot{I}_n^{W+}], [\dot{F}_n^{W-}, \dot{F}_n^{W+}]) >] \quad (12)$$

where $q_j^{B-} = Max_i\, q_{ij}^-$, $q_j^{B+} = Max_i\, q_{ij}^+$, $\dot{T}_j^{B-} = Max_i\, \dot{T}_{ij}^-$, $\dot{T}_j^{B+} = Max_i\, \dot{T}_{ij}^+$, $\dot{I}_j^{B-} = Min_i\, \dot{I}_{ij}^-$, $\dot{I}_j^{B+} = Min_i\, \dot{I}_{ij}^+$, $\dot{F}_1^{B-} = Min_i\, \dot{F}_{ij}^-$, $\dot{F}_1^{B+} = Min_i\, \dot{F}_{ij}^+$;

q_j^{W-} = $Min_i\, q_{ij}^-$, q_j^{W+} = $Min_i\, q_{ij}^+$, $\dot{T}_j^{W-}$ = $Min_i\, \dot{T}_{ij}^-$, $\dot{T}_j^{W+}$ = $Min_i\, \dot{T}_{ij}^+$, $\dot{I}_j^{W-}$ = $Max_i\, \dot{I}_{ij}^-$, $\dot{I}_j^{W+}$ = $Max_i\, \dot{I}_{ij}^+$, $\dot{F}_j^{W-}$ = $Max_i\, \dot{F}_{ij}^-$, $\dot{F}_j^{W+}$ = $Max_i\, \dot{F}_{ij}^+$.

Step 3. Determine the neutrosophic grey relational coefficient of each alternative from PIS and NIS

The grey relational coefficient of each alternative from PIS is defined as follows:

$$\Omega_{ij}^+ = \frac{\underset{i}{Min}\,\underset{i}{Min}\,\rho_{ij}^+ + \sigma\,\underset{i}{Max}\,\underset{i}{Max}\,\rho_{ij}^+}{\rho_{ij}^+ + \sigma\,\underset{i}{Max}\,\underset{i}{Max}\,\rho_{ij}^+}, \quad (13)$$

where $\rho_{ij}^+ = D\,(q_{ij}, q_{ij}^B)$, (i = 1, 2, …, m; j = 1, 2, …, n)

and the grey relational coefficient of each alternative from NIS is defined as given below

$$\Omega_{ij}^- = \frac{\underset{i}{Min}\,\underset{i}{Min}\,\rho_{ij}^- + \sigma\,\underset{i}{Max}\,\underset{i}{Max}\,\rho_{ij}^-}{\rho_{ij}^- + \sigma\,\underset{i}{Max}\,\underset{i}{Max}\,\rho_{ij}^-}, \quad (14)$$

where $\rho_{ij}^- = D\,(q_{ij}, q_{ij}^W)$, (i = 1, 2, …, m; j = 1, 2, …, n).

Here, $\sigma \in [0, 1]$ represents the distinguishing coefficient and generally, $\sigma = 0.5$ is considered in the decision making context.

Step 4. Determination of weights of the attributes

The main idea of GRA method is that the chosen alternative should have the maximal degree of grey relation from the PIS. So, the maximal grey relational coefficient presents the most suitable alternative for the given weight vector. Here, we assume that the weight vector of the attributes is partially known to the DM. Now, the grey relational coefficient between PIS and itself is (1, 1, …, 1), similarly, grey relational coefficient between NIS and itself is also (1, 1, …, 1). The corresponding comprehensive deviations are given below.

$$D_i^+(\omega) = \sum_{j=1}^{n}(1-\Omega_{ij}^+)\omega_j \quad (15)$$

$$D_i^-(\omega) = \sum_{j=1}^{n}(1-\Omega_{ij}^-)\omega_j \quad (16)$$

Smaller values of $D_i^+(\omega)$ and $D_i^-(\omega)$ represent the better alternative. Now we use the max-min operator of Zimmermann and Zysco [37] to integrate all the distances $D_i^+(\omega)$ and $D_i^-(\omega)$, i = 1, 2, …, m separately. Then, we construct the following programming model [29] for incompletely known weight information as:

$$\text{(M-1A)}\begin{cases}\text{Min } \alpha^+ \\ \text{subject to} \\ \sum_{j=1}^{n}(1-\Omega_{ij}^+)\omega_j \le \alpha^+, i=1,2,...,m \\ \omega \in X.\end{cases} \quad (17)$$

$$\text{(M-1B)}\begin{cases}\text{Min } \alpha^- \\ \text{subject to} \\ \sum_{j=1}^{n}(1-\Omega_{ij}^-)\omega_j \le \alpha^-, i=1,2,...,m \\ \omega \in X.\end{cases} \quad (18)$$

where $\alpha^+ = \underset{i}{Max}\sum_{i=1}^{n}(1-\Omega_{ij}^+)\omega_j$; $\alpha^- = \underset{i}{Max}\sum_{i=1}^{n}(1-\Omega_{ij}^-)\omega_j$, i = 1, 2, …, m.

By solving the model (M-1A) and model (M-1B), we get the optimal solutions $\omega^+ = (\omega_1^+, \omega_2^+, \ldots, \omega_n^+)$ and $\omega^- = (\omega_1^-, \omega_2^-, \ldots, \omega_n^-)$ respectively.

Finally, we obtain the weight vector (ω) by combining the above two optimal solutions as follows:

$$\omega = \tau\,\omega^+ + (1-\tau)\,\omega^-;\ \tau \in [0, 1] \quad (19)$$

However, if the information about weights of the attributes are completely unknown, we can formulate another programming model [29] as follows:

$$\text{(M-2)}\begin{cases}\text{Min } D_i^+(\omega) = \sum_{j=1}^{n}\{(1-\rho_{ij}^+)\omega_j\}^2 \\ \text{subject to} \\ \sum_{j=1}^{n}\omega_j = 1, i=1,2,...,m.\end{cases} \quad (20)$$

Now we can aggregate the above multiple objective optimization models with same weights into the single objective optimization model as follows:

$$\text{(M-3)}\begin{cases}\text{Min } D_i^+(\omega) = \sum_{i=1}^{m}D_i^+(\omega) = \sum_{i=1}^{m}\sum_{j=1}^{n}\{(1-\Omega_{ij}^+)\omega_j\}^2 \\ \text{subject to} \\ \sum_{j=1}^{n}\omega_j = 1.\end{cases} \quad (21)$$

In order to solve the above model, we formulate the Lagrange function as given below.

$$L(\omega, \zeta) = \sum_{i=1}^{m}\sum_{j=1}^{n}\{(1-\Omega_{ij}^+)\omega_j\}^2 + 2\zeta(\sum_{j=1}^{n}\omega_j - 1) \quad (22)$$

Here, ζ is the Lagrange multiplier.

Now we differentiate the Eq. (22) with respect to ω_j (j = 1, 2, ..., n) and ζ. Then, by equating the partial derivatives to zero, we obtain the set of equations as follows:

$$\frac{\partial L(\omega_j, \zeta)}{\partial \omega_j} = 2\sum_{i=1}^{m}(1-\Omega_{ij}^+)^2\omega_j + 2\zeta = 0,$$

$$\frac{\partial L(\omega_j, \zeta)}{\partial \zeta} = \sum_{j=1}^{n}\omega_j - 1 = 0$$

By solving the aboveequatins, we obtain

$$\omega^+ = \frac{\left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\{1-\Omega_{ij}^+\}^2\right)^{-1}\right]^{-1}}{\sum_{i=1}^{m}\{1-\Omega_{ij}^+\}^2} \quad (23)$$

Similarly, we can get the attribute weight ω^- by considering NIS as follows:

$$\omega^- = \frac{\left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left\{1-\Omega_{ij}^-\right\}^2\right)^{-1}\right]^{-1}}{\sum_{i=1}^{m}\left\{1-\Omega_{ij}^-\right\}^2} \quad (24)$$

Finally, we can calculate the j-th attribute weight by using the Eq. (19).

Step 5. Determine the degree of neutrosophic grey relational coefficient

The degree of neutrosophic grey relational coefficient of each alternative from PIS and NIS are obtained by the equations (25) and (26) respectively.

$$\Omega_i^+ = \sum_{j=1}^{n}\omega_j\ \Omega_{ij}^+;\ i = 1, 2, ..., m \quad (25)$$

$$\Omega_i^- = \sum_{j=1}^{n}\omega_j\ \Omega_{ij}^-;\ i = 1, 2, ..., m \quad (26)$$

Step 6. Determine the neutrosophic relative relational degree

We compute the neutrosophic relative relational degree of each alternative from PIS by using the following Eq.

$$\Re_i = \frac{\Omega_i^+}{\Omega_i^+ + \Omega_i^-},\ i = 1, 2, \ldots, m. \quad (27)$$

Step 7. Rank the alternatives

The ranking order of the alternatives is obtained according to the decreasing order of the neutrosophic relative relational degree. The maximal value of $\Re_i$, i = 1, 2, …, m reflects the most desirable alternative.

4 Proposed GRA based algorithm for MADM problems with interval neutrosophic uncertain linguistic information

In the following steps**,** we develop a new GRA based algorithm for solving MADM problems under interval neutrosophic uncertain linguistic information

Step 1. Assune v_{ij} = < [x_{ij}^-, x_{ij}^+], ([T_{ij}^-, T_{ij}^+], [I_{ij}^-, I_{ij}^+], [F_{ij}^-, F_{ij}^+]) >; (i = 1, 2, …, m; j = 1, 2, …, n) be an interval neutrosophic uncertain linguistic decision matrix provided by the DM, for the alternative G_i with respect to the attribute H_j, where [x_{ij}^-, x_{ij}^+] denotes uncertain linguistic variable.

Step 2. If the attributes are benefit-type, then we normalize the decision matrix by using the Eq. (9), or we utilize the Eq. (10) in case of cost-type attributes.

Step 3. Identify PIS (Q^B) and NIS (Q^W) from the decision matrix by using Eqs (11) and (12) respectively.

Step 4. Use the distance measures to determine the distances of all alternatives from PIS and NIS.

Step 5. Compute neutrosophic grey relational coefficient of each alternative from PIS and NIS by using the equations. (13) and (14) respectively.

Step 6. If the attribute weights are partially known to the DM, then we solve the models (M-1A) and (M-1B) to find the optimal solutions ω^+ = (ω_1^+, ω_2^+, …, ω_n^+) and ω^- = (ω_1^-, ω_2^-, …, ω_n^-) respectively. Then, weight vector (ω) is obtained by utilizing the Eq. (19). If the information about attribute weights are completely unknown, we solve the model (M-3) to determine ω^+ and ω^-. Finally the weight vector (ω) is calculated by employing the Eq. (19).

Step 7. Find the degree of neutrosophic grey relational coefficient of each alternative from PIS and NIS by employing the equations (25) and (26) respectively.

Step 8. Determine the neutrosophic relative relational degree ($\Re_i$) of each alternative from PIS by using the Eq. (27).

Step 9. Rank all the alternatives G_i (i = 1, 2, …, m) based on $\Re_i$ and choose the best alternative.

Step 10. End.

5 Numerical example

A MADM problem with interval neutrosophic uncertain linguistic information studied by Broumi et al. [36] has been considered in this Section to show the applicability and the effectiveness of the proposed extended GRA approach. Assume that an investment company desires to invest a sum of money in the best option. Suppose there are four possible alternatives to invest the money: (1) G_1 is a car company; (2) G_2 is a food company; (3) G_3 is a computer company; (4) G_4 is an arm company. The company must take a decision based on the following attributes: (1) H_1 is the risk; (2) H_2 is the growth analysis; (3) H_3 is the environmental impact analysis. The rating of performance values of the four alternatives with respect to the three attributes are presented by the DM in terms of INULVs under the linguistic term set P= {p_0 = extremely poor; p_1 = very poor; p_2 = poor; p_3 = medium; p_4 = good; p_5 = very good; p_6 = extremely good [36]. The decision matrix with interval neutrosophic uncertain linguistic variables is presented in Table 1 as follows:

Table 1. The decision matrix in terms of interval neutrosophic uncertain linguistic variables [36]

$$\begin{bmatrix} <[p_4,p_5],([0.4,0.5],[0.2,0.3],[0.3,0.4])> & <[p_5,p_6],([0.4,0.6],[0.1,0.2],[0.2,0.4])> \\ <[p_5,p_6],([0.5,0.7],[0.1,0.2],[0.2,0.3])> & <[p_4,p_5],([0.6,0.7],[0.1,0.2],[0.2,0.3])> \\ <[p_5,p_6],([0.3,0.5],[0.1,0.2],[0.3,0.4])> & <[p_5,p_6],([0.5,0.6],[0.1,0.3],[0.3,0.4])> \\ <[p_3,p_4],([0.7,0.8],[0.0,0.1],[0.1,0.2])> & <[p_3,p_4],([0.5,0.7],[0.1,0.2],[0.2,0.3])> \\ \multicolumn{2}{c}{<[p_4,p_5],([0.2,0.3],[0.1,0.2],[0.5,0.6])>} \\ \multicolumn{2}{c}{<[p_4,p_5],([0.5,0.7],[0.2,0.2],[0.1,0.2])>} \\ \multicolumn{2}{c}{<[p_4,p_4],([0.5,0.6],[0.1,0.3],[0.1,0.3])>} \\ \multicolumn{2}{c}{<[p_5,p_6],([0.3,0.4],[0.1,0.2],[0.1,0.2])>} \end{bmatrix}$$

Now the proposed approach is described in the following steps.

Step 1. Normalization
The attributes of the given MADM problem are considered as benefit types. Therefore, we don't require the normalization of the decision matrix.

Step 2. Identify the PIS and NIS from the given decision matrix
The PIS (Q^B) is obtained from the decision matrix as follows:
Q^B = (<[p_5, p_6], [0.7, 0.8], [0.0, 0.1], [0.1, 0.2]>; <[p_5, p_6], [0.6, 0.7], [0.1, 0.2], [0.2, 0.3]>; <[p_5, p_6], [0.5, 0.7], [0.1, 0.2], [0.1, 0.2]>)
The NIS (Q^W) is obtained from the decision matrix as follows:
Q^W =(<[p_3, p_4], [0.3, 0.5], [0.2, 0.3], [0.3, 0.4]>; <[p_3, p_4], [0.4, 0.6], [0.1, 0.3], [0.3, 0.4]>; <[p_4, p_4], [0.2, 0.3], [0.2, 0.3], [0.5, 0.6]>)

Step 3. Determination of neutrosophic grey relational coefficient of each alternative from PIS and NIS
We calculate the Hamming distance between each alternative and PIS by utilizing the Eq. (7). Then, the neutrosophic grey relational coefficient of each alternative from PIS can be obtained by using the Eq. (13) as follows:

$$\Omega_{ij}^{+} = \begin{bmatrix} 0.5294 & 0.9755 & 0.5051 \\ 0.7699 & 0.9917 & 1.0000 \\ 0.5414 & 0.8956 & 0.9024 \\ 0.7745 & 0.7065 & 0.8956 \end{bmatrix}$$

We also evaluate the Hamming distance between each alternative and NIS by using the Eq. (7). Then, the neutrosophic grey relational coefficient of each alternative from NIS can be determined with the help of the Eq. (14) as follows:

$$\Omega_{ij}^{-} = \begin{bmatrix} 0.8314 & 0.7103 & 0.9333 \\ 0.6000 & 0.7444 & 0.4510 \\ 0.6995 & 0.5670 & 0.5134 \\ 0.5343 & 1.0000 & 0.5534 \end{bmatrix}$$

Step 4. Determination of the weights of the attributes

Case 1. The partially known weight information is presented as follows:
$0.25 \le \omega_1 \le 0.4,\ 0.2 \le \omega_2 \le 0.35,\ 0.4 \le \omega_3 \le 0.5$ such that $\sum_{j=1}^{3} \omega_j = 1$ and $\omega_j \ge 0$, j = 1, 2, 3.
Now we construct the single objective programming model by using the model (M-1A) and model (M-1B) as given below.
Model (M-1A).
Min α^+
subject to
$0.4706\,\omega_1 + 0.0245\,\omega_2 + 0.4949\,\omega_3 \le \alpha^+$,
$0.2301\,\omega_1 + 0.0083\,\omega_2 \le \alpha^+$,
$0.4586\,\omega_1 + 0.1044\,\omega_2 + 0.0976\,\omega_3 \le \alpha^+$,
$0.2255\,\omega_1 + 0.2935\,\omega_2 + 0.1044\,\omega_3 \le \alpha^+$,
$0.25 \le \omega_1 \le 0.4,\ 0.2 \le \omega_2 \le 0.35,\ 0.4 \le \omega_3 \le 0.5$,
$\sum_{j=1}^{3} \omega_j = 1$ and $\omega_j \ge 0$, j = 1, 2, 3.
Model (M-1B).
Min α^-
subject to
$0.1686\,\omega_1 + 0.2897\,\omega_2 + 0.0667\,\omega_3 \le \alpha^-$,
$0.4\,\omega_1 + 0.2556\,\omega_2 + 0.549\,\omega_3 \le \alpha^-$,
$0.3005\,\omega_1 + 0.433\,\omega_2 + 0.4866\,\omega_3 \le \alpha^-$,
$0.4657\,\omega_1 + 0.4466\,\omega_3 \le \alpha^-$,
$0.25 \le \omega_1 \le 0.4,\ 0.2 \le \omega_2 \le 0.35,\ 0.4 \le \omega_3 \le 0.5$,
$\sum_{j=1}^{3} \omega_j = 1$ and $\omega_j \ge 0$, j = 1, 2, 3.
Solving the above two models (M-1A and M-1B), we get the weight vectors respectively as given below.
ω^+ = (0.25, 0.35, 0.40) and ω^- = (0.294, 0.306, 0.40)
For τ = 0.5, the combined weight vector of the attributes is obtained as ω = (0.272, 0.328, 0.4).

Case 2. Consider the information about the attribute weights be completely unknown to the DM. Then, we can get the unknown weights of the attributes by using the relations (23) and (24). The weights of the attributes are obtained respectively as follows:

$\omega^+ = (0.118, 0.645, 0.237)$ and $\omega^- = (0.318, 0.468, 0.213)$

Therefore, the resulting weight vector of the attributes by taking $\tau = 0.5$ is $\omega = (0.218, 0.557, 0.225)$.

Step 5. Calculate the degree of neutrosophic grey relational coefficient

The degree of neutrosophic grey relational coefficient of each alternative from PIS for Case 1 and Case 2 are presented as follows:

Case 1: $\Omega_1^+ = 0.6660$, $\Omega_2^+ = 0.9347$, $\Omega_3^+ = 0.8020$, $\Omega_4^+ = 0.8000$

Case 2: $\Omega_1^+ = 0.7724$, $\Omega_2^+ = 0.9452$, $\Omega_3^+ = 0.8199$, $\Omega_4^+ = 0.7639$.

Similarly, the degree of neutrosophic grey relational coefficient of each alternative from NIS for Case 1 and Case 2 are demonstrated as follows:

Case 1: $\Omega_1^+ = 0.8324$, $\Omega_2^+ = 0.5878$, $\Omega_3^+ = 0.5816$, $\Omega_4^+ = 0.6947$

Case 2: $\Omega_1^+ = 0.7869$, $\Omega_2^+ = 0.6469$, $\Omega_3^+ = 0.5838$, $\Omega_4^+ = 0.7980$.

Step 6. Evaluate the neutrosophic relative relational degree

We calculate the neutrosophic relative relational degree of each alternative from PIS for Case 1 and Case 2 are presented as follows:

Case 1: $\Re_1 = 0.4448$, $\Re_2 = 0.6139$, $\Re_3 = 0.5796$, $\Re_4 = 0.5354$

Case 2: $\Re_1 = 0.4954$, $\Re_2 = 0.5937$, $\Re_3 = 0.5841$, $\Re_4 = 4891$.

Step 7. Rank the alternatives

The ranking order of the alternatives for Case 1 and Case 2 are presented according to the values of the neutrosophic relative relational degrees as given below.

Case 1: $\Re_2 > \Re_3 > \Re_4 > \Re_1$

Case 2: $\Re_2 > \Re_3 > \Re_1 > \Re_4$

We observe that the Arms Company is the best alternative for investment purpose for both the cases (see Table 2).

Note 1. Broumi et al. [36] consider the weight vector $\omega = (0.35, 0.25, 0.4)$ and use TOPSIS method to rank the alternatives. If we consider the same weight structure i.e. $\omega = (0.35, 0.25, 0.4)$, then the ranking order of the alternatives based on the proposed GRA method is obtained as follows:

$G_2 > G_3 > G_4 > G_1$ and obviously, G_2 would be the best choice.

Note 2. If we consider the proposed Euclidean measure to calculate the distance between two INULVs, then (0.25, 0.35, 0.4) and (0.232, 0.559, 0.209) would be the obtained weight vectors for Case 1 and Case 2 respectively. If we follow the same procedure as described above, the neutrosophic relative relational degree of each alternative from PIS for Case 1 and Case 2 are computed as follows:

Case 1: $\Re_1 = 0.4213$, $\Re_2 = 0.6174$, $\Re_3 = 0.5508$, $\Re_4 = 0.496$;

Case 2: $\Re_1 = 0.4657$, $\Re_2 = 0.599$, $\Re_3 = 0.5556$, $\Re_4 = 4686$.

Therefore, the ranking order of the alternatives for Case 1 and Case 2 are shown as given below.

Case 1: $\Re_2 > \Re_3 > \Re_4 > \Re_1$

Case 2: $\Re_2 > \Re_3 > \Re_4 > \Re_1$

So, the Arms Company G_2 would be the best choice for investment purpose.

6 Conclusion

In the paper we have presented a solution method for MADM problems with interval neutrosophic uncertain linguistic information through extended GRA method. Interval neutrosophic uncertain linguistic variables are suitable for dealing with incomplete and inconsistent information which exist in real world problems. In this paper, we have proposed Euclidean distance between two INULVs. Also, we have addressed the incomplete or completely unknown weights of the attributes to the decision maker.

Table 2. Comparison of the proposed method with other existing method

Method	weight vector	ranking results	best option
Proposed method (Case 1) (using Hamming distance)	(0.272, 0.328, 0.4)	$G_2 > G_3 > G_4 > G_1$	G_2
Proposed method (Case 2) (using Hamming distance)	(0.218, 0.557, 0.225)	$G_2 > G_3 > G_1 > G_4$	G_2
Proposed method (Case 1) (using Euclidean distance)	(0.25, 0.35, 0.4)	$G_2 > G_3 > G_4 > G_1$	G_2
Proposed method	(0.232, 0.559, 0.209)	$G_2 > G_3 > G_4 > G_1$	G_2

(Case 2)
(using Euclidean distance)

Broumi et al. [36] (0.35, 0.25, 0.4) $G_2 > G_4 > G_3 > G_1$ G_2

We have developed two different optimization models to recognize the weights of the attributes in two different cases. Then, extended GRA method has been developed to identify the ranking order of the alternatives. Finally, a numerical example has been solved to demonstrate the feasibility and applicability of the proposed method and compared with other existing methods in the literature. We hope that the proposed method can be helpful in the field of practical decision making problems such as school selection, teacher selection, medical diagnosis, pattern recognition, supplier selection, etc.

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