

## Zeno Paradox, Unexpected Hanging Paradox (Modeling of Reality & Physical Reality, A Historical-Philosophical view)

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Farzad Didehvar

[didehvar@aut.ac.ir](mailto:didehvar@aut.ac.ir)

Department of Mathematics and Computer Science, Amir Kabir University

**Abstract.** In our research about Fuzzy Time and modeling time, "Unexpected Hanging Paradox" plays a major role. Here, we compare this paradox to the Zeno Paradox and the relations of them with our standard models of continuum and Fuzzy numbers. To do this, we review the project "Fuzzy Time and Possible Impacts of It on Science" and introduce a new way in order to approach the solutions for these paradoxes. Additionally, we have a more general discussion about paradoxes, as Philosophical back ground of the subject and in this way, we introduce the concepts of *General View*, *the first picture*, *BBF-General View*.

**Keywords.** Zeno Paradox, Unexpected Hanging Paradox, Fuzzy Time, Fuzzy Time-Particle interpretation of Quantum Mechanics, *General View*, *the first picture*, *BBF-General View*, P vs NP, TC\*, Complexity Theory, Cryptography, Quantum Mechanics

**Introduction.** Zeno Paradox is an ancient paradox and Unexpected Hanging paradox a new one. Here, we show that the impact of Zeno Paradox on Mathematics and Science could be repeated somewhat by "Unexpected Hanging Paradox". Actually, we use a modeling of time as a solution of "Unexpected Hanging Paradox" [5]. We show this solution is a convenient one to solve some problems in Theory of Computation and Physics [3], [6], [10]. Also, fuzzy time could be considered as a solution of Zeno Paradox.

In the first and the second chapter, we discuss about "*General View*". Actually, *the General View* is the space which the problem or the paradox is proposed there. We show that we have different *General Views*. For Zeno paradox, we introduce the associated *General View*, additionally we show in brief, how the concept of "*General View*" has been changed in history, as time passes.

In the shadow of the above and Zeno Paradox, we explain about the models of continuum in the third chapter as we see in nowadays Mathematics. There, we define *the first picture*.

In section 4, we discuss about solving a paradox in general. We claim that solving paradoxes depends on the *General View*, Model and theory we choose. So, by changing them, probably we should try to find the new solutions. As a conclusion, the file of a paradox and its solutions seems open, forever.

In section 5, we discuss about the differences between Zeno paradox and Liar Paradox. As a Consequence, the paradoxes have many different varieties. The question which arise is:

What about the "Unexpected Hanging Paradox" in this comparison?

Answering the above question, is the subject of the sixth chapter. Although, historically this paradox is considered as either Logical or Epistemological Paradox but in the new approach we know the situation of this paradox mostly similar to the Zeno Paradox. At first, we discuss about the history of this paradox, in the next step, we introduce our approach and the conclusions of that. This approach involves a solution respect to our "*General View*" (The same as Zeno Paradox) and our Model which is, a Fuzzy Model of Time. We show, how Physics and more specifically a novel interpretation of Quantum Mechanics supports this idea successfully. We call this interpretation "Fuzzy Time-Particle interpretation of Quantum Mechanics". By the above explanations, we know the purposed solution to this paradox a successful one, which probably will be supported by experiments, in Physical sense. In addition, it solves some major problems in Theory of Computation and Complexity Theory. In addition, it gives a new interpretation of Quantum Mechanics in Physics. By accepting this solution of paradox and the shift in the theory provided by it, many problems have a much better situation than before in Physics, Theory of Complexity of Algorithms and Cryptography. Our last and major claim is: We reach to the new models of continuum, which are much intuitive and solve more problems technically.

By "*much intuitive*" in above, the author intends a novel intuition which involves Quantum Mechanics, as one of the major pillars of Science in modern era. So, more precisely, it is noteworthy to mention that, the intuition has been changed, as time passes in history.

## **1. General View**

Here, first we present the way of approaching to paradoxes in general. To do that, firstly we introduce the concept of *General View*. The *General View* is somewhat the perspective and window which we see the problem and paradox from that, as it is explained in below. Simultaneously, it is considered as our background of the whole body of discussion.

Actually, we accept there is a reality out of our Mind, but we don't know about and how it is. Any understanding of us from reality is after interference of mind and our environment. So here, pure reality has no shape. As it is shown in figure 1, it is considered as a "black box".

Anyway, it is possible to make a hierarchy. In each level of this hierarchy after the first level, we know something as reality, (which is not black box as we see in the first level), it is stuffed by things which is considered as real objects out of our mind, but if we pay attention, it is mind which recognizes and classifies them from the previous levels, by employing different types of similarities, abstraction and corrections in the process of feedback to improve our model.

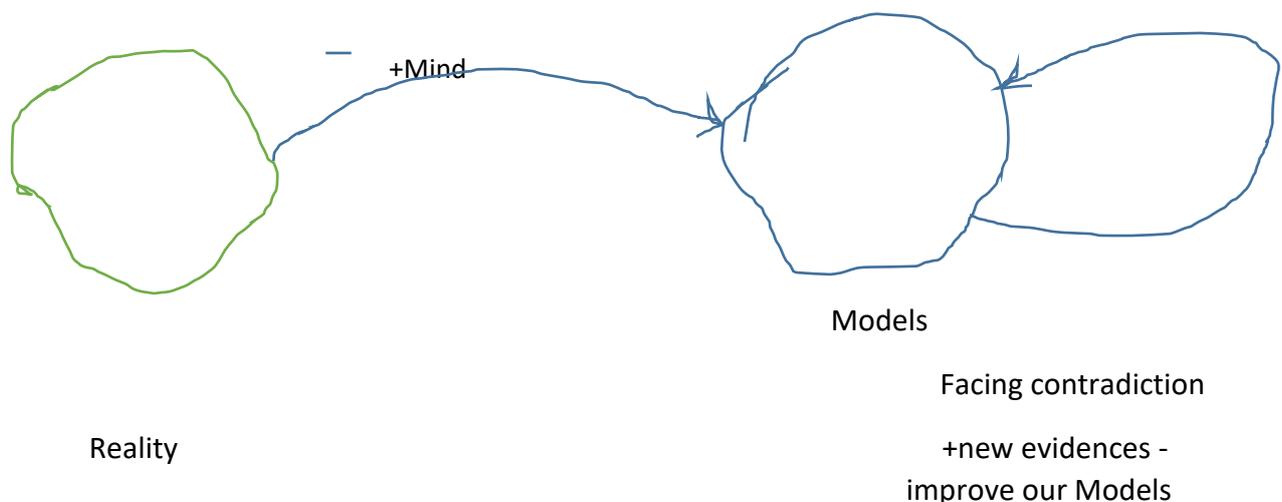


Figure 1

It is notable to say, the second level of hierarchy is taking shape mostly unconsciously. It is what we usually know as intuition. By higher levels, we would be more and more far from intuition and our models usually are more artificial.

After generating our models, it is possible to fall in some troubles. Three types of these troubles are

1. Falling in contradictions,
2. By new evidences as time passes, we face the problems and contradictions,
3. We discover counterintuitive matters, (in comparison to our past experiences)

The author tries to modify our models based on the above, to refute contradictions and contra intuitive matters.

All above is "*The General View*" which we consider in this article and is illustrated by the above figure. We call this General View, "*Black Box- Feedback -General view*" or in brief, "*BBF General View*".

## **2. About Zeno Paradox**

The Zeno paradox is a famous paradox which was known by the ancient Greeks [20], [21]. Seemingly, during that era (even now) people accepted two statements about the distance and time as facts, as follows

First, between any two places there is another place, in the middle of them. (At least in a line). The same is true for two instants of time.

Second, we have finite places and points in any finite part of space, surfaces, lines and time.

Zeno paradox in our view, shows these two statements are contradictory and the necessity of having a better modeling for this paradox. But Zeno as the follower of Parmenides, used it to defend his teacher's view.

Parmenides believed motion (in general "change") is an illusion, and Zeno as his follower considered the Zeno Paradox as a proof for that.

But historically, science and prevailing view of people do not accept the way of Zeno and Parmenides. It is believed here, either in the first place the idea of motion is not an illusion or if it is, this illusion is so important and it should be studied and considered much more!

Why? Probably because people in general, has goals, desires and theories related to the motion and this does not agree with the way Parmenides purposed.

Also, people in that era, probably do not believe and were not familiar with something illustrated in fig1. They were far from that. Possibly, they were

familiar to some idea like: reality and there are ways to know and to approach  
There is something as reality.

Reality didn't consider as a black box, it is considered as something understandable. It is possible we make mistakes in our understanding of reality but they are corrigible.

Anyway, we pass a long path from the elder view to our modern view, which is called "*BBF-General View*" in this article. In addition, a long path was paved from ancient view in homer era to the era of Greek Philosophers, consequently we have at least three types of "*General view*", that is mainly about the concept of Mind and consciousness. Seemingly by some theories, in Homeric age, they didn't consider consciousness [16].

### **3. How did we cope with this paradox?**

Based on our idea in fig 1, we do not observe the pure reality in Physics, but around the problems of the real world, we have intuitions and we create a Model. About this problem, seemingly Greeks and possibly all ancient people did not believe a finite part of the world contains infinite number of points, on the other hand they did not observe the points and time discrete and disjoint elements, they knew them as continuous concepts. We call this picture and Model, *the first picture*. It is certain that, this model is not an exact Mathematical Model but the most natural and intuitive picture of the situation, as we know.

In history, our understanding of time and space changes to the models less intuitive, but much logical and simultaneously easy to work with, mathematically. By logic, we mean here, a simple logical system to work with, like Mathematical logic. More specifically, by some models of continuum, like Dedekind Model and Cauchy Model. These works are results of centuries Mathematical and Logical working and thinking. More precisely, it is logical and works very well with Mathematical and logical systems. Nevertheless, it is manipulated and somewhat artificial. Since, in this model, infinite numbers of points putting side by side and they are disjoint. This manipulated system, contradicts our intuition, (*the first picture*). But Mathematically, for Mathematicians and for Modeling purposes, this is a heaven and a great success.

Anyway, by accepting these types of Models, we successfully solve the Zeno Paradox.

Here, according to the "*BBF-General View*", we have pure reality as a black box (The first level), the second level is what we called it "*the first picture*", and the mathematical models of continuum in the next levels.

From the first, along with the development of this model and a Theory like Set Theory in nineteenth century in Europe and the first decades of twenty century, we see the resistances.

The ideas of Gauss about infinity, the confrontations of Kronecker and Cantor, Poincare and Brouwers ideas and Philosophies, in contrast to Hilbert Suggestions and Philosophy show some part of these resistances.

It is notable to mention, the last example in above (the Poincare and Brouwer ideas) the streamline continues by Heyting in a logical form (Intuitionism).

Classical way continues from Cantor and Frege's activities to the successes and Failures of Hilbert's program, in the subject of Mathematics, Logic and Theoretical Computer Science.

. These activities involve people like Gödel, Gentzen, Turing... .

By Finitism, Hilbet tries to approach one of the aspects of the first picture.

In all above, we see a commitment toward *the first picture*, as a background and the most natural and intuitive picture.

Later on, by changing the center of Science from Europe to United States after Second War, people stress more on "practical purposes" rather Philosophical aspects. In Engineering we see the reviving of these ideas in Fuzzy Mathematics and Fuzzy Logic started by Zadeh. But this time, the Mathematical structures made by Dedekind cut or Cauchy sequence (Equivalent systems by accepting Aristotle-Frege Logic) is accepted as the foundation. Based on them, Fuzzy Mathematicians tried to approach *the first picture* again. The concept of vagueness and approximability play a central role in this issue.

It is proved that their attempts are somehow parallel to the attempts of People in Intuitionism.

Simultaneously, the intuitionism and Constructivism were alive, especially in Netherland and little by little it found its place in United states. Along with that, we have the other logical approaches which are based on the nature of Sciences like human Science and Quantum Mechanics.

A noticeable example among many others is "Para Consistent Logic", should be highlighted, which after more than sixty years become a so rich and fruitful subject.

In all above, clearly, some part of problem is related to the subject of Vagueness, but that's not the whole story. The discussions around proof and the concept of infinity play a crucial role here. Also, we should add the above list, understanding of the concepts of space and time.

In sum, seemingly, these five concepts are in the center of the above discussion.

Anyway, we have two clear approaches which grow parallelly,

1. Concept Based Approaches, similar to the Brouwer's works,
2. Logical Approaches, as an example in the contrast to the Brouwer's works in 1, we have Heyting's works,

These points could impact "*the first picture*", as we can see in the Philosophical works by Brouwer or development of Analysis by Paraconsistent logic.

Two other Philosophical approaches, Finitism, Ultrafinitism show how the other aspect of "*The First Picture*", (the first aspect), is considered.

In above, briefly we describe the historical and Philosophical situation about and around Numbers, Continuum, the confrontations of discrete issues and Continuum subjects.

Zeno paradox plays a key role in this matter. As it is said, our solutions are related to the model which we considered. Consequently, it is not true to say we solve the problem decisively. It is rather to say, we reach to a coexisting with this solution, between the paradox and our Model. Usually, by choosing these types of Models, we obtain some benefits and loose some others. In the next chapter we explain this, more precisely.

What we wish to show here is about a Novel Paradox "Unexpected Hanging Paradox". Our claim, in brief is:

These Models doesn't work here, and we need a new model of continuum to solve this problem. The novel model is constructed based on the sets of a special type of fuzzy numbers as instants of time.

#### 4. Solving a Paradox

Suppose  $S_p$  is a solution for paradox P, this solution is given by using a background which consisted of three major parts, associated Theory, Model and "General View", as our example in above. So, we show this deduction as below

$$(P, (v_i, T_j, M_k)) \text{ deduces } (S_p \text{ is a solution of } P)$$

Here,  $v_i$  is our *General View*,  $T_j$  is our theory which we consider,  $M_k$  is our model.

If the Theory is not at the center of our discussion, we have the following definition from above

$$(P, (v_i, M_k)) \text{ deduces } (S_p \text{ is a solution of } P)$$

Or

$$(P, (v_i, M_k)) \text{ deduces } (S_p \text{ is a solution of } P)$$

Also, if the model is not at the center of our discussion, we have the following definition from above

$$(P, (v_i, T_j, M_k)) \vdash (S_p \text{ is a solution of } P)$$

As an example, in above discussions about Zeno Paradox, the discussed view is the "*BBF-General View*" which is illustrated in fig 1, and the model is Dedekind Cut. So, our solution depends on our view, theory and model.

The fact is, as time passes, the view, the theory and the model could be changed. So, a paradox is an open question forever.

In more details, by considering a Theory and Model based on *The first picture*, the Zeno solution for Zeno Paradox is a plausible one. More exactly, accepting a contradiction in *The first picture*, and refuting the concept of motion. As we said, we didn't accept this solution, since we feel to need the concept of motion. Historically, after renaissance we work more on the concept of motion, and Mathematical models associated to it. As a conclusion, gradually we are going toward the models introduced by Cauchy and Dedekind in 19 century. Now, we have a new solution for new models and theories.

It is shown here, dealing with "Unexpected Hanging Paradox" we need some new changes in our Theories and Models about Continuum. It should be mentioned that, not only theories and models but also our *General views* has changed as time passes in history.

As a conclusion, seemingly the files of paradoxes are forever open to new solutions. By probable changing of views and models and theories, our solutions will be changed.

Remark. Seemingly based on the above, when we reach to a contradiction in paradoxes, we have different ways to face it,

1. To stop!
2. To change the logic,
- 3,4. Changing Model and Theory,
5. Changing the General View,
6. Considering the possible shifts and changes of our prevailing intuition in history,

It is certain that we have number seven of conditions, that is a combining of 1,..,6.

## **5. Liar Paradox**

In the "Zeno Paradox", we focus on the reality and how mind recognizes and classifies reality.

What about paradoxes like liar paradoxes? They seem different, since these types of paradoxes do not discuss about Physical world like above. The central subject in Liar paradox, is not physical world as we see in picture one and "*BBF-General View*", of "Zeno Paradox".

This shows that we have different types of paradoxes, depends on the variety of our "*General Views*".

## **6. Unexpected hanging Paradox**

One of the most famous paradoxes which is introduced by Lennart Ekbom between 1943-1944 is "Unexpected Hanging Paradox" (or Surprise test paradox). The version of Ekbom of this paradox was based on the reality and

announcement of a civil defense drill. Daniel John o'Conner introduced it in Mind [13]. Martin Gardner introduced it in Public in Scientific American magazine in 1963. Later on, he published a book around this subject [18]. There is a rather vast literature about. W.V Quine (On a so called paradox, Mind 62, 1953, (245:65-67)) [8] and S.A kripke (Philosophical Troubles) [11], are two eminent logicians who payed attention to this paradox.

Shira Krichman & Ran Rose employs this paradox to prove the second Incompleteness of Gödel. [17]

In [7], the author shows a modified version of it. The important point in this modified version is

*"This version is exactly a contradiction".*

It is done somehow and in general as the continuation of author's PhD thesis [14], in order to find a new logical strategy to shed a light on P vs NP problem.

In [5] a solution of paradox is given, considering time as a Fuzzy Concept. Actually, the solution is based on changing our model. Usually, they face this paradox in two major approaches, Logical approach and Epistemological approach. Here, we see it as a Physical paradox and our problem is how to model the situation and time, similar to Zeno paradox by the associated *"General View", "BBF-General View"*.

Again, the story goes toward modeling continuum like Zeno Paradox. The known models for continuum do not work and we need a new one.

Intuitively, the instants of time are not disjoint, as it is mentioned in the first decades of Twenty century by Brower and Husserl [9]. In the purposed continuum model for Fuzzy Time Computation (TC\*), this point is satisfied.

The second point which should be considered for this model is:

*"Any two instants of time are overlapping"*.

This is done by considering the support of fuzzy function associated to any instant of time, equal to the set of real numbers. We need this point to solve *"Unexpected Hanging Paradox"*, as it is shown in [5]. On the face of it, this idea seems contra-intuitive, but we show the Physical Theories, more precisely Quantum Mechanics support it. To the best of author's knowledge, nowhere

this model has presented before. Possibly, the major reason is the mentioned contra-intuitive aspect.

Although at the first glance, the second point seems somewhat contra intuitive, nevertheless it is shown the second point could be supported by Quantum Mechanics, as one of the major pillars of Science in our era. To do that, the author purposes a new interpretation of this Theory, so called "Fuzzy Time-Particle interpretation of Quantum Mechanics" [1], [3]. En passant, by searching the literature, the most closed subject to ours is:

"Probabilistic Time" by C.Wetterich [12].

There are the other works about in literature, but they usually consider the vagueness of time and space simultaneously. But as it is explained in [3], [4], [19] they are completely different points.

The other major positive point of this model is verifiability of it by experience.

More exactly, Fuzzy Time-Particle interpretation and Fuzziness of time could be checked by experiment [3]. In the case that the result of experience be positive and supports the idea, by acceptance of the interpretation, we should accept any two instants of time are overlapping.

Actually, the positive answer to these experiments, leads us to new understanding of the concept of time. As a result, many points in this subject would be changed.

One of the major aspect is the point that, we have two types of time in this theory, *real time and abstract time* [1], [2], [3], [19]. The real time which taken by a particle is related to the system we consider and besides others it shows the role of observer. Although, "abstract time" is a theoretical concept but based on the theory, it is approximately the "real time" of some systems.

In [2], the fuzzy time function is computed. In [19], we see that the fuzzy time function couldn't be symmetric and a reason is provided for anti-symmetry of time.

Anti-symmetry has different aspects, first of all, anti-symmetries between time and space which is shown as the impact of Schrodinger Equation or Dirac Equation, the second is the possible anti-symmetries among different dimensions of space. This possibility means the possibility of changing

the Schrodinger and Dirac equations, since they are symmetric respect to the space axes. The second case seems unlikely, unless a very slight differences among different axis's are possible. The third aspect of symmetricity, is the symmetricity of changing  $s \rightarrow -s$ ,  $s \in \{x, y, z, t\}$  in the formulas.

It is important to mention that the work didn't start by Logic, Paradox or Physical theories. The work starts to find a new way to shed a light on the problems of Complexity Theories, specially P vs NP problem.

Among the other possible ways, the author considers the possibility of employing Paradoxes in solving Complexity Theory problems, analogues to employing liar paradox in Gödel Theorem.

The first step in this way is choosing the convenient paradox. The Unexpected Hanging Paradox has many elements in common with the elements in Theory of Computation. By considering the modified version of this paradox, the author reaches to a contradiction as it is said in above. Considering fuzzy time model, helps us to solve some of these problems in Complexity Theory successfully [10], [15].

In this way, the literature of Theory of Computation and Cryptography changes dramatically and many theorems will be solved in the new theory (TC\*), for instance, it is proved that  $P^*$  is not equal  $NP^*$  and  $P^* \neq BPP^*$  in  $TC^*$ . [10]

$P^*$ ,  $NP^*$ ,  $BPP^*$  are the new classes which are defined in  $TC^*$ , respect to the Complexity Classes P, NP, BPP in TC.

The central point and idea here is the existence of random generator in  $TC^*$ , but an exact proof needs a little more effort (it is done). We have more conclusions which are briefly discussed in [15].

On the other hand, time is a Physical concept, so we need a Theory in Physics based on fuzzy time. As we said in above paragraphs, we introduce

Fuzzy Time-Particle interpretation of Quantum Mechanics [1], [3], [5].

Note that, by considering fuzzy time, we are not able to repeat the argument of Zeno Paradox. Since in this paradox, the arrangement of steps in the axis of time is not as we see in Classical model of time. For instance, here it is possible that the 1001-th step happens before 1000-th step. This model agrees more to the first picture, as it is described in the third chapter, since the instants of

time are not disjoint sets. So associated to this model, we have no more a paradox!

One of the major points here is the ability of considering a novel modeling based on the above. This is by discretizing the real line and in the second step, considering the fuzzy numbers instead of the discrete points. To the best of author's knowledge, this model is the closest model to our intuition, the first picture, respect to the previous picture. We call it "*Discrete Fuzzy Model of continuum*".

In a nutshell, we have an intuition about continuum (it is possible that our intuition about continuum is related to the function of the right hemisphere of the brain) and we need a language and logic to theorize the problem and having the ability of communication.

But language is a discrete issue and seemingly related to the left hemisphere of the brain. Consequently, in order to have a theory about continuum, we need to manipulate our intuition, since we express our intuition about continuum by discrete matters. So, our goal is to consider a manipulation which harm less our intuition and technically works well.

As we mentioned above, even by accepting Physicalism (or by some fragments of it) and by considering mind as brain, the above explanation is a plausible one. Since, our brain has two hemispheres, which play their roles in two different ways. These different ways, approve our different intuitions about continuum and discrete issues.

**Remark.** Applying the fuzzy model for instants of time and "*Discrete Fuzzy Model of continuum*" agree more to our intuition and Physical Theories

(More exactly Quantum Mechanics). Simultaneously it provides a shift in theory, which causes a better situation in Physics, Theory of Computation and Cryptography. Nevertheless, this is a theoretical success and we need experimental support to accept fuzziness of the instants of time and the above mentioned models. In [3], the designing of this experiment is done.

It is noteworthy to mention, even if the experiment fails to show the fuzziness of time which seems so unlikely to the author, we will have theoretical successes in this way [10].

**Conclusion.** In brief and as a conclusion, seemingly "Unexpected Hanging Paradox" is much important than we have thought!

Actually, this paradox was investigated in two approaches, logical approach and epistemological approach. Here, instead of these two approaches, we suggest to consider this paradox similar to Zeno Paradox. So, we consider this paradox as a paradox related to Physics and the concept of time. The proposed model is the model of Fuzzy numbers.

To do the above, we explained more about Zeno Paradox, the way which we see that and the impact of it on science. We follow a parallel line about "Unexpected Hanging Paradox", and we show the results in Theory of Computation and Physics. As a conclusion, the results could change dramatically the situation of some problems and paradoxes in Physics, Theory of Computation and Cryptography [6], [10].

Finally, as we see in the remark, the changes cause an improvement in modeling, by solving some of our problems and it agrees more with our intuition. Throughout this article, by intuition, we mean considering Quantum Mechanics too. For sure, we haven't such an intuition in ancient time, or among people which has no knowledge about Quantum Mechanics. In this way, by solving "Zeno Paradox" and "Unexpected Hanging Paradox", and in consequence applying either Fuzzy time instants model or "*Discrete Fuzzy Model of continuum*", we reach to a better situation in modeling and solving problems in the mentioned sciences in the above paragraph.

To explain more deep and exact the subject, we introduce three concepts in Philosophical background of the subject, "*General view*", "*BBF-General View*" and "*The first picture*".

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