Quantum Mechanics: an Intelligible Description of Objective Reality?

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Abstract

Jim Cushing emphasized that physical theory should tell us an intelligible and objective story about the world, and concluded that the Bohm theory is to be preferred over the Copenhagen interpretation. We argue here, however, that the Bohm theory is only one member of a wider class of interpretations that can be said to fulfil Cushing's desiderata. We discuss how the pictures provided by these interpretations differ from the classical one. In particular, it seems that a rather drastic form of perspectivalism is needed if accordance with special relativity is to be achieved.

1 Intelligibility and objectivity

In his book "Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony" [7], Jim Cushing lucidly defended the position that one of the main aims of physical theory is to provide an intelligible picture of objective physical reality. He argued that the Copenhagen interpretation of quantum mechanics fails to yield such a picture, whereas the Bohm interpretation succeeds. This led him to the historical thesis that the wide acceptance of the Copenhagen interpretation must be understood as a result of external, contingent influences.

Jim Cushing's thesis that not many physicists have given sufficient thought to the interpretational problems of quantum theory in general, and the difficulties of the Copenhagen interpretation in particular, is doubtlessly correct.

It has also to be conceded that objectivity and intelligibility are traditional goals of physics that should not easily be abandoned. However, one can legitimately ask what the exact meaning of these concepts is. In particular, ideas about what is understandable and what is not can be shown to vary over time and to depend on context [8]; similarly, objectivity does not necessarily imply use of the concepts of classical physics. Therefore, it is not obvious that the ideals of objectivity and intelligibility uniquely single out the Bohm theory.

In this paper we will investigate the possibilities to interpret quantum theory in accordance with Jim Cushing's philosophical desiderata. That is, we will attempt to determine what ways there are to interpret the usual mathematical formalism of quantum mechanics in terms of properties possessed by physical systems, independently of consciousness and measurements (in the sense of human interventions). As we will see, Bohm's interpretation is only one of a number of possibilities.

2 Objectivity, no-collapse and modality

The problem of the interpretation of quantum mechanics revolves around two related issues: the status of measurements and the question of whether the quantum state can be interpreted in terms of objective states of affairs, i.e. definite properties possessed by physical systems.

The measurement problem is first of all a consequence of the assumption, made in many presentations of the theory, that a separate evolution mechanism exists—the collapse of the wave function—that applies exclusively to measurements. The idea that measurements possess a special status goes back to the early days of quantum theory, when Bohr spoke about the finite and uncontrollable disturbances caused by interactions with measuring devices. Von Neumann developed the idea in the context of his Hilbert space formalism [20]; according to von Neumann unitary evolution alone would in general not lead to one definite measurement outcome; collapses are needed to break the unitary chain.

However, the lack of a clear-cut, objective, demarcation between measurements and ordinary physical interactions creates a tension between this assumption of measurement collapses and the aim of objectivity. If we aim for an interpretation according to which quantum theory is about a world that exists independently of human thought and experiments, we had better avoid a special status of measurements (in the sense of human interventions).

This thought is reinforced by experimental evidence gathered in the last decades. The evidence in question strongly suggests that the coherence of superpositions is never lost; in other words, that collapses do not occur.

Accordingly, we will concentrate on "no-collapse interpretations" of quantum mechanics. These interpretations aim at dissolving the measurement problem by allowing only unitary evolution: there are no collapses, and measurements are treated just like other interactions, by the Schrödinger equation (or a generalization of it). Because measurements thus lose their special status, it becomes unnatural to construe the content of the theory in terms of outcomes of experiments. The alternative that we will consider is to interpret the formalism as providing information about *properties* of physical systems. As we will see, this addresses the measurement problem and the objectivity issue at one stroke.

In the no-collapse scheme the need to assign a special status to macroscopic devices disappears; there is consequently no longer a reason to introduce a split between a classical and a quantum domain. This makes it possible to think of quantum theory as a universal theory, valid for all systems (of course, this is not meant to involve the belief that present-day quantum theory is the final physical theory; it is sufficient to view quantum mechanics as a *possibly* correct universal theory).

Summing up, the interpretations that we will consider accept the standard mathematical formalism of Hilbert space, but without collapses, and regard it as encoding information about objective physical characteristics of the represented systems. For this, it is important to distinguish between the state as it is defined in Hilbert space (the mathematical state, a vector or a density operator) on the one hand and *physical properties* on the other. By a physical property of a system we mean a definite value of a physical quantity belonging to this system; i.e./ a feature of physical reality, the *physical state* of the system. It is not an a priori obvious matter what the relation is between mathematical and the physical states; i.e., exactly in what way the mathematical state represents the physical properties possessed by the system. Indeed, it is the main task of no-collapse interpretations to unambiguously and consistently specify to which physical properties a state in Hilbert space corresponds.

Physical properties, i.e. values of physical magnitudes, correspond to yes/no propositions (asserting whether or not the properties in question are possessed) and can be represented by orthogonal projection operators in Hilbert space (projecting on the eigenspaces of the observables that represent the definite-valued magnitudes). The task of no-collapse interpretations is thus to make clear what is the set of definite-valued projection operators, once the state in Hilbert space is given.

Now, indeterminism is a notorious feature of the usual interpretation of quantum mechanics. It is a feature grounded in experience: repeated measurements performed on a system that is reproduced in the same quantum state lead to a probability distribution of results. Without collapses, the final stage of such a measurement is described by one entangled total state of object system and measuring device: the same initial object and device states will always lead, in view of the deterministic character of the Schrödinger equation, to the same final mathematical state. Apparently, we must assume that one quantum state can correspond to different possible physical properties (in this example, to different possible measurement results, e.g. pointer positions on a dial). This means that there is not a one-to-one correspondence between the quantum state and physical reality. Rather, the quantum state will determine what *may* be the case, what the *possible* physical situations are.

Thus, the sought-after link between quantum state and physical properties must have a probabilistic character. The quantum state will fix possibilities. In philosophical jargon, the interpretation of the quantum state must contain a *modal* aspect (possibility and necessity are "modalities"). Interpretations along this line have been proposed by various authors. The best known "modal interpretation" is the one put forward by Bohm [3]. Later, other modal no-collapse interpretations have been developed [17, 18, 15, 9, 10, 14]. The term "modal interpretation" itself was coined by B. van Fraassen [17].

As we will presently see, the just-described characteristics of no-collapse interpretations, together with the requirement that they should not rely on things not represented in the quantum formalism, uniquely determine them. All such no-collapse interpretations fall under one general mathematical scheme.

3 The uniqueness of no-collapse interpretations

What we want is a prescription for finding out which properties a system may possess, if the quantum state is given. That is, we want to construct a set of definite-valued observables; the possibilities will then be represented by a probability distribution over value assignments to these observables. Now, it is a notorious feature of the Hilbert space formalism that not all observables can have definite values simultaneously (because of the Kochen-Specker theorem and similar results). The question therefore becomes: what is the maximum set of observables that are definable from the quantum state and can jointly be assigned definite values without getting into contradictions? If the no-collapse idea is to work, we should be able to reproduce the usual quantum probabilities as classical probability measures on value assignments to the observables in this set.

In order to make our results general, we will allow for the possibility that there is a preferred observable R that is always definite, for all quantum states. This will make it possible to include theories like the Bohm theory in our collection of no-collapse interpretations (in the Bohm theory position is a preferred observable). The situation in which no privileged observable exists then becomes a special case.

Consider an arbitrary pure quantum state represented by a ray ψ in a Hilbert space $\mathcal{H}$ and the Boolean algebra or lattice, $\mathcal{B}(R)$, generated by the eigenspaces of the observable R. The usual quantum mechanical probabilities defined by ψ for the values of R can be represented by a probability measure over the 2-valued homomorphisms (consistent assignments of truth values 0 and 1) on $\mathcal{B}(R)$. We now ask for the maximal lattice extension $\mathcal{D}(\psi, R)$ of $\mathcal{B}(R)$, generated by eigenspaces of observables other than R, on which there exist 2-valued homomorphisms such that we can represent in the same way the quantum mechanical probabilities defined by ψ for values of R plus these additional observables.

Since we want to stay within the quantum formalism, and refrain from introducing mathematical structures not present there, we require the definite-valued observables to be definable in Hilbert space in terms of ψ and R. We therefore demand that each element of $\mathcal{D}(\psi, R)$ be invariant under all automorphisms of Hilbert space that preserve the ray ψ and R (note that this requirement is different, and in fact stronger, than the one made in previous work on this subject, [4, 5, 6]—we will come back to the difference below).

We consider an n-dimensional Hilbert space $\mathcal{H}$ ($n < \infty$), and an observable R with $m \leq n$ distinct eigenspaces r_i of $\mathcal{H}$. Let $\psi_{r_i} = (\psi \vee r_i^{\perp}) \wedge r_i$, $i = 1, 2, \ldots, k \leq m$, denote the orthogonal projections of ψ onto the eigenspaces r_i. Now, the set of automorphisms that leave ψ and R invariant includes all automorphisms that are equal to the unity operator everywhere outside of one of the spaces r_i (i.e., when they work on vectors orthogonal to r_i), and are rotations around ψ_{r_i} or reflections with respect to ψ_{r_i} inside r_i. Consider a projection operator P that is to correspond to a definite-valued property. That is, it projects on one of the eigenspaces of a definite-valued observable and represents a definite yes-no proposition. If the subspace of $\mathcal{H}$ on which P projects is contained in one of the r_i, it should be invariant under rotations and reflections with respect to ψ_{r_i}. This leaves four possibilities for the subspace in question: it can be the null-space, ψ_{r_i}, $\psi_{r_i}^{\perp} \wedge r_i$, or r_i.

In the general case, the subspace on which P projects will not be contained in one of the r_i spaces, but will have non-zero projections on a number of them. The requirement that it remains invariant under the above-mentioned automorphisms now implies that its projection on r_i is either null, ψ_{r_i}, $\psi_{r_i}^{\perp} \wedge r_i$, or r_i. All possible subspaces on which P may project are therefore found by taking one of these latter spaces for each value of i, and taking their span.

The lattice of subspaces that correspond to definite propositions is therefore generated by all sublattices $\{0, \psi_{r_i}, \psi_{r_i}^{\perp} \wedge r_i, r_i\}$. In the case that r_i is one-dimensional, ψ_{r_i} is equal to r_i and $\psi_{r_i}^{\perp} \wedge r_i$ equals 0, so that the sublattice then reduces to $\{0, r_i\}$.

It is clear from this construction that the resulting set of definite-valued projection operators is indeed a lattice: it is closed under the lattice operations of disjunction and conjunction (corresponding to taking the span or intersection of the associated eigenspaces). Moreover, the lattice is Boolean: all projection operators in it commute with each other, as is clear from inspection of the generators of the lattice. Therefore, no Kochen and Specker-type paradoxes can arise, and the quantum mechanical probabilities (including joint probabilities) can be represented by means of a classical probability distribution on the lattice.

The above construction made use of the existence of a preferred observable, namely R. The assumption that there is such an observable was inspired by the example of the Bohm theory, which is recovered when R is taken to be the position operator. (Strictly speaking we have not really dealt with the Bohm case, because there the preferred observable is continuous—but using the Dirac formalism with the vectors $|q\rangle$ spanning one-dimensional eigenspaces of the position operator immediately leads to the result in an informal way.) In discussions about hidden-variable theories the objection is often made that the selection of a preferred observable "is made by hand"; indeed, the existence of a privileged observable is not a natural part of the quantum mechanical formalism. It is therefore worth-while to see what happens if no additional structural elements are introduced—if the state $|\psi\rangle$, and the Hilbert space structure, are the only things that are used to define the definite properties.

A possible way of implementing this idea is to take the projection on $|\psi\rangle$ itself for R. If we denote the subspace orthogonal to ψ by $\psi^{\perp}$, we obtain the definite lattice consisting of the subspaces $\{0, \psi, \psi^{\perp}, \mathcal{H}\}$. Exactly the same result is obtained if we take the unity operator on $\mathcal{H}$ for R. These choices therefore lead to the "orthodox" property assignment: only observables of which $|\psi\rangle$ is an eigenvector qualify as definite-valued ([5]). This traditional way of assigning properties at first sight returns us to the measurement problem, because after a measurement the combined system of measuring device and object system ends up in an entangled state which does not correspond to a definite "pointer state" property. However, on second thoughts the situation is not so obvious. The projection operator $|\psi\rangle\langle\psi|$ is an observable of the *total* system, and the property assignment pertains likewise to this total system; whereas we are really interested in the *individual* properties of device and object after the measurement. Therefore, we need substitutes for $|\psi\rangle\langle\psi|$ that represent the states of these individual systems. In the context of standard quantum mechanics such operators readily suggest themselves, namely the *density operators* for the partial systems. This prompts a consideration of the definite lattices that result if the operators $W_1 \otimes I$ and $I \otimes W_2$ are taken for R (here we assume that the total Hilbert space is the tensor product of Hilbert spaces belonging to the partial systems, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$).

The eigenspaces of $W_1 \otimes I$ are $w_i \otimes \mathcal{H}_2$. We can write $|\psi\rangle$ as a biorthogonal decomposition

$$|\psi\rangle = \sum_{k,j} c_{k,j} |\alpha_{k,j}\rangle \otimes |\beta_{k,j}\rangle, \tag{1}$$

with $|\alpha_{k,j}\rangle$ in $\mathcal{H}_1$, $|\beta_{k,j}\rangle$ in $\mathcal{H}_2$, $\langle\alpha_{l,i}|\alpha_{m,j}\rangle = \delta_{lm}.\delta_{ij} = \langle\beta_{l,i}|\beta_{m,j}\rangle$. The index j takes possible degeneracies into account: $|c_{k,j}|^2$ depends only on k, not on j. The projection of $|\psi\rangle$ on $w_i \otimes \mathcal{H}_2$ is now given by $|\psi_i\rangle = \sum_j c_{i,j} |\alpha_{i,j}\rangle \otimes |\beta_{i,j}\rangle$. The lattice of definite properties is generated by the sublattices $\{0, \psi_i, \psi_i^{\perp} \wedge (w_i \otimes \mathcal{H}_2), w_i \otimes \mathcal{H}_2\}$. We can now restrict this lattice to a lattice of definite properties of the first system alone (represented in $\mathcal{H}_1$) by looking for all definite projections of the form $P \otimes I$; the projection operators P then represent properties of system 1 by itself. Inspection of the lattice shows that all projections of the sought form are generated by the projectors $P_{w_i} \otimes I$. The restriction of the lattice of definite properties of the combined system to a lattice of definitive properties of system 1 is therefore the Boolean lattice generated by the projections P_{w_i}. These are the same properties as assigned by modal interpretations of the type discussed in [9, 10, 19]. In measurement situations of the kind first discussed by von Neumann ([20]) these definite properties correspond to pointer positions, so that the measurement problem disappears: when the measurement has been completed, the measuring device indicates a definite result.

The analysis just given is similar to the one worked out by Bub, Clifton, and Goldstein [4, 5, 6]. The essential difference is that Bub and Clifton required the *lattice* of definite properties to be definable from $|\psi\rangle$ and R, whereas we have imposed the stronger demand that the *individual definite properties* themselves be so definable (in both cases this is a "no-hidden variables demand" in the sense that we do not want to accept properties that remain hidden from view once we have specified $|\psi\rangle$ and R). Our stronger requirement makes the analysis considerably simpler. As was to be expected, the lattice of definite properties that we found above on the basis of the stronger requirement is included in the lattice determined by Bub and Clifton. The latter is given by $\{p : \psi_i \leq p \text{ or } \psi_{r_i} \leq p^{\perp}, \text{ for all } i = 1, \ldots, k\}$, or equivalently, the commutant of the set of all the non-null projectors on ψ_i, for all $i = 1, \ldots, k$. The definite projection operators that we have determined above also commute with this set of projectors, so that our set of definite properties indeed forms a sublattice of the Bub-Clifton lattice. The latter lattice possesses more "fine structure", namely projection operators that cannot be defined individually but still belong to the lattice, which is defined as a whole. But although the two approaches are distinct, they lead to very similar results. For the measurement situation we have just discussed, the only final difference is that in the Bub-Clifton approach all individual one-dimensional projections within the null-space of W_1 are definite, whereas in our approach it is only the projector on this null-space as a whole that is definite.

In our derivation of the "modal lattice" we made use of the notion that W_1 can be considered the *state* of system 1. Although this assumption is usually made in standard quantum mechanics, it is not uncontroversial in foundational discussions—it is not self-evident that it is justified in the present context. It would therefore be helpful if we could give an independent derivation that does not presuppose that W_1 is a preferred observable.

In order to do so we make use of the biorthogonal way of writing $|\psi\rangle$, Eq. (1). As stated, our aim is to determine those properties of system 1 that can be defined from this state plus the structure of Hilbert space. In particular, we will use that $\mathcal{H}$ is the tensor product of the Hilbert spaces of the individual systems 1 and 2, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. We want the definite properties of system 1 to be invariant under automorphisms that leave $|\psi\rangle$ the same; but clearly there are no non-trivial such automorphisms that operate in $\mathcal{H}_1$ alone (of the form $U_1 \otimes I$). However, there are such automorphisms that have the form $U_1 \otimes U_2$, with U_1 and U_2 defined on $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively [12]. These automorphisms with product form define individual automorphisms in the two Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$. So we focus on automorphisms $U_1 \otimes U_2$ that leave $|\psi\rangle$ invariant, and ask which projectors in $\mathcal{H}_1$ remain the same under their operation (in other words, under the operation of the associated U_1). Now, the biorthogonal form (1) is unique up to unitary transformations within the subspaces spanned by the vectors $\{|\alpha_{k,j}\rangle\}_j$ (and $\{|\beta_{k,j}\rangle\}_j$). These spaces are labelled by values of k. Subspaces contained *within* these spaces are not invariant under all transformations that preserve $|\psi\rangle$; but the spaces themselves, spanned by $\{|\alpha_{k,j}\rangle\}_j$ ($\{|\beta_{k,j}\rangle\}_j$), are. These spaces are exactly the eigenspaces of the reduced density operator W_1. So we arrive at precisely the same conclusion as before: the lattice of those properties of system 1 that can be defined on the basis of $|\psi\rangle$ alone, is generated by the projection operators P_{w_i}. Since this lattice is Boolean, definite values can be assigned to its elements without contradictions, and the quantum mechanical probabilities on the lattice can be represented in a classical Kolmogorovian probability space.

4 Definite properties without collapses

Let us sum up: the no-collapse schemes that we have discussed are about definite properties (definite measurement outcomes, for example) even if the quantum state $|\psi\rangle$ is a superposition of the eigenvectors associated with the properties in question. To review how this is achieved, consider a composite physical system, consisting of two parts. Let us assume for simplicity that there is no degeneracy, so that the bi-orthonormal decomposition is unique and has he following form:

$$|\psi\rangle = \sum_k c_k |\psi_k\rangle \otimes |R_k\rangle, \tag{2}$$

with $|\psi_k\rangle$ in $\mathcal{H}_1, |R_k\rangle$ in $\mathcal{H}_2$, $\langle\psi_i|\psi_j\rangle = \delta_{ij}$ and $\langle R_i|R_j\rangle = \delta_{ij}$.

The modal no-collapse interpretation that we have dealt with at the end of the foregoing section gives the following physical meaning to this mathematical state. The projection operators $\{|\psi_k\rangle\langle\psi_k|\}$ represent definite properties of the system described in $\mathcal{H}_1$: exactly one of the mentioned projectors is assigned the value 1, the others get the value 0. The interpretation thus selects, on the basis of the form of the state $|\psi\rangle$, the set of quantities that are definite-valued. All observables that are functions of these projectors, namely hermitian operators with spectral resolution given by $\Sigma a_k|\psi_k\rangle\langle\psi_k|$, are also definite-valued and possess one of their possible values (their eigenvalues).

The Born probabilities are reproduced as an ordinary probability distribution on the Boolean lattice generated by $\{|\psi_k\rangle\langle\psi_k|\}$. The probability that the *l-th* possibility is actually realized (that $|\psi_l\rangle\langle\psi_l|$ has the value 1) is given by $|c_l|^2$.

In the case of degeneracy, that is $|c_j|^2 = |c_i|^2$, for $i, j \in I_l$ (with I_l a set of indices), the one-dimensional projectors have to be replaced by multi-dimensional projectors $P_l = \sum_{i\in I_l} |\psi_i\rangle\langle\psi_i|$; the physical properties now correspond to these projectors. The class of definite-valued physical quantities in this case contains only non-maximal hermitian operators characterized by their spectral resolution $\Sigma a_k P_k$. The probability of value a_l is given by $\sum_{i\in I_l} |c_i|^2$.

The relation between mathematical state and actual physical properties is therefore probabilistic. The probabilities materialize as relative frequencies in repetitions of the situation described by the same state. In the individual case, the probabilities quantify the information provided by the state about the actual state of affairs. In general, these probabilities will not be 1 or 0. This expresses the fact that the physical situation generally could have been different from what it actually is, given the mathematical state.

The situation after an ideal (von Neumann) *measurement* will also be described by a superposition of the form (2), with $|\psi_k\rangle$ denoting states of the object system and $|R_k\rangle$ states of the measuring device ("pointer position states"). The physical meaning of this mathematical state according to our no-collapse interpretation is that *one* of the pointer positions is actually realized. We thus see that the traditional argument for the occurrence of a collapse of the wave function no longer has force. The argument in question says that if a definite result is obtained in a measurement, the state immediately after the measurement should reflect the presence of this result: and this is taken to mean that it should be the corresponding eigenstate. Accordingly the measurement must induce a transition—the collapse—from a superposition of eigenstates to one of the terms in the superposition. By contrast, in our scheme a system can possess a definite property even if the state in Hilbert space is not an eigenstate of the associated observable.

It therefore becomes possible to consistently assume that the evolution of the mathematical state is unitary at all times. Measurements are treated as ordinary physical interactions between measuring device and object system, both treated by quantum mechanics.

5 Perspectivalism

The interpretational scheme that we have outlined salvages the idea of an objective reality that is characterized by definite values of physical quantities. But the ensuing picture is nevertheless very different from what we have become used to in classical physics. In our scheme the definite-valued quantities are defined by the *total* quantum state, representing both the object system and the rest of the universe. By contrast, classical particles possess properties like position and momentum quite independently of whether there is any interaction with the rest of the world. If good measurements are made on such classical objects, the results reflect these pre-existing values. However, in our quantum scheme the outcome of a measurement is in general not the reflection of a pre-existing object system property.

The latter point is illustrated by experiments of the Einstein-Podolsky-Rosen type. In a modern version, two electrons whose total spin state is the singlet state, with vanishing total spin, fly apart until their mutual distance has become very great. Subsequently, spin measurements are made on the individual particles. For each particle, there is the choice of measuring the spin in either one of two directions. The experiment can be repeated with different choices of these directions, so that four combinations of directions will be measured in the series of repetitions. Correlations between outcomes in these four pairs of directions are predicted by quantum mechanics (and verified in actual experiments). As is well known, for some choices of the spin directions these correlations violate Bell inequalities.

It is a mathematical fact that the Bell inequalities remain satisfied as long as the spin values found in the measurements on the individual particles can be regarded as coming from one joint probability distribution [13]. The latter would be the case if the measurement results were determined by spin values already jointly possessed by the electrons, independently of which—or whether—measurements are made. If that were true, there would be well-defined, definite spin values in the four directions under discussion in each run of the experiment; in repetitions these values would vary and form an ensemble that defines a joint distribution of the four spin quantities. Only two of them could actually be measured in any single experimental run (one direction for each particle); but the measured values would evidently be samples from a joint distribution. The violation of Bell's inequalities by the predictions of quantum mechanics, and by the experimental results, therefore shows that we cannot think of the EPR situation in classical terms—the measurements do not reveal pre-existing jointly defined quantities.

See Figure 1 for a schematic representation of the situation: either σ_1 or σ_1' is measured on electron 1, and similarly for electron 2. The two horizontal and two diagonal lines symbolize the four possible combinations of measurements. The vertical double lines represent the electrons.

This result is shocking for the classical intuition, but it is in complete accordance with the mathematical structure of quantum mechanics and with the way properties are assigned in our modal no-collapse interpretation; in this sense it is intelligible, and it seems not unreasonable to expect that intuition can adapt itself to the situation [8]. That the violation of Bell inequalities has been empirically verified is of course strong evidence that a completely classical account cannot be maintained anyway. Either non-local interactions should be allowed (this happens in Bohm's interpretation) or it should be accepted that the choice of which observables are definite-valued is made through the interaction with the rest of the universe (see [11] for more details about how this works out for the modal scheme in the EPR situation). However, it turns out that an argument that is analogous to Bell's reasoning can be applied to a new situation, with consequences that appear even more drastic [16].

Consider two well-localized systems, S_i, $i = 1, 2$. Let α and β be two hyperplanes of simultaneity for some reference frame Σ. Let E_i be the places

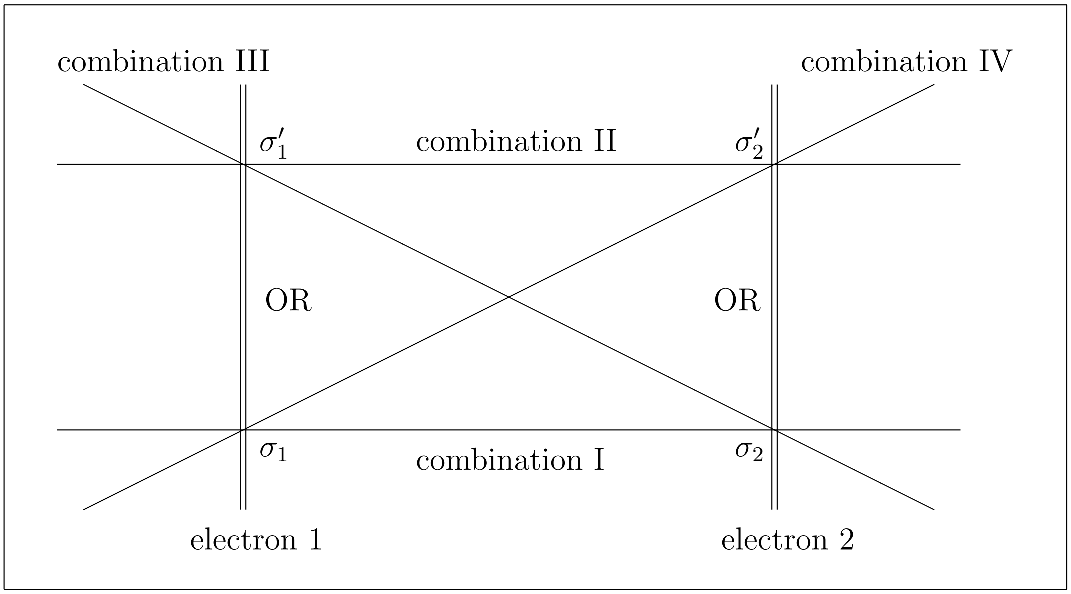

Figure 1. The four possible combinations of spin measurements.

where the systems S_i are located on α, and let F_i be the corresponding regions on β (see Figure 2). We assume that the two systems are sufficiently far apart that E_1 is spacelike separated from F_2, and E_2 is spacelike separated from F_1. Let γ be a spacelike hypersurface containing F_1 and E_2, and let δ be a spacelike hypersurface containing E_1 and F_2.

If S_1 and S_2 are isolated during their evolution between α and β there will be unitary operators U_i such that the state ρ of the combined system $S_1 \oplus S_2$ on β will be related to its state on α by

$$\rho(\beta) = U_1 \otimes U_2 \ \rho(\alpha) \ U_1^\dagger \otimes U_2^\dagger. \tag{3}$$

If the regions E_1, E_2, F_1, F_2 are sufficiently small, they may be treated as points, and we may regard γ and δ as hyperplanes of simultaneity for reference frames Σ', Σ'', respectively. Let $\rho(\gamma)$ be the state on hypersurface γ, and let $\rho(\delta)$ be the state according on δ. On the basis of the assumption of unitary evolution between α and β, the states on the hyperplanes γ and δ can easily be related to $\rho(\alpha)$. We find:

$$\rho(\gamma) = U_1 \otimes I_2 \ \rho(\alpha) \ U_1^\dagger \otimes I_2, \tag{4}$$

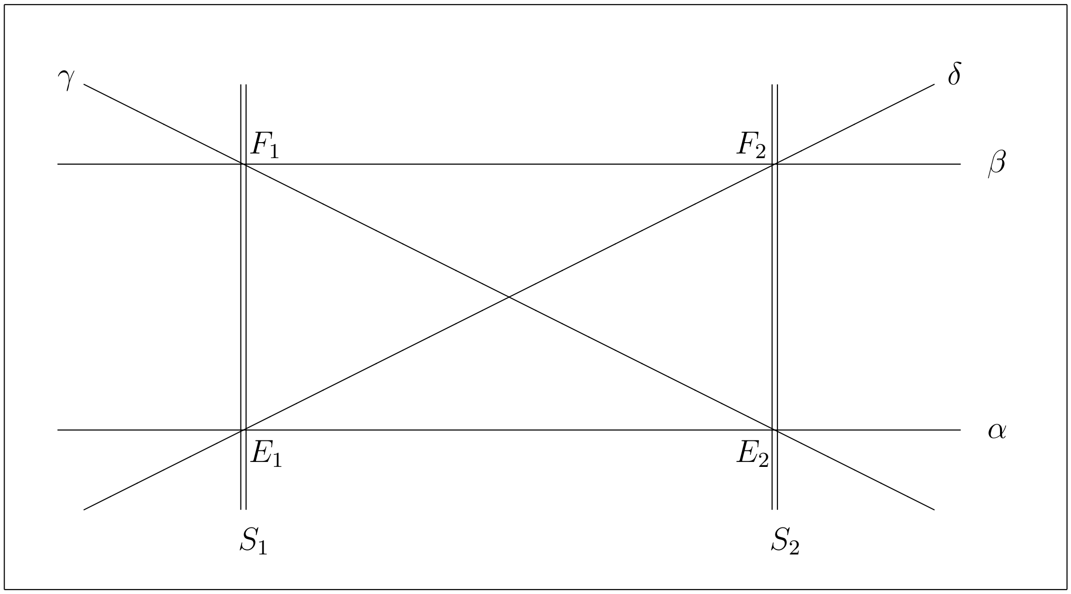

Figure 2. The four simultaneity hyperplanes α, β, γ and δ.

and similarly

$$\rho(\delta) = I_1 \otimes U_2 \; \rho(\alpha) \; I_1 \otimes U_2^\dagger. \tag{5}$$

Now suppose that A_1 and A_2 are definite properties of S_1 and S_2, respectively, on α, and B_1 and B_2 are definite properties on β, as determined by purely local interactions at their positions via the prescriptions of the previous sections [16]. What we would expect is that the value of A_1 possessed by S_1 at E_1 is possessed by it without reference to the hypersurface containing E_1 that is contemplated, and similarly for the other points of intersection E_2, F_1, F_2. It seems natural to assume that what happens at these four space-time points are events that do not depend for their description on the way the space-time is sliced up into simultaneity hyperplanes. But if this were true, there would be a joint probability distribution over the values of our four observables that yields as marginals the quantum mechanical Born probabilities on all four hyperplanes. In this we have assumed the central tenet of special relativity, namely that the different frames of reference are equivalent; in our case that the Born probability rule applies equally on α, β, γ and δ.

But the states on the various hyperplanes are interrelated, as indicated in Eqs. (4, 5). By inspection of these relations we find that the existence of such a joint distribution is equivalent to the existence of a joint distribution calculated in *one* state, namely $\rho(\alpha)$, and yielding, as marginals, the statistics for the observables $A_1 \otimes A_2$, $A_1 \otimes C_2$, $C_1 \otimes A_2$, $C_1 \otimes C_2$, where

$$C_i = U_i^\dagger\, B_i\, U_i. \tag{6}$$

However, as we have explained for the case of the EPR-experiment, such a joint distribution of four non-commuting observables, yielding the quantum mechanical Born marginals for the pairs of observables, cannot exist in general [13]. Bell inequalities will be violated in some states and for some observables; see [16] for the construction of an explicit example. This violation of a Bell inequality entails the nonexistence of a joint distribution. Therefore, if $\rho(\alpha)$ is a state such that a Bell inequality is violated for the observables A_1, C_1, A_2, C_2, then it cannot be the case that A_1 at E_1, A_2 at E_2, B_1 at F_1, and B_2 at F_2 constitute space-time events that exist independently of the context, i.e. the spacelike separated events with which they are correlated.

The argument here mimics the earlier Bell argument: the mathematics is the same. The structural identity of the two arguments can also be seen clearly from the similarity between Figure 1 and Figure 2. The symbols have different meanings, but the mutual relations are the same. In the original Bell case locality was at issue: in that case one could argue that the distant measurement setting has a non-local influence on the situation at the nearby side of the experiment. However, only one distant setting is actually realized in any run of the experiment, so that the differences are differences with respect to *what could have been the case*. By contrast, in the new case *both* hyperplanes on which the event we consider lies are actually present. We can therefore no longer argue in terms of what *would* have happened *if* some other situation *were* real, and the non-local disturbances that *would* take place. We are now compelled to conclude that it must make a difference whether we view what happens in E_1, e.g., from the perspective of E_2 or from the perspective of F_2. In other words, events are not just there, but are different depending on the hyperplane of which they are considered a part! Apparently we must relativize the description of events, and the properties of physical systems, to a *perspective* from which the description takes place (see [1, 2] for attempts along these lines).

This result is even more perplexing than the conclusions drawn from the original violations of Bell inequalities. In the original Bell case property assignments, and measurement outcomes, were shown to be contextual, or subject to non-local influences. But since only one measurement can actually be made, no conflict arises with the idea that the property in question is part of a unique and objective space-time event. In our new case, the different contexts, i.e. the different hyperplanes, are jointly actual. So events can in general no longer be unique and objective in themselves.

6 Conclusion

Jim Cushing stood up for the idea that physics should strive for an intelligible picture of a world that exists independently of consciousness and measurements. That idea can indeed be implemented, even in the quantum mechanical context. The Bohm interpretation, Jim Cushing's favorite, provides an example whose intelligibility standard stays close to the classical norm. An objection to the Bohm theory is that it achieves its aims by introducing additional theoretical structure (the preferred observable) that does not figure in the standard quantum formalism. There is another possibility, motivated by the same philosophy and falling under the same general category of interpretations, that stays closer to the structure of the Hilbert space formalism. Like the Bohm theory, this is a no-collapse interpretation with a "modal" character: it predicts possibilities rather than one unique course of events.

Admittedly, this interpretation deviates more from classical ideas than the Bohm picture with its particles and trajectories. However, it is only natural that the standards of what are intelligible physical pictures change with scientific developments [8]. That quantum theory gives rise to new conceptions of intelligibility is something to be expected, especially since the violations of Bell inequalities have shown that not all classical explanatory ideals can be maintained together. That it is not fixed a priori what the definite properties of physical systems are, but that this depends on the total quantum state and therefore on the context, i.e. the relation with the rest of the world, is therefore not something to be rejected out of hand. In fact, ideas of this kind come up naturally within quantum mechanics: some of the founding fathers of the theory, in particular Bohr, already suggested conceptions of this kind (though not on the formal basis presented here).

However, it appears that the quantum theory may require a change in conceptual standards that goes even further. As the analysis of the previous section shows, we cannot have a Lorentz covariant interpretation of quantum theory if we hold on to the idea that events are hyperplane-independent. One response would be to drop the requirement of complete Lorentz covariance—this is the course taken by most adherents of the Bohm interpretation. Here one accepts that there exists a preferred frame of reference, even though this frame cannot be determined by empirical means. The alternative, which is more in line with the ideas underlying relativity theory, is to insist on Lorentz covariance; but then hyperplane independence must go. This means that what properties a physical system possesses depends not only on its interaction with its direct environment, but also on the simultaneity hyperplane on which the system is taken to lie. That would imply a much more radical relativizing of properties than we have seen before. Suggestions to implement this in the modal no-collapse scheme have already been made [1, 2], but more work remains to be done.

The various ways in which the quantum theory can be interpreted can, as we have illustrated, be analyzed and categorized in a systematic way. Work in the foundations of quantum mechanics is clearly making progress. Without the persistence and originality of determined and enthusiastic researchers like Jim Cushing, this would never have happened.

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