Hindawi
Complexity
Volume 2018, Article ID 5139840, 10 pages
https://doi.org/10.1155/2018/5139840

Research Article

Estimation and Synthesis of Reachable Set for Singular Markovian Jump Systems

Yucai Ding and Hui Liu

School of Science, Southwest University of Science and Technology, Mianyang 621010, China

Correspondence should be addressed to Yucai Ding; dycyer@163.com

Received 31 January 2018; Accepted 28 March 2018; Published 26 June 2018

Academic Editor: Sing Kiong Nguang

Copyright © 2018 Yucai Ding and Hui Liu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The problems of reachable set estimation and state-feedback controller design are investigated for singular Markovian jump systems with bounded input disturbances. Based on the Lyapunov approach, several new sufficient conditions on state reachable set and output reachable set are derived to ensure the existence of ellipsoids that bound the system states and output, respectively. Moreover, a state-feedback controller is also designed based on the estimated reachable set. The derived sufficient conditions are expressed in terms of linear matrix inequalities. The effectiveness of the proposed results is illustrated by numerical examples.

1. Introduction

The research on singular systems has attracted significant attention in the past years due to the fact that singular systems can better describe a larger class of physical systems such as robotic systems, electric circuits, and mechanical systems. When singular systems experience abrupt changes in their structures, it is natural to model them as singular Markovian jump systems [1, 2]. The analysis and synthesis of such class of systems have gained considerable attention because of their importance in applications (see, e.g., the literature [3–10] and the references therein).

Reachable set is one of the important techniques for parameter estimation or state estimation problems [11]. Reachable set for a dynamic system is the set containing all the system states starting from the origin under bounded input disturbances. However, the exact shape of reachable sets of a dynamic system is very complex and hard to obtain; for this reason a number of researchers began to turn their attention to the reachable set estimation problem. The common strategies for reachable set estimation are ellipsoidal method [12] and polyhedron method [13]. The main idea of these methods is to detect simple convex shapes like ellipsoid or polyhedron, which contains all the system states. Compared with polyhedron method, the primary advantage of ellipsoidal method is that the ellipsoid structure is simple and directly related to quadratic Lyapunov functions. As a result, linear matrix inequalities (LMIs) techniques can be used to determine bounding ellipsoids. In the framework of bounding ellipsoid, the reachable set estimation problem for linear time delay systems has received significant research attention in recent years. In [14] sufficient conditions for the existence of bounding ellipsoids containing the reachable set of continuous-time linear systems with time-varying delays were derived by using the Lyapunov-Razumikhin function. In [15], by using Lyapunov-Krasovskii functional method, the author derived some less conservative conditions than those in [14]. In [16] the reachable set of delayed systems with polytopic uncertainties was investigated by using the maximal Lyapunov-Krasovskii functional approach, and some new conditions bounding the set of reachable states are derived. Interesting results on reachable set of delayed systems with polytopic uncertainties can also be found in [17–20]. In addition, some other strategies without using Lyapunov-Krasovskii functional have been provided to estimate the reachable set of continuous-time linear time-varying systems [21] and nonlinear time delay systems [22, 23]. The authors in [24] extended the ideas of reachable set estimation of continuous-time systems to discrete-time systems, wherein a fundamental result (Lemma 2.1 [24]) for the reachable set estimation of discrete-time systems was proposed. The authors in [25] improved the fundamental result obtained in [24] and provided a basic tool (Lemma 4 [25]) for the reachable set estimation of discrete-time systems. On the basis of the general ideas proposed in [25], the reachable set estimation problem was also extended to some classes of complicated systems, such as singular systems [26], Markovian jump systems [27], switched linear systems [28], and T-S fuzzy systems [29, 30]. For the reachable set estimation of discrete-time systems, the other important contributions can be found in [31, 32]. On the other hand, the problem of controller design for specifications involved with the reachable set of a control system is also a very important issue [33]. The controller design problems concerning reachable set were studied in [34] and [35] by using ellipsoidal method and polyhedron method, respectively. Two issues were raised in [34]: the first one is to design a controller such that the reachable set of the closed-loop system is contained in an ellipsoid, and the admissible ellipsoid should be as small as possible; the second one is to design a controller such that the reachable set of the closed-loop system is contained in a given ellipsoid. By constructing suitable Lyapunov-Krasovskii functional, LMI-based sufficient conditions for the existence of controller guaranteeing the ellipsoid bounds as small as possible have been derived for continuous-time delay systems [34] and discrete-time periodic systems [36]. It is obvious that the LMI-based controller design is quite simple and numerically tractable. However, it should be pointed out that the reachable set estimation and synthesis problems of singular Markovian jump systems are much more difficult and challenging than that for nonsingular Markovian jump systems since the ellipsoid containing the reachable set is not directly related to quadratic Lyapunov functions. To the best of the authors' knowledge, no related results have been established for reachable set estimation and synthesis of singular Markovian jump systems, which has motivated this paper.

In this paper, we consider the problems of reachable set estimation and synthesis of singular Markovian jump systems. By using the Lyapunov approach, the estimation conditions on state reachable set and output reachable set are derived, respectively. Moreover, the desired state-feedback controller is designed based on the estimated reachable set.

Notation. Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; A^T represents the transpose of A; $\mathrm{Sym}(M)$ stands for $M + M^T$; $X > 0$ (<0) means X is a symmetric positive (negative) definite matrix; $\mathbb{E}\{\cdot\}$ refers to the expectation; $X_{l\times m}$ denotes the matrix composed of elements of first l rows and m columns of matrix X; $\|\cdot\|$ refers to the Euclidean vector norm; the symbol "$*$" in LMIs denotes the symmetric term of the matrix; I is the unit matrix with appropriate dimensions.

2. Problem Formulation

Consider the following singular Markovian jump system:

$$\begin{aligned} E\dot{x}(t) &= A(r_t)x(t) + B(r_t)u(t) + D(r_t)\omega(t) \\ y(t) &= C(r_t)x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the measured output, and $\omega(t) \in \mathbb{R}^q$ is the exogenous disturbance which satisfies

$$\omega^T(t)\omega(t) \leq \overline{\omega}^2. \tag{2}$$

E, $A(r_t)$, $B(r_t)$, $C(r_t)$, and $D(r_t)$ are real constant matrices with appropriate dimensions and $\mathrm{rank}(E) = l < n$. $\{r_t, t \geqslant 0\}$ is a continuous-time Markovian process with transition rate matrix $\Pi = [\pi_{ij}]$ ($i \in \mathcal{S} = \{1, 2, \dots, N\}$) and the evolution of Markovian process is governed by the following transition rate:

$$\Pr\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & j \neq i \\ 1 + \pi_{ii}\Delta + o(\Delta), & j = i, \end{cases} \tag{3}$$

where $\Delta > 0$ and $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$; $\pi_{ij} \geqslant 0$ for $i \neq j$ is the transition rate from mode i to mode j and $\pi_{ii} = -\sum_{j=1,j\neq i}^{N}\pi_{ij}$.

For notational simplicity, in the sequel, for each possible $r_t = i$, $i \in \mathcal{S}$, matrices $A(r_t)$, $B(r_t)$, $C(r_t)$, and $D(r_t)$ will be denoted by A_i, B_i, C_i, and D_i, respectively. When $u(t) = 0$, the system $E\dot{x}(t) = A_i x(t) + D_i\omega(t)$ is referred to as a free system.

In this paper we are interested in determining ellipsoids that contain, respectively, the state reachable set and output reachable set. In the reachable set analysis, it is required that systems should be asymptotically stable. When this requirement is not met, we will further design a state-feedback controller such that the reachable set of the closed-loop system is contained in the smallest ellipsoid.

The state reachable set of the free system in (1) is defined by

$$\begin{aligned} \mathcal{R}_x = \{x(t) \in \mathbb{R}^n \mid x(t),\ t \\ \geq 0,\ \text{is a solution of (1) for } x(0) = 0\}. \end{aligned} \tag{4}$$

An ellipsoid $\mathcal{E}(X)$ bounding the reachable set can be always represented as follows:

$$\mathcal{E}(X) = \{x \in \mathbb{R}^n \mid x^T X x \leq 1,\ X > 0\}. \tag{5}$$

Particularly, when $X = \kappa I$ for $\forall\kappa > 0$, the ellipsoid $\mathcal{E}(X)$ will become a ball which is denoted by $\mathcal{B}(X)$.

Since $\mathrm{rank}(E) = l < n$, there exist two nonsingular matrices M and N such that

$$\begin{aligned} MEN &= \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \\ MA_iN &= \begin{bmatrix} A_{11i} & A_{12i} \\ A_{21i} & A_{22i} \end{bmatrix}, \\ MD_i &= \begin{bmatrix} D_{1i} \\ D_{2i} \end{bmatrix}. \end{aligned} \tag{6}$$

Let $N^{-1}x(t) = \tilde{x}(t) = [\tilde{x}_1^T(t)\ \ \tilde{x}_2^T(t)]^T$. Then the free system can be rewritten as the following differential-algebraic form:

$$\dot{\tilde{x}}_1(t) = A_{11i}\tilde{x}_1(t) + A_{12i}\tilde{x}_2(t) + D_{1i}\omega(t) \tag{7}$$

$$0 = A_{21i}\tilde{x}_1(t) + A_{22i}\tilde{x}_2(t) + D_{2i}\omega(t). \tag{8}$$

The following definition and lemma are also useful in deriving the main results.

Definition 1 (see [2]). (I) The free system is said to be regular if $\det(sE - A_i)$ is not identically zero for each $i \in \mathcal{S}$.

(II) The free system is said to be impulse-free if $\deg(\det(sE - A_i)) = \mathrm{rank}(E)$ for each $i \in \mathcal{S}$.

Lemma 2 (see [2]). *For any matrices U and $V \in \mathbb{R}^{n\times n}$ with $V > 0$, one has $UV^{-1}U^T \geq U + U^T - V$.*

Lemma 3 (see [12]). *Let $V(x(t))$ be a Lyapunov function and $V(x(0)) = 0$. If*

$$\dot{V}(x(t)) + \alpha V(x(t)) - \frac{\alpha}{\overline{\omega}^2}\omega^T(t)\omega(t) \leq 0, \quad \alpha > 0, \tag{9}$$

then $V(x(t)) \leq 1$, $\forall t \geq 0$.

3. Main Results

3.1. State Reachable Set Estimation. In this subsection, we will focus our attention on determining a ball which contains the state reachable set of the free system.

Theorem 4. *If there exist nonsingular matrices $X_i \in \mathbb{R}^{n\times n}$ and a scalar $\alpha > 0$ such that the following LMIs hold for each $i \in \mathcal{S}$,*

$$E^T X_i = X_i^T E \geq 0, \tag{10}$$

$$\mathcal{H} = \begin{bmatrix} \mathrm{Sym}\left(A_i^T X_i\right) + \sum_{j=1}^{N}\pi_{ij}E^T X_j + \alpha E^T X_i & X_i^T D_i \\ * & -\frac{\alpha}{\overline{\omega}^2} I \end{bmatrix} < 0, \tag{11}$$

then the state reachable set of free system starting from the origin is mean-square bounded within the following set:

$$\bigcap_{i=1}^{N}\mathcal{E}\left(\widehat{X}_i\right) = \bigcap_{i=1}^{N}\left\{x \in \mathbb{R}^n \mid x^T\widehat{X}_i x \leq 1\right\}, \tag{12}$$

where

$$\begin{aligned} \widehat{X}_i &= \frac{1}{r_i^2}N^{-T}N^{-1} \\ r_i &= \frac{1}{\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}\left(\widetilde{X}_i\right)}} + \left\|A_{22i}^{-1}\right\|\left(\frac{\left\|A_{21i}\right\|}{\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}\left(\widetilde{X}_i\right)}} + \left\|D_{2i}\right\|\overline{\omega}\right) \\ \widetilde{X}_i &= \left(M^{-T}X_i N\right)_{l\times l}. \end{aligned} \tag{13}$$

Proof. We first prove the regularity and nonimpulsiveness of the free system. Let $M^{-T}X_i N = \left[\begin{smallmatrix} X_{11i} & X_{12i} \\ X_{21i} & X_{22i}\end{smallmatrix}\right]$. Then, by (10), we obtain that $X_{12i} = 0$. From (11), it is easy to show that

$$\mathrm{Sym}\left(A_i^T X_i\right) + \sum_{j=1}^{N}\pi_{ij}E^T X_j + \alpha E^T X_i < 0. \tag{14}$$

Pre- and postmultiplying (14) by N^T and N, respectively, we get

$$\begin{bmatrix} \star & \star \\ * & A_{22i}^T X_{22i} + X_{22i}^T A_{22i} \end{bmatrix} < 0, \tag{15}$$

where $\star$ will be irrelevant to the results of the following discussion; thus the real expressions of these two variables are omitted. It follows from (15) that

$$A_{22i}^T X_{22i} + X_{22i}^T A_{22i} < 0 \tag{16}$$

which implies that A_{22i} is nonsingular for each $i \in \mathcal{S}$. Therefore, by Definition 1, we have that the free system is regular and nonimpulsive.

Next, we will show the state reachable set of free system is mean-square bounded within the set $\mathcal{E}(\widehat{X})$. Consider the following Lyapunov function:

$$V\left(x(t), r_t\right) = x^T(t)E^T X_i x(t). \tag{17}$$

Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{x(t), r_t\}$. Calculating the difference of $V(x(t), r_t)$ along the trajectories of the free system, we get

$$\begin{aligned} \mathcal{L}V\left(x(t), r_t\right) &= \dot{x}^T(t)E^T X_i x(t) + x^T(t)X_i^T E\dot{x}(t) \\ &\quad + x^T(t)\sum_{j=1}^{N}\pi_{ij}E^T X_j x(t). \end{aligned} \tag{18}$$

Defining the augmented system variable as $\xi(t) = [x^T(t)\ \omega^T(t)]^T$ and using conditions (10) and (11), we have

$$\begin{aligned} &\mathcal{L}V\left(x(t), r_t\right) + \alpha V\left(x(t), r_t\right) - \frac{\alpha}{\overline{\omega}^2}\omega^T(t)\omega(t) \\ &\quad = \xi^T(t)\mathcal{H}\xi(t) < 0. \end{aligned} \tag{19}$$

Then we can deduce from Lemma 3 that $\mathbb{E}\{x^T(t)E^T X_i x(t)\} \leq 1$, which infers that

$$\mathbb{E}\left\{\widetilde{x}^T(t)N^T E^T M^T M^{-T}X_i N\widetilde{x}(t)\right\} \leq 1. \tag{20}$$

Recalling that $X_{12i} = 0$, it follows from (20) that

$$\mathbb{E}\left\{\widetilde{x}_1^T(t)\widetilde{X}_i\widetilde{x}_1(t)\right\} \leq 1. \tag{21}$$

From (21) we have $\min_{i\in\mathcal{S}}\lambda_{\min}(\widetilde{X}_i)\mathbb{E}\{\|\widetilde{x}_1(t)\|^2\} \leq \mathbb{E}\{\widetilde{x}_1^T(t)\widetilde{X}_i\widetilde{x}_1(t)\} \leq 1$, which implies that $\mathbb{E}\{\|\widetilde{x}_1(t)\|\} \leq 1/\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}(\widetilde{X}_i)}$.

Since A_{22i} is nonsingular for each $i \in \mathcal{S}$, (8) can be rewritten as

$$\tilde{x}_2(t) = -A_{22i}^{-1}\left(A_{21i}\tilde{x}_1(t) + D_{2i}\omega(t)\right). \quad (22)$$

Then we can deduce that

$$\mathbb{E}\left\{\left\|\tilde{x}_2(t)\right\|\right\} \leq \left\|A_{22i}^{-1}\right\|\left(\frac{\left\|A_{21i}\right\|}{\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}\left(\widetilde{X}_i\right)}} + \left\|D_{2i}\right\|\overline{\omega}\right). \quad (23)$$

It follows from the fact $\tilde{x}(t) = [\tilde{x}_1^T(t)\ 0]^T + [0\ \tilde{x}_2^T(t)]^T$ that

$$\mathbb{E}\left\{\left\|\tilde{x}(t)\right\|\right\} \leq \mathbb{E}\left\{\left\|\tilde{x}_1(t)\right\| + \left\|\tilde{x}_2(t)\right\|\right\} \leq r_i, \quad (24)$$

where $r_i = 1/\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}(\widetilde{X}_i)} + \|A_{22i}^{-1}\|(\|A_{21i}\|/\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}(\widetilde{X}_i)} + \|D_{2i}\|\overline{\omega})$.

By (24), it can be seen that

$$\frac{1}{r_i^2}\mathbb{E}\left\{\tilde{x}^T(t)\,\tilde{x}(t)\right\} \leq 1 \quad (25)$$

which implies that the trajectories of (7)-(8) are mean-square bounded within the set $\bigcap_{i=1}^{N}\mathcal{B}((1/r_i^2)I)$. Moreover, notice that $\tilde{x}(t) = N^{-1}x(t)$, and (25) can be rewritten as $(1/r_i^2)\mathbb{E}\{x^T(t)(N^{-T}N^{-1})x(t)\} \leq 1$. By denoting $(1/r_i^2)N^{-T}N^{-1} = \widehat{X}_i$, the state reachable set of free system is mean-square bounded within the set $\bigcap_{i=1}^{N}\mathcal{E}(\widehat{X}_i)$. □

It should be noted that inequality (10) represents a nonstrict LMI. This may lead to numerical problems since equality constraints are usually not satisfied perfectly. Below, we will develop a numerically tractable and nonconservative LMI condition.

Theorem 5. *If there exist symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, nonsingular matrices Q_i, and a scalar $\alpha > 0$ such that the following LMI holds for each $i \in \mathcal{S}$,*

$$\begin{bmatrix} \Psi_i & \left(P_iE + R^TQ_iS^T\right)^T D_i \\ * & -\dfrac{\alpha}{\overline{\omega}^2}I \end{bmatrix} < 0, \quad (26)$$

then the state reachable set of free system is mean-square bounded within the following set:

$$\bigcap_{i=1}^{N}\mathcal{E}\left(\widehat{P}_i\right) = \bigcap_{i=1}^{N}\left\{x \in \mathbb{R}^n \mid x^T\widehat{P}_ix \leq 1\right\}, \quad (27)$$

where

$$\begin{aligned}
\Psi_i &= \operatorname{Sym}\left(A_i^T\left(P_iE + R^TQ_iS^T\right)\right) + \sum_{j=1}^{N}\pi_{ij}E^TP_jE + \alpha E^TP_iE \\
\widehat{P}_i &= \frac{1}{\tilde{r}_i^2}N^{-T}N^{-1} \\
\tilde{r}_i &= \frac{1}{\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}\left(\widetilde{P}_i\right)}} + \left\|A_{22i}^{-1}\right\|\left(\frac{\left\|A_{21i}\right\|}{\sqrt{\min_{i\in\mathcal{S}}\lambda_{\min}\left(\widetilde{P}_i\right)}} + \left\|D_{2i}\right\|\overline{\omega}\right) \\
\widetilde{P}_i &= \left(M^{-T}P_iM^{-1}\right)_{l\times l},
\end{aligned} \quad (28)$$

$R \in \mathbb{R}^{(n-l)\times n}$ is any matrix with full row rank and satisfies $RE = 0$; $S \in \mathbb{R}^{n\times(n-l)}$ is any matrix with full column rank and satisfies $ES = 0$.

Proof. Let $X_i = P_iE + R^TQ_iS^T$ in (26); it is easy to obtain (10) and (11). In this case, inequality (20) will be replaced by $\mathbb{E}\{\tilde{x}^T(t)N^TE^TM^TM^{-T}P_iM^{-1}MEN\tilde{x}(t)\} \leq 1$, which infers that $\mathbb{E}\{\tilde{x}_1^T(t)\widetilde{P}_i\tilde{x}_1(t)\} \leq 1$ with $\widetilde{P}_i = (M^{-T}P_iM^{-1})_{l\times l}$. Then, following the same lines as (22)–(25) in Theorem 4, we can get $\mathbb{E}\{x^T(t)\widehat{P}_ix(t)\} \leq 1$ for any $i \in \mathcal{S}$. Therefore, the state reachable set of free system is mean-square bounded within the set $\bigcap_{i=1}^{N}\mathcal{E}(\widehat{P}_i)$. □

Remark 6. In order to make the ellipsoid $\mathcal{E}(\widehat{P}_i)$ as small as possible, we require trace$(\widehat{P}_i) \rightarrow$ max. For this purpose, we can add the additional requirement $\widetilde{P}_i > \epsilon I$ and then maximize a positive scalar ϵ, which is equivalent to the following minimization problem:

$$\begin{aligned}
&\min \quad \overline{\epsilon} \\
&\text{s.t.} \quad (26), \\
&\qquad \begin{bmatrix} -\overline{\epsilon}I & I \\ I & -\widetilde{P}_i \end{bmatrix} < 0,
\end{aligned} \quad (29)$$

where $\overline{\epsilon} = 1/\epsilon$.

3.2. Output Reachable Set Estimation

Theorem 7. *If there exist symmetric positive definite matrices $P_i \in \mathbb{R}^{n\times n}$ and $Y_i \in \mathbb{R}^{n\times n}$, nonsingular matrices Q_i, and a scalar $\alpha > 0$ such that the following LMIs hold for each $i \in \mathcal{S}$,*

$$\begin{bmatrix} \Psi_i & \left(P_iE + R^TQ_iS^T\right)^T D_i \\ * & -\dfrac{\alpha}{\overline{\omega}^2}I \end{bmatrix} < 0 \quad (30)$$

$$C_i^T Y_i C_i \leq \widehat{P}_i, \tag{31}$$

then the output reachable set of free system is mean-square bounded within the following set:

$$\bigcap_{i=1}^{N} \mathscr{E}(Y_i) = \bigcap_{i=1}^{N} \left\{ y \in \mathbb{R}^p \mid y^T Y_i y \leq 1 \right\}, \tag{32}$$

where Ψ_i, R, S, and $\widehat{P}_i$ are defined in Theorem 5.

Proof. By Theorem 5, LMI (30) ensures

$$\mathbb{E}\left\{ x^T(t) \widehat{P}_i x(t) \right\} \leq 1, \quad \forall i \in \mathcal{S}. \tag{33}$$

With this and (31), we obtain that

$$\mathbb{E}\left\{ x^T(t) C_i^T Y_i C_i x(t) \right\} \leq \mathbb{E}\left\{ x^T(t) \widehat{P}_i x(t) \right\} \leq 1. \tag{34}$$

Due to the fact that $y(t) = C_i x(t)$, (34) can be rewritten as $\mathbb{E}\{y^T(t) Y_i y(t)\} \leq 1$. Thus, the output reachable set of free system is mean-square bounded within the set $\bigcap_{i=1}^{N} \mathscr{E}(Y_i)$. □

Remark 8. The output reachable set is also expected to be as small as possible. To achieve this goal, we first solve LMI (30) and get $\widehat{P}_i$ satisfying trace$(\widehat{P}_i) \rightarrow$ max, which can be implemented by using (29). Then we add the additional requirement $C_i^T Y_i C_i > \delta I$ and maximize a positive scalar δ, which is equivalent to the following minimization problem:

$$\begin{aligned} &\min \quad \bar{\delta} \\ &\text{s.t.} \quad (31), \\ &\qquad \begin{bmatrix} -\bar{\delta} I & I \\ I & -C_i^T Y_i C_i \end{bmatrix} < 0, \end{aligned} \tag{35}$$

where $\bar{\delta} = 1/\delta$.

3.3. State-Feedback Controller Design. In this section, we turn our attention to the state-feedback control problem. Our goal here is to find a state-feedback controller, which not only stabilizes the closed-loop system, but also makes the ellipsoid bound on the reachable set of closed-loop system as small as possible.

Now, consider the state-feedback controller $u(t) = K_i x(t)$, where K_i is a gain matrix to be determined later. By using this controller, the closed-loop system can be obtained as

$$\begin{aligned} E\dot{x}(t) &= \widetilde{A}_i x(t) + D_i \omega(t) \\ y(t) &= C_i x(t), \end{aligned} \tag{36}$$

where $\widetilde{A}_i = A_i + B_i K_i$.

Theorem 9. *Consider singular Markov jump system (1). If there exist nonsingular matrices $P_i \in \mathbb{R}^{n\times n}$, matrices S_i, and scalars $\alpha > 0$, $\delta_i > 0$ such that the following LMIs hold for each $i \in \mathcal{S}$,*

$$P_i^T E^T = E P_i \geq 0 \tag{37}$$

$$P_i^T E^T \leq \delta_i I \tag{38}$$

$$\begin{bmatrix} \Phi_i & D_i & \mathscr{W}_i \\ * & -\dfrac{\alpha}{\bar{\omega}^2} I & 0 \\ * & * & -\mathscr{J}_i \end{bmatrix} < 0, \tag{39}$$

then the reachable set of system (1) is mean-square bounded within the set $\bigcap_{i=1}^{N} \mathscr{E}(\overline{P}_i) = \bigcap_{i=1}^{N} \{x \in \mathbb{R}^n \mid x^T \overline{P}_i x \leq 1\}$, and the desired controller gain matrix is given by $K_i = S_i P_i^{-1}$, where

$$\begin{aligned} \Phi_i &= \operatorname{Sym}\left(P_i^T A_i^T + S_i^T B_i^T\right) + (\alpha + \pi_{ii}) P_i^T E^T \\ \mathscr{W}_i &= \left[\sqrt{\pi_{i1}} P_i^T \ \cdots \ \sqrt{\pi_{i(i-1)}} P_i^T \ \sqrt{\pi_{i(i+1)}} P_i^T \ \cdots \ \sqrt{\pi_{iN}} P_i^T \right] \\ \mathscr{J}_i &= \operatorname{diag}\left(P_1 + P_1^T - \delta_1 I, \ldots, P_{i-1} + P_{i-1}^T - \delta_{i-1} I, P_{i+1} \right. \\ &\quad \left. + P_{i+1}^T - \delta_{i+1} I, \ldots, P_N + P_N^T - \delta_N I \right) \\ \overline{P}_i &= \frac{1}{\hat{r}_i^2} N^{-T} N^{-1} \\ \hat{r}_i &= \sqrt{\max_{i\in\mathcal{S}} \lambda_{\max}\left(\check{P}_i\right)} + \left\| A_{22i}^{-1} \right\| \left(\left\| A_{21i} \right\| \sqrt{\max_{i\in\mathcal{S}} \lambda_{\max}\left(\check{P}_i\right)} \right. \\ &\quad \left. + \left\| D_{2i} \right\| \bar{\omega} \right) \\ \check{P}_i &= \left(N^{-1} P_i M^T \right)_{l\times l}. \end{aligned} \tag{40}$$

Proof. Denote $X_i = P_i^{-1}$ and $S_i = K_i P_i$ for each $i \in \mathcal{S}$. Then, pre- and postmultiplying (39) by diag(X_i^T, I, I) and its transpose, respectively, we obtain

$$\begin{bmatrix} \widetilde{\Phi}_i & X_i^T D_i & \widetilde{\mathscr{W}}_i \\ * & -\dfrac{\alpha}{\bar{\omega}^2} I & 0 \\ * & * & -\mathscr{J}_i \end{bmatrix} < 0, \tag{41}$$

where $\widetilde{\Phi}_i = \operatorname{Sym}(\widetilde{A}_i^T X_i) + (\alpha + \pi_{ii}) E^T X_i$, $\widetilde{\mathscr{W}}_i = [\sqrt{\pi_{i1}} I \ \cdots \ \sqrt{\pi_{i(i-1)}} I \ \sqrt{\pi_{i(i+1)}} I \ \cdots \ \sqrt{\pi_{iN}} I]$.

Using Lemma 2, we have

$$\begin{aligned} &\delta_i^{-1} P_j P_j^T \geq P_j + P_j^T - \delta_i I, \\ &\qquad j = 1, 2, \ldots, i-1, i+1, \ldots, N. \end{aligned} \tag{42}$$

From (41) and (42), it is easy to obtain that

$$\begin{bmatrix} \widetilde{\Phi}_i & X_i^T D_i & \widetilde{\mathscr{W}}_i \\ * & -\dfrac{\alpha}{\bar{\omega}^2} I & 0 \\ * & * & -\widetilde{\mathscr{J}}_i \end{bmatrix} < 0, \tag{43}$$

where $\widetilde{\mathcal{J}}_i = \text{diag}(\delta_1^{-1}P_1P_1^T, \ldots, \delta_{i-1}^{-1}P_{i-1}P_{i-1}^T, \delta_{i+1}^{-1}P_{i+1}P_{i+1}^T, \ldots, \delta_N^{-1}P_NP_N^T)$.

By Schur complement, the previous matrix inequality becomes

$$\begin{bmatrix} \widehat{\Phi}_i & X_i^T D_i \\ * & -\dfrac{\alpha}{\overline{\omega}^2} I \end{bmatrix} < 0, \tag{44}$$

where $\widehat{\Phi}_i = \text{Sym}(\widetilde{A}_i^T X_i) + (\alpha + \pi_{ii})E^T X_i + \sum_{j=1,j\neq i}^{N} \delta_j \pi_{ij} X_j^T X_j$.

Since $j \in \mathcal{S}$, (38) infers that $P_j^T E^T \leq \delta_j I$. Pre- and postmultiplying the previous matrix inequality by X_j^T and X_j, respectively, we have $E^T X_j \leq \delta_j X_j^T X_j$. This together with (44) implies

$$\begin{bmatrix} \overline{\Phi}_i & X_i^T D_i \\ * & -\dfrac{\alpha}{\overline{\omega}^2} I \end{bmatrix} < 0, \tag{45}$$

where $\overline{\Phi}_i = \text{Sym}(\widetilde{A}_i^T X_i) + \sum_{j=1}^{N} \pi_{ij} E^T X_j + \alpha E^T X_i$.

Pre- and postmultiplying (37) by X_i^T and X_i, respectively, we obtain

$$E^T X_i = X_i^T E \geq 0. \tag{46}$$

From the above discussion, we show that if (37)–(39) hold, then (45) and (46) hold. Thus, it follows from Theorem 4 that the closed-loop system can be stabilized by the designed state-feedback controller.

Next, we show that the reachable set of the closed-loop system (36) is mean-square bounded within the set $\bigcap_{i=1}^{N} \mathcal{E}(\overline{P}_i)$. From (45) and (46), it is easy to show that there exists a Lyapunov function $V(x(t), r_t) = x^T(t)E^T X_i x(t)$ such that $\mathcal{L}V(x(t), r_t) + \alpha V(x(t), r_t) - (\alpha/\overline{\omega}^2)\omega^T(t)\omega(t) < 0$, where $\mathcal{L}V(x(t), r_t)$ denotes the difference of $V(x(t), r_t)$ along the trajectories of (36). It follows from Lemma 3 that $\mathbb{E}\{x^T(t)E^T X_i x(t)\} \leq 1$, which implies

$$\mathbb{E}\left\{\widetilde{x}^T(t) N^T E^T M^T M^{-T} X_i N \widetilde{x}(t)\right\} \leq 1. \tag{47}$$

Noting that $X_i = P_i^{-1}$, then (47) can be rewritten as

$$\mathbb{E}\left\{\widetilde{x}^T(t) N^T E^T M^T M^{-T} P_i^{-1} N \widetilde{x}(t)\right\} \leq 1. \tag{48}$$

Recalling that $M^{-T}P_i^{-1}N$ is a lower triangular matrix, (48) infers that

$$\mathbb{E}\left\{\widetilde{x}_1^T(t)\, \check{P}_i^{-1} \widetilde{x}_1(t)\right\} \leq 1, \tag{49}$$

where $\check{P}_i^{-1} = (M^{-T}P_i^{-1}N)_{l\times l}$.

By (49), it can be seen that

$$\begin{aligned} \min_{i\in\mathcal{S}} \lambda_{\min}\left(\check{P}_i^{-1}\right) \mathbb{E}\left\{\left\|\widetilde{x}_1(t)\right\|^2\right\} &\leq \mathbb{E}\left\{\widetilde{x}_1^T(t)\, \check{P}_i^{-1} \widetilde{x}_1(t)\right\} \\ &\leq 1. \end{aligned} \tag{50}$$

Using the fact that $\min_{i\in\mathcal{S}} \lambda_{\min}(\check{P}_i^{-1}) = 1/\max_{i\in\mathcal{S}} \lambda_{\max}(\check{P}_i)$, we get

$$\mathbb{E}\left\{\left\|\widetilde{x}_1(t)\right\|\right\} \leq \sqrt{\max_{i\in\mathcal{S}} \lambda_{\max}\left(\check{P}_i\right)}. \tag{51}$$

This together with (24) yields

$$\begin{aligned} &\mathbb{E}\left\{\left\|\widetilde{x}_2(t)\right\|\right\} \\ &\quad \leq \left\|A_{22i}^{-1}\right\| \left(\left\|A_{21i}\right\| \sqrt{\max_{i\in\mathcal{S}} \lambda_{\max}\left(\check{P}_i\right)} + \left\|D_{2i}\right\| \overline{\omega}\right). \end{aligned} \tag{52}$$

From (24), (51), and (52), we have that

$$\mathbb{E}\left\{\left\|\widetilde{x}(t)\right\|\right\} \leq \widehat{r}_i, \tag{53}$$

where $\widehat{r}_i = \sqrt{\max_{i\in\mathcal{S}} \lambda_{\max}(\check{P}_i)} + \|A_{22i}^{-1}\|(\|A_{21i}\| \sqrt{\max_{i\in\mathcal{S}} \lambda_{\max}(\check{P}_i)} + \|D_{2i}\|\overline{\omega})$. This implies that $(1/\widehat{r}_i^2)\mathbb{E}\{\widetilde{x}^T(t)\widetilde{x}(t)\} \leq 1$. Recalling that $\widetilde{x}(t) = N^{-1}x(t)$, we have

$$\frac{1}{\widehat{r}_i^2} \mathbb{E}\left\{x^T(t)\left(N^{-T}N^{-1}\right)x(t)\right\} \leq 1. \tag{54}$$

By denoting $(1/\widehat{r}_i^2)N^{-T}N^{-1} = \overline{P}_i$, the reachable set of closed-loop system (36) is mean-square bounded within the set $\bigcap_{i=1}^{N} \mathcal{E}(\overline{P}_i)$. □

Remark 10. In order to make the ellipsoid $\mathcal{E}(\overline{P}_i)$ as small as possible, we shall carry out the following minimization problem:

$$\begin{aligned} &\min \quad \epsilon \\ &\text{s.t.} \quad (37), (38), (39), \\ &\qquad\quad \check{P}_i \leq \epsilon I. \end{aligned} \tag{55}$$

4. Numerical Examples

In this section, two numerical simulation examples are given to show the effectiveness of the main results derived above.

Example 11. Consider the free system in (1) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -0.8695 & -1.5760 \\ -0.2389 & 1.8258 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.5043 & 0.1206 \\ -1.1634 & -1.4435 \end{bmatrix},$$

$$D_1 = D_2 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$

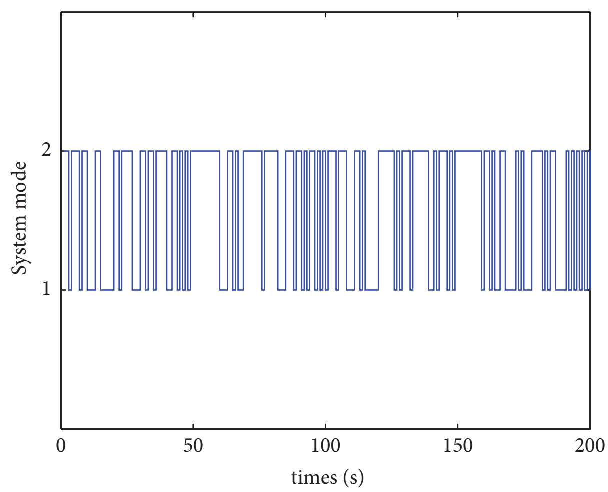

FIGURE 1: The switching between two modes.

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}. \tag{56}$$

The switching between two modes is described by the following transition rate matrix:

$$\Pi = \begin{bmatrix} -3 & 3 \\ 6 & -6 \end{bmatrix}. \tag{57}$$

In this example, we choose $M = I$, $N = I$. By solving optimization problem (29) with the aid of `fminsearch`, the minimal $\bar{\epsilon}$ and the corresponding α are 0.1312 and 1.7419, respectively. Using the above parameter values, we can obtain $\widehat{P}_1 = (1/\bar{r}_1^2)I = 2.1414I$ and $\widehat{P}_2 = (1/\bar{r}_2^2)I = 0.9992I$ by solving (26). Owing to Theorem 5, the state reachable set $\mathscr{R}_x$ is mean-square bounded within the set $\mathscr{B}(\widehat{P}_1) \cap \mathscr{B}(\widehat{P}_2) = \mathscr{B}(\widehat{P}_1)$.

By applying Theorem 7, we have the following results:

$$Y_1 = \begin{bmatrix} 1.6561 & 0 \\ 0 & 1.6561 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 0.9975 & -0.9975 \\ -0.9975 & 1.9950 \end{bmatrix}. \tag{58}$$

Therefore, the output reachable set of free system is mean-square bounded within the set $\mathscr{B}(Y_1) \cap \mathscr{E}(Y_2)$.

For simulation we assume that $x_0 = [0.2\ \ 0.1303]^T$ and the disturbance is chosen as $\omega(t) = \sin(t)$. A case for stochastic variation with transition rate matrix Π is shown in Figure 1. The state reachable set $\mathscr{R}_x$ and the ball $\mathscr{B}(\widehat{P}_2)$ are depicted in Figure 2. Figure 2 shows that the trajectory of the system is mean-square bounded within the region $\mathscr{B}(\widehat{P}_2)$. The output reachable set $\mathscr{R}_y$, the ball $\mathscr{B}(Y_1)$, and the ellipsoid $\mathscr{E}(Y_2)$ are depicted in Figure 3. Figure 3 shows that the output reachable set $\mathscr{R}_y$ is mean-square bounded within the region $\mathscr{B}(Y_1) \cap \mathscr{E}(Y_2)$.

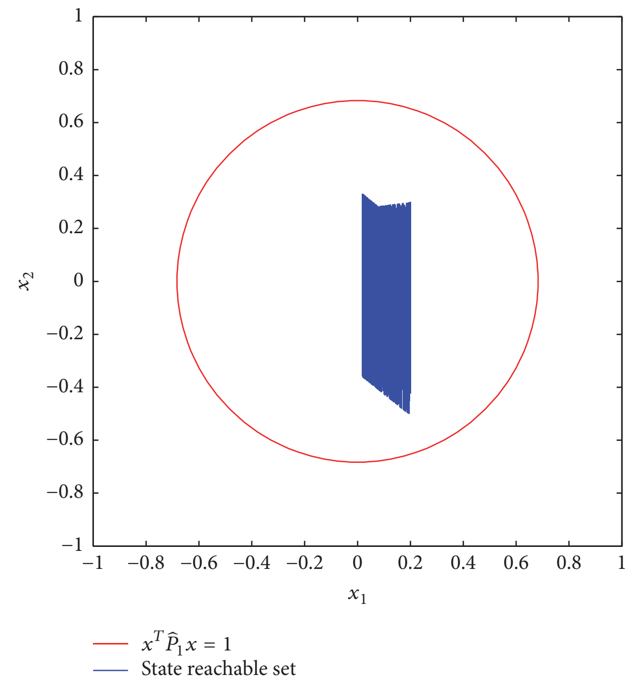

FIGURE 2: The state reachable set $\mathscr{R}_x$ and the bounding ball $\mathscr{B}(\widehat{P}_1)$.

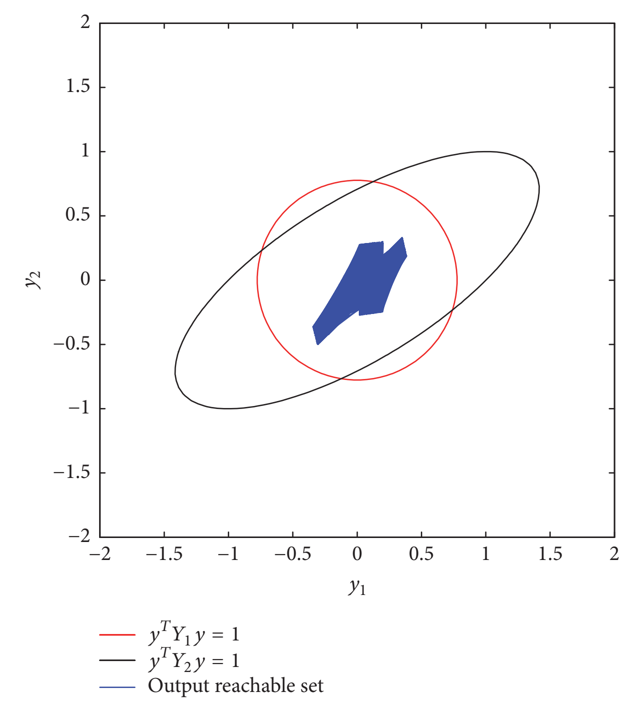

FIGURE 3: The output reachable set $\mathscr{R}_y$ and the bounding ellipsoids.

Example 12. Consider system (1) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} 0.25 & 1.97 \\ 0.44 & 2.31 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1.90 & -1.72 \\ 1.87 & 0.64 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.14 \\ 0.79 \end{bmatrix}, \quad (59)$$
$$B_2 = \begin{bmatrix} 0.34 \\ -0.50 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.50 \\ 0.50 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0.70 \\ 0.70 \end{bmatrix}.$$

The switching between two modes is described by the following transition rate matrix:

$$\Pi = \begin{bmatrix} -3 & 3 \\ 6 & -6 \end{bmatrix}. \quad (60)$$

By Theorem 9, we get the following results:

$$\begin{aligned} \alpha &= 0.1997, \\ \delta_1 &= 22.1356, \\ \delta_2 &= 18.0362, \\ P_1 &= \begin{bmatrix} 18.0211 & 0 \\ 3.8045 & 102.2184 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 14.7029 & 0 \\ -8.6240 & 67.2193 \end{bmatrix}. \end{aligned} \quad (61)$$

Therefore, the gain matrices of state-feedback controller can be obtained as

$$\begin{aligned} K_1 &= \begin{bmatrix} -12.7027 & -5.3634 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -7.1957 & 4.6404 \end{bmatrix}. \end{aligned} \quad (62)$$

The corresponding parameter values $\hat{r}_1^2$ and $\hat{r}_2^2$ are, respectively, 27.7747 and 314.7993, which imply $\overline{P}_1 = (1/\hat{r}_1^2)I = 0.0360I$ and $\overline{P}_2 = (1/\hat{r}_2^2)I = 0.0032I$. Applying this controller makes the state reachable set of closed-loop system (36) mean-square bounded within the region $\mathscr{B}(\overline{P}_1)$.

For the purpose of the simulation, we assume the initial condition $x_0 = [0.3 \ \ 0.9763]^T$ and the disturbance is chosen as $\omega(t) = \sin(0.2t)$. Figure 4 shows one possible switching between two modes. Figure 5 depicts the state reachable set of closed-loop system (36).

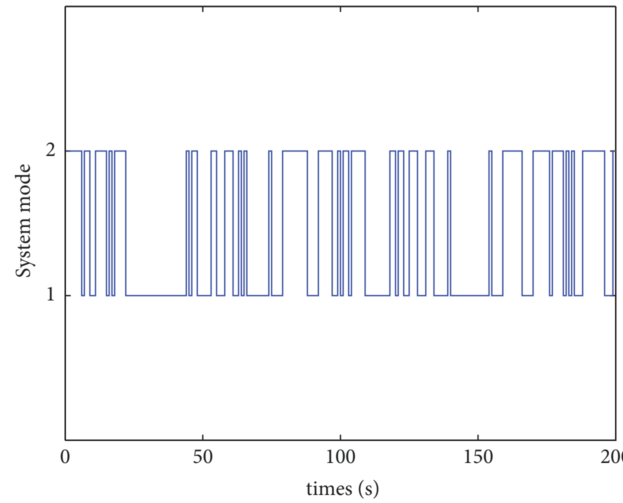

FIGURE 4: The switching between two modes.

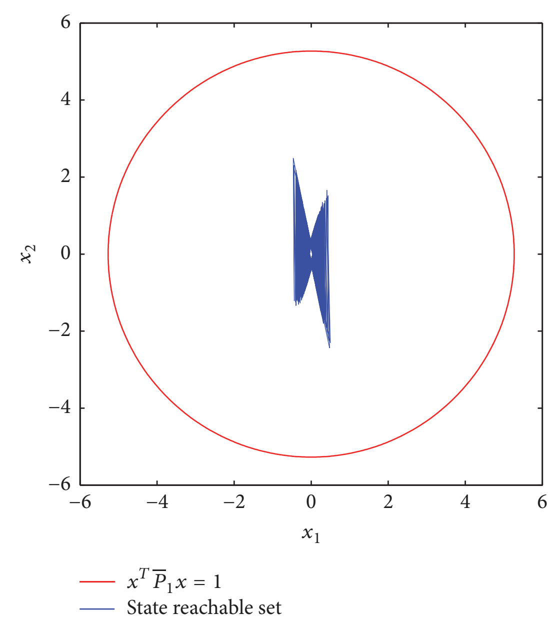

FIGURE 5: The state reachable set $\mathscr{R}_x$ and the bounding ball $\mathscr{B}(\overline{P}_1)$.

5. Conclusions

This paper has dealt with the problems of reachable set estimation and state-feedback controller design for singular Markovian jump systems. New sufficient conditions for the state reachable set estimation and output reachable set estimation have been, respectively, derived in terms of linear matrix inequalities. Based on the estimated reachable set, the state-feedback controller has also been designed. Numerical examples and simulation results have been provided to demonstrate the effectiveness of the proposed methods.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by Applied Basic Research Project of Science and Technology Department of Sichuan Province (no. 2017JY0336), a project supported by Scientific Research Fund of Sichuan Provincial Education Department (no. 16ZA0146), the National Natural Science Foundation of China (no. 11501474), the Longshan Academic Talents Research Support Program of the Southwest University of Science and Technology (no. 17LZX537 and no. 17LZXY11), and the Doctoral Research Foundation of Southwest University of Science and Technology (no. 13zx7141).

References

[1] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*, Springer, Berlin, Germany, 2006.

[2] E.-K. Boukas, *Control of Singular Systems with Random Abrupt Changes*, Communications and Control Engineering, Springer, Berlin, Germany, 2008.

[3] Y. Xia, E.-K. Boukas, P. Shi, and J. Zhang, "Stability and stabilization of continuous-time singular hybrid systems," *Automatica*, vol. 45, no. 6, pp. 1504–1509, 2009.

[4] L. Wu, P. Shi, and H. Gao, "State estimation and sliding-mode control of Markovian jump singular systems," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 55, no. 5, pp. 1213–1219, 2010.

[5] Z.-G. Wu, J. H. Park, H. Su, and J. Chu, "Stochastic stability analysis for discrete-time singular Markov jump systems with time-varying delay and piecewise-constant transition probabilities," *Journal of The Franklin Institute*, vol. 349, no. 9, pp. 2889–2902, 2012.

[6] Y. Kao, J. Xie, and C. Wang, "Stabilization of singular Markovian jump systems with generally uncertain transition rates," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 59, no. 9, pp. 2604–2610, 2014.

[7] Y. Ding, H. Liu, and J. Cheng, "H∞ filtering for a class of discrete-time singular Markovian jump systems with time-varying delays," *ISA Transactions®*, vol. 53, no. 4, pp. 1054–1060, 2014.

[8] Q. Zhang, L. Li, X.-G. Yan, and S. K. Spurgeon, "Sliding mode control for singular stochastic Markovian jump systems with uncertainties," *Automatica*, vol. 79, pp. 27–34, 2017.

[9] Y. Ding, S. Zhong, and S. Long, "Asymptotic stability in probability of singular stochastic systems with Markovian switchings," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 18, pp. 4312–4322, 2017.

[10] Y. Ma, X. Jia, and D. Liu, "Finite-time dissipative control for singular discrete-time Markovian jump systems with actuator saturation and partly unknown transition rates," *Applied Mathematical Modelling: Simulation and Computation for Engineering and Environmental Systems*, vol. 53, pp. 49–70, 2018.

[11] B. T. Polyak, S. A. Nazin, C. Durieu, and E. Walter, "Ellipsoidal parameter or state estimation under model uncertainty," *Automatica*, vol. 40, no. 7, pp. 1171–1179, 2004.

[12] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory*, vol. 15 of *SIAM Studies in Applied Mathematics*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1994.

[13] E. K. Kostousova, "On tight polyhedral estimates for reachable sets of linear differential systems," in *Proceedings of the 9th International Conference on Mathematical Problems in Engineering, Aerospace and Sciences, ICNPAA 2012*, pp. 579–586, Austria, July 2012.

[14] E. Fridman and U. Shaked, "On reachable sets for linear systems with delay and bounded peak inputs," *Automatica*, vol. 39, no. 11, pp. 2005–2010, 2003.

[15] J.-H. Kim, "Improved ellipsoidal bound of reachable sets for time-delayed linear systems with disturbances," *Automatica*, vol. 44, no. 11, pp. 2940–2943, 2008.

[16] Z. Zuo, D. W. Ho, and Y. Wang, "Reachable set bounding for delayed systems with polytopic uncertainties: the maximal Lyapunov-Krasovskii functional approach," *Automatica*, vol. 46, no. 5, pp. 949–952, 2010.

[17] O. M. Kwon, S. M. Lee, and J. H. Park, "On the reachable set bounding of uncertain dynamic systems with time-varying delays and disturbances," *Information Sciences*, vol. 181, no. 17, pp. 3735–3748, 2011.

[18] Z. Zuo, Y. Chen, Y. Wang, D. W. Ho, M. Z. Chen, and H. Li, "A note on reachable set bounding for delayed systems with polytopic uncertainties," *Journal of The Franklin Institute*, vol. 350, no. 7, pp. 1827–1835, 2013.

[19] B. Zhang, J. Lam, and S. Xu, "Relaxed results on reachable set estimation of time-delay systems with bounded peak inputs," *International Journal of Robust and Nonlinear Control*, vol. 26, no. 9, pp. 1994–2007, 2016.

[20] Y. Sheng and Y. Shen, "Improved reachable set bounding for linear time-delay systems with disturbances," *Journal of The Franklin Institute*, vol. 353, no. 12, pp. 2708–2721, 2016.

[21] L. V. Hien and H. M. Trinh, "A new approach to state bounding for linear time-varying systems with delay and bounded disturbances," *Automatica*, vol. 50, no. 6, pp. 1735–1738, 2014.

[22] P. T. Nam, P. N. Pathirana, and H. Trinh, "Reachable set bounding for nonlinear perturbed time-delay systems: the smallest bound," *Applied Mathematics Letters*, vol. 43, pp. 68–71, 2015.

[23] Y. Sun and F. Meng, "Reachable set estimation for a class of nonlinear time-varying systems," *Complexity*, Article ID 5876371, Art. ID 5876371, 6 pages, 2017.

[24] N. D. That, P. T. Nam, and Q. P. Ha, "Reachable set bounding for linear discrete-time systems with delays and bounded disturbances," *Journal of Optimization Theory and Applications*, vol. 157, no. 1, pp. 96–107, 2013.

[25] J. Lam, B. Zhang, Y. Chen, and S. Xu, "Reachable set estimation for discrete-time linear systems with time delays," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 2, pp. 269–281, 2015.

[26] Z. Feng and J. Lam, "On reachable set estimation of singular systems," *Automatica*, vol. 52, pp. 146–153, 2015.

[27] Z. Feng and W. X. Zheng, "On reachable set estimation of delay Markovian jump systems with partially known transition probabilities," *Journal of The Franklin Institute*, vol. 353, no. 15, pp. 3835–3856, 2016.

[28] Y. Chen, J. Lam, and B. Zhang, "Estimation and synthesis of reachable set for switched linear systems," *Automatica*, vol. 63, pp. 122–132, 2016.

[29] Z. Feng, W. X. Zheng, and L. Wu, "Reachable set estimation of T-S fuzzy systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, 2016.

[30] W. Xiang, H.-D. Tran, and T. T. Johnson, "Output reachable set estimation for switched linear systems and its application in safety verification," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 62, no. 10, pp. 5380–5387, 2017.

[31] L. Van, N. Thanh, and H. Trinh, "New results on state bounding for discrete-time systems with interval time-varying delay and bounded disturbance inputs," *IET Control Theory & Applications*, vol. 8, no. 14, pp. 1405–1414, 2014.

[32] L. V. Hien, N. Trung, and H. Binh, "A novel approach to state bounding for discrete-time Markovian jump systems with interval time-varying delay," *IMA Journal of Mathematical Control and Information*, vol. 33, no. 2, pp. 293–307, 2016.

[33] H. Trinh and L. V. Hien, "On reachable set estimation of two-dimensional systems described by the Roesser model with time-varying delays," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 1, pp. 227–246, 2018.

[34] B. Zhang, J. Lam, and S. Xu, "Reachable set estimation and controller design for distributed delay systems with bounded disturbances," *Journal of The Franklin Institute*, vol. 351, no. 6, pp. 3068–3088, 2014.

[35] E. K. Kostousova, "On feedback target control for uncertain discrete-time bilinear systems with state constraints through polyhedral technique," in *Proceedings of the 9th International Conference for Promoting the Application of Mathematics in Technical and Natural Sciences, AMiTaNS 2017*, Bulgaria, June 2017.

[36] Y. Chen and J. Lam, "Estimation and synthesis of reachable set for discrete-time periodic systems," *Optimal Control Applications and Methods*, vol. 37, no. 5, pp. 885–901, 2016.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
The Scientific World Journal
Journal of Probability and Statistics
International Journal of Mathematics and Mathematical Sciences
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of Optimization
International Journal of Engineering Mathematics
International Journal of Analysis
Journal of Complex Analysis
Advances in Numerical Analysis
Mathematical Problems in Engineering
International Journal of Differential Equations
Discrete Dynamics in Nature and Society
International Journal of Stochastic Analysis
Journal of Mathematics
Journal of Function Spaces
Abstract and Applied Analysis
Advances in Mathematical Physics