The Reduction of Necessity to Essence

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Abstract

In 'Essence and Modality', Kit Fine proposes that for a proposition to be metaphysically necessary is for it to be true in virtue of the nature of all objects whatsoever. Call this view *Fine's Thesis*. This paper is a study of Fine's Thesis in the context of Fine's logic of essence (LE). Fine himself has offered his most elaborate defense of the thesis in the context of LE. His view will be a central focus of this paper. Fine's defense rests on the widely shared assumption that metaphysical necessity obeys the laws of the modal logic S5. In order to get S5 for metaphysical necessity, he assumes a controversial principle about the nature of all objects. I will show that the addition of this principle to his original system E5 leads to inconsistency with an independently plausible principle about essence. In response, I develop a theory that avoids this inconsistency while allowing us to maintain S5 for metaphysical necessity. However, I conclude that our investigation of Fine's Thesis in the context of LE motivates the revisionary conclusion that metaphysical necessity obeys the principles of the modal logic S4, but not those of S5. I argue that this constitutes a distinctively essentialist challenge to the received view that the logic of metaphysical necessity is S5.

Introduction

In 'Essence and Modality', Kit Fine proposes that instead of explaining the notion of essence in terms of metaphysical necessity, we should understand metaphysical necessity as a special case of essence:

> For each class of objects, be they concepts or individuals or entities of some other kind, will give rise to its own domain of necessary truths, the truths which flow from the nature of the objects in question. The metaphysically necessary truths can then be identified with the propositions which are true in virtue of the nature of all objects whatever. (Fine, 1994, p.9)

Call the view that for a proposition to be metaphysically necessary is for it to be true in virtue of the nature of all objects whatsoever *Fine's Thesis*.[1]

On its intended interpretation, the thesis takes for granted a notion of essence that is not analyzable in terms of metaphysical necessity. It can thus be understood as an analysis, or reduction, of metaphysical necessity in terms of an independently understood notion of essence. So understood, Fine's Thesis crucially depends on the theory of essence that underlies it. In order to make progress with the question of whether the thesis is true, we need to make explicit what general principles guide our reasoning with the notion of essence. In other words, we need to know what the logic of essence is.

The only extant theory of essence that is sufficiently systematic and general for the present inquiry is Fine's own logic of essence (LE): his system E5 and its extensions.[2] This paper provides a detailed study of Fine's Thesis in the context of LE. I consider different ways in which the view might be developed and investigate their philosophical tenability. An investigation of Fine's Thesis in LE helps us to get a better grip on what the thesis amounts to by making explicit what essentialist principles it depends on. Moreover, it enables us to investigate to what extent endorsing the thesis has revisionary consequences for our thinking about metaphysical necessity.

While the thesis has received some attention in the literature[3], it has—with the important exception of Fine (1995a)—not yet been systematically discussed in the context of LE. However, LE is a natural starting point

[1] The qualification 'whatsoever' is meant to stress the fact that the quantifier should be understood as absolutely unrestricted.

[2] The two relevant works are Fine (1995a, 2000). Fine (1995a) develops a proof system and Fine (2000) gives a semantics for this proof system and proves soundness and completeness of this system with respect to the semantics. I will use 'LE' as an umbrella term to refer to Fine's system E5 or its extensions, as well as to the language of LE. Whenever I speak about a specific system, I will refer to this system explicitly.

[3] See, for instance, Hale (1996), Correia (2006, 2012), Cameron (2010), Vetter (2011), Wildman (2018) and Teitel (2019). Cameron (2010) lists the thesis as one of the three most promising options for explaining the nature and source of modality.

for an investigation of Fine's Thesis, since apart from being the only fully developed logic of essence available, it is in the context of LE that Fine himself has offered his most elaborate defense of the thesis. Fine's own development and defense of the thesis will be a central focus of this paper.

Fine's defense rests on the widely shared assumption that metaphysical necessity obeys the laws of the modal logic S5. In order to get S5 for metaphysical necessity, Fine assumes a controversial principle about the nature of all objects. I will show that the addition of this principle to his original system E5 leads to inconsistency with an independently plausible principle about essence. I argue that this inconsistency can be avoided by adopting a weakened version of the controversial principle that still allows us to maintain S5 for metaphysical necessity. However, I will conclude that our investigation of Fine's Thesis in the context of LE motivates the revisionary conclusion that metaphysical necessity obeys the principles of the modal logic S4, but not those of S5. In my view, this result does not cast doubt on either LE or Fine's Thesis. Rather, it constitutes a distinctively essentialist challenge to the received view that the logic of metaphysical necessity is S5. But even if this conclusion is not granted, the investigation of Fine's Thesis in LE affords us an important and hitherto neglected perspective from which to assess our antecedently held views about metaphysical necessity.

One of the most distinctive features of Fine's system E5 is that it validates the *essentiality of being* (EB), the claim that for any object, it is essential to it to be something. Moreover, in conjunction with Fine's Thesis, E5 validates *necessitism*, the view that necessarily everything is necessarily something. Both the essentiality of being and necessitism are controversial philosophical theses. But I will not question them in this paper. It is of great interest to develop a logic of essence that invalidates EB and that, in conjunction with Fine's Thesis, invalidates necessitism.[4] But it is also worth exploring the system E5 despite its commitment to them, since neither EB nor necessitism is obviously false.[5]

The paper will be structured as follows. §1 introduces some of the tech-

[4]I explore various ways of doing this in 'Essence and Contingency' (unpublished manuscript).

[5]Teitel (2019) argues that there is a tension between Fine's Thesis, *contingentism*, i.e. the negation of necessitism, and the view that S5 is the correct logic for metaphysical necessity. The present paper is concerned with the question of whether Fine's Thesis is true while taking necessitism for granted, so I will not explicitly challenge Teitel's diagnosis of the tension here.

nical and philosophical background that will be needed for the discussion. §2 provides a detailed examination of two salient developments of Fine's Thesis in which metaphysical necessity obeys S5 and argues that they are subject to weighty objections. §3 develops and examines a theory that avoids these problems while still preserving S5 for metaphysical necessity. The paper ends with the conclusion that the best version of Fine's Thesis in the context of LE involves the adoption of a theory in which the logic of metaphysical necessity is exactly S4. An appendix establishes some of the technical results appealed to in the main text.

1

I will start with a brief presentation of the language of LE and Fine's system E5.[6] The vocabulary of the language of LE consists of a denumerable infinity of individual variables, individual constants, n-place predicate symbols ($n = 0, 1, 2, ...$), a special collection of designated 1-place 'rigid' predicate symbols, a primitive 2-place dependence predicate $\geq$ (I sometimes write $x \leq y$ instead of $y \geq x$), as well as the logical predicate of identity $=$, the logical constants $\neg, \vee, \forall$, the essentialist operator symbol $\Box$ and the abstraction operator λ, which allows us to form complex one-place predicates from arbitrary formulas: if ϕ is a formula and x is a variable, then $\lambda x \phi$ is a one-place predicate.

The only non-standard formation rule concerns the essentialist operator $\Box$: if ϕ is a formula and F is a 1-place predicate, then $\Box_F \phi$ is a formula. Informally, we can read $\Box_F \phi$ as 'It is true in virtue of the nature of the objects which F that ϕ', or as 'It is essential to the objects which F that ϕ'.[7] For example, $\Box_{\lambda x(Socrates=x)}$Socrates $\in$ {Socrates}[8] may be read as 'It is true in virtue of the nature of Socrates that Socrates is a member of singleton Socrates', or more simply 'It is essential to Socrates to be a member of singleton Socrates'. The predicate subscript allows us to pick out any plurality of objects as the subject of an essentialist attribution.

[6] A more rigorous presentation of the language used here can be found in Fine (2000).

[7] The notion of essence that Fine intends to capture in LE is what he calls the 'constrained consequential notion'. See Fine (1995b) for a general explication of what this notion amounts to.

[8] '{Socrates}' is used as a proper name here; there are no complex singular terms in the language of LE.

Another non-standard device of the language of LE concerns the use of rigid predicate symbols. Informally, we can think of the rigid predicate symbols as standing for pluralities. A more standard way to achieve the same effect would be to use the logical predicate 'is one of' and plural terms instead of rigid predicate symbols. Note, however, that the rigid predicate symbols in LE may have empty extensions, whereas standard logics of plurals do not allow for empty pluralities. The rigid predicate *symbols* are a subset of the class of rigid predicates of the language: a predicate is said to be *rigid* if it is either a rigid predicate symbol or is of the form $\lambda x \bigvee_{1 \leq i \leq n} \phi_i, n \geq 0$, where each formula $\phi_i, i = 1, ...n$, is either of the form Px, where P is a rigid predicate symbol, or of the form $x = y$, where y is a variable distinct from x, or of the form $x = c$, where c is an individual constant.

The following terminology and abbreviations, adopted from Fine (2000), will be frequently used in what follows:

(i) $\top := \forall x(x = x)$, where x is the first variable under some fixed ordering of the variables;

(ii) $\bigvee := \lambda x(\top)$;[9]

(iii) $\bigwedge := \lambda x(\neg\top)$;[10]

(iv) When x is a variable or a constant, we write $\Box_x$ for $\Box_{\lambda y(x=y)}$, where y is the first variable (under some standard ordering) distinct from x;

(v) $(F_1, ..., F_n) := \lambda x(\bigvee_{1 \leq i \leq n} F_i x)$, x the first variable not to occur free in any of $F_1, ..., F_n$. When used as a subscript to an essentialist operator, we will usually omit the outer parentheses and write $\Box_{F_1,...,F_n}$ instead of $\Box_{(F_1,...,F_n)}$;

(vi) Let E be a formula or predicate. Suppose $x_1, ..., x_m$ are the free variables of E in order of appearance and that $P_1, ..., P_n$ are the rigid predicate symbols of E, likewise in order of appearance.

Then $x\eta E := \bigvee_{1 \leq i \leq m} x = x_i \vee \bigvee_{1 \leq j \leq n} P_j x$. Informally: the object x occurs in the proposition (or relation) expressed by E;

[9]Note that this is a predicate that applies to absolutely everything. The essentialist operator subscripted with $\bigvee$, i.e. $\Box_{\bigvee}$, thus expresses truth in virtue of the nature of all objects.

[10]Note that this predicate applies to nothing. The essentialist operator subscripted with $\bigwedge$, i.e. $\Box_{\bigwedge}$, may be understood as expressing 'truth regardless of the nature of any objects'. Cf. Fine (1995a, p. 246).

(vii) If E is a predicate or a formula, $|E| := \lambda x(x\eta E)$, x the first variable not free in E. Intuitively, $|E|$ is to be understood as the *objectual content* of E;

(viii) $x \leq F := \exists y(Fy \wedge x \leq y)$, y the first variable distinct from x and not free in F. Informally: some F depends on x;

(ix) $cF := \lambda x(x \leq F)$, x the first free variable not free in F. cF applies to the objects upon which the F's depend. Intuitively, cF can thus be thought of as the *dependence closure* of the objects which F;

(x) $\Diamond_F \phi := \neg\Box_F\neg\phi$. Informally: it is compatible with the nature of the Fs that ϕ.

Fine's system E5 is based on a classical (non-free) quantificational logic.[11] The essentialist operator obeys the following modal axioms and rules:[12]

K $\Box_F(\phi \to \psi) \to (\Box_F\phi \to \Box_F\psi)$;

T $\Box_F\phi \to \phi$;

5 $\neg\Box_F\phi \to \Box_{F,|\phi|}\neg\Box_F\phi$, F rigid;

RN If ϕ is a theorem, then $\Box_{|\phi|}\phi$ is a theorem;

MON $\forall x(Fx \to Gx) \to (\Box_F\phi \to \Box_G\phi)$;[13]

In the context of LE we can ask, for any subscripted essentialist operator, what modal principles the operator obeys. The operator that will be of special interest here is $\Box_{\bigvee}$, which we can informally read as 'it is true in virtue of the nature of all objects that'. Fine's Thesis says that metaphysical necessity should be identified with truth in virtue of the nature of all objects whatsoever. On the intended interpretation of Fine's Thesis, the

[11]By the system E5 I mean the system presented in Fine (2000). Fine calls both the system in Fine (1995a) and that in Fine (2000) "E5", even though they are strictly speaking not the same system. The differences between them are the following two. First, the language of the system in Fine (1995a) contains an existence predicate, whereas the language of the system in Fine (2000) does not. Second, the system in Fine (2000) contains an additional rule of *modal predicate elimination*, which, as Fine notes, "seems necessary for the proof of completeness" (ibid., p. 547). The differences between these systems never matter for our discussion here, and I will continue to refer to the proofs and results in Fine (1995a) as proofs in E5.

[12]For a full axiomatization of E5 see Fine (2000).

[13]On the intended interpretation, the quantifiers should be understood as unrestricted.

quantifier should be understood as absolutely unrestricted. The question whether Fine's Thesis is correct can thus be formally expressed as the question whether $\Box_{\bigvee}$ expresses metaphysical necessity. A necessary condition for this to be the case is that $\Box_{\bigvee}$ has the formal properties of the metaphysical necessity operator, that is, it should obey the same logical principles as metaphysical necessity.

It turns out that in E5, $\Box_{\bigvee}$ satisfies exactly the propositional modal principles of the modal logic S4[14]: the K-schema, $\Box_{\bigvee}(\phi \to \psi) \to (\Box_{\bigvee}\phi \to \Box_{\bigvee}\psi)$, the T-schema, $\Box_{\bigvee}\phi \to \phi$, the 4-schema, $\Box_{\bigvee}\phi \to \Box_{\bigvee}\Box_{\bigvee}\phi$, as well as a rule of $\Box_{\bigvee}$-necessitation to the effect that $\Box_{\bigvee}\phi$ is a theorem whenever ϕ is a theorem.

Moreover, E5 validates the following controversial first-order modal principles:

EB $\forall x \Box_x \exists y(x = y)$

CBF $\Box_{\bigvee}\forall x\phi(x) \to \forall x\Box_{\bigvee}\phi(x)$

EB expresses the essentiality of being, the claim that for any object, it is essential to it to be something. CBF is the Converse Barcan formula. Relatedly, E5 validates NNE:[15]

NNE $\Box_{\bigvee}\forall x\Box_{\bigvee}\exists y(x = y)$

Given Fine's Thesis, NNE expresses necessitism. In the context of E5, Fine's Thesis is thus incompatible with contingentism.[16] All of these principles are controversial, to be sure, but as I have mentioned, I will not question them in this paper.

Fine (1995a) himself embraces these consequences, although he doesn't mention either of them explicitly. However, along with a majority of metaphysicians, Fine takes the modal system S5 to be the correct propositional logic of metaphysical necessity. Since in E5, $\Box_{\bigvee}$ satisfies exactly the principles of S4, E5 needs to be extended in order for $\Box_{\bigvee}$ to satisfy S5. S5 results

[14]See the appendix for a proof.

[15]The proofs of NNE and CBF in E5 are exactly analogous to their proofs in a normal modal system with classical quantificational axioms and rules.

[16]For a book-length defense of necessitism see Williamson (2013). See also Goodman (2016). Fine has recently argued that necessitism is obviously true (cf. Fine (2016)).

from S4 by the addition of the B-schema, $\phi \to \Box_{\bigvee}\Diamond_{\bigvee}\phi$.[17] So one way to get S5 for $\Box_{\bigvee}$ would be to simply add the B-schema to E5. It would seem quite arbitrary, however, to single out one operator and add a special axiom for it; and it is in fact not the approach Fine takes. Rather, Fine proposes to add the following axiom to E5:[18]

DOM $\forall x Px \to \Box_P \forall x Px$, where P is a rigid predicate

Informally, DOM says that it is true in virtue of the nature of all the objects that there are that they are all of the objects that there are; it lies in their nature to be exhaustive.[19]

Let S5π denote the modal system of quantified S5 with constant domain and let E5+DOM be the logic that results from E5 by adding DOM as an axiom. Fine (1995a, p. 267) shows that if we translate any formula ϕ in the language of S5π into a formula ϕ' in the language of LE by replacing each occurrence of $\Box$ with $\Box_{\bigvee}$, then for any theorem ϕ of S5π, ϕ' is a theorem of E5+DOM. The addition of DOM to E5 is thus sufficient for $\Box_{\bigvee}$ to satisfy S5.[20]

The result mentioned in the previous paragraph implies that E5+DOM not only validates S5 for $\Box_{\bigvee}$ but also the Barcan formula:

BF $\forall x \Box_{\bigvee}\phi(x) \to \Box_{\bigvee}\forall x \phi(x)$

This is no surprise, since just like in the case of quantified modal logic, BF can be derived from NNE, given S5. In conjunction with Fine's Thesis, E5+DOM thus involves a commitment to the view that there could neither be fewer nor more objects.

Let me now turn to Fine's justification for DOM. Before introducing DOM, Fine notes: 'I have tried to maintain a neutral position on what

[17]For a prominent critique of S5 as the correct logic for metaphysical necessity, see Salmon (1989). Salmon argues that both the 4- and the 5-schemas are not valid for metaphysical necessity. His arguments are thus arguments against both S4 and S5.

[18]This is "domain" axiom (V)(ii) in Fine (1995a, p. 250). See also Fine (2000, p. 583).

[19]Cf. Fine (1995a, p. 250). In contrast to Fine (2000), Fine (1995a) formulates DOM using a "rigidifier" $\langle\ \rangle$. If rigidifiers are present in the language, the logic is no longer closed under the rule of necessitation RN; cf. Fine (1995a, p.264). The language of LE in Fine (2000) does not contain rigidifiers. I will continue to use this language for the present discussion.

[20]Cf. Theorem 4 in Fine (1995a). Strictly speaking, Fine's result only shows that $\Box_{\bigvee}$ obeys S5 if it occurs in a formula of LE that is in the image of the translation $'$. I will later show that $\Box_{\bigvee}$ obeys S5 in E5+DOM for all formulas of LE and that it satisfies exactly the principles of S5.

kind of object belongs to the domain of quantification (...) However, if the domain is taken to consist of all metaphysically possible objects, then two further axioms should be added' (Fine, 1995a, p. 250).[21] The second of these axioms is DOM. The first of them is stated in terms of an existence predicate E that is taken as primitive, and for which no further axioms are given. It says that each object is such that its existence, in the sense that the predicate E applies to it, is compatible with the nature of all objects; formally, $\Diamond_{\bigvee} Ex$. This axiom itself is neutral about whether E should just be interpreted as equivalent to $\lambda x \exists y(x = y)$; so interpreted, it is a theorem of E5. Other interpretations need not concern us here.

What exactly does the appeal to 'metaphysically possible objects' amount to, and how does it support DOM? Here is what Fine says:

> [DOM] is plausible if the domain is taken to include the concept of a metaphysically possible object. For if $x_1, x_2, ...$ are all of the metaphysically possible objects, then it is presumably true in virtue of the nature of those objects and of the concept of being a metaphysically possible object that any metaphysically possible object is one of $x_1, x_2, ...$ Even if the domain consists only of the metaphysically possible individuals we can still guarantee

[21]Does this assumption differ from the assumption that the quantifiers are absolutely unrestricted? I claim that it does not. In his response to Williamson (2013), Fine notes: 'I am perfectly happy to concede that the unrestricted quantifiers range over all possible objects and so am happy to accept that necessarily everything necessarily has being' (Fine, 2016, p. 3). According to Fine, necessitism is obviously true on an unrestricted reading of the quantifiers. This leaves open the possibility to understand quantification over all metaphysically possible objects as being less inclusive than unrestricted quantification. One might, for example, think on actualist grounds that the metaphysically possible objects do not include sets of incompossible objects—objects that could not even be possibly actual—although these objects are still something. (The details of such a view are beyond the scope of this paper; see, for instance, Fine (2016).) But there is a simple argument for the claim that metaphysical necessity cannot be truth in virtue of the nature of any restricted class of objects, so the interpretation of Fine which takes quantification over the metaphysically possible objects to be tantamount to unrestricted quantification is more charitable in the present context. The argument goes as follows. Let $\Box$ express metaphysical necessity, M stand for any predicate (for example, 'is a metaphysically possible object') that does not apply to absolutely everything, and let x be any object for which $\neg Mx$ holds. By necessitism, which Fine is committed to, we have $\Box \exists y x = y$. But plausibly, $\neg \Box_M \exists y x = y$, since otherwise by the principles of LE there is a z such that Mz which depends on x. (This assumption is especially plausible if we read M as 'is a metaphysically possible object', since it is plausible to think that no metaphysically possible object depends on any metaphysically impossible object, or at the very least that not every metaphysically impossible object is such that some metaphysically possible object depends on it.) So $\Box$ cannot be $\Box_M$.

> the truth of the axiom by taking the minimal necessity $\Box_\wedge$ to be conceptual necessity, i.e., truth in virtue of the nature of all concepts. (Fine, 1995a, p. 250)

Fine's argument here is extremely condensed and not straightforward. The idea seems to be that even if it does not lie in the nature of all the metaphysically possible objects that they are all the metaphysically possible objects, it *does* lie in their nature that they are all the metaphysically possible objects once we take into account the nature of the concept of a metaphysically possible object. Fine suggests in the passage that this can be done in two ways: either we explicitly add the concept of a metaphysically possible object to the subject of the relevant essentialist attribution, or we implicitly add it via the 'minimal necessity' $\Box_\wedge$.

The role of the concept of a metaphysically possible object in this argument requires some interpretation. In a recent paper, Fine maintains that a necessary condition for the intelligibility of the concept of a possible object, for any sense of possibility, is that the possibility in question conforms to the Barcan formula under an unrestricted reading of the quantifier (cf. Fine (2016, p. 20)). He there argues that while logical possibility does not conform to the Barcan formula and that there can therefore be no intelligible concept of a logically possible object, it is evident that the concept of a metaphysically possible object is intelligible and that metaphysical possibility conform to the Barcan formula.[22]

On this assumption, the concept of a metaphysically possible object tacitly introduces the assumption that metaphysical possibility conforms to the Barcan formula. Fine's argument for DOM can then be interpreted as an attempt to provide an essentialist explanation of the presumed modal fact—implied by the Barcan formula—that it is metaphysically impossible for there to be more objects. An analogy might help to illustrate this. Suppose, plausibly, that it is metaphysically necessary that the (actual) natural numbers are all and only the natural numbers there are. Then one might think that a satisfying essentialist explanation of this fact is that it lies in the nature of the natural numbers (and perhaps the concept of being a natural

[22]Fine (1995a, p. 250f.) advances a similar, though more tentative, argument in the context of LE against using a conception of possible object where the possibility is broader than metaphysical possibility. This argument also presupposes that the corresponding broad conception of possibility should satisfy a Barcan-type principle.

number) that these natural numbers are all and only the natural numbers that there are.[23] Analogously, it may seem plausible that it lies in the nature of all metaphysically possible objects $x_1, x_2, ...$ (and perhaps the concept of a metaphysically possible object) that every metaphysically possible object is one of $x_1, x_2, ...$, since this might provide a satisfying essentialist explanation of the Barcan formula for metaphysical necessity. In both cases, the essentialist claim doesn't *follow* from the modal claim; yet, the truth of the latter may render the former more plausible.

Teitel (2019) has recently argued that Fine's appeal to metaphysically possible objects in justifying Fine's Thesis in LE amounts to a failure to meet the ambition of reducing modality to something inherently non-modal, unless the ideology of possible objects can itself be reduced to something non-modal (cf. ibid., p. 59). But it is not clear that this charge is warranted. It is correct that Fine's argument for DOM seems to rely on distinctively modal assumptions, namely the truth of the Barcan formula for metaphysical necessity and his corresponding understanding of the concept of a metaphysically possible object. However, it is not clear that this prevents the reduction from being 'non-modal' in a metaphysically significant sense. In general, the use of modal assumptions in the justification of essentialist principles does not show that necessity cannot be reduced to essence; it shows at most that our understanding of essence is not completely independent of our understanding of modality—a claim that is not disputed by Fine.

Still, we can (and I think we should) avoid using the concept of a metaphysically possible object in discussing Fine's Thesis and carry out the debate in more neutral terms, thus circumventing Teitel's objection. As pointed out at the beginning of this section, we can unambiguously formulate Fine's Thesis in LE as the claim that $\Box_{\bigvee}$ expresses metaphysical necessity if the quantifiers are understood as unrestricted. We can furthermore ask whether DOM is true on an unrestricted reading of the quantifiers, i.e. whether it is true in virtue of the nature of all objects that they are all the objects there are. As far as I can see, DOM, just like the Barcan formula for metaphysical necessity, is neither obviously true nor obviously false. As highlighted by Fine's argument, these principles are intimately connected in the present dialectic: accepting the latter may provide one with a good, even if not decisive, reason to accept the former, while rejecting the latter

[23]This analogy was suggested to me by Kit Fine.

prevents one from accepting the former. DOM thus has a special status in this dialectic, since, at least pending further independent arguments, it is the only principle of E5+DOM whose justification is intimately bound up with modal assumptions—all the principles of E5 appear to be justifiable on purely essentialist grounds (cf. Fine (1995a)).

However, it is best not to rely on these modal assumptions alone in deciding whether to accept DOM. The remainder of this paper will be concerned with a systematic investigation of the consequences of accepting DOM, offering more principled grounds for evaluating the prospects of maintaining Fine's Thesis in conjunction with the view that metaphysical necessity obeys S5.

2

I will now introduce Fine's (2000) semantics for LE, identify two natural classes of models in which E5+DOM is sound and study some of their properties that will be useful for our further investigation. I begin with some definitions.

A *structure* $\mathcal{S}$ is a triple $\langle W, I, \succeq \rangle$, where (i) W is a non-empty set; (ii) I is a function taking each $w \in W$ into a non-empty set I_w; (iii) $\succeq$ (dependence) is a reflexive and transitive relation on $\bigcup_{w \in W} I_w$ with respect to which each world is closed ($a \in I_w$ and $a \succeq b$ implies $b \in I_w$).

A *model* $\mathcal{M}$ is a quadruple $\langle W, I, \succeq, V \rangle$, where (i) $\langle W, I, \succeq \rangle$ is a structure; (ii) V (valuation) is a function that takes each constant a into an individual $V(a)$ of some I_w ($w \in W$), each rigid predicate symbol P into a subset $V(P)$ of some I_w, and each world w and non-rigid n-place predicate symbol F into a set $V(F, w)$ of n-tuples of I_w.

Given any subset J of $\bigcup I_w$, the *closure* $c(J)$ of J (in $\mathcal{M}$) is defined as $\{b : a \succeq b \text{ for some } a \in J\}$.

Let $\mathcal{M}$ be a model and ϕ a sentence or closed predicate whose constants are $a_1, ..., a_m$ and whose rigid predicate symbols are $P_1, ..., P_n$. Then the *objectual content* $|\phi|^{\mathcal{M}}$ of ϕ (in $\mathcal{M}$) is taken to be $\{V(a_1), ..., V(a_m)\} \cup V(P_1) \cup ... \cup V(P_n)$. ϕ is *defined* (in $\mathcal{M}$) at $w \in W$ if $|\phi|^{\mathcal{M}} \subseteq I_w$.

For simplicity, Fine defines truth and extension for *full* models. A model $\mathcal{M}$ is *full* if for each $\mathbf{a} \in \bigcup I_w$ there is a constant a for which $V(a) = \mathbf{a}$. Given a full model $\mathcal{M}$, we define $\mathcal{M}, w \vDash \phi$, the *truth* of the sentence ϕ at

w, and F_w, the *extension* of the predicate F at w. Note that $\mathcal{M}, w \vDash \phi$ and F_w are defined just in case ϕ and F are defined at w in $\mathcal{M}$. The clauses are as follows:

(i) $\mathcal{M}, w \vDash Fa_1, ...a_n$ iff $\langle V(a_1), ..., V(a_n)\rangle \in F_w$, where F is any non-logical predicate;

(ii) $\mathcal{M}, w \vDash a = b$ iff $V(a) = V(b)$;

(iii) $\mathcal{M}, w \vDash a \geq b$ iff $V(a) \succeq V(b)$;

(iv) $\mathcal{M}, w \vDash \neg\phi$ iff not $w \vDash \phi$;

(v) $\mathcal{M}, w \vDash (\phi \vee \psi)$ iff $\mathcal{M}, w \vDash \phi$ or $\mathcal{M}, w \vDash \psi$;

(vi) $\mathcal{M}, w \vDash \forall x\phi(x)$ iff $\mathcal{M}, w \vDash \phi(a)$ whenever $V(a) \in I_w$;

(vii) $\mathcal{M}, w \vDash \Box_F\phi$ iff (a) $|\phi|^{\mathcal{M}} \subseteq c(F_w)$ and (b) $\mathcal{M}, v \vDash \phi$ whenever $I_v \supseteq F_w$.

(viii) $G_w = V(G, w)$, where G is a (non-rigid) predicate symbol;

(ix) $P_w = V(P)$, where P is a rigid predicate symbol;

(x) $\lambda x\phi(x)_w = \{\mathbf{a} \in I_w : \mathcal{M}, w \vDash \phi(a), \text{where } \mathbf{a} = V(a)\}$.

Notice that the semantic clause for $\Box_{\bigvee}$ simplifies to

$\mathcal{M}, w \vDash \Box_{\bigvee}\phi$ iff $\mathcal{M}, v \vDash \phi$ whenever $I_v \supseteq I_w$.

A sentence ϕ is *valid* if for every model $\mathcal{M}$ and every world w of $\mathcal{M}$ at which ϕ is defined, ϕ is true at w in $\mathcal{M}$; and a formula $\phi(x_1, ..., x_n)$ is *valid* if each of its closed instances $\phi(a_1, ..., a_n)$ is valid. A sentence ϕ is *valid in a model* $\mathcal{M}$ if it is true at every world at which ϕ is defined in that model; ϕ is *valid in a class of models* $\mathfrak{S}$ if it is valid in every model $\mathcal{M} \in \mathfrak{S}$.

A sentence ϕ is *valid in a structure* $\mathcal{S}$ iff for every model $\mathcal{M}$ based on $\mathcal{S}$ and every world w of $\mathcal{M}$ at which ϕ is defined (in $\mathcal{M}$), ϕ is true at w; ϕ is said to be *valid in a class of structures* $\mathfrak{S}$ just in case ϕ is valid in every $\mathcal{S} \in \mathfrak{S}$. If Γ is a set of formulas and $\mathfrak{S}$ is a class of structures, we say that Γ *defines* $\mathfrak{S}$ iff for all structures $\mathcal{S}$, $\mathcal{S}$ is in $\mathfrak{S}$ just in case all members of Γ are valid in $\mathcal{S}$.

We are now ready to define two classes of structures that will be of special interest in what follows.

A *constant domain structure* is a structure $\langle W, I, \succeq\rangle$ in which I is a constant function. In a constant domain structure, the domain does not vary across worlds. A *constant domain model* is a model $\langle W, I, \succeq, V\rangle$, where $\langle W, I, \succeq\rangle$ is a constant domain structure.

An *equivalence structure* is a structure $\langle W, I, \succeq\rangle$ in which I satisfies the following condition: For all $w, v \in W$, if $I_w \subseteq I_v$, then $I_w = I_v$. In an equivalence structure, no domain of a world is properly included in any other domain of a world. An *equivalence model* is a model $\langle W, I, \succeq, V\rangle$, where $\langle W, I, \succeq\rangle$ is an equivalence structure.

We know from the soundness and completeness result in Fine (2000) that the logic of the class of all models is E5. Fine also notes without proof that the logic of the class of equivalence models is E5+DOM. It is thus natural to investigate these models in the present context. Another natural class of models that validates E5+DOM is the class of constant domain models, a subclass of the class of equivalence models. These models have a particularly simple structure and closely resemble the constant domain models of quantified modal logic. The naturalness and simplicity of constant domain models constitutes a good reason for a closer examination of them.

I will begin by justifying some of the claims made in the previous section. The first result concerns the validity of the 4-schema for $\Box_{\bigvee}$ in E5:

Proposition 1. *$\Box_{\bigvee}\phi \to \Box_{\bigvee}\Box_{\bigvee}\phi$ is a theorem schema of E5.*[24]

Corollary 1. *In E5, $\Box_{\bigvee}$ satisfies all valid schemas of S4.*[25]

We can in fact show something stronger, namely that $\Box_{\bigvee}$ satisfies *exactly* the S4-valid schemas. In other words, whenever ϕ is a propositional modal schema that is not valid in S4, there is an instance of ϕ in the language of LE that is not valid in E5. A proof of Proposition 2 is given in the appendix.

Proposition 2. *In E5, the logic of $\Box_{\bigvee}$ is exactly S4.*

The next result concerns the important relationship between the schemas DOM, BF and B and the class of equivalence structures.

[24] *Proof*:The claim immediately follows from the semantic clause for $\Box_{\bigvee}$ and completeness.

[25] *Proof*: The claim follows from the fact that (a) all axiom schemas of S4 are satisfied by $\Box_{\bigvee}$ in E5 and (b) the fact that the rules of modus ponens (MP) and necessitation (RN$_{\bigvee}$): if ϕ is a theorem, then $\Box_{\bigvee}\phi$ is a theorem, both preserve theoremhood in E5.

Proposition 3. *The schemas BF, DOM and B each define the class of equivalence structures.*[26]

Moreover, the logics E5+S, where S is DOM, BF or B, are the same.[27]

Corollary 2. *DOM, BF and B are valid in the class of constant domain models.*

Corollary 3. $\Box_{\bigvee}$ *satisfies all valid schemas of S5 in the class of equivalence models.*[28]

In analogy to Proposition 2, we can show that the logic of $\Box_{\bigvee}$ is exactly propositional S5 in the class of equivalence models, and thus in E5+DOM. Every propositional modal schema that is not valid in S5 has an instance in the language of LE that is not valid in E5+DOM. A proof of Proposition 4 is given in the appendix.

Proposition 4. *In E5+DOM, the logic of* $\Box_{\bigvee}$ *is exactly S5.*

We now turn to an interesting feature of the logic of constant domain structures. Constant domain models validate the following principle that allows us to identify the specific 'minimal source' of any attribution of an essential property:

DEL $\Box_{F,G}\phi \wedge \Box_F(\phi \vee \neg\phi) \rightarrow \Box_F\phi$

Proposition 5. *DEL is valid in the class of constant domain models.*[29]

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[26] *Proof:* I give a proof for BF. The proofs for DOM and B are similar.

For the right-to-left direction suppose BF is valid in a structure $\mathcal{S}$. We show that $\mathcal{S}$ is an equivalence structure. Let $w, v \in W$ and suppose that $I_w \subseteq I_v$. Let $\mathcal{M}$ be a model based on $\mathcal{S}$ such that the extension of the rigid predicate P is I_w, i.e. $V(P) = I_w$. Since BF is valid in $\mathcal{S}$, the instance $\forall x \Box_{\bigvee} Px \rightarrow \Box_{\bigvee}\forall x Px$ is true at w. But $\mathcal{M}, w \vDash \forall x \Box_{\bigvee} Px$, since $\bigvee_w = I_w = V(P)$. Hence, $\mathcal{M}, w \vDash \Box_{\bigvee}\forall x Px$. Thus, $\mathcal{M}, v \vDash \forall x Px$, since $I_w \subseteq I_v$. From this it follows that every $\mathbf{a} \in I_v$ is in I_w, because $V(P) = I_w$; hence, $I_w = I_v$.

For the converse suppose that $\mathcal{M}$ is an equivalence model and $w \in W$. Suppose $\mathcal{M}, w \vDash \forall x \Box_{\bigvee}\phi(x)$. By definition, $\mathcal{M}, w \vDash \Box_{\bigvee}\phi(a)$ whenever $V(a) \in I_w$. Now let $\mathbf{a} = V(a) \in I_w$ be arbitrary. Then $\mathcal{M}, v \vDash \phi(a)$ for all $v \in W$ such that $I_w \subseteq I_v$. But $\mathcal{M}$ is an equivalence model, so $I_w = I_v$. From this and the fact that $\mathbf{a}$ was arbitrary, we have $\mathcal{M}, v \vDash \forall x\phi(x)$ for all $v \in W$ such that $I_w \subseteq I_v$. Thus, by the semantic clause for $\Box_{\bigvee}$ we obtain $\mathcal{M}, w \vDash \Box_{\bigvee}\forall x\phi(x)$.

[27] See the paragraph after Theorem 1 in the appendix.

[28] *Proof*: Similar to the proof of Corollary 1.

[29] The straightforward proof is omitted here. Note that we can in fact show something stronger, namely that DEL *defines* the class of constant domain models. DEL is thus not valid in the class of equivalence models. As a consequence, the logic of constant domain models is stronger than the logic E5+DOM.

An interesting special case of DEL is the case where F is $\bigwedge$. It says that whenever ϕ has no objectual content and is essential to some things, then it is true 'regardless of the nature of any objects'. If we take the *purely qualitative* propositions to be those without objectual content, this special case of DEL entails that every purely qualitative proposition true in virtue of the nature of some objects is true 'regardless of the nature of any objects'. This principle clearly doesn't hold in E5. For suppose that $\Box_s \exists x Hx$. It is then easy to find a countermodel to $\Box_{\bigwedge} \exists x Hx$: just pick a world where the denotation of s is not in the domain of that world and where the extension of H is empty.

The counterexample to DEL above is purely formal, however. It therefore doesn't yet give us any philosophical reasons for doubting DEL. The following argument is philosophically more significant. Consider the proposition that *Socrates is a member of some set*, formally $\exists y(s \in y)$. If we assume that it is metaphysically necessary that Socrates is a member of singleton Socrates, then it is also metaphysically necessary that Socrates is a member of some set.[30] So $\Box_{\bigvee} \exists y(s \in y)$. But the following is a theorem of E5: $\Box_s(\exists y(s \in y) \vee \neg\exists y(s \in y))$. Hence, by DEL, $\Box_s \exists y(s \in y)$.

DEL thus allows us to derive that it is essential to Socrates to be a member of some set from the assumption that it is metaphysically necessary that he is a member of some set. But this result is problematic if we adopt a Finean conception of essence. For a more general reason for thinking that it is not essential to Socrates that he be a member of singleton Socrates is that it is not essential to him that there be any sets; Socrates' nature is, plausibly, not concerned with the existence of sets at all. As Fine himself puts it, 'There is nothing in the nature of a person, if I may put it this way, which demands that he belongs to this or that set or which demands, given that the person exists, that there even be any sets' (Fine, 1994, p. 5).[31] These considerations constitute a significant argument against constant domain models.

DEL fails to be valid if we drop domain constancy; in particular, it may fail in equivalence models. E5+DOM is thus not subject to the problem raised above for constant domains. I will now argue that it faces another,

[30]Note that we can formulate the argument independently of Fine's Thesis by using $\Box_{s,\{s\}}$ in place of metaphysical necessity.
[31]See also (Fine, 1995a, p. 241).

equally serious problem instead. Consider the following principle: for any object x, it is compatible with x's nature that there be nothing except the entities x depends on, including x itself. Formally, $\forall x \Diamond_x \forall y (y \leq x)$. Call this the *independence principle.* (The dual of the independence principle says that there is no object whose nature requires there to be an object it doesn't depend on; formally, $\neg \exists x \Box_x \exists y \neg (y \leq x)$.) Fine's counterexamples to the definition of essence in terms of metaphysical necessity are a consequence of this general principle.[32] One way of making explicit the intuition behind the view that it is not essential to Socrates that he be a member of singleton Socrates is that Socrates' nature only requires there to be those objects that Socrates depends on; and neither singleton Socrates nor any other set is such an object.

Even though for present purposes, the principle is not required to hold in full generality in order to constitute a problem for E5+DOM, it is still worthwhile considering what the consequences of its acceptance are in the context of LE. Let IP be the formula $\forall x \Diamond_x \forall y (y \leq x)$. We can show that IP defines the class of structures satisfying the following condition: for all $a \in \bigcup I_w$, there is a $w \in W$ such that $I_w = c(\{a\})$. Call a structure satisfying this condition an *IP structure.*[33] IP structures have the interesting property that the dependence relation $\succeq$ in the structure is completely determined by the *existence sets* of the individuals in the structure, where an existence set of an individual a is the set of worlds $I_a = \{w : a \in I_w\}$. More concisely, the following principle holds for IP structures:

DEP $\forall a, b \in \bigcup I_w$: $b \succeq a$ iff $I_b \subseteq I_a$.

The "only if" direction of DEP holds in all structures by the closure condition on structures. The "if" direction, by contrast, does not hold in general. In IP structures, we can drop the dependence relation $\succeq$. DEP is interesting because it allows us to state in the metalanguage that b depends on a just in case a is something in all worlds in which b is something. DEP is thus a close cousin to the definition of dependence in terms of existence and metaphysical necessity that Fine rejects. The difference between DEP and

[32] This is not meant to be an exegetical claim about Fine's counterexamples. While the original motivation for these counterexamples may have had nothing to do with the independence principle, it is still instructive to see that the counterexamples can be derived from a more general principle about essence.

[33] It is worth noting that E5+IP is complete with respect to the class of IP structures.

the rejected definition is that the worlds in DEP need not be metaphysically possible, where metaphysical possibility is understood as compatibility with the nature of all objects. The fact that the addition of IP to E5 or its extensions induces such natural properties on structures adds to the plausibility of IP.

For the present purpose, we can be noncommital about whether the independence principle is true in full generality, however. In order to create a problem for E5+DOM we only need a single true instance of the independence principle.[34] So even someone who thinks that only one instance of the independence principle is true will have to give up E5+DOM.

Now to the problem. Suppose for the sake of argument (and for simplicity) that Socrates depends only on himself. The following two claims are inconsistent in E5+DOM.

(1) It is compatible with Socrates' nature that he be the only object there is. ($\Diamond_s \forall y(y = s)$)

(2) There are objects that are distinct from Socrates. ($\exists x(x \neq s)$)

We can easily derive a contradiction semantically. By (1), there is a world v that contains Socrates and nothing else. By (2), the actual world w contains objects distinct from Socrates. So the domain of v is properly included in the domain of w, which is impossible in equivalence models, because no domain of a world is properly included in any other domain of a world those models.

The following syntactic derivation shows how we can derive the negation of (2) from an instance of (1) in E5+DOM. This derivation gives us more explicit insight into which principles of E5+DOM lead to the inconsistency.[35]

[34] Something like the independence principle seems to have been widely accepted at least for individual substances by scholastic philosophers and many early modern philosophers, including Descartes and Leibniz.

[35] In addition to DOM, the following axioms and rules of E5+DOM are used in the derivation:

K $\Box_F(\phi \to \psi) \to (\Box_F\phi \to \Box_F\psi)$;

T $\Box_F\phi \to \phi$;

5 $\neg\Box_F\phi \to \Box_{F,|\phi|}\neg\Box_F\phi$, F rigid;

MP If ϕ is a theorem and $\phi \to \psi$ is a theorem, then ψ is a theorem;

RN If ϕ is a theorem, then $\Box_{|\phi|}\phi$ is a theorem;

λ $\lambda x\phi y \leftrightarrow \phi(y)$, where $\phi(y)$ is the result of substituting y freely for the free occurrences of x in ϕ.

(1)	$\Diamond_s \forall x(x = s)$	Premise
(2)	$\forall x(\lambda y(y = s)x) \to \Box_s \forall x(\lambda y(y = s)x)$	DOM
(3)	$\Box_s(\forall x(\lambda y(y = s)x) \to \Box_s \forall x(\lambda y(y = s)x))$	(2), RN
(4)	$\Diamond_s \forall x(\lambda y(y = s)x) \to \Diamond_s \Box_s \forall x(\lambda y(y = s)x)$	K, (3)
(5)	$\Diamond_s \forall x(\lambda y(y = s)x)$	(1), λ
(6)	$\Diamond_s \Box_s \forall x(\lambda y(y = s)x)$	(4), (5), MP
(7)	$\Diamond_s \Box_s \forall x(\lambda y(y = s)x) \to \Box_s \forall x(\lambda y(y = s)x)$	5
(8)	$\Box_s \forall x(\lambda y(y = s)x)$	(6), (7), MP
(9)	$\forall x(x = s)$	(8), T, λ

In words, if it is compatible with Socrates' nature that he be the only object there is, then it is essential to Socrates that he be the only object there is; as a consequence, Socrates is actually the only object there is. Thus, contrapositively: if Socrates is not the only object there is, then it is essential to Socrates that he not be the only object there is.

I take the fact that E5+DOM is incompatible with even a single true instance of the independence principle in the way shown above to constitute a weighty objection against E5+DOM.[36]

3

The problems for Fine's Thesis posed in the previous section can be fixed. In order to block the derivation above, one of the principles it depends on has to be given up. Giving up the premise is not an attractive option, because we are now interested in maintaining the possibility that there is a true instance of the independence principle. The logical principles K, T, 5 and λ are axioms of E5. Giving up any of these principles would amount to a radical change in the logic of essence. Such a move seems to be unjustified in this context. Likewise, giving up MP would be completely unmotivated. So the only remaining options are to give up DOM or RN. Giving up DOM would amount to giving up S5 for $\Box_{\bigvee}$, however. This may ultimately be an option, but it is not yet forced on us. A more conservative option that is worth exploring first is to see if we can weaken RN. The derivation above

[36]Strictly speaking, the relevant instance has to be one that concerns an object that doesn't depend on absolutely everything. I believe this doesn't significantly improve the position of a proponent of E5+DOM.

crucially depends on an application of RN to an instance of DOM. A natural suggestion would thus be to weaken RN to the following rule:

RN$^-$ If $\vdash_{E5} \phi$, then $\vdash \Box_{|\phi|}\phi$

RN$^-$ says that only theorems of E5 can be necessitated, thus excluding its application to DOM and theorems derived with the help of DOM. Let E5+DOM$^-$ be the logic that is just like E5+DOM except that RN is replaced by RN$^-$. This section examines Fine's Thesis in the context of E5+DOM$^-$.

The new rule of necessitation requires us to slightly modify our model theory. We define a *centered structure* to be a quadruple $\langle W, I, \succeq, w\rangle$, where $W, I, \succeq$ are as before, and $w \in W$; w is the *actual world* of the structure. A *centered model* is a quintuple $\langle W, I, \succeq, w, V\rangle$, where $\langle W, I, \succeq, w\rangle$ is a centered structure and V is a valuation function as before. The truth relation $\mathcal{M}, w \vDash \phi$ is, as before, defined for sentences that are defined in $\mathcal{M}$ at the world of evaluation. A sentence is *true in a centered model* iff it is true at the actual world of the model. A sentence is called *A-valid in a centered structure* iff, for every centered model $\mathcal{M}$ based on this centered structure, it is true at the actual world whenever it is defined at the actual world. A sentence is *A-valid* iff it is A-valid in all centered structures. A-validity for formulas is again understood as A-validity of all of their closed instances. Note that the class of A-valid formulas coincides with the class of formulas valid in the original sense if no further condition is imposed on the structures.

With these definitions in hand, we may now define a class of centered structures for which E5+DOM$^-$ is sound. An *A-equivalence structure* is a centered structure $\mathcal{S} = \langle W, I, \succeq, w^*\rangle$ such that for all $v \in W$, if $I_{w^*} \subseteq I_v$, then $I_{w^*} = I_v$. In an A-equivalence structure, no domain of a world properly contains the domain of the actual world. An *A-equivalence model* is a centered model based on an A-equivalence structure. If Γ is a set of formulas and $\mathfrak{S}$ is a class of centered structures, we say that Γ *A-defines* $\mathfrak{S}$ iff for all centered structures $\mathcal{S}$, $\mathcal{S}$ is in $\mathfrak{S}$ just in case all members of Γ are A-valid in $\mathcal{S}$.

We have the following analogue to Proposition 3 for A-equivalence structures.

Proposition 6. *The schemas DOM, BF and B each A-define the class of A-equivalence structures.*

We also have the following two important results:

Proposition 7. *If ϕ is A-valid in an A-equivalence structure $\mathcal{S}$, then $\Box_\bigvee \phi$ is A-valid in $\mathcal{S}$.*[37]

Corollary 4. *$\Box_\bigvee$ satisfies all theorems of S5 in the class of A-equivalence structures.*

Notice that Proposition 7 does not hold for centered structures in general—$\text{RN}_\bigvee$ does not in general preserve A-validity in a centered structure. I will show in the appendix that E5+DOM$^-$ is also complete with respect to the class of A-equivalence structures.[38] We can thus infer from Corollary 4 and the completeness result that $\Box_\bigvee$ satisfies all the valid schemas of S5 in E5+DOM$^-$; and it turns out that it satisfies only those schemas, so that the logic of $\Box_\bigvee$ is exactly S5 in E5+DOM$^-$. Moreover, it is clear that DEL is not A-valid in A-equivalence structures. So the first problem raised for constant domain models does not arise. As to the second problem from the previous section, Proposition 6 entails that E5+DOM$^-$ is compatible with the independence principle, because A-equivalence structures admit of worlds whose domains are a proper subset of the domain of the actual world. E5+DOM$^-$ thus avoids the problems that befell the logic of constant domain and equivalence structures while allowing us to retain S5 for $\Box_\bigvee$.

In A-equivalence structures, we can informally think of the worlds with the same domain as the actual world as the *sphere of metaphysically possible worlds*. The laws of the modal logic S5 hold throughout this sphere for $\Box_\bigvee$. On this picture, the metaphysically possible worlds just are the worlds that contain the same objects as the actual world. The worlds outside the sphere of metaphysically possible worlds may be called the *merely essence-possible worlds*, understood as not including the metaphysically possible worlds.

It is important to note that $\Box_\bigvee$ only obeys the characteristic principles of S5 within the sphere of the metaphysically possible worlds. These principles will not in general hold for $\Box_\bigvee$ outside this sphere. On this view, the fact that metaphysical necessity obeys S5 is a special feature of the collective

[37] *Proof.* Let $\mathcal{S} = \langle W, I, \succeq, w^* \rangle$ be an A-equivalence structure in which $\Box_\bigvee \phi$ is not A-valid. Then there is a model $\mathcal{M} = \langle W, I, \succeq, w^*, V \rangle$ based on $\mathcal{S}$ such that for some $v \in W$, $I_v = I_{w^*}$ and $\mathcal{M}, v \nvDash \phi$. We define a bijective function f from W to itself as follows: (i) $f(w^*) = v$; (ii) $f(v) = w^*$; (iii) $f(w) = w$ for all $w \neq v, w^*$. Define a model $\mathcal{M}' = \langle W, I, \succeq, w^*, V' \rangle$ as follows: (i) $V(\tau) = V'(\tau)$ for all constants and rigid predicates τ; (ii) $V'(F, f(w)) = V(F, w)$ for every predicate symbol F. Then a straightforward induction on the complexity of formulas shows that for all formulas ψ and all $w \in W : \mathcal{M}', f(w) \vDash \psi$ iff $\mathcal{M}, w \vDash \psi$. Hence, $\mathcal{M}', w^* \nvDash \phi$, so ϕ is not A-valid in $\mathcal{S}$.

[38] See Theorem 1.

nature of all objects. For it is consistent to assume that it is compatible with the nature of some objects that some laws of S5, namely those that depend on the B-schema, may fail for metaphysical necessity. In this sense, the B-schema has a special status among the laws governing metaphysical necessity in this theory.[39] Although this might to some extent count against Fine's Thesis in the context of E5+DOM$^-$, I don't think it constitutes a decisive objection against it.[40]

Let me now turn to another objection to Fine's Thesis in the context of E5+DOM$^-$.[41] The objection might go something like this: 'What I mean by "metaphysically possible" includes your merely essence-possible worlds, because metaphysical possibility is supposed to be absolutely unrestricted possibility. But the restriction to your metaphysically possible worlds is illegitimate, because it imposes a restriction on the quantification over possibilities and our interest is in unrestricted, not relative, possibility. In short, what you call "metaphysical possibility" is not metaphysical possibility, but some sort of restricted possibility.'

I think that this objection draws attention to a number of important aspects of the view elaborated here. It is useful to distinguish between two different versions of this objection.

The first version of the objection draws on an understanding of necessity operators as operators that act like restricted universal quantifiers over

[39] The view that the B-schema has a special status among the laws governing metaphysical necessity is not unprecedented in the literature. Salmon (1989), for example, argues that 'even if the B principle is necessarily true, its alleged status as a logical (or analytic) truth remains in need of justification' (p.4).

[40] There is a close variant of E5+DOM$^-$ that is worth mentioning here. Instead of restricting RN, we could extend the language of LE with a rigid predicate constant A and add the axiom DOMc: $\forall x A x$ to E5. The resulting logic has the usual rule of necessitation RN. As a consequence, we get the following categorical instance of DOM as a theorem: $\Box_A \forall x A x$. It may be shown that E5+DOMc is sound and complete with respect to the class of A-equivalence models in which A is always interpreted as having as extension the domain of the actual world of the model, if we understand validity in the original sense, i.e. not as A-validity. $\Box_\bigvee$ obeys the same logical principles in E5+DOMc as in E5+DOM$^-$. In E5+DOMc there is another candidate for an operator expressing metaphysical necessity: $\Box_A$. This operator is also an S5 modal operator validating both Barcan formulas. However, this 'non-qualitative' operator is arguably not a good candidate for expressing metaphysical necessity, because no facts about metaphysical necessity or possibility would be essential to any object, unless that object depends on absolutely everything.

[41] The objection could be made, *mutatis mutandis*, against Fine's Thesis in the context of E5 or E5+DOM, though not in the context of the logic of constant domain structures. The replies I give apply to all these cases.

worlds.[42] Accordingly, metaphysical necessity can be understood as absolute necessity in the sense of absolutely unrestricted quantification over all worlds. Any restriction on the quantification over worlds should, on this view, seem illegitimate if metaphysical necessity is at issue. So whatever worlds are singled out by the operator $\Box_{\bigvee}$, one might hold that they are not legitimately called *the* metaphysically possible worlds.

In response to this objection, it should be pointed out that the unrestricted sense of necessity the objection trades on is characterized by quantification in the extensional metalanguage. However, an operator of the kind alluded to in the objection is not syntactically definable in the language of LE. In the context of this logic, there is no syntactically definable operator O such that $O\phi$ is true at a world if and only if ϕ is true at every world, because ϕ may not be defined at every world,[43] and an operator expressing the condition of a formula being defined at every world is likewise not definable in the language. From the perspective of the logic of essence, necessity as truth at absolutely every world might simply be deemed unintelligible. The illegitimacy of an understanding of necessity along these lines might be attributed to the priority of the object-language. The semantics in Fine (2000) is not supposed to be understood as a reductive account of our understanding of the essentialist operator. Rather, the role of the semantics is largely instrumental. Thus, the fact that the 'absolute necessity' operator definable in the metalanguage is not expressible in the object-language is not obviously a defect of this logic.

However, even if such an operator were available in the language, it would not be a good candidate for expressing metaphysical necessity, because it would not in general obey the widely accepted principle that essence implies necessity: whenever something is essential to some things it is metaphysically necessary. For let $\mathcal{M}$ be a model and a be an object that exists in some but not all worlds in the model. Let w be a world at which a exists. Then $\Box_a a = a$ is true at w, but $Oa = a$ is not, since $a = a$ is not true at every world, because a does not exist at every world.

The second version of the objection does not invoke worlds. Rather, it is based on a characterization of metaphysical necessity as necessity *in the*

[42]See, for example, Lewis (1973, p.5).

[43]Note that a formula ϕ may be true at every world of a model without it being the case that $\Box_{\bigwedge}\phi$ is true at any world, since ϕ may have objectual content.

highest degree, necessity in the strongest or broadest sense, or necessity *tout court*.[44] Given Fine's Thesis, the objection goes, metaphysical necessity does not fit this description, because there are necessity operators in LE that are stronger than $\Box_{\bigvee}$. The strongest kind of necessity expressible in the object language would seem to be $\Box_{\bigwedge}$, since for every formula ϕ, $\Box_{\bigwedge}\phi \to \Box_{\bigvee}\phi$; but the converse may fail if ϕ has non-empty objectual content. Another candidate operator for expressing metaphysical necessity is the operator that, given any ϕ, makes $\Box_{|\phi|}\phi$. Let us abbreviate this operator $\Box_{|\cdot|}$. Why is $\Box_{\bigvee}$ a better candidate for expressing metaphysical necessity than either of those two?

In response, it is worth pointing out that the characterization of metaphysical necessity the objection is based on is not uncontroversial.[45] However, neither $\Box_{\bigwedge}$ nor $\Box_{|\cdot|}$ is a good candidate for expressing metaphysical necessity, quite independently of whether metaphysical necessity is in some sense the broadest necessity.

First, consider $\Box_{\bigwedge}$. This operator is not a candidate for expressing metaphysical necessity on the grounds that it does not apply to any ϕ with objectual content. For example, $\Box_{\bigwedge}(\phi \vee \neg\phi)$ is false if ϕ has objectual content. As to $\Box_{|\cdot|}$, we may have $\Box_{|\phi|,|\psi|}(\phi \wedge \psi)$ without either of $\Box_{|\phi|}\phi$ or $\Box_{|\psi|}\psi$. So $\Box_{|\cdot|}$ is not a normal modal operator. But metaphysical necessity is plausibly a normal modal operator.[46] Moreover, neither of these operators generally satisfies the principle that essence implies necessity. For we may have $\Box_{\bigvee}\phi$ without either of $\Box_{|\phi|}\phi$ or $\Box_{\bigwedge}\phi$. By contrast, $\Box_{\bigvee}$ satisfies all of these desiderata.

In the context of LE, Fine's Thesis constitutes a simple and compelling account of the relation between essence and necessity. Someone who endorses any system of LE but denies Fine's Thesis would have to come up

[44]See, for instance, Kripke (1980) as a canonical reference, and Williamson (2016) for a more recent example. See also Hale (1996, 2013).

[45]See Bacon (2018) for some forceful objections to the idea that metaphysical necessity is the broadest necessity.

[46]See Williamson (2016) for a recent argument that every 'objective' necessity should obey this constraint. The question whether metaphysical necessity is the broadest necessity is often asked against the background of a theory of propositions according to which propositions form a Boolean algebra. See, for instance, Bacon (2018) and Williamson (2016). This assumption is implausible in the present context, however. An exact definition of what it means to be the broadest necessity in the context of LE would require discussing a higher-order extension of LE that is beyond the scope of this paper. The definition of 'broadest necessity' suggested in Bacon (2018) would have to be modified in any case. A higher-order theory of essence is developed in Ditter (unpublished).

with an alternative account of the connection between essence and necessity. It is worth mentioning some of the potential challenges that any such account would face. As mentioned above, an important principle that any account would have to respect is the principle that essence implies necessity, and thus in particular the schema $\Box_{\bigvee}\phi \to \Box\phi$, where $\Box$ expresses metaphysical necessity. A denial of Fine's Thesis will typically involve the rejection of the converse of this principle, claiming that there are metaphysical necessities that are not essential to anything.[47] This entails that $\Box_{\bigvee}$ is broader than $\Box$, on any plausible way of understanding what it means for one operator to be broader than another. But this is problematic if metaphysical necessity is supposed to be the broadest 'objective' necessity. Moreover, whereas Fine's Thesis represents the metaphysical necessities as a unified class, a denial of Fine's Thesis involves a fractured view of the metaphysical necessities that cries out for further explanation. Finally, any denial of Fine's Thesis in LE would need to provide a combined semantics of the language of LE supplemented with $\Box$. The most obvious way of doing this would be to add a truth clause for $\Box$ to the semantics of LE that takes $\Box$ to be a restricted universal quantifier over worlds. Given the validity of $\Box_{\bigvee}\phi \to \Box\phi$, the set of worlds that $\Box$ quantifies over must be included in the set of worlds that $\Box_{\bigvee}$ quantifies over. But since the denial of Fine's Thesis usually goes along with a denial of the schema $\Box\phi \to \Box_{\bigvee}\phi$, the inclusion should be allowed to be proper, for otherwise we would have $\Box_{\bigvee}\phi \leftrightarrow \Box\phi$. The challenge is to come up with a well motivated semantic clause for $\Box$.

I am not suggesting that these challenges cannot be met. But until they are, Fine's Thesis constitutes the most attractive available account of the relation between essence and metaphysical necessity in LE.

4 Conclusion

Our discussion has revealed that in LE, Fine's Thesis can be consistently maintained together with the assumption that S5 is the correct logic of metaphysical necessity by endorsing the logic E5+DOM^{-}.

That said, I would like to conclude by suggesting that a proponent of LE should question the received view that metaphysical necessity obeys S5 and

[47] I say 'typically' because one may in principle deny that to be metaphysically necessary *is* to be true in virtue of the nature of all objects while still accepting that the two are equivalent in some weaker sense such as necessary or material equivalence.

instead endorse the logic E5 in conjunction with Fine's Thesis, according to which metaphysical necessity obeys exactly S4. For while there are strong independent reasons for believing that the logic of metaphysical necessity is at least S4, there is in fact very little direct argumentative support for the validity of the B-schema; its validity is usually either taken for granted or assumed on the basis of simplicity considerations. But in the context of LE, considerations of simplicity and elegance speak in favor of E5, in which metaphysical necessity obeys the modal logic S4. E5 is a simpler and to some extent more natural system than E5+DOM$^-$. Moreover, the validity of the B-schema in the context of LE requires DOM; however, as pointed out in §1, while all the principles of E5 seem to be justifiable on purely essentialist grounds, any justification of DOM that has been considered so far relies on the Barcan formula for metaphysical necessity. Thus, in the absence of further independent argumentative support for DOM, we have good reason to be at least as skeptical about DOM as we are about the Barcan formula for metaphysical necessity. Finally, the conjunction of E5 and Fine's Thesis has considerable appeal and gives rise to an S4-theory of metaphysical necessity that is justified on distinctively essentialist grounds. It therefore provides an important essentialist case for endorsing the revisionary consequence that the logic of metaphysical necessity is not S5, but S4.[48]

Appendix

In this section, I prove the completeness of E5+DOM$^-$ with respect to the semantics given in the main text. Moreover, I provide proofs of Propositions 2 and 4 from the main text.

The notions of a set of sentences being E5+DOM$^-$-, or E5-consistent are understood in the usual way. Given that E5 is a proper sublogic of E5+DOM$^-$ in the sense that every theorem of E5 is a theorem of E5+DOM$^-$, it is clear that any set of sentences that is E5+DOM$^-$-consistent is also E5-consistent.

[48]Many thanks to Andrew Bacon, Peter Fritz, Jeremy Goodman, Mark Jago, Kathrin Koslicki, Harvey Lederman, Jon Litland, Annina Loets, Chris Scambler, David Schroeren, Trevor Teitel, audiences at the ECAP 9 in Munich, the CUNY Logic and Metaphysics Workshop, the GAP.10 Congress in Cologne and an NYU Lunch Talk, as well as two anonymous referees and the editors for this journal for comments on earlier drafts and helpful discussions. Special thanks to Cian Dorr, Kit Fine and Marko Malink for detailed comments on earlier drafts and many helpful discussions of this material.

To prove completeness, we show that every E5+DOM$^-$-consistent set is satisfiable in some A-equivalence model. The fact that every E5+DOM$^-$-consistent set is satisfiable follows from the fact that every such set is E5-consistent and thus has a model, by the completeness result for E5. The only thing that remains to be shown is that some such model is an A-equivalence model. The following rule of *predicate elimination* will be needed for the proof:

> PE If $\vdash \forall x(Px \leftrightarrow Fx) \rightarrow \phi$, then $\vdash \phi$, where P is a rigid predicate symbol that does not occur in F or ϕ.

PE is a rule of both E5 and E5+DOM$^-$ (cf. Fine (2000, p. 547)).

Lemma 1. *Let Γ be an E5+DOM$^-$-consistent set of sentences. If P is a rigid predicate symbol that does not occur in Γ, then $\Gamma \cup \forall x Px$ is E5+DOM$^-$-consistent.*

Theorem 1 (Completeness)**.** *Every E5+DOM$^-$-consistent set is satisfiable in some A-equivalence model.*

Proof. Use Lemma 1 and the completeness result for E5 from Fine (2000). □

It is worth noting that we can prove completeness for E5+BF$^-$ and E5+B$^-$ with respect to the class of A-equivalence models in an exactly analogous manner.

Next, I proceed to prove Propositions 2 and 4 from the main text. Proposition 2 follows from Corollary 1 from the main text and Theorem 2 below, and Proposition 4 follows from Corollary 3 from the main text and Theorem 3 below. In what follows, $\mathcal{L}_M$ is the propositional modal language with a fixed set of denumerably many propositional variables $\{p_1, p_2, ...\}$, and the logical constants $\neg, \Box$ and $\vee$ (the other constants are treated as metalinguistic abbreviations in the usual way). By a *propositional modal schema* I mean a formula of $\mathcal{L}_M$. An instance of a propositional modal schema ϕ is either (i) the result of uniformly substituting formulas of $\mathcal{L}_M$ for the variables in ϕ, or (ii) the result of uniformly substituting formulas of the language of LE for the variables in ϕ, if, in addition, all symbols of the form $\Box$ occuring in ϕ are replaced by symbols of the form $\Box_{\bigvee}$. Where S designates a propositional modal logic, a propositional modal schema is an *S-schema* just in case all of its $\mathcal{L}_M$-instances are S-theorems. Thus, if ϕ is not an S-schema, then there is an $\mathcal{L}_M$-instance of ϕ that is not an S-theorem. Where E is any logic of essence containing E5, we say that $\Box_{\bigvee}$ satisfies exactly the principles of a propositional modal logic S in E just in case for any propositional modal schema ϕ, every LE-instance of ϕ is an E-theorem iff ϕ is an S-schema.

Definition 1. An *S4-frame* is a pair $\langle W, R\rangle$, where W is a non-empty set and R is reflexive and transitive relation on W. A *propositional S4-model* is a triple $\langle W, R, V\rangle$, where $\langle W, R\rangle$ is an S4-frame, and V is a *valuation function* mapping each propositional variable into a subset of W.

Definition 2. An *LE$_R$-structure* is a tuple $\langle W, R, I, \succeq\rangle$, where $\langle W, I, \succeq\rangle$ is a structure, and R is a relation on W. A model based on an LE$_R$-structure is defined in the same way as a model based on a structure.

Lemma 2. *Every S4-frame $\langle W, R\rangle$ can be extended to an LE$_R$-structure $\langle W, R, I, \succeq\rangle$ in which $\forall w, v \in W : wRv$ iff $I_w \subseteq I_v$.*

Proof. Given an S4-frame $\langle W, R\rangle$, we define the corresponding LE$_R$-structure as follows: (i) Let I be the function taking each $w \in W$ to the set of its R-predecessors, i.e. $I_w = \{v : vRw\}$; (ii) Let $\succeq$ be the converse of R. It is clear that $\langle W, R, I, \succeq\rangle$ so construed is indeed an LE$_R$-structure. In particular, every I_w is closed under $\succeq$. It remains to show that $\forall w, v \in W : wRv$ iff $I_w \subseteq I_v$, which is straightforward.

□

We call the LE$_R$-structure defined in the proof above the *corresponding LE$_R$-structure* for a given S4-frame. It is evident that this structure is unique. Next we define a mapping between $\mathcal{L}_M$ and the language of LE. For definiteness, let $\{p_i : i \in \omega\}$ be the set of propositional variables of $\mathcal{L}_M$, and $\{F_i : i \in \omega\}$ the set of 0-place (non-rigid) predicate symbols of LE. We recursively define a translation $'$ from the formulas of $\mathcal{L}_M$ to the formulas of LE as follows:

(i) $p_i' = F_i$, for all $i \in \omega$;

(ii) $(\neg\phi)' = \neg\phi'$;

(iii) $(\phi \vee \psi)' = (\phi' \vee \psi')$;

(iv) $(\Box\phi)' = \Box_{\bigvee}\phi'$

Note that every formula in the image of $'$ has empty objectual content when interpreted in an LE-structure and is thus defined at every world. Moreover, each such formula contains only 0-place predicate symbols and (possibly) logical constants. We associate with each 0-place predicate F (in LE) a subset of W relative to a LE- or LE$_R$-model $\mathcal{M} = \langle W, R, I, \succeq, V \rangle$: $T(F) := \{w : V(F, w) = \{\emptyset\}\}$. By the truth definition for LE, $T(F)$ is the set of worlds at which F is true relative to $\mathcal{M}$.

Lemma 3. *Let $\langle W, R \rangle$ be an S4-frame and $\langle W, R, I, \succeq \rangle$ its corresponding LE$_R$-structure. If $\mathcal{M} = \langle W, R, V \rangle$ and $\mathcal{E} = \langle W, R, I, \succeq, V' \rangle$ are models such that $V(p_i) = T(F_i)$, for all $i \in \omega$, then for all formulas ϕ in $\mathcal{L}_M$, and $w \in W$: $\mathcal{M}, w \vDash \phi$ iff $\mathcal{E}, w \vDash \phi'$.*

Proof. By an easy induction on the complexity of formulas in $\mathcal{L}_M$. □

Theorem 2. *Let ϕ be a propositional modal schema that is not an S4-schema. Then ϕ has an LE-instance that is not an E5-theorem.*

Proof. Suppose that ϕ is an invalid schema of propositional S4. Then ϕ has an $\mathcal{L}_M$-instance ψ that fails on some S4-frame $\langle W, R \rangle$. So there is an S4-model $\mathcal{M} = \langle W, R, V \rangle$ and some $w \in W$ such that $\mathcal{M}, w \nvDash \psi$. Let $\langle W, R, I, \succeq \rangle$ be the corresponding LE$_R$-structure and $\mathcal{E} = \langle W, R, I, \succeq, V' \rangle$ be such that $\mathcal{M}$ and $\mathcal{E}$ satisfy the condition of Lemma 3. Let $'$ be the translation function defined above. It follows from Lemma 3 that $\mathcal{E}, w \nvDash \psi'$. ψ' is clearly an LE-instance of ϕ. Since R does not affect the valuation of any formula in $\mathcal{E}$, $\mathcal{E}$ and $\mathcal{E}^* = \langle W, I, \succeq, V' \rangle$ satisfy exactly the same formulas at every world. It follows that $\mathcal{E}^*, w \nvDash \psi'$. □

Notice that the model $\mathcal{E}^*$ in the proof above is an IP-model. Theorem 2 thus holds if we replace E5 in it with E5+IP. The proof that $\Box_{\bigvee}$ satisfies exactly the principles of propositional S5 in E5+DOM proceeds similarly, though it is somewhat more straightforward and is therefore omitted here.

Theorem 3. *Let ϕ be a propositional modal schema that is not an S5-schema. Then ϕ has an LE-instance that is not an E5+DOM-theorem. Moreover, ϕ has an LE-instance that is invalid in the class of constant domain models.*

Theorem 3 also holds if we replace E5+DOM with E5+DOM$^-$.

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