Deleuzian Haecceity and Derridean Arche-Writing as a Stackified ∞-Exigency

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Abstract

We construct a mathematization of Derridian "arche-writing" and Deleuzian "haecceity." We posit an ∞-categorification of exigency (∞-exigency), a higher-dimensional visual epistemology (∞-visual epistemology), and (∞)-stack Wittgenstein ladder. We reframe haecceities in terms of diamonds, in the sense of Scholze, and mathematize the haecceity-and-arche-writing reflection as a pro-diamond. As an exercise in ∞-visual epistemology, we validate a diamond ∞-stack time signature and ∞-stack harmony.

Keywords: diamonds, haecceity, ∞-stack, arche-writing, condensed sets, pro-objects, pro-diamond, visual epistemology

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1 Introduction

Where does mathematics begin? It is at best difficult to assign any ontic or epistemic claim to such a question. But for the sake of mathematical exigency, we shall assume this question has merit and proceed. The notion of a beginning, even a mathematical beginning, is intimately tied up with a notional "point", some designation of occurrence with a duration. Points are usually defined as geometric objects, specifically zero-dimensional objects, and are taken axiomatically and specific to geometry. The notion of a point in number theory, for example, has an entirely different incarnation. To introduce a point in number theory, a topological space $Spec(R)$ is introduced for a commutative ring R, which is defined to be the set of all prime ideals of R. So the points in $Spec(R)$ are prime ideals and not zero-dimensional objects. Clearly, in the language of $Spec(R)$, this notion of a point is not "as such" at all (Derrida (1998)), for it refers to a prime ideal which in tern refers to a subset condition which in tern refers to another. Derrida will call this endless play of dually present/absent signfiers and referents the "trace" of writing, or "arche-writing." Because of this stratification [1] of continuing referents, we might as well ask if the apropos question is not where mathematics begins but where does mathematics stop? At what "point"?

In concert with Derrida, we agree that the notion of a point in mathematics is at best heterogneous and is never "as such." We go further and claim that any "as such" is infinitely aporetic, the ∞-aporetic "as". In essence, meaning is multiple and we can stackify this notion to geometrize its meaning [2], all the way to an ∞-stack which is an $(\infty, 1)$-sheaf on an $(\infty, 1)$-site taking values in ∞-groupoids. Whatever the framework used to represent multiplicity, we are to assume that a point exists, that satisfies a self-identity, an object-persistence, with a definition ad infinitum or that terminates axiomatically. What is worse, most mathematics is impredicative, and hence self-referential in its heterogeneity. If the very notion of origin is so simulacral (Baudrillard (1983)), then we contend that the very space of mathematics is likewise dislocated in a grand "arche-writing".

It could be said that to orient this dislocation, mathematicians attempt to construct reciprocity laws to merge branches of mathematics towards a grand unified theories, and locally towards self-consistent definitions. For example, arithmetic geometry is a beautiful branch of mathematics which uses algebraic geometry to solve number theoretic problems. Predicative mathematics is another attempt at creating a mathematics that is not self-referential. Of course it still remains whether merging branches ever solves the issue of "as such."

1.1 Haecceity

(Deleuze & Guattari (1987)) introduce the notion of "haecceity" in the section *Memories of a Haecceity.*

> A body is not defined by the form that determines it nor as a determinate substance or subject nor by the organs it possesses or the functions it fulfills. On the plane of consistency, a body is defined only by a longitude and a latitude...Nothing but affects and local movements, differential speeds.

[1] in the sense of constructible sheaves

[2] where an n-stack is an $((n+1), 1)$-sheaf on an $((n+1), 1)$-site taking values in $(n+1)$-groupoids. A site is a *small* category with a Grothendieck topology

> There is a mode of individuation very different from that of a person, subject, thing, or substance. We reserve the name *haecceity* for it. A season, a winter, a summer, an hour, a date have a perfect individuality lacking nothing, even though this individuality is different from that of a thing or a subject.
>
> They are haecceities in the sense that they consist entirely of relations of movement and rest between molecules or particles, capacities to affect and be affected...Tales must contain haecceities that are not simply emplacements, but concrete individuations that have a status of their own and direct the metamorphosis of things and subjects.
>
> ...That awful five in the evening! We say, "What a story!" "What heat!" "What a life!" to designate a very singular individuation. The hours of the day in Lawrence, in Faulkner. A degree of heat, an intensity of white, are perfect individualities; and a degree of heat can combine in latitude with another degree to form a new individual, as in a body that is cold here and hot there depending on its longitude.

So there exists a cosmic tension between a Derridian "as such" ontically precluded and a Deleuzain haecceity granted ontic existence. To mathematize these concepts, is to reframe them in the language of mathematics. Can we sheafify "arche-writing"? Can we sheafify a haecceity? If so, how can we? Clearly, the (a?) question arises as to whether a truly singular object could combine with another truly singular object and what the mechanics of that combination looks like, "as" like, if only by syllogisms, forbidding another self-repeating false sorities (Kierkegaard (1838)).

We now consider what sort of visual epistemic claims can be made about haecceities. As detailed below, we claim that we can visualize these haecceities as mathematical impurities [3]. As later detailed with a "point as pro-'etale site", labeling the haecceity as a singular point, does not diminish the many serious arithemtic geometric definitions and references underlying that "point." These invisible mathematical impurities/geometric points are only made visible upon pullback of a quasi-pro-'etale cover, revealing profinitely many copies

[3]geometric points: $Spa(C) \to \mathcal{D}$ for C an algebraically closed curve and D a mathematical diamond, which is a pro-'etale sheaf on the category of perfectoid spaces of characteristic p Scholze (2017)

of $Spa(C)$. So the reflections here are double. "A degree of heat, an intensity of white" are perfectly singular points never attainable, only attainable through their profinitely many copies. So, while haecceities are defined "as such," subject to visual epistemology, their very representation is "arche-writing" ripe with minimal repeatability; haecceity as geometric point in a diamond. There is no truer definition of aporia.

We do our best to represent singular objects like haecceities with singular mathematical structures, which greatly help in visualizing these singular haecceities and keeping them on the plane of immanence and self-identical whilst simultaneously being so phenomenologically reflective. We further claim that the only way to visualize haecceities is through our new notion of a ∞-visual epistemology, also detailed below.

We now begin our mathematization of "arche-writing" and haecceity. We use ∞-categories and sheaves on categories of perfectoid spaces of characteristic p towards an analogical ∞-exigency. We reinterpret haecceity as a pro-diamond (Dobson (2021a), Dobson (2021b)).

Our main conjectures are:

Proposition 1. *"As such" is ∞-aporetic "as."*

Proposition 2. *"Origin as the heterogenous "as" " is a condensed set.*

Proposition 3. *Haecceities are geometric points in a mathematical diamond.*

Proposition 4. *Arche-writing is a pro-diamond.*

Proposition 5. *Haecceities are v-stacks taking values in the category of self-singularities.*

Proposition 6. *Arche-writing is an ∞-stack taking values in $\mathcal{D}^{\diamond}$.*

Definition 1. ∞-*Exigency*

Definition 2. ∞-*Visual Epistemology*

Proposition 7. *Diamond N Time Signature*

2 Categorify Exigency

The exigencies of mathematics, what we will call mathematical exigency, are complete and consistent dialectical proofs. It is unresolved whether mathematics knows a negative dialectics (Adorno (1990)). We can further fine-tune this to say that mathematical exigency requires completeness, consistency, and object self-consistency. To categorify exigency is to reframe exigency in terms of categories. Category theory exalts the relations between objects in proportion to the definition of objects.

We construct a 1-category with objects exigencies and 1-morphisms entropic categorizations (Dobson & Fields (2021)), requiring that the morphisms compose associatively and the existence of an identity morphism for each object. Our entropic categorizations satisfy the first requirement, but the second is a little trickier. What would it mean to have an identity morphism on an exigency? As alluded to above, "self-identity" for empty variables such as the canonical "X" cannot be guaranteed. We have already conjectured that there is no personal identity "over" time (Dobson & Prentner (2021a)), leaving no reason why any privilege should be accorded this particular "x", or this particular "x" in that particular axiom. There is undeniably (uncertainly so) an "x-ness" about this particular "x" (**?**), but it cannot be proven that it is the "same" x that is never "x" "as such." Nevertheless, this object "x" is taken in object self-consistency. Likewise, we will assume that the identity arrow on exigency preserves self-identify and self-consistency.

We now construct a 2-category with the same exigency objects and 1-morphisms, but now we have 2-morphisms between the 1-morphisms, which are morphisms between the entropic categorizations. We can continue to construct an $(\infty, 0)$-category where all k-morphisms for $k \geq 1$ are invertible, landing us in the realm of ∞-groupoids. The notion of simultaneity in representations of representations...of representations was constructed in (Dobson & Prentner (2021a), Dobson & Prentner (2021a)), giving way to a nice model of subjective awareness. That same nestedness can mathematically describe the urgency of dislocation of time/space in the arche-writing.

2.1 ∞-Exigency

We can stackify this construction as an extension to higher category theory, as alluded to previously. For example, to stackify exigency, we can represent exigency as a 1-stack, which is a $(2, 1)$-sheaf on a $(2, 1)$-site taking values in a $(2, 1)$-category rather than sets. Let us go further and represent exigency as a 2-stack, which is a $(3, 1)$-sheaf on a $(3, 1)$-site taking values in 3-categories. Let us go further and represent exigency as a 3-stack, which is a $(4, 1)$-sheaf on a $(4, 1)$-site taking values in 4-categories. Let us go further still and define an ∞-exigency:

Definition 3. *An ∞-exigency is an ∞-stack of an $(\infty, 1)$-sheaf on an $(\infty, 1)$-site taking values in ∞-groupoids.*

Imagining ∞-exigency as an ∞-stack of an $(\infty, 1)$-sheaf taking values in ∞-groupoids, we somehow lose exigency. It seems that exigency breaks down once it is geometrized to n-stacks for too high of n. However, in proportion as we lose canonical visual epistemology as ordinary understanding breaks down, we gain a vastly richer understanding of the geometry of exigency, that exigency could be geometerized; that something so grammatological could be geometerized. This space fails ordinary "sight" and canonical comprehension, and something akin to a higher affine visualization must take over to handle these infinities. It is a moduli space pre-visualization and before all logocentric grammatologies. It is as immediated "as" the Deleuzian plane of immanence.

As the language of sheaves is much befitting of the complexities of representations of exigency, we recall the definitions of a presheaf and a sheaf:

Definition 4. (Hartshorne, 1977) *Let X be a topological space. A* presheaf of sets *on X is a contravariant functor $F : Op(X) \to Sets$ on the category $Op(X)$ of open sets of X.*

Definition 5. (Hartshorne, 1977) *Let X be a topological space. A sheaf $\mathcal{F}$ on X is a presheaf satisfying two axioms:*

- *Let U be an open subset of X and U_i an open cover of U. Given a collection of sections s_i on U_i, with $s_i|U_{ij} = s_j|U_{ij}$, then there exists a section s on U such that $s|U_i = s_i$.*
- *Let U be an open subset of X and U_i an open cover of U. If s is a section on U such that $\forall i,\ s|U_i = 0$, then s is zero.*

Sheaves provide a hierarchical structure which allows for an easier visual understanding of the geometries of multiple reflecting reference points. The space of relations of the reference points takes a particular form, a profinite form. Recall: [4]

Definition 6. *A* profinite set *is a compact, Hausdorff, totally disconnected topological space that is a formal cofiltered limit of a collection of finite sets.*

[4]All standard definitions not otherwise referenced are from `https://ncatlab.org/nlab/show/HomePage`.

3 Mathematical "Arche-Writing"

(Derrida (1998)) introduces "arche-writing":

> ...experience as the experience of the present is never a simple experience of something present over and against me, right before my eyes as in an intuition; there is always another agency there. Repeatability contains what has passed away and is no longer present and what is about to come and is not yet present. The present therefore is always complicated by non-presence. Derrida calls this minimal repeatability found in every experience "the trace." Indeed, the trace is a kind of proto-linguisticality (Derrida also calls it "arche-writing"), since language in its most minimal determination consists in repeatable forms (Stanford (2019)).

So, repeatability, the very act of consecution, precludes the existence of a pure referent singularly self-identical called "now." Any "now" is jointly, simultaneously a past and a prophecy, a present without advent, a time without happening. Derridean arche-writing is a trace, an always proto-present. It is a haunting.

We will later attempt a mathematization of this arche-writing, by representing the endless reflections of a proto-present in a mathematical diamond, for a diamond has no beginning, and, as such is befitting the haecceity.

> ...Haecceity, fog, glare. A haecceity has neither beginning nor end, origin nor destination: it is always in the middle. It is not made of points, only of lines. It is a rhizome (Deleuze & Guattari (1987)).

Perfectoid spaces are ripe for representing this radical beginning in the middle, without beginning. The state of haecceity is a duality, yet not totalized as such, akin to being fully conscious during dreamless sleep, where it is highly contentious whether those states could ever share the same temporality (Dobson (2021c)).

Is not mathematics plagued with the same trace of an endless construction of present/absent references that doubly fails the signatory and the referent? Mathematics is, after all, a writing system multiply prone to referent slippage. Perhaps this issue could be alleviated if we knew where mathematics begins. So again we ask where does mathematics begin? Specifically, from what origin does this beginning begin?

(Derrida (1998)) claims, antiphrastically, that

> Writing is not a sign of a sign, except if one says it of all signs, which would be more profoundly true.

If we can at least hold on to the claim, for "now", then what is implied is that either writing indeed has an origin, or the origin is precisely what is forbidden to semiology, as is any sign as an origin. This makes the origin a multiplicity, so that no signified has an origin.

> ...if the origin is always heterogeneous, then nothing is ever given as such in certainty. Whatever is given is given as other than itself, as already past or as still to come. What becomes foundational therefore in Derrida is this "as": origin as the heterogeneous "as." The "as" means that there is no knowledge as such, there is no truth as such, there is no perception, no intuition of anything as such (Stanford (2019)).

What does such a claim mean for mathematical exigency? In our words:

Proposition 8. *"As such" is ∞-aporetic "as."*

Because of this endlessly, without beginning, heterogenous-origin "inside" this haunting, the space and time of signatory advent are forever dislocated and at the "same time," phenomenologically so. We have previously proffered a notion of pro-emergence and emergent time (Dobson (2021b)) and condensed time (Dobson & Fields (2021)) to handle singularity peculiarities in various semantics. We merely mention that it seems the exigency of this slippage in time demands a more proper representation by a time more capable of handling singularities; a time more nonarchimedean; a p-adic time, $Time_{\mathbf{Q_p}}$. It is this dislocated but non-derivative space and time which is the (non-unique) precondition for:

> This should be read without a pause: the animal-stalks-at-five-o'clock. The becoming-evening, becoming-night of an animal, blood nuptials. Five o'clock is this animal! This animal is this place!.

> Spatiotemporal relations, determinations, are not predicates of the thing but dimensions of multiplicities.

By reframing haecceities as geometric points with geometrized referents, we are stackifying the notion of meaning "as" multiplicity. By calling the dimensions of multiplicities a 1-stack on the topological space of spatiotemporal relations, we are framing it as a $(2,1)$-sheaf on a $(2,1)$-site taking values in the $(2,1)$-category of groupoids of dimensions of multiplicities, and thus allowing more multiplicity in meaning. We can go further and stackify these dimensions as an ∞-stack of an $(\infty,1)$-sheaf on an $(\infty,1)$-site taking values in ∞-groupoids. We mathematize these "dimensions of multiplicities" in the form of a "diamond" detailed below.

The notion of an origin point in mathematics is both canonical and contentious. Canonical in the sense that mathematical proofs proceed dialectially and iteratively, beginning with axioms/postulates and culminating with theorems and proofs. There is implied and expected an order on the mathematical process. Regarding contentious, affine space is a geometric structure that functions independently of a canonical choice of coordinate system and has no designated "origin." However, this space does not quite capture, epistemically, the "trace" in the arche-writing, "as" origin is supervened by displacement vector. Perhaps affine is the mathematical equivalent of "as" in the "origin as the heterogenous 'as'" Stanford (2019).

Perhaps what is novel to mathematics is the many incarnations of a particular construct. The point in mathematics, being "as" a Derridian "trace," can take a notational form or a highly complex form of a morphism of schemes, as in the "diamond" construct of Scholze, which we describe below.

Proposition 9. *Arche-writing is "as" a Pro-Diamond as an incarnation of the "as" in "origin as heterogenous as."*

From there, we can posit the ∞-categorification of arche-writing using $\mathcal{D}^{\diamond}$a large, stable ∞ category of diamonds (Dobson (2021a)).

We now briefly discuss a profinite version of this heterogenous origin.

4 Condensed Origin

We claim that a mathematical representation of a dislocated present, an origin as heterogenous "as", an instantiation of arche-writing, takes the form of a condensed set. Recall the definition of a condensed set (Dobson & Fields (2021), Clausen & Scholze (2021)):

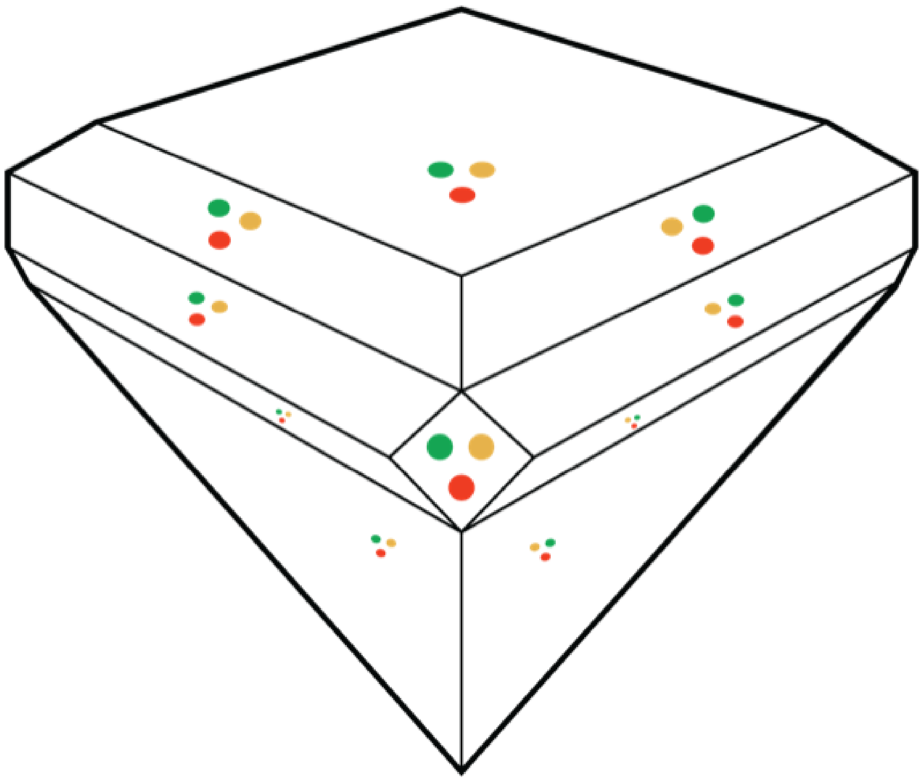

Figure 1: Diamond $SpdQ_p = Spa(Q_p^{cycl})/\underline{Z_p^{\times}}$ with geometric point $Spa\ C \to \mathcal{D}$ (Dobson (2021a))

Definition 7. *Let C be a category, and let $Cond(C)$ denote the category of "condensed" objects of C. Clausen & Scholze (2021) show that $Cond(C)$ can be represented as the category of small sheaves on C, or equivalently as a representable functor $F : C^{op} \to Set$.*

More formally, we have:

Definition 8. *The* pro-étale site $*_{proét}$ *of a point is the category of profinite sets ProFinSet, with finite jointly surjective families of maps as covers. A* condensed set *is a sheaf of sets on $*_{proét}$. Similarly, a condensed ring/group/object is a sheaf of rings/groups/objects on $*_{proét}$.*

In essence, while the point is seemingly singular, it is actually a pro-'etale site equipped with a sheaf of sets. So the notional point refers past self-identify to this higher structure, leading us to ""arche-writing"-writing." We will analogize the totality of ""arche-writing"-writing" as a certain profinite condition, the profinitely many copies of $Spa(C)$ upon pullback along a quasi-pro-'etale cover, represented as a pro-diamond.

We now introduce haecceities as geometric points in a diamond.

5 Haecceity as Mathematical Impurity

The contention of arche-writing is that meaning is multiple. Our contention goes further: that meaning over a singularity is multiple, for singularity in the sense of haecceity. Even when we speak of a degree of heat on the plane of immanence, we invoke thermodynamics and a mathematical singularity.

> Climate, wind, season, hour are not of another nature than the things, animals, or people that populate them, follow them, sleep and awaken within them.

While Derrida contends that no sign is ever given as such, in essence, Deleuze contends that the haecceity is given as such, as it only is, as it is self-identical.

> For you will yield nothing to haecceities unless you realize that its what you are, and that you are nothing but that Deleuze & Guattari (1987).

The haecceity is so singular, that even the time of the haecceity abides no canonical cardinality.

> Even when times are abstractly equal, the individuation of a life is not the same as the individuation of the subject that leads it or serves as its support. It is not the same Plane: in the first case, it is the plane of consistency or of composition of haecceities, which knows only speeds and affects; and in the second case, it is the altogether different plane of forms, substances, and subjects. And it is not in the same time, the same temporality. *Aeon*: the indefinite time of the event, the floating line that knows only speeds and contiunually divides that which transpires into an already-there that is at the same time not-yet-here, a simultaneous too-late and too-early, a something that is both going to happen and has just happened. *Chronos*: the time of measure that situates things and persons, develops a form, and determines a subject...the "pulsed time" of a formal and functional music based on values versus the "nonpulsed time" of a floating music, both floating *and* machinic, which has nothing but speeds or differences in dynamic. IN short, the difference is not at all between the ephemeral and the durable, nor even between the regular and the irregular, but between two modes of individuation, two modes of temporality.

"Aeon" is the time of the trace; a profinite time, a p-adic time, $\mathrm{Time}_{\mathbf{Q_p}}$. Our contention is that haecceity and trace are dual reflections sharing the same temporality of the Aeon. To stackify a shared temporality, is to consider the Aeon as a 1-stack taking values in $(2,1)$-categories. For example, to model Aeon as a 1-stack on haecceities, is to attach to every open set of the $(2,1)$-site of haecceities a $(2,1)$-category of Aeon, with objects Aeon, 1-morphisms between Aeons, and 2-morphisms between the 1-morphisms. To stackify Aeon as a 2-stack is to represent it as a $(3,1)$-sheaf on the $(3,1)$-site of haecceities taking values in $(3,1)$-categories. For example, to model Aeon as a 2-stack on the $(3,1)$-site of haecceities is to attach to every open set of the $(3,1)$-site of haecceities a $(3,1)$-category of Aeon, with objects Aeon, 1-morphisms between Aeons, 2-morphisms between the 1-morphisms, and 3-morphisms between the 2-morphisms. Let us go further. To ∞-stackify Aeon as an ∞-stack on the $(\infty,1)$-site of haecceities is to attach to every open set of the $(\infty,1)$-site an ∞-groupoid of Aeon, with its objects and invertible ∞-morphisms. Have we lost canonical time?

Mathematics can help assuage this tension because it was created, grammatologically, to handle the daunting infinities which frighten our fragile ontologies and make us lose time. We can use mathematics to structure this tension as a duality and posit haecceties as diamonds and arche-writing as a pro-diamond, structures capable of obtaining both views.

Proposition 10. *A haecceity is a geometric point in a diamond.*

Let us unravel this proposition. A geometric point is a morphism of schemes. Recall:

Definition 9. (Hartshorne, 1977) *An affine scheme is a locally ringed space $(X, \mathcal{O}_{\mathcal{X}}$ which is isomorphic (as a locally ringed space) to the spectrum of some ring (the pair consisting of a topological space Spec A together with the sheaf of rings $\mathcal{O}$. A scheme is a locally ringed space $(X, \mathcal{O}_{\mathcal{X}}$ in which every point has an open neighborhood U such that the topological space U, together with the restricted sheaf $\mathcal{O}_{\mathcal{X}}|_U$ is an affine scheme.*

Quotienting a scheme by an étale equivalence relation creates an algebraic space. The diamond construction mirrors this quotient.

5.1 V-topology

Recall the pro-'etale and v topologies:

Definition 10. *(Scholze (2017)) Let X be an analytic adic space X on which a fixed prime p is topologically nilpotent. We associate an étale site $X_{ét}$ for any X.*

- *The Grothendieck pro-étale topology, where a cover $\{f_i : X_i \to X\}$ consists of pro-étale maps $X_i \to X$ such that for any quasicompact open subset $U \subset X$, there are finitely many indices i and quasicompact open subsets $U_i \subset X_i$ such that the U_i jointly cover U.*

- *The Grothendieck v-topology, where a cover $\{f_i : X_i \to X\}$ consists of any maps $X_i \to X$ such that for any quasicompact open subset $U \subset X$, there are finitely many indices i and quasicompact open subsets $U_i \subset X_i$ such that the U_i jointly cover U. The v-topology is generated by open covers and all surjective maps of affinoids.*

5.2 Diamond

Now we recall the definition of a diamond and give a few of their many incarnations (Scholze (2017), Dobson (2021a)):

Definition 11. *Let Perfd be the category of perfectoid spaces. Let Perf be the subcategory of perfectoid spaces of characteristic p. Let Y be a pro-étale sheaf on Perf. Then Y is a diamond if Y can be written as the quotient X/R with X a perfectoid space of characteristic p and R a pro-étale equivalence relation $R \subset X \times X$.* (Scholze, 2017)

Definition 12. *(Scholze (2017))*

Let Y be an analytic adic space over Z_p. The diamond associated to Y is the v-sheaf defined by

- $Y^\diamond : X \to \{((X^\#, \iota), f : X^\# \to Y)\}/ \simeq$,
- *where $X^\#$ is a perfectoid space with an isomorphism $\iota : (X^\#)^b \simeq X$.*

Examples of diamonds are the following (Scholze (2017), Dobson (2021a)):

Example 1. *For $X = Spa(R, R^+)$, we say $Spd(R, R^+) = Spa(R, R^+)^\diamond$.*

Example 2. *The Fargues-Fontaine Curve X_{FF}*

Example 3. $X^\diamond_{FF} \cong (SpdC \times SpdQ_p)/(\phi \times id)$.

Example 4. *The diamond Faruges-Fontaine Curve: $\mathcal{Y}^\diamond_{S,E} = S \times (Spa\mathcal{O}_E)^\diamond$*

Example 5. *Let $\mathcal{D}$ and $\mathcal{D}'$ be diamonds. Then the product sheaf $D \times_\diamond D'$ is also a diamond.*

Example 6. $SpdQ_p = Spd(Q_p^{cycl})/\underline{Z^{x_p}}$ *where* $\underline{Z^{x_p}}$ *is the profinite group* $Gal(Q_p^{cycl}/Q_p)$.

Example 7. $SpdQ_p \times_\diamond SpdQ_p$.

Example 8. $Sht_{\mathcal{G},b,\{\mu_i\}}$*: moduli spaces of mixed-characteristic local G-shtukas is a locally spatial diamond.*

Example 9. *All Banach-Colmez spaces are diamonds.*

Example 10. *Any closed subset of a diamond is a diamond.*

These geometric objects are named "diamonds" because their geometric points resemble mathematical mineralogical impurities. For C be an algebraically closed affinoid field and $\mathcal{D}$ a diamond, a geometric point $Spa(C) \to \mathcal{D}$ is made "visible" by pulling it back through a quasi-pro-étale cover $X \to \mathcal{D}$, the consecution of which is profinitely many copies of $Spa(C)$. So the impurity, the geometric point, is invisible, is never seen in itself or "as itself", or, what is more true, is always seen "as" another, "as" a reflection. Multiple representations of the geometric points of $\mathcal{D}$ can be made based on multiple quasi-pro-étale covers $X \to \mathcal{D}$.

5.3 Perfectoid Spaces

We quickly recall the definition of perfectoid spaces and give a few of their many incarnations (Scholze (2017)).

Definition 13. *A perfectoid space is an adic space covered by affinoid adic spaces of the form Spa*(R, R^+) *with* R *a perfectoid ring.*

Examples of perfectoid spaces are the following (Scholze (2017), Dobson (2021a)):

Example 11. *The perfectoid Shimura variety*

Example 12. $S_{K^p} \sim \varprojlim_{K^p} (S_{K^p K_p} \bigotimes_E E_p)^{ad}$

Example 13. *Any completion of an arithmetically profinite extension (APF) extension, in the sense of Fontaine and Wintenberger, is perfectoid.*

Example 14. *If* K *is a perfectoid field and* $K+ \subset K$ *is a ring of integral elements, then Spa*$(K, K+)$ *is a perfectoid space.*

Example 15. *Any locally Zariski closed subset of a perfectoid space is a perfectoid space.*

Example 16. *The nonarchimedean field* Q_p *is not perfectoid as there is no topologically nilpotent element* $\xi \in Z_p$ *whose pth power divides p.*

Through the diamond formalism, we now see that $\mathrm{Time}_{\mathbf{Q_p}}$ is time constructed of haecceities. As such, the sentence "the animal-stalks-at-five-o'clock" can indeed be read without a pause, and, moreover, all at once, as it can be read as reflections of haecceities in a diamond, in the same sense that all reflections can be seen at once in the diamond. What is singular is combined into its many incarnations.

6 Pro-Diamond

We analogize to pro-diamond the proto-language arche-writing.

Proposition 11. *An incarnation of arche-writing is the profinitely many copies of Spa*(C) *from pullback along the quasi-pro-'etale cover* $X \to \mathcal{D}$*.*

Recall, our previous pro-diamond conjecture (Dobson (2021a), Dobson (2021b)):

Conjecture 4.1.1. Let $\mathcal{D}^{\diamond\diamond}$ be a small cofiltered category of diamonds with morphisms the diamond product Scholze (2017). Two objects in $\mathcal{D}^{\diamond\diamond}$ are diamonds (v-sheaves) and spatial v-sheaves. The pro-diamond pro-object in the category of pro-objects of $\mathcal{D}^{\diamond\diamond}$ is the formal cofiltered limit of objects of $\mathcal{D}^{\diamond\diamond}$. The $Hom_{\mathcal{D}^{\diamond\diamond}}\ (F(-), G(-))$ for pro-objects $F : D \to C$ and $G : E \to C$ is given by the pro-diamond functor, the pro version of the diamond functor. Recall that

> a pro-object of a category C is a formal cofiltered limit of objects of C.

A cofiltered category has the property that

> for every pair of objects c_1 and c_2 of C, there is an object c_3 of C such that there exists an arrow $c_3 \to c_1$ and there exists an arrow $c_3 \to c_2$.

Recall the category of pro-objects in C is defined as:

Definition 4.1.2 Let C be a category. The category of pro-objects in C is the category defined as follows.

- The objects are pro-objects in C.
- The set of arrows from a pro-object $F : D \to C$ to a pro-object $G : E \to C$ is the limit of the functor $D^{op} \times E \to Set$ given by $Hom_C(F(-), G(-))$.
- Composition of arrows arises, given pro-objects $F : D_0 \to C$, $G : D_1 \to C$, and $H : D_2 \to C$ of C, by applying the limit functor for diagrams $D^{op} \times E \to Set$ to the natural transformation of functors $HomC(F(-), G(-)) \times HomC(G(-),H(-)) \to Hom_C(F(-), H(-))$ given by composition in C.
- The identity arrow on a pro-object $F : D \to C$ arises, using the universal property of a limit, from the identity arrow $Hom_C(F(c), F(c))$ for every object c of C.

As we note in Dobson (2021a), we could also construct the pro-diamond pro-object of the category of diamonds by taking the isomorphism classes of diamonds under the diamond equivalence relation as pro-objects.

As arche-writing is a totality without advent of endless referents and arche-meaning is only multiple, the pro-diamond combines haecceities in a profinite condition.

We now develop our notion of ∞-visual epistemology.

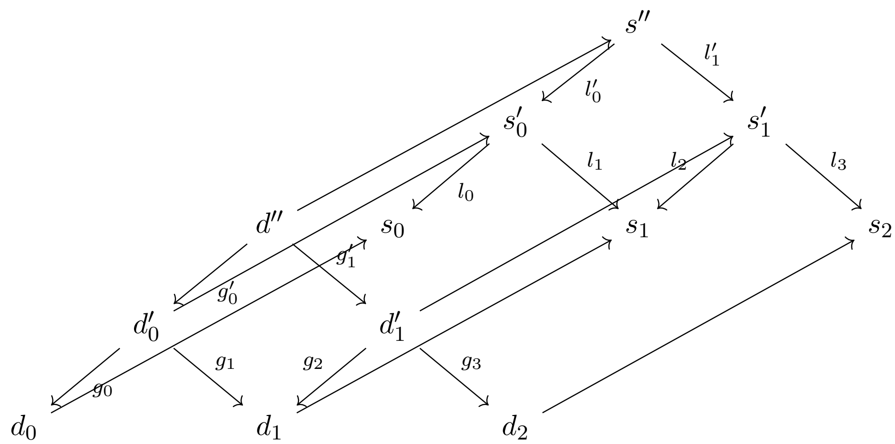

Figure 2: Pro-diamond in $\mathcal{D}^{\diamond\diamond}$ (Dobson (2021a))

7 Higher Visual Epistemology

Where is the meaning in a mathematical proof, postulate, and/ or conjecture? Visual epistemology, by definition, holds that truth and validity are "in" the visualization, or that any epistemic claim about knowledge can be grounded in a visualization. If we exalt visual epistemology, we could claim that, speaking categorically, the mathematical meaning is in the visualized morphisms, the connections, their analytical equivalent in verbal epistemology being definitions. When we "see" a diagrammatic proof and we claim "Oh, I get it now. I know it now!", what does that actually entail and what sort of epistemic claims can be made under such an apostrophe? Whether proof begets existence or the contrary, we can easily extend our categorifcation of exigency to the categorification of epistemology; claiming that the category of proof begets the category of existence, and the contrary. However, it is highly unclear to what extent we have served epistemic truth by claiming that a visual proof is sufficiently so. For clearly, we cannot say that a theorem is untrue because it is highly non-renderable as a "visualization." Also, clearly, no visual epistemic claim could be made of an image without a referent. So, we must place conditions on the state of the image, else epistemic meaning is forever joint visual and verbal.

7.1 Looking-Glass Epistemology of Mirrors

Let us briefly consider the class of visual epistemic claims that can be made about the image in a mirror. A mirror is a one-sided object, phenomenologically. Clearly, upon looking "into" a normal mirror, I see a chiral-reversed, two-dimensional compressed image of my three-dimensional "self." But do they share the same temporality? So, the image in the mirror is a chiral reflection of the referent, a veritable looking-glass image. What is the epistemology of that image and where does the image exist exactly? There is indeed a "mirror-ness" about the mirror Wittgenstein (1953), a mirrored haecceity, but what are the conditions and sufficiencies of that "mirror-ness", of the space of a mirror image? To the extent that this reflected image is untrue to the referent, we can claim that a mirror image is an image without a referent, a phenomenological looking-glass, or rather, what we will define as a Wittgenstein ∞-mirror-ladder (Wittgenstein (1953)).

Definition 14. *A Wittgenstein ∞-mirror-ladder is an "∞-trace" ladder without any ontic steps. It is a ladder of ∞-mirrors which infinitely mirrors all supervening propositions.*

What further becomes of the image without a referent upon the creation of an $(\infty,1)-$ category of mirrors, $(\infty,1)$-mirrors, to account for reflections of reflections... of reflections, and what is meant by the invertibility of $n > 1$ morphisms between the mirror objects? Simply, what and where is the referent in an $(\infty,1)$-mirror, if there is any?

To the extent that the mirror image fails to properly reflect its referent, it is doubtful that any epistemic claim can be made about the image. Attaching a mirror interpretation to every referent, we could claim that, in the vein of "as such," epistemology is mirrors all the way down; [5] thus begetting a new mirror exigency.

7.2 ∞-Visual Epistemology

7.2.1 $(\infty,1)$-Looking-Glass Cake

In visualizing "an intensity of white" or "an hour", we are at minimal using a 1-stack representation [6]. So imagining "an intensity of white" as a 1-stack means we are assigning,

[5] Apropos to an (∞)-stack Wittgenstein ladder, wherein propositions are ascended ∞-stack reflections.

[6] A 1-stack is a $(2,1)$-sheaf on an $(2,1)$-site taking values in the $(2,1)$-category of groupoids rather than sets

subject to the sheaf condition, to every open set of our $(2,1)$-site the $(2,1)$-category "intensity of white", subject to the particular Grothendieck construction, if we are claiming that this is the universal haecceity.

Now let us stackify a second haecceity example, an hour, specifically the hour of "5-o'clock." For example, a 1-stack interpretation of a "5-o'clock" haecceity is a $(2,1)$-sheaf on a $(2,1)$-site taking values in $(2,1)$-categories. This means to every open set of the $(2,1)$-site of haecceities, there is attached a $(2,1)$-category of "5-o'clock". This $(2,1)$-category of "5-o'clock" has objects "5-o'clocks", 1-morphisms between the "5-o'clocks", and 2-morphisms between the 1-morphisms. What are the preconditions to visualize a representational $(2,1)$-site of haecceities [7]? Phenomenologically, we can ask what is the meaning of a time-morphism, a morphism between temporal hours. What is more, we can ask what is the meaning of a 2-morphism "between" the time-morphism, where this "between" is indubitably ∞-aporetic, and what is its functionality? Carrying on, to ∞-stackify a haecceity is to represent the haecceity as an $(\infty,1)$-sheaf on an $(\infty,1)$-site taking values in ∞-groupoids.

Secondly, let us consider how we could visualize this example. Suppose I wanted to hand round an $(\infty,1)$-looking-glass cake. Or, what is more profound, suppose I wanted to hand round an ∞-stack-looking-glass cake! Recall, an ∞-stack is a $(\infty,1)$-sheaf on an $(\infty,1)$-site taking values in ∞-groupoids. So the looking-glass cake of this ∞-stack-looking-glass cake takes values in ∞-groupoids! How do we even begin to visualize this and its accompanying $(\infty,1)$-site? What is more, how can we visualize its universal construction, if we are claiming that either this ∞-stack-looking-glass cake is the universal construction or this ∞-stack looking-glass cake satisfies a universal construction. We can of course reduce the complexity using some aporetic "as" reference, stating that the ∞-stack looking-glass cake is "as" a multilayered scoop of a supernumerary rainbow or spray bow ice-cream, to make "visualization" easier on us. We have here a few possibilities:

- Either this is a simple case of mathematics as metaphor, as mathematics as metaphor of/for visualization. Or

[7] What are the preconditions to visualize a 1-morphism between 5-o'clocks, and what is more, a 2-morphism between the 1-morphism?

- The $(\infty, 1)$-looking-glass cake is simply visualized "as" another image without a referent. Or

- It is such that ∞-stacks break visualization and all visual epistemology.

7.3 Wittgenstein (∞)-Stack Ladder

We are in equal confusion if I were to say that "I think at the event horizon of thoughts" or any glowing permutation: "I live/think at the event horizon of thoughts/life"; I live/think at the Cauchy horizon of the event horizon of thoughts/life." I live/think at the Cauchy horizon of thoughts/life." What sort of epistemic claim can be made in these cases? Using an (∞)-stackification visual epistemology can have two opposing results: in one way, we could become more confident in validating any epistemic claims in these statements because we can "see it;" in the second way, using an (∞)-stackification visual epistemology which breaks visualization therein breaks any claims to epistemology by providing an (∞)-stackification of Wittgenstein's ladder (Wittgenstein (1953)); mathematical propositions are used to step beyond them, as they themselves are looking glass. It seems rather, ad infinitum, that we again find ourselves, (*whatever that means and however that works*), asking what/where is the meaning in this statement, how is there meaning in this statement, what is its validation, and can mathematics as metaphor help to validate any epistemic claims in this statement.

We are forthcoming developing a theory of "thoughts/thinking" as ∞-stack-haecceities that are not ontological in that they are beyond ontology [8]. What is infinitely more complicating is that it is highly questionable whether we have immediate access to the processes that cause thoughts (*how does that work?*) (Dobson & Fields (2021a)) [9] [10]. So the incarnation of ∞-stacks breaking visualization in ∞-haecceities that are not ontological are many[11] [12].

[8] What is a thought? What does it mean to "have" thoughts or have we confused the directionality of that relation? How can we specifically define what a thought is (ontic/epistemic/quantum information theoretic/evolutionary biologic/classical physics/.../definition) and, moreover, what "thinking" is, for all species?

[9] due to problems with the asymptotic entanglement of "I", and "identity" being only instantaneously "something we can get away with" (Dobson & Fields (2021a))

[10] There are no ontic boundaries (Dobson & Fields (2021) Dobson & Fields (2021a) Dobson (2021a))

[11] Thoughts as ∞-sorcerer holograms.

[12] There are no ontic thoughts.

At any rate, you "have" them (Deleuze & Guattari (1987)).

Invariably, to "make visual" an object this complex, requires symmetrizing this object and forgetting its higher structure morphisms in a grand data compression schema. We ask how can we assign epistemic truth to the mental image before it is rendered and compressed "as renderable image."

We posit a stronger definition of visual epistemology, called ∞-visual epistemology:

Definition 15. *An ∞-visual epistemology is truth and validity accorded a mentally visualized image of an n-stackified mathematical concept that is not privy to rendering as a physicalized visualization for high values of n. Any visual epistemology is a stratification of ∞-visual epistemology* [13].

Proposition 12. *∞-exigency is a precondition for ∞-visual epistemology.*

As such, this object is privy to higher-dimensional visualization solely in the imagination and validates those mathematicians driven by intuition, whose proofs are vastly intuitive.

As a simple exercise in ∞-visual epistemology, let us consider how to make sense of the claim that "home is yesterday" (BBC (1988)). Clearly, such a claim would fail truth according to a verbal epistemology, as "home" has a grammatology of associations with a notional sense of space. But we could also argue that the claim fails visual epistemology, as once again, "home" has a grammatology of dimensionality referents not accorded or afforded to the temporal realm. But, according to our definition of ∞-visual epistemology, extended to the non-mathematical realm, we can indeed visualize a moduli stack to support a temporalized space, and therefore validate the claim.

As a second simple exercise, consider the buffalo sentence, the consecuted phrase "Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo" (Wikipedia (2021)). This claim is supported via verbal epistemology through the grammatology of its "lexical ambiguity" and glossematics, where such ambiguity is a glowing incarnation of "arche-writing."

The sentence employs three distinct meanings of the word buffalo:

- as a proper noun to refer to a specific place named Buffalo, the city of Buffalo, New York, being the most notable;

[13] in the sense of constructible sheaves.

- as a verb to buffalo, meaning (in American English[1]) "to bully, harass, or intimidate" or "to baffle"; and
- as a noun to refer to the animal, bison (often called buffalo in North America). The plural is also buffalo.

An expanded form of the sentence which preserves the original word order is:

"Buffalo bison that other Buffalo bison bully also bully Buffalo bison" (Wikipedia (2021)).

Like the mirror, this buffalo sentence provides an image without a referent. Moreover, clearly, this sentence fails visual epistemology for the very consecution of buffalo. However, our ∞-visual epsitemology could validate such a claim by modeling buffalo as profinitely many copies of geometric points as morphisms of schemes, stackifying consecution as profinite.

Before beginning our pro-diamond analogical construction, we begin with two simple (perhaps not simple) questions: *how can we classify mathematical thinking? and what is a re-interpretation?*, by which we mean, *where is the meaning when interpretation is twice?* These questions are the (pre) conditions for mathematical exigency.

If we ask what begets visual truth and what is the exigency of visual truth, we are lead invariably to ask if mathematical truth have a singular referent? Our answer will lead us either to "as such" or to haecceity.

If we indeed find a solution "x" in a constructive proof, it is contentious where this "x" exists. We proudly claim let "x" be this, "x" being empty and only self-identical yet having properties and conditions of the power of duration befitting any qualifier. We are to suppose this "x" is the same "x" "as" posited previously, thus again begetting the simulacrum of an idea of self-consistency. We leave open the concern of how any 'object' in object persistence could persist in a nonarchimedean emergent time (Dobson (2021a), Dobson (2021b)).

8 Diamond ∞-Time signature

As an exercise in higher visual epistemology, we attempt an non-renderable visualization of the slippage at play between haecceity and arche-writing, happening without happening, in a time without advent; a visualization "as" thought experiment.

If haecceities can be read all at once due to their arithmetic geometry, we extend the stack-simultaneity to the nature of "thoughts" and penultimately conclude with a discussion and interrogatory on the ontic and epistemic nature of "thoughts."

Are some thoughts haecceities and others arche-writing or are thoughts both simultaneously? If they are both simultaneously, does this imply that there exists a scaled model of $\mathrm{Time}_{\mathbf{Q_p}}$ that could structurally hold both haecceities and arche-writing? Are thoughts "as" thoughts leaving us no access to the mechanism which produces thoughts, therein leaving thoughts to be more like geometric points?

8.1 ∞-Stack Harmony

If we model thoughts as ∞-stacks taking values in ∞-categories, then scaling "the animal-stalks-at-five-o'clock" to all arche-writing has a neuro-equivalent of n-stack thought, where n varies as the number of haecceities, assuming they are countable, or ∞-stack for countably infinite. So if one diamond is $\mathrm{Time}_{\mathbf{Q_p}}$, or the ability to think in p-adic time signatures or "as" p-adic time signatures, for a $\mathrm{Time}_{\mathbf{Q_p}}$ without origin or advent, then n-stack thought could be thinking per a diamond n time signature:

Definition 16. *A diamond n time signature is a time signature with two n-stack-clefs representing haecceities and the proto-linguisticality arche-writing, respectively, and with the "notes" as diamonds.*

We can extend this and categorify the very essence of a time signature.

Definition 17. *A diamond ∞-time signature, by which we mean an $(\infty, 1)$-time signature, is a category containing objects time signatures, 1-morphisms between the objects, 2-morphisms between the 1-morphisms, ... ∞-morphisms between the $(\infty - 1)$-morphisms, where the morphisms are invertible for $n > 1$.*

Definition 18. *In $(\infty, 1)$-time signature, an n-morphism is an n-harmony.*

Definition 19. *In $(\infty, 1)$-time signature, an ∞-morphism between $(\infty - 1)$-morphisms is an ∞-harmony.* [14]

[14] What is the meaning of the invertibility of an ∞-harmony?

Definition 20. *An ∞-stack harmony is $(\infty,1)$-sheaf on an $(\infty.1)$-site where harmonies take values in ∞-groupoids.*

Definition 21. *A diamond ∞-stack time signature is $(\infty,1)$-sheaf on an $(\infty.1)$-site where the time signatures take values in ∞-groupoids.*

Definition 22. *A diamond ∞ time signature is a time signature with two ∞-stack clefs representing haecceities and the proto-linguisticality arche-writing, respectively, and with the "notes" $\mathcal{D}^{\diamond}$s.*

For example,

- A diamond 1-time signature consists of two 1-stack clefs, respectively haecceities and arche-writing, each taking time values in (2,1)-categories of groupoids with the notes diamonds.
- Categorifying, we could create the 2-category of 1-time signatures, where objects are 1-time signatures, 1-morphisms are 1-harmonies, and 2-morphisms are 2-harmonies.
- We could create the $(\infty,1)$-category of 1-time signatures, where objects are 1-time signatures, 1-morphisms are 1-harmonies, and 2-morphisms are 2-harmonies, ... up to ∞-harmonies.
- Stackifying, we could create the ∞-stack of 1-time signatures, which is an $(\infty,1)$-sheaf on an $(\infty.1)$-site where the 1-time signatures take values in ∞-groupoids.
- Lastly, we could create the ∞-stack of ∞-time signatures, which is an $(\infty,1)$-sheaf on an $(\infty.1)$-site where the ∞-time signatures take values in ∞-groupoids.

9 Conclusions and Extensions

We have posited a geometric structure capable of holding haecceities and arche-writing in a multiplicitous incarnation of what is constructed as singular. Our construction can be

[16] 1-stack as a v-stack. Recall, a v-stack is a diamond stack on a $(2,1)$-site with Grothendieck topology the v-topology, where attached to every open set is a diamond $Spa(C) \to \mathcal{D}$ for C an algebraically closed curve and $\mathcal{D}$ a diamond.

Table 1: ∞-Stackification of Haecceity and Meaning as Multiplicity [15]

Haecceity	∞-Stackification
∞-Exigency	∞-stack
Haecceity	$(\infty, 1)$-sheaf on $\mathcal{D}^{\diamond}$ [16]
A degree of heat	$(\infty, 1)$-sheaf on $\mathcal{D}^{\diamond}$
5 o'clock	$(\infty, 1)$-sheaf on $\mathcal{D}^{\diamond}$
An intensity of white	$(\infty, 1)$-sheaf on $\mathcal{D}^{\diamond}$
An hour	$(\infty, 1)$-sheaf on $\mathcal{D}^{\diamond}$
A winter	$(\infty, 1)$-sheaf on $\mathcal{D}^{\diamond}$
Home is Yesterday	$(\infty, 1)$-sheaf in time ∞-groupoid

scaled to the language of n-stacks, which are $(n+1)$-sheafs on $(n+1)$-sites taking values in $(n+1)$-category of groupoids. In this language we end with our grand scaled conclusion:

Proposition 13. *∞-Aporetic "as such" is a stack condition.*

Proposition 14. *Haecceities are diamond v-stacks* [17].

Proposition 15. *Arche-writing is a $\mathcal{D}^{\diamond}$ ∞-stack.*

[17] taking values in the $(2, 1)$-category of "diamond" self-singularity groupoids

Conflict of Interest

The author declares that they have no conflicts of interest with the reported work.

References

Adorno, T. (1990). Negative Dialectics. London: Routledge. p. xix. ISBN 0-415-05221-1

Baudrillard, J. (1983) *Simulacra and Simulation.* Los Angeles: Semiotext(e), 1983 (Translation by Foss, P., Batton, P. & Beitchman, P. of *Simulacres et Simulation*, Paris: Éditions Galilée, 1981).

BBC (1998).*The Mind, BBC series, Part 1.*

Clausen, D. & Scholze, P.(2021). Lectures on Condensed Mathematics.

Deleuze, G. & Guattari, F. (1987) *A Thousand Plateaus: Capitalism and Schizophrenia.* Minneapolis: University of Minnesota, 2007 (Translation by Massumi, B. of *Mille Plateaux*, Paris: Les Éditions de Minuit, 1980).

Derrida, Jacques. (1998). Of Grammatology. Baltimore :Johns Hopkins University Press.

Dobson, S. (2021) Efimov K-theory of Diamonds. https://philarchive.org/archive/DOBEKO.

Dobson, S. (2021) Pro-Generative Adversarial Network and V-Stack Perceptron, Diamond Holographic Principle, and Pro-Temporal Emergence, https://philpapers.org/archive/DOBPAN-3.

Dobson, S. (2021) Dobson, S. (2021). *Artemis Blu II: In Diamonds (of Diamond Hourglasses)*, in preparation.

Dobson, S. & Fields, C. (2021) *Events and Memory in Functorial Time I: Localizing Temporal Logic to Condensed, Event-dependent Memories.* https://philpapers.org/rec/DOBEAM.

Dobson, S. & Fields, C. (2021) *Six Operations in Diamond Holography.* in preparation.

Dobson, S. & Prentner, R. (2021) *Perfectoid Diamonds and n-Awareness. A Meta-Model of Subjective Experience.* arXiv:2102.07620 [math.GM].

Dobson, S. & Prentner, R. (2021) *Pluralist-Monism. Derived Category Theory as the Grammar of nAwareness.* arXiv:2009.01691 [math.GM].

Hartshorne, R. (1977) *Algebraic geometry.* (Graduate Texts in Mathematics, No. 52) New York: Springer-Verlag.

Kierkegaard, S. (1838) *Journal entry from August 17, 1838.*

Scholze, P. (2017). *'Etale cohomology of Diamonds.*arXiv:1709.07343 [math.AG].

Stanford Encyclopedia of Philosophy, Jacques Derrida. (2019). https://plato.stanford.edu/entries/derrida/.

Wikipedia Authors.*"Buffalo" Sentence.*

Wittgenstein, L. (1953) *Philosophical Investigations.* (Macmillan Publishing Company).