

Does non-measurability favour imprecision?

CIAN DORR

New York University, U.S.A.

cian.dorr@nyu.edu

1 An argument for the permissibility of imprecise credence

Consider the following case, inspired by Isaacs, Hájek, and Hawthorne (2021: henceforth IHH):¹

Strange Spinner: God— who turns out to be quite the connoisseur of unusual games of chance — shows you an interesting contraption He has made. It consists of a disc with a pointer attached to the centre so that it is free to spin, and a lamp attached. He explains that He has bestowed a special blessing on some of the points on the disc’s circumference, and decreed that when the spinner is spun, the lamp will come on if and only if the pointer comes to rest when pointing to a blessed point. He chose the set of blessed points to be *non-measurable*, with respect to the standard Lebesgue measure determined by angular distance on the disc’s circumference.² In fact, it is as ill-behaved as possible as far as measurability is concerned: all its measurable subsets, and all its complement’s measurable subsets, have Lebesgue measure 0.³ He

¹Hoek (2021) has similar examples, though they are used for very different dialectical purposes.

²The *outer Lebesgue measure* $\lambda^*(X)$ of a set of n -tuples $X \subseteq [0, 1]^n$ is defined as the greatest lower bound of $\{\sum_{n < \omega} m(B_n) \mid X \subseteq \bigcup_n B_n \text{ and each } B_i \text{ is a box}\}$, where a *box* B is a product of n intervals and $m(B)$ is the product of those interval’s lengths. Its *inner Lebesgue measure* $\lambda_*(X)$ can be defined as $1 - \lambda^*([0, 1]^n - X)$. X is Lebesgue-measurable with Lebesgue measure $y = \lambda(X)$ when $y = \lambda^*(X) = \lambda_*(X)$. It is easy to show that the outer [inner] Lebesgue measure of a set is the greatest lower [least upper] bound of the Lebesgue measures of its measurable supersets [subsets]. We apply these functions to sets of points on the circumference of the spinner by arbitrarily choosing a point y and mapping each $x \in [0, 1)$ to the point x rotations clockwise from y : since the functions are rotation-invariant, it makes no difference which y we choose.

³Equivalently: the set has inner Lebesgue measure 0 and outer Lebesgue measure 1.

1 invites you to experiment with the spinner, assuring you that when you spin
2 the pointer, it will randomly select a point on the circumference in a way that is
3 *completely fair*. God then departs. You call after Him—‘Wait! Can You explain
4 exactly what You mean by ‘completely fair’?’—but He is gone. You wonder
5 what doxastic attitude you should take to the proposition that the lamp will
6 come on the first time you spin the pointer.

7 The question is hard because the case suggests no principled considerations in favour
8 of having credence x , for any real number $x \in [0, 1]$. Indeed, for each such x , the idea
9 of responding to your situation by having credence x that the lamp will come on seems
10 unsettlingly *arbitrary*, in a way that carries a whiff of irrationality.

11 Some philosophers theorize about doxastic states using a generalization of the familiar
12 framework of real-valued credences that seems tailor-made to help with puzzling cases
13 like *Strange Spinner*. This theory posits two relations each of which can obtain between
14 an agent, a proposition A , and at most one real number: the number can be the agent’s
15 *lower credence* in A , $Cr_*(A)$, or the agent’s *upper credence* in A , $Cr^*(A)$. We always have
16 $Cr_*(A) \leq Cr^*(A)$. When $Cr_*(A) = Cr^*(A)$, that number is one’s *credence* in A , $Cr(A)$. When
17 $Cr_*(A) < Cr^*(A)$, one is said to be *credally imprecise* with respect to A ; in that case, no real
18 number is one’s credence in A .⁴

19 IHH use cases like *Strange Spinner* to argue that credal imprecision is sometimes *rational-*
20 *ally permissible*. Assuming for simplicity that it is rationally obligatory to have credence 1
21 in what God tells you, their argument in fact implies that it is rationally permissible for
22 your lower and upper credences that the lamp will come on when you spin the spinner to

⁴In assuming credal imprecision to be incompatible with real-valued credence, I am ruling out a vagueness-theoretic interpretation which identifies being credally imprecise with respect to A with being such that no real number is *definitely* one’s credence in A . For given classical logic, we cannot take ‘no F is definitely G ’ to be incompatible with ‘some F is G ’ without trivializing ‘definitely’. Some fans of imprecision (van Fraassen 1990, Rinard 2013) favour a “supervaluationist interpretation” which suggests this identification; but IHH (§6) firmly reject it. I will explain in §3.2 how my argument can be adapted to fit the vagueness-theoretic interpretation.

1 be 0 and 1, respectively.

2 Before describing their argument, I should admit that I do not really understand its
3 conclusion, since—despite the valiant efforts of a long philosophical tradition (Keynes
4 1921, Koopman 1940, Levi 1974, van Fraassen 1990, Walley 1991, Kaplan 1996, Joyce 2010,
5 Sturgeon 2020)—I do not understand what it *is* to be credally imprecise with respect to a
6 proposition. Although various answers have been suggested, they diverge so much that I
7 am doubtful that there is any single phenomenon (or putative phenomenon) that theorists
8 of credal imprecision have been talking about.

9 There are indeed definitions of lower and upper credence that are widely accepted in
10 the literature, but they invoke other imprecision-theoretic terminology that raises similar
11 interpretative challenges. A common approach is to generalize the use of ‘credence’ so
12 that we say that in some cases, one’s credence in a proposition is not a real number, but
13 rather a *set* of real numbers. One’s lower and upper credences can then be defined as the
14 greatest lower and least upper bounds of this set. If anything, this just exacerbates the
15 difficulty. When $x < y$, many different sets have lower bound x and upper bound y ; thus
16 even if we had a good account of what it would be for one’s lower and upper credences
17 in A to be x and y , it is not obvious that it would extend to an account of what it would
18 be for one rather than another of the sets with greatest lower bound x and least upper
19 bound y to be one’s credence in A .⁵ Harder still are the questions raised by the popular
20 analysis of set-valued credence according to which each agent has a *credal representor*, a set
21 of real-valued functions (probability functions, if the agent is ideally rational), such that
22 one’s credence in A is the set containing exactly the numbers to which some member of
23 one’s credal representor maps A .⁶ Even if we had a good account of what it is to have a
24 certain set-valued credence function, it is not obvious that it would extend to an account

⁵It might be proposed that only closed intervals $[x, y]$ can be credences: this makes the set-valued terminology a notational variant of the terminology of lower and upper credences.

⁶See Bradley 2019 and the many references therein. IHH express some scepticism about this framework.

1 of what it is to have a certain credal representor, since two distinct sets of probability
 2 functions can agree as regards the set to which they map every proposition.⁷

3 More informative analyses of lower and upper credence are certainly possible, though
 4 much more controversial. One simple candidate is what I will call the “inner-outer
 5 analysis”, explored by Fagin and Halpern (1991). According to it, one’s lower credence in
 6 A is one’s *inner* credence in A : that is, the least upper bound of the set of real numbers that
 7 are one’s credences in propositions that entail A . Likewise, one’s upper credence in A is
 8 one’s *outer* credence in A : the greatest lower bound of one’s credences in propositions that
 9 A entails. The inner-outer analysis is appealingly simple, but has some consequences that
 10 few proponents of imprecise credence accept. For example, it implies that anyone who
 11 assigns a credence to one necessity and one impossibility automatically has lower and
 12 upper credences in every proposition. It also implies that if the range of one’s credence
 13 function is a finite set, then all one’s lower and upper credences must belong to it.⁸ So I
 14 will not be assuming the inner-outer analysis, or any other analysis of lower and upper
 15 credence; instead, I will treat these notions as primitive, bracketing my doubts about their
 16 interpretation in order to engage with IHH’s arguments.

⁷An example from Walley (1991: §2.7.3) shows that this can happen even when both sets are closed and convex. Suppose we have three mutually exclusive, jointly exhaustive, contingent propositions A, B, C . Define probability functions $\text{Pr}_1, \text{Pr}_2, \text{Pr}_3$ as follows:

	\perp	A	B	C	$A \vee B$	$A \vee C$	$B \vee C$	\top
Pr_1	0	$2/3$	$1/3$	0	1	$2/3$	$1/3$	1
Pr_2	0	$1/3$	0	$2/3$	$1/3$	1	$2/3$	1
Pr_3	0	$2/3$	0	$1/3$	$2/3$	1	$1/3$	1

Let X be the smallest convex set containing Pr_1 and Pr_2 , and Y the smallest convex set containing Pr_1, Pr_2 , and Pr_3 . These are different sets, since Pr_3 is not a linear combination of Pr_1 and Pr_2 . Both are closed. And both determine the same set-valued function, since the value of Pr_3 is always in the interval spanned by the values of Pr_1 and Pr_2 .

⁸Fagin and Halpern (1991) show that the inner-outer analysis has another controversial consequence, namely that your unconditional lower credences obey the “complete monotonicity” conditions characteristic of Dempster-Shafer belief functions (Shafer 1976): a special case is that $\text{Cr}_*(A \vee B) \geq \text{Cr}_*(A) + \text{Cr}_*(B) - \text{Cr}_*(AB)$. For an argument that rational lower credences need not obey this constraint, see Walley 1991: §5.13.4.

1 IHH in fact have at least two lines of argument for the claim that credal imprecision is
2 rationally permissible in cases like *Strange Spinner*, but the one I want to focus on (until
3 §5) turns on another potentially controversial notion, that of *imprecise objective chance*. We
4 assume that at a time t , a proposition A can have an *lower chance* $\text{Ch}_{*t}(A)$ and an *upper*
5 *chance* $\text{Ch}^*_t(A)$, where $0 \leq \text{Ch}_{*t}(A) \leq \text{Ch}^*_t(A) \leq 1$. If the two are equal, this number is also
6 $\text{Ch}_t(A)$, A 's *chance* at t . Otherwise, no real number is A 's chance at t .

7 Although the concept of imprecise objective chance is not in wide use, there is reason to
8 think that it is in good standing, assuming that the concepts of imprecise credence and of
9 (precise) objective chance both are. Those who understand objective chance primarily in
10 terms of its relationship with rational credence will plausibly be able to exploit their grasp
11 of imprecise credence to understand imprecise chance in some parallel way. Furthermore,
12 the inner-outer analysis of imprecise chance (which identifies a proposition's lower and
13 upper chances with its inner and outer chances, i.e. the least upper bound of the chances
14 of propositions that entail it and the greatest lower bound of the chances of propositions
15 it entails) looks more plausible than the inner-outer analysis of imprecise credence. For
16 example, there is no obvious reason to think it possible for a proposition to lack any lower
17 or upper chance at a time, or for some finite set to contain all real-valued chances but not
18 all lower and upper chances.⁹

19 To express the key premise linking imprecise chance to imprecise credence, we will
20 need to avail ourselves of the notion of *conditional* imprecise credence. By contrast with
21 the familiar ratio formula $\text{Cr}(A | B) = \text{Cr}(AB)/\text{Cr}(B)$ for real-valued conditional credence,
22 there is no natural candidate definition of conditional lower or upper credence in terms of
23 the unconditional notions. Nevertheless, the conditional concepts seem no more obscure
24 than the unconditional ones, and they are helpful for the purposes of systematic theorizing
25 (cf. Hájek 2003).¹⁰

⁹IHH in fact seem to endorse the inner-outer analysis of imprecise chance, although it is not needed for their argument.

¹⁰The inner-outer analysis in the form given above does not generalize straightforwardly

1 Once the concepts of imprecise chance and conditional imprecise credence are on the
 2 table, we can use them to state a generalization of the “Principal Principle” (Lewis 1980).
 3 That principle says, roughly, that one’s credences, conditional on a hypothesis about the
 4 chances, should match the chances ascribed by that hypothesis. More precisely:

PP If Cr is a rational prior credence function, and E is admissible at t , then for
 any proposition A and $x \in [0, 1]$:

$$Cr(A \mid E \wedge Ch_t(A) = x) = x$$

5 so long as this is defined.¹¹

6 IHH’s proposed generalization says the corresponding thing about lower and upper cre-
 7 dences and lower and upper chances. In our notation, it can be stated as follows:

GEN-PP If Cr_* and Cr^* are a rational pair of a lower and upper prior credence
 function, and E is admissible at t , then for any proposition A and $x \in [0, 1]$:

$$Cr_*(A \mid E \wedge Ch_{*t}(A) = x) = x$$

$$Cr^*(A \mid E \wedge Ch^*_t(A) = x) = x$$

8 so long as these are defined.¹²

to a definition of conditional lower and upper credence in terms of conditional real-
 valued credence. But Ruspini (1987) shows that, assuming one’s credence function Cr
 is probabilistically coherent, the inner-outer analysis is equivalent to defining $Cr_*(A)$
 and $Cr^*(A)$ as the infimum and supremum of $\{Pr(A) \mid Pr \text{ is a probabilistically coherent}$
 extension of Cr defined on an algebra including $A\}$. This reformulated analysis does
 generalize straightforwardly: $Cr_*(B \mid A)$ and $Cr^*(B \mid A)$ can be defined as the infimum and
 supremum of $\{Pr(B \mid A) \mid Pr \text{ is a probabilistically coherent extension of } Cr, \text{ defined on } A$
 and $B \text{ and such that } Pr(A) > 0\}$.

¹¹‘ E is admissible at t' ’ might be glossed as ‘ E is entirely about the intrinsic states of the
 world at, and the chances at, times at and before t ’: see Lewis 1980. ‘Rational prior credence
 function’ raises tricky issues, but could defensibly be glossed as ‘credence function that
 could be that of a rational agent at a time before t' ’.

¹²IHH write their version of GEN-PP as $Cr(A \mid Ch(A) = [x, y]) = [x, y]$. This differs from
 mine in several ways that are not important here: it conditionalizes on a proposition that

1 This yields PP as a special case: since $\text{Ch}_{*t}(A) = x$ and $\text{Ch}^*_t(A) = x$ are admissible at t ,
 2 we can substitute the former for E in the second equation and the latter for E in the first
 3 equation to derive that

$$\text{Cr}_*(A \mid \text{Ch}_{*t}(A) = \text{Ch}^*_t(A) = x) = \text{Cr}^*(A \mid \text{Ch}_{*t}(A) = \text{Ch}^*_t(A) = x) = x.$$

4 To apply GEN-PP to *Strange Spinner*, we will need to cash out the “fairness” of the
 5 spinner in a way that links facts about the measures of sets of points to facts about lower
 6 and upper chances. This is naturally done as follows. Where X is any subset of $[0, 1)$ and
 7 y is any point on the circumference of the disc, say that X is *y-selected* iff the next time the
 8 pointer is spun, the clockwise angular distance (measured in revolutions) from y to its tip
 9 belongs to X . Then we take the fairness claim to imply that for any X and y , the lower and
 10 upper chances that X is *y-selected* are respectively $\lambda_*(X)$ and $\lambda^*(X)$, the Lebesgue inner
 11 and outer measures of X (see note 2). (Note that this can consistently hold for all y , since
 12 Lebesgue measure is translation-invariant.) Since the set of blessed points has Lebesgue
 13 inner measure 0 and outer measure 1, and with chance 1 the lamp turns on iff the selected
 14 point is blessed, it follows that the lower and upper chances that the lamp will light when
 15 the pointer is spun are 0 and 1, respectively.¹³ Thus GEN-PP implies that conditional on
 16 the description of the case, you should have lower credence 0 and upper credence 1 that
 17 the lamp will light after the first spin. Given our simplifying assumption that you should
 18 have credence 1 in the description of the case, it plausibly follows that you should have
 19 *unconditional* lower credence 0 and upper credence 1 in this proposition.

20 IHH do not actually endorse GEN-PP. They do say that it ‘is plausible in much the
 21 same way that the original [Principal] principle is’, but are careful not to rule out a kind of
 22 extreme permissivism which would rule out both principles. Since they are only arguing

specifies both a lower and an upper chance; uses the ideology of set-valued credence and chance; does not provide for the inclusion of an arbitrary “admissible” proposition E ; and does not explicitly relativize to time or restrict to rational *prior* credences.

¹³I assume for simplicity that there is zero chance that it will never be spun.

1 for the *permissibility* of imprecise credence, the following weaker premise is all they need:

2 PERMISSIVE GEN-PP It is always rationally permissible to conform to GEN-PP.

3 This entails the permissibility of credal imprecision, given the further premise that there
4 are cases where one should have nonzero credence that a proposition has imprecise chance.

5 I will grant—for the sake of argument, and *pace* Hoek 2021—that there are such cases, and
6 that *Strange Spinner* is one of them. But I will argue that PERMISSIVE GEN-PP is false.

7 **2 Against PERMISSIVE GEN-PP**

8 Let's continue the story:

Two Spinners: The next day, God returns with two identical-looking spinners. He explains that one of them—the *interesting* spinner—is the same one you already saw. The other one—the *boring* spinner—is also set up such that its lamp will come on if the pointer comes to rest on a blessed point. But on it, the blessed points form a contiguous interval comprising one-third of the disc's circumference. God then hands you one of the spinners, telling you that He made the choice by tossing a fair coin: if it came up Heads, you have the boring spinner, while if it came up Tails, you have the interesting spinner. Departing, He reminds you that spinning the pointer will randomly select a point on the circumference in a way that is *completely fair*. Left alone with your spinner, you begin to experiment. After the first spin, the lamp turns on after the pointer comes to rest. The second time, it stays off. You keep at it, keeping records as you go: a 1 when the lamp comes on, a 0 when it doesn't. After thirty spins, your tally looks like this:

1 0 1 0 1 1 0 1 0 0 0 1 0 1 0 0 0 0 0 0 0 1 0 0 1 0 0 0 1 0

9 That's ten 1s and twenty 0s: just the proportion that would be most likely if

1 you had the boring spinner. You wonder how you should react.

2 I say you should react by becoming *more confident* that you have the boring spinner than
3 you were before your experiments.

4 I hope readers will share my sense that this is obviously the right reaction. For those
5 who demand an argument, I offer the following: your observations should leave you *pretty*
6 *confident* that you have the boring spinner. Beforehand, you should not have been pretty
7 confident. But necessarily, if you are pretty confident afterwards and not before, you are
8 more confident afterwards than before.¹⁴

9 If you still do not feel the pull of the judgment, bear with me; I will say more in its
10 defence in §3.4. First, let's see how it can be used to argue against PERMISSIVE GEN-PP.

11 We will of course need some auxiliary premises. The first is a bridge principle linking
12 the ordinary expression 'more confident than' with the theoretical jargon of imprecise
13 credence. No such principle is beyond dispute: part of what makes the jargon difficult is
14 the lack of uncontroversial connections between it and such ordinary locutions. But the
15 following rather weak principle will suffice for our purposes:

16 CONFIDENCE If x at t_1 is more confident in A than y at t_2 is in B , then x 's lower
17 credence at t_1 in A is not less than y 's lower credence at t_2 in B .

18 Given CONFIDENCE, our starting judgment implies that your lower credence that you have
19 the boring spinner should not be lower after your observations than it was initially.¹⁵

20 The second auxiliary premise concerns how imprecise credences should change in
21 response to new evidence. This will let us transform our claim about how your credal
22 state should evolve over time into one about your credal state at the initial time. Since we

¹⁴For a defence of the final premise, see the discussion of 'Strong Modified Monotonicity' in Dorr, Nebel, and Zuehl 2023: §4.

¹⁵CONFIDENCE can be strengthened without appreciable loss of plausibility to the claim that if x at t_1 is *at least* as confident in A as y is at t_2 in B , then x 's lower credence at t_1 in A is greater than or equal to y 's lower credence at t_2 in B and x 's upper credence at t_1 in A is greater than or equal to y 's upper credence at t_2 in B . The converse of this stronger conditional also has some appeal, though it is considerably more controversial.

1 are helping ourselves to the concepts of *conditional* lower and upper credence, there is a
2 very natural candidate principle of this sort, namely the imprecise analogue of orthodox
3 Bayesian conditionalization:

4 GENERALIZED CONDITIONALIZATION Suppose you are rational and acquire total
5 evidence E between t and t^+ . Then for any propositions A and B , your lower
6 and upper credences at t^+ in A given B are equal, respectively, to your lower and
7 upper credences at t in A given BE , provided these are all well-defined.

8 Assuming that one should have unconditional credence x in A just in case one has cre-
9 dence x in A conditional on some tautology \top , GENERALIZED CONDITIONALIZATION implies
10 the standard Bayesian conditionalization principle for real-valued credence. Familiar ob-
11 jections to that principle thus carry over to GENERALIZED CONDITIONALIZATION. However,
12 these objections seem orthogonal to our present concerns. For example, there are worries
13 about cases where one’s memory might fail (Arntzenius 2003), and worries related to the
14 difficulty of finding an interpretation of “evidence” on which it’s plausible both that one
15 should always have credence 1 in one’s evidence and that one’s credential state should
16 only change when one’s evidence does (Jeffrey 1965). While these worries are important,
17 they do not impugn the use of standard conditionalization (at least as an approximation)
18 in examples where it is clear what proposition should play the role of one’s “total evid-
19 ence”. In the imprecise setting, GENERALIZED CONDITIONALIZATION plausibly deserves a
20 similar status.

21 Given CONFIDENCE and GENERALIZED CONDITIONALIZATION, our starting judgment im-
22 plies that initially, your unconditional lower credence in the proposition that you have the
23 boring spinner should not exceed your lower credence in that proposition conditional on
24 the proposition that the first 30 spins have the given sequence of outcomes. To see why this
25 is problematic for PERMISSIVE GEN-PP, we will need to further flesh out our interpretation
26 of God’s claim about the “fairness” of your spinner. In the previous section, we took that
27 claim to imply that for any $X \subseteq [0, 1)$ and any point y on the circumference, the inner

1 and outer chances that the next spin will y -select X (i.e. pick out a point whose clockwise
2 angular distance from y belongs to X) equal the Lebesgue inner and outer measures of X .
3 Now we must somehow extend this to propositions about where the pointer lands after
4 the first n spins. Since the Lebesgue inner and outer measures λ_* and λ^* can be defined on
5 all subsets of $[0, 1)^n$, there is a very natural extension. Say that $X \subseteq [0, 1)^n$ is y -selected at
6 t iff X contains the n -tuple of numbers corresponding to the angular distances from y of
7 the points where the pointer will land the next n times it is spun after t . Then the fairness
8 claim is naturally understood to imply that for any such X , the lower and upper chances
9 at t of X being y -selected are respectively $\lambda_*(X)$ and $\lambda^*(X)$.

10 Importantly, this interpretation secures an imprecise analogue of probabilistic inde-
11 pendence for distinct spins. Just as the Lebesgue measure of the Cartesian product of some
12 measurable sets is the product of their individual Lebesgue measures, so the Lebesgue
13 inner and outer measures of the Cartesian product of any sets are the products of their
14 individual Lebesgue inner and outer measures: $\lambda_*(X_1 \times \cdots \times X_n) = \lambda_*(X_1) \cdots \lambda_*(X_n)$ and
15 $\lambda^*(X_1 \times \cdots \times X_n) = \lambda^*(X_1) \cdots \lambda^*(X_n)$.¹⁶ So the lower and upper chance at t of the proposition
16 that $X_1 \times \cdots \times X_n$ is y -selected at t are respectively $\lambda_*(X_1) \cdots \lambda_*(X_n)$ and $\lambda^*(X_1) \cdots \lambda^*(X_n)$,
17 the products of the lower and upper chances of the propositions that the i th spin will
18 y -select X_i . When each X_i has inner measure 0 and outer measure 1, these lower and

¹⁶We prove this for inner measure; the case of outer measure is analogous. We need the following lemma: for every $X \subseteq [0, 1)^k$, there is a Lebesgue-measurable $Y \subseteq X$ such that $\lambda(Y) = \lambda_*(X)$. (Proof: By definition of λ_{k*} , for every $\varepsilon > 0$ there is a $Y_\varepsilon \subseteq X$ such that $\lambda(Y_\varepsilon) > \lambda_*(X) - \varepsilon$; so we can take $Y = \bigcup_n Y_{1/n}$.) So, let Y_1, \dots, Y_n be Lebesgue-measurable subsets of $[0, 1)$ such that $Y_i \subseteq X_i$ and $\lambda(Y_i) = \lambda_*(X_i)$, and let Z be a Lebesgue-measurable subset of $[0, 1)^n$ such that $Z \subseteq \prod_i X_i$ and $\lambda(Z) = \lambda_*(\prod_i X_i)$. For any $X \subseteq [0, 1)$, let $C_i^n[X]$ denote the set of n -tuples in $[0, 1)^n$ whose i th co-ordinate is in X . Since $\prod_i X_i = \bigcap_i C_i^n[X_i]$, we have $Z \subseteq C_i^n[X_i]$ for each i . Also, since $Y_i \subseteq X_i$, $C_i^n[Y_i] \subseteq C_i^n[X_i]$. Thus $Z \cup C_i^n[Y_i] \subseteq C_i^n[X_i]$, and so $\lambda(Z \cup C_i^n[Y_i]) \leq \lambda_*(C_i^n[X_i]) = \lambda_*(X_i) = \lambda(Y_i) = \lambda(C_i^n[Y_i])$. It follows that $\lambda(Z \cap \overline{C_i^n[Y_i]}) = 0$ for every i . Thus $\lambda(Z \cap \overline{\prod_i Y_i}) = \lambda(Z \cap \overline{\bigcap_i C_i^n[Y_i]}) = \lambda(Z \cap \bigcup_i \overline{C_i^n[Y_i]}) = \lambda(\bigcup_i (Z \cap \overline{C_i^n[Y_i]})) \leq \sum_i \lambda_n(Z \cap \overline{C_i^n[Y_i]}) = 0$, and hence $\lambda_*(\prod_i X_i) = \lambda(Z) = \lambda(Z \cap \prod_i Y_i) \leq \lambda(\prod_i Y_i)$. But of course since each $Y_i \subseteq X_i$ we also have $\lambda(\prod_i Y_i) \leq \lambda_*(\prod_i X_i)$. We can conclude that $\lambda_*(\prod_i X_i) = \lambda(\prod_i Y_i) = \prod_i \lambda(Y_i) = \prod_i \lambda_*(X_i)$, using the fact that the Lebesgue measure of a product is the product of Lebesgue measures.

1 upper chances will be 0 and 1 respectively. This holds in particular when for some y , each
2 X_i is either $\{z \in [0, 1): \text{the point } z \text{ rotations from } y \text{ is blessed}\}$ or its complement. The
3 upshot is that if you have the interesting spinner, *every finite sequence of possible observations*
4 *has inner chance 0 and outer chance 1.*

5 This interpretation of the fairness claim provides chance-theoretic propositions that can
6 be plugged into GEN-PP or PERMISSIVE GEN-PP to derive propositions about the lower and
7 upper conditional credences you are required or permitted to have in various outcomes,
8 conditional on having the boring and interesting spinners. However, these claims do not
9 immediately yield what we are looking for, namely a value for your lower credence that
10 you have the boring spinner, conditional on a particular sequence of outcomes. To derive
11 such a claim, we will need to appeal to some basic *coherence principles* about lower and
12 upper conditional credence.

13 We will need two families of such principles, each generalizing a familiar coherence
14 principle for real-valued conditional probability. The first family generalizes the following
15 *multiplicative axiom*, basic to the theory of real-valued conditional probability:

$$\mathbf{M} \quad \text{Cr}(AB \mid C) = \text{Cr}(A \mid BC) \text{Cr}(B \mid C)$$

When C is a tautology and $\text{Cr}(B) > 0$, this is equivalent to the familiar ratio formula $\text{Cr}(A \mid B) = \text{Cr}(AB)/\text{Cr}(B)$, assuming that unconditional credence can be equated with credence conditional on a tautology. In theories like that of Popper (1955, 1959) that treat conditional probability as primitive, (M) plays a fundamental role in co-ordinating probabilities conditional on different propositions.¹⁷ Proponents of imprecise credence need some principles that can do a similar job. And in fact, all extant formal treatments

¹⁷(M) also holds in Kolmogorov's theory of "regular conditional probability" (see Easwaran 2019), where claims of conditional probability must always be relativized to a *sub- σ -algebra*, provided that we interpret all three conditional probabilities as relativized to the same sub- σ -algebra.

of imprecise conditional probability imply the following generalizations of (M):

$$\begin{array}{ll}
 \mathbf{M}_{***} & \text{Cr}_*(AB|C) \geq \text{Cr}_*(A|BC) \text{Cr}_*(B|C) & \mathbf{M}^{***} & \text{Cr}^*(AB|C) \leq \text{Cr}^*(A|BC) \text{Cr}^*(B|C) \\
 \mathbf{M}_{**^*} & \text{Cr}_*(AB|C) \leq \text{Cr}_*(A|BC) \text{Cr}^*(B|C) & \mathbf{M}^{**^*} & \text{Cr}^*(AB|C) \geq \text{Cr}^*(A|BC) \text{Cr}_*(B|C) \\
 \mathbf{M}_{*^*} & \text{Cr}_*(AB|C) \leq \text{Cr}^*(A|BC) \text{Cr}_*(B|C) & \mathbf{M}^{*^*} & \text{Cr}^*(AB|C) \geq \text{Cr}_*(A|BC) \text{Cr}^*(B|C)
 \end{array}$$

1 For example, all six inequalities can easily be seen to fall out of the “representor” approach,
 2 given that each member of the representor is (or determines via the ratio formula) a condi-
 3 tional probability function obeying (M).¹⁸ Other analyses of lower and upper conditional
 4 probability (e.g. that of Walley 1991, based on a concept of ‘prevision’ for random vari-
 5 ables) also validate these inequalities. Together, they do quite a lot to constrain $\text{Cr}_*(\cdot | BC)$
 6 and $\text{Cr}^*(\cdot | BC)$ given $\text{Cr}_*(\cdot | C)$ and $\text{Cr}^*(\cdot | C)$, though by contrast with the real-valued
 7 case, they do not completely pin them down even when $\text{Cr}_*(B | C)$ is positive.

When one of the three relevant pairs of propositions has a real-valued conditional credence (i.e. has equal lower and upper conditional credences), we can combine the above inequalities to derive some useful equations:

$$\begin{array}{ll}
 \mathbf{M}_{**=} & \text{Cr}_*(AB|C) = \text{Cr}_*(A|BC) \text{Cr}(B|C) & \mathbf{M}^{**=} & \text{Cr}^*(AB|C) = \text{Cr}^*(A|BC) \text{Cr}(B|C) \\
 \mathbf{M}_{*=*} & \text{Cr}_*(AB|C) = \text{Cr}(A|BC) \text{Cr}_*(B|C) & \mathbf{M}^{*=*} & \text{Cr}^*(AB|C) = \text{Cr}(A|BC) \text{Cr}^*(B|C) \\
 \mathbf{M}^{=*} & \text{Cr}(AB|C) = \text{Cr}^*(A|BC) \text{Cr}_*(B|C) & \mathbf{M}^{=*} & \text{Cr}(AB|C) = \text{Cr}_*(A|BC) \text{Cr}^*(B|C)
 \end{array}$$

8 In each case, the two inequalities got by replacing the equals sign with an upper or lower
 9 asterisk imply that the equality holds whenever Cr is defined (i.e., whenever Cr_* and Cr^*
 10 agree) on the relevant two propositions.¹⁹

11 The second family of coherence principles generalize the following *finite additivity*

¹⁸They relevant mathematical fact is that for any indexed families $\{a_i\}_{i \in I}$ and $\{b_i\}_{i \in I}$, $\inf\{a_i\} \inf\{b_i\} \leq \inf\{a_i b_i\} \leq \inf\{a_i\} \sup\{b_i\} \leq \sup\{a_i b_i\} \leq \sup\{a_i\} \sup\{b_i\}$.

¹⁹These equations entail in turn that (M) holds whenever all three of $\text{Cr}(B | C)$, $\text{Cr}(AB | C)$, and $\text{Cr}(A | BC)$ exist. Moreover, if $\text{Cr}(A | BC)$ and one of $\text{Cr}(B | C)$ and $\text{Cr}(AB | C)$ exist, the other must also exist; and if both $\text{Cr}(B | C)$ and $\text{Cr}(AB | C)$ exist and $\text{Cr}(B | C) > 0$, $\text{Cr}(A | BC)$ must exist.

1 axiom from the theory of real-valued conditional probability:

$$\mathbf{A} \quad \text{Cr}(A | C) = \text{Cr}(AB | C) + \text{Cr}(A\bar{B} | C)$$

This plays a similarly fundamental role to (M).²⁰ The following inequalities can play an analogous role in the theory of imprecise conditional credence:²¹

$$\mathbf{A}_{**} \quad \text{Cr}_*(A|C) \geq \text{Cr}_*(AB|C) + \text{Cr}_*(A\bar{B}|C) \quad \mathbf{A}^{**} \quad \text{Cr}^*(A|C) \leq \text{Cr}^*(AB|C) + \text{Cr}^*(A\bar{B}|C)$$

$$\mathbf{A}_{**}^* \quad \text{Cr}_*(A|C) \leq \text{Cr}_*(AB|C) + \text{Cr}^*(A\bar{B}|C) \quad \mathbf{A}^{**}_* \quad \text{Cr}^*(A|C) \geq \text{Cr}^*(AB|C) + \text{Cr}_*(A\bar{B}|C)$$

$$\mathbf{A}^*_* \quad \text{Cr}_*(A|C) \leq \text{Cr}^*(AB|C) + \text{Cr}_*(A\bar{B}|C) \quad \mathbf{A}^*_* \quad \text{Cr}^*(A|C) \geq \text{Cr}_*(AB|C) + \text{Cr}^*(A\bar{B}|C)$$

These follow from the “representor” analysis for exactly the same reason as the analogous multiplicative inequalities; they also fall out from other approaches such as that of Walley (1991). And as before, pairs of inequalities can be combined to yield useful equations in cases where one of the three proposition-pairs receives a real-valued conditional credence:

$$\mathbf{A}_{**} = \quad \text{Cr}_*(A|C) = \text{Cr}_*(AB|C) + \text{Cr}(A\bar{B}|C) \quad \mathbf{A}^{**} = \quad \text{Cr}^*(A|C) = \text{Cr}^*(AB|C) + \text{Cr}(A\bar{B}|C)$$

$$\mathbf{A}_{**}^* \quad \text{Cr}_*(A|C) = \text{Cr}(AB|C) + \text{Cr}_*(A\bar{B}|C) \quad \mathbf{A}^*_* \quad \text{Cr}^*(A|C) = \text{Cr}(AB|C) + \text{Cr}^*(A\bar{B}|C)$$

$$\mathbf{A}^*_* \quad \text{Cr}(A|C) = \text{Cr}^*(AB|C) + \text{Cr}_*(A\bar{B}|C) \quad \mathbf{A}^*_* \quad \text{Cr}(A|C) = \text{Cr}_*(AB|C) + \text{Cr}^*(A\bar{B}|C)$$

2 These give us back **A** in the case where all three (or indeed any two) of $\text{Cr}(A | C)$,
3 $\text{Cr}(AB | C)$, and $\text{Cr}(A\bar{B} | C)$ exist.

We now have all we need to complete our argument against PERMISSIVE GEN-PP. Assume you start out with lower and upper credence functions Cr_* and Cr^* conforming to GEN-PP.

Let B be *you got the boring spinner*. Let E be some proposition that specifies the outcome of the first n spins for some $n \geq 1$, and entails that the lamp stayed off m times. (In the case

²⁰Popper (1959) makes an exception to (A) to allow for special propositions C (e.g. contradictions) with the property that $\text{Cr}(A | C) = 1$ for all A ; I prefer to let $\text{Cr}(\cdot | C)$ go undefined in such cases.

²¹This presentation is redundant: the two inequalities on the last line follow from the ones above them by substituting \bar{B} for B and appealing to double negation elimination.

we have been focusing on, $n = 30$ and $m = 20$.) Applying GEN-PP to the time before the coin-toss, we have

$$\text{Cr}(B) = \text{Cr}(\bar{B}) = 1/2$$

Applying it to the time after the coin toss conditional on your getting the boring spinner, we have

$$\text{Cr}(E | B) = 2^m / 3^n$$

(Note that for large n , this is small for all $m \leq n$.) Finally and most importantly, the fact that any sequence of outcomes on the interesting spinner has inner chance 0 and outer chance 1 gives us the following equations, applying GEN-PP to the time after the toss conditional on your getting the interesting spinner:

$$\text{Cr}_*(E | \bar{B}) = 0$$

$$\text{Cr}^*(E | \bar{B}) = 1$$

Given (M), (M_{*=}), and (M^{**=}), the above equations imply:

$$\text{Cr}(BE) = \text{Cr}(E | B) \text{Cr}(B) = 2^m / 3^n \times 1/2 = 2^{m-1} / 3^n$$

$$\text{Cr}_*(\bar{B}E) = \text{Cr}_*(E | \bar{B}) \text{Cr}(\bar{B}) = 0 \times 1/2 = 0$$

$$\text{Cr}^*(\bar{B}E) = \text{Cr}^*(E | \bar{B}) \text{Cr}(\bar{B}) = 1 \times 1/2 = 1/2$$

By (A_{*=}) and (A^{*=}) (setting $A := E$, $B := \bar{B}$, $C := \top$), we get

$$\text{Cr}_*(E) = \text{Cr}(BE) + \text{Cr}_*(\bar{B}E) = 2^{m-1} / 3^n + 0 = 2^{m-1} / 3^n$$

$$\text{Cr}^*(E) = \text{Cr}(BE) + \text{Cr}^*(\bar{B}E) = 2^{m-1} / 3^n + 1/2 = (2^m + 3^n) / (2 \times 3^n)$$

Finally we can determine your conditional upper and lower credences in B given E by

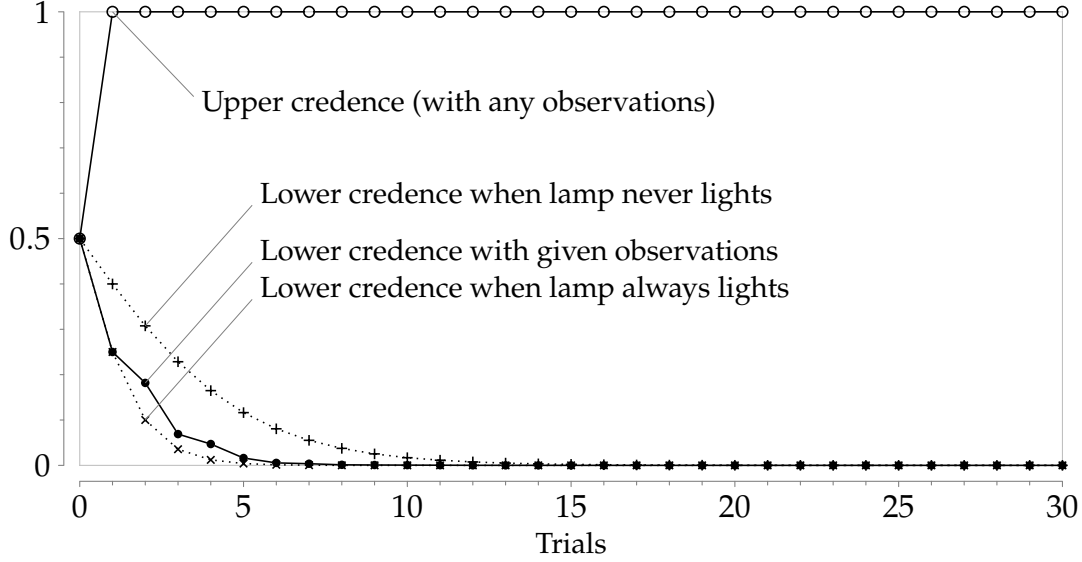


Figure 1. Evolution of your attitude to the proposition that you have the boring spinner

applying $(M=^*_*)$ and $(M=^*_*)$ (setting $A := B, B := E, C := \top$):

$$\begin{aligned} \text{Cr}_*(B | E) &= \frac{\text{Cr}(BE)}{\text{Cr}^*(E)} = \frac{2^{m-1}/3^n}{(2^m + 3^n)/(2 \times 3^n)} = \frac{2^m}{2^m + 3^n} \\ \text{Cr}^*(B | E) &= \frac{\text{Cr}(BE)}{\text{Cr}_*(E)} = \frac{2^{m-1}/3^n}{2^{m-1}/3^n} = 1 \end{aligned}$$

1 Given GENERALIZED CONDITIONALIZATION, it follows that your upper credence that you
 2 have the boring spinner will jump from $\frac{1}{2}$ to 1 after your first observation and stay at 1
 3 thereafter. Meanwhile, your lower credence that you have the boring spinner will decrease
 4 monotonically from its starting value of $\frac{1}{2}$. Figure 1 depicts what this looks like for the
 5 sequence of outcomes specified in *Two Spinners*, as well as a sequence in which the lamp
 6 comes on every time and one where it never comes on. Whatever you see, your lower
 7 credence will approach 0 as you make more observations. It will never again be $\frac{1}{2}$ or
 8 above. Given CONFIDENCE, this means that nothing you could observe would make you
 9 more confident that you have the boring spinner.

10 I conclude that you should *not*, in this case, have the credences mandated by GEN-PP.
 11 Thus PERMISSIVE GEN-PP is false.²²

²²Evolutions where one's lower credence in some proposition decreases while one's

1 3 Responses

2 This section will discuss four possible responses to the argument of §2. The first rejects
3 my interpretation of the case; the second rejects CONFIDENCE; the third rejects GENERALIZED
4 CONDITIONALIZATION; and the fourth denies the judgment that the given observations
5 should make you more confident that you have the boring spinner.

6 3.1 *Rejecting the interpretation of the case*

7 In conversation, a popular response has been to reject my interpretation of the “fairness”
8 of the spinners, on which for any $X \subseteq [0, 1)^n$ and any reference point y , the lower
9 and upper chances of X being y -selected equal the Lebesgue inner and outer measures
10 of X . It has been suggested that the lower and upper chances of a sequence of lamp
11 on/lamp off outcomes on the interesting spinner should instead be determined from
12 a representor which contains all and only the probability functions that treat the spins
13 as probabilistically independent events with some fixed chance. To see the difference
14 between the two interpretations, consider the proposition that the interesting spinner’s
15 lamp comes on after the first spin and not after the second. On my interpretation, its lower
16 and upper chances are 0 and 1. On the alternative interpretation, its lower chance is still
17 0, but its upper chance is only $\frac{1}{4}$, since among the relevant probability functions, the ones
18 that maximize its probability are those that treat the spins as independent events with
19 probability $\frac{1}{2}$.

20 So far, this proposal is rather ad hoc, since it only concerns the chances of propositions
21 about when the lamp comes on. But there is a natural way of generalizing it to arbitrary

upper credence increases are called *dilations*. White (2010) (building on Seidenfeld and Wasserman 1993) considers a case where proponents of imprecise probability counsel dilation with respect to the proposition that a coin landed heads, and deems this implausible. IHH’s reply to White strikes me as convincing. But my reasons for thinking you should not dilate in *Two Spinners* are unrelated to White’s reasons for opposing dilation in his case: his worry is entirely about the *predictability* of dilation, whereas I simply think that your lower credence should increase rather than decrease.

1 propositions about where the spinner lands after each of the first n spins. We can assign
2 lower and upper chances to all these propositions using a representor that contains all
3 and only the total (finitely additive) probability functions \Pr for which there exist a point
4 y and a finitely additive extension of Lebesgue measure λ^+ defined on $\mathcal{P}[0, 1)$, such that
5 for all $X_1, \dots, X_n \subseteq [0, 1)$, $\Pr(X_1 \times \dots \times X_n \text{ is } y\text{-selected}) = \lambda^+(X_1) \cdots \lambda^+(X_n)$.²³

6 For several reasons, this does not seem to me to be a promising line of response. First:
7 a key desideratum for IHH is that the lower and upper chances (and the corresponding
8 credences) should respect relevant symmetries of the setup. In our case, such symmetries
9 include not just *global* rotations which apply to every spin, but also *local* rotations which ap-
10 ply to only one spin. Mathematically, using $+$ to stand for addition mod 1, global rotations
11 correspond to functions on $[0, 1)^n$ that map each $\langle x_1, \dots, x_n \rangle$ to $\langle x_1 + z, \dots, x_n + z \rangle$ for some
12 z , while local rotations correspond to functions that map $\langle x_1, \dots, x_{i-1}, x_i, x_{i+1}, \dots, x_n \rangle$ to
13 $\langle x_1, \dots, x_{i-1}, x_i + z, x_{i+1}, \dots, x_n \rangle$ for some z and i . Both kinds of symmetries seem legit-
14 imate: since the spinner doesn't have a memory, in the same sense in which we are free to
15 choose any point to assign to zero when mapping from $[0, 1)$ to propositions about a single
16 spin, we should also be free to choose a different zero point each time when mapping from
17 $[0, 1)^n$ to propositions about the first n spins. But while the alternative lower and upper
18 chances are invariant under global rotations, they are not invariant under local rotations.
19 To see this, let V and W be two non-overlapping subsets of $[0, 1)$ with inner measure 0
20 and outer measure 1 such that W is the result of applying some rotation to V .²⁴ Let A
21 be the proposition that the next two spins will y -select V and \bar{V} (the complement of V),

²³Alternatively, we could limit the representor to probability functions associated in this way with total *rotation-invariant* extensions of Lebesgue measure. Note that either way, the proposal will require giving up the inner-outer analysis of imprecise chance. For it implies that for $X \subseteq [0, 1)^n$, X has a real-valued chance of being y -selected only if X is Lebesgue-measurable, so that inner and outer chances still correspond to inner and outer Lebesgue measures in $[0, 1)^n$, even though lower and upper chances do not. For example, the *outer* chance of *lamp on, then lamp off* is 1, although its *upper* chance is $\frac{1}{4}$.

²⁴For example, V could be a Vitali set with outer measure 1 (see IHH, §2) and W its rotation through any nonzero rational angle.

1 and let B be the proposition that the next two spins will y -select V and \overline{W} : B is thus the
2 image of A under a local rotation. A 's upper chance is $\frac{1}{4}$, since among the probability
3 functions in the representor, its probability is maximized by those associated with λ^+ s for
4 which $\lambda^+(V) = \lambda^+(\overline{W}) = \frac{1}{2}$. B 's upper chance, however, is 1. For since V 's outer measure is
5 1, the representor includes probability functions \Pr whose associated measures
6 λ^+ have $\lambda^+(V) = 1$, in which case also $\lambda^+(\overline{W}) = 1$ (because $V \subseteq \overline{W}$), and thus $\Pr(V \times \overline{W}$ is
7 y -selected) = 1.

8 Second: insofar as the concepts of lower and upper chance are in good standing, we
9 should be able to simply elaborate the case to have God explicitly tell you that the lower
10 and upper chances work in accordance with my interpretation. Or if we find this too
11 theory-laden, we could specify that if the coin lands heads, God will supply you with a
12 whole series of interesting spinners, each of which you get to spin just once. No two of
13 these spinners are duplicates, although the set of blessed points on each of them has inner
14 Lebesgue measure 0 and outer Lebesgue measure 1. In this case, there are no grounds
15 whatsoever for limiting the chance-representor in the relevant way—for all you know, the
16 first two spinners could be alignable in such a way that a point on the second spinner is
17 blessed if and only if its counterpart on the first spinner is unblessed. But these variations
18 do little to diminish the intuitive plausibility of the judgment that the given sequence of
19 observations should make you more confident that you have the boring spinner.

20 Third: the argument against PERMISSIVE GEN-PP can anyway be made using the al-
21 ternative interpretation! Consider your lower and upper credences after you have made
22 29 spins and seen the lamp come on ten times. The members of the proposed chance-
23 representor that maximize the probability of *that* outcome are the ones that treat spins
24 of the interesting spinner as independent events with probability $\frac{10}{29}$. These probability
25 functions treat the observed outcomes as slightly favouring the hypothesis that you have
26 the interesting spinner. Thus after 29 spins, your lower credence that you have the bor-
27 ing spinner will be slightly below $\frac{1}{2}$. Given CONFIDENCE, this rules out your being more

1 confident that you have the boring spinner.

2 3.2 *Rejecting CONFIDENCE*

3 Another possible response involves accepting the judgment that you should get more
4 confident that you have the boring spinner, while also accepting that your lower and
5 upper credences should evolve in the way depicted in Figure 1. This requires rejecting
6 CONFIDENCE.

7 Some discussions of the relation between imprecise credence and comparisons of
8 confidence suggest independent motivations for giving up CONFIDENCE. Some authors
9 (van Fraassen 1990, Joyce 2010, Rinard 2013) posit a close connection between imprecise
10 credence and *vagueness* in words like ‘confident’. According to Rinard, for example, ‘An
11 agent is determinately more confident of P than Q if and only if every function $[Pr]$
12 in her representor has $Pr(P) > Pr(Q)$ ’ (Rinard 2013). Since ‘more confident’ is vague,
13 proponents of this biconditional should not also accept the corresponding biconditional
14 without the ‘determinately’. And insofar as one rejects that biconditional, the motivation
15 for CONFIDENCE within the representor approach looks shaky. If one does not rule out an
16 agent’s being more confident in A than B while having a representor containing probability
17 functions for which $Pr(A) \leq Pr(B)$, why would one rule it out in the special case where
18 the agent has a higher lower credence in B than in A ?

19 Rinard’s biconditional does not fit naturally with the view we have been assuming
20 (following IHH), on which being credally imprecise with respect to a proposition is incon-
21 sistent with there being a real number that is one’s credence in it. It seems more suited
22 to a view on which credal imprecision is only inconsistent with there being a unique real
23 number that *determinately* is one’s credence in the relevant proposition. Indeed, it even
24 suggests that such imprecision might in fact entail that there *is* a unique real number that
25 is one’s credence in that proposition.²⁵ That alternative view weakens the prima facie case

²⁵The latter claim would hold if the vagueness view is cashed out in the following way: for any partial function Pr from propositions to reals, Pr is in one’s representor iff it is not

1 for GEN-PP, since it suggests that the original Principal Principle does not in fact fall silent
2 about the relevant cases. But it doesn't really matter. For the vagueness-theoretic worries
3 about CONFIDENCE do not get a grip if we weaken that principle by adding 'determinately'
4 to its antecedent, yielding:

5 WEAK CONFIDENCE If x is *determinately* more confident at t_1 that A than y is at t_2
6 that B , then x 's lower credence at t_1 in A is not less than y 's lower credence at t_2
7 in B .

8 This is enough for a variant of our argument against PERMISSIVE GEN-PP, since your obser-
9 vations in *Two Spinners* should plausibly leave you not only more confident, but *determin-*
10 *ately* more confident, that you have the boring spinner. A response that accepts that you
11 should be more confident while denying that you should be determinately more confident
12 seems quite bizarre and unpalatable, given any reasonable foundational account of the
13 phenomenon of vagueness.

14 Giving up even WEAK CONFIDENCE would require a radical rethinking of the aspirations
15 of the theory of imprecise credence. Bayesian epistemology has traditionally been centrally
16 concerned with questions about confidence, e.g. whether certain observations should
17 make you more or less confident in certain hypotheses. If WEAK CONFIDENCE is rejected,
18 there seems to be nothing left to connect claims about how observations should affect
19 your imprecise credal state with these classic questions. If we care about the confidence-
20 theoretic questions, we will need some separate theory to answer them; it is unclear why
21 we would even be interested in the theory of credal imprecision.

22 The strategy of rejecting CONFIDENCE (especially if it also involves rejecting WEAK CON-
23 FIDENCE) tends moreover to exacerbate the challenge of explaining what it *is* to be credally
24 imprecise, since it undercuts various explanations that turn on the familiar notion of

determinately false that (for all A and x , one has credence x in A iff $\Pr(A) = x$). One could also argue for the claim from the premise that for all \Pr , \Pr is in one's representor iff it is not determinately false that: for all A and B , $\Pr(A) \leq \Pr(B)$ iff one is at least as confident in B as in A . (See the discussion of "Credence Existence" in Dorr, Nebel, and Zuehl 2021.)

1 confidence. Of course, there are approaches to this challenge that do not appeal to that
2 notion, some of which seem *prima facie* friendly to failures of CONFIDENCE. For example,
3 the time-honoured idea that credence can be understood in terms of *betting dispositions*
4 has been developed by Walley (1991) into a versatile account of imprecise credence, where
5 differences in lower and upper credence boil down to differences between the most one
6 would pay to buy a certain gamble and the least one would accept to sell it.²⁶ Another time-
7 honoured idea identifies credences with certain all-or-nothing *beliefs* or *judgments* about
8 probability (in some broadly epistemic sense of ‘probability’); this can also be extended to
9 an account of upper and lower credence either in terms of less-than-maximally-specific be-
10 liefs about the original notion of probability, or specific beliefs about some independently
11 understood notion of imprecise probability, or both.²⁷ Both of these ideas look well-suited
12 to making sense of failures of CONFIDENCE, and maybe even WEAK CONFIDENCE. There is no
13 *obvious* reason why some evidence could not render one (determinately) more confident
14 in a while simultaneously decreasing the maximum one would pay for a one-dollar bet
15 on *A*, or making one less opinionated about its epistemic probability, or even causing one
16 to believe that its lower epistemic probability has decreased. But neither idea seems to
17 do much to make the evolution prescribed by GEN-PP in *Two Spinners* more palatable.
18 For after your observations, it seems intuitively reasonable for you to be disposed to pay
19 considerably more than 50 cents for a bet that pays \$1 if you have the boring spinner, and
20 for you to sincerely make speeches like ‘Probably, this is the boring spinner’ or ‘I think this
21 is likely to be the boring spinner’ (which plausibly express beliefs that the all-or-nothing

²⁶Mahtani (2016) suggests a different account of credal imprecision in terms of betting behaviour, on which it is characterized by a kind of *instability* in one’s betting dispositions.

²⁷IHH (§6) suggest that having lower credence x and upper credence y in a proposition might ‘amount to the belief that it has inner probability x and outer probability y ’. One might alternatively identify this state with being such that x is the infimum of $\{x' \mid \text{one believes that } A\text{'s lower epistemic probability is at least } x'\}$ and y the supremum of $\{y' \mid \text{one believes that } A\text{'s upper epistemic probability is at most } y'\}$. This will make it possible to have lower and upper credences in a proposition without being fully opinionated about its lower and upper epistemic probabilities, and even while believing them to be identical.

1 theorist would take to be inconsistent with having lower credence less than $\frac{1}{2}$). The chal-
 2 lenge to PERMISSIVE GEN-PP can thus be raised not just from pre-theoretic judgments about
 3 *confidence*, but from pre-theoretic judgments about many other topics which one might
 4 wish to draw on in an explanation of credal imprecision.

5 3.3 *Rejecting GENERALIZED CONDITIONALIZATION*

6 A third possible response to the argument is to give up on GENERALIZED CONDITIONALIZ-
 7 ATION. There is some precedent for this in the literature on imprecise credence. Several
 8 authors (Wilson 2001, Cattaneo 2008, Bradley 2022) propose that when you gain new
 9 evidence E , your new representor should be derived from your old one by first throwing
 10 away those probability functions that gave E especially low probabilities, and then con-
 11 ditionalizing each remaining probability function on E . This can easily lead to failures of
 12 GENERALIZED CONDITIONALIZATION.²⁸

13 However, we can specify favourable circumstances where the “throwing away” pro-
 14 cedure these authors favour will agree with GENERALIZED CONDITIONALIZATION as regards
 15 the lower or upper credence of some proposition. Suppose you have a real-valued cre-
 16 dence x in HE : i.e., $\Pr(HE) = x$ for every \Pr in your representor. Then the members \Pr of
 17 your representor that maximize $\Pr(E)$ are exactly those that maximize $\Pr(\overline{HE})$, and hence

²⁸For example, suppose that you are to successively draw two balls from an urn. Each ball can be red or black, so there are four possibilities: R_1R_2 , R_1B_2 , B_1R_2 , and B_1B_2 . Your initial representor contains three probability functions:

	R_1R_2	R_1B_2	B_1R_2	B_1B_2
Pr ₁	0.25	0.25	0.25	0.25
Pr ₂	0.01	0	0	0.99
Pr ₃	0	0.01	0.99	0

Thus your lower credence in R_2 conditional on R_1 is 0 (thanks to Pr₃) and your upper credence is 1 (thanks to Pr₂). But assuming the 0.01 Pr₂ and Pr₃ assign to R_1 counts as “extreme” enough (relative to the 0.5 assigned by Pr₁) for them to be discarded after you observe R_1 , this observation will leave you with the singleton representor $\{\Pr_1(\cdot | R_1)\}$, and thus with credence 0.5 in R_2 .

1 also maximize $\Pr(\overline{H} | E)$, i.e. $\Pr(\overline{H}E)/(\Pr(\overline{H}E) + x)$.²⁹ Since these \Pr will not be discarded
2 after you learn E , we have the following:

3 WEAK GENERALIZED CONDITIONALIZATION Suppose you are rational and acquire
4 total evidence E between t and t^+ , and have a real-valued credence in HE at
5 t . Then your upper credence at t^+ in \overline{H} equals your upper credence at t in \overline{H}
6 conditional on E , and your lower credence at t^+ in H equals your lower credence
7 at t in H conditional on E .

8 This principle is strong enough to substitute for GENERALIZED CONDITIONALIZATION in our
9 argument. Taking H to be the proposition that you have the boring spinner and E to be
10 some sequence of observed outcomes, the hypothesis of WEAK GENERALIZED CONDITION-
11 ALIZATION is satisfied; so we can conclude that your lower credence that you have the
12 boring spinner diminishes as depicted in Figure 1. This is enough, given CONFIDENCE, to
13 rule out your becoming more confident. The “throwing away” procedure just blocks the
14 argument that your *upper* credence goes to 1. Indeed, on natural precisifications of the
15 view, sufficiently many observations will leave your upper credence *also* far below $\frac{1}{2}$, as
16 the only probability functions left in your representor will be descendants of probability
17 functions that assigned high probability to the interesting spinner generating exactly the
18 outcomes you observed. This plausibly implies that you will end up *less* confident that
19 you have the boring spinner. Thus, far from helping with our example, the strategy of
20 ‘learning by ignoring the most wrong’ (Bradley 2022) actually makes the problem worse.

21 Of course, this strategy is not the only possible way to give up GENERALIZED CONDITION-
22 ALIZATION, and some might hope to handle the case by developing some novel alternative
23 updating method.³⁰ But the difficulties facing this project should not be underestimated.

²⁹Since for positive x, y, z , $\frac{y}{x+y} \leq \frac{z}{x+z}$ iff $xy + yz \leq xz + yz$, iff $xy \leq xz$, iff $y \leq z$, iff $x + y \leq x + z$.

³⁰Weatherson (2007) defends a “dynamic Keynesianism” on which, as on the “throwing away” view discussed above, one should sometimes discard some members of the representor before conditionalizing the remainder on the evidence. On Weatherson’s view,

1 True, the most widely-discussed arguments for conditionalization—Dutch book argu-
2 ments (Lewis 1999) and arguments relating to the value of accuracy (Greaves and Wallace
3 2006)—are tricky to generalize to imprecise credence, due to a lack of consensus about the
4 decision-theoretic upshot of imprecise credences and about how to measure their accur-
5 acy.³¹ But quite apart from these arguments, there is also a simple normative intuition that
6 supports both ordinary conditionalization and its imprecise generalization: namely, that
7 when you are trying to gather evidence to compare two hypotheses H_1 and H_2 , you will
8 need evidence that at least one of the two fails to entail. If what you learn is entailed by
9 both hypotheses, it is irrelevant to the comparison. This strikes me as a deeply plausible
10 thought even if it is not backed up by any further argument. And it implies GENERALIZED
11 CONDITIONALIZATION, assuming ‘You learnt nothing relevant to the comparison between
12 H_1 and H_2 ’ implies ‘Your lower and upper credences in H_1 and H_2 conditional on their
13 disjunction should not change’.³²

14 3.4 *Biting the bullet*

15 A final response is to reject the judgment that you should respond to your observations by
16 becoming more confident that you got the boring spinner. This might be tolerable if the
17 surprise could be limited to far-fetched thought experiments. Unfortunately, it generalizes

however, the functions to be discarded are identified not by the low probability they assign the evidence, but by some mysterious factors for which he provides no formal model. Such changes are supposed to represent a special phenomenon of “learning about fundamental evidential relationships”. But it is not promising to attribute the failure of GENERALIZED CONDITIONALIZATION in *Two Spinners* to this supposedly special kind of learning. We can imagine our agent as already equipped with a detailed take on evidential relationships, formed during a long career of hypothesis-testing.

³¹Although Walley (1991: ch. 6), defining imprecise credence in terms of betting dispositions, is able to derive GENERALIZED CONDITIONALIZATION using a kind of Dutch Book argument.

³²By setting $H_1 = AE$ and $H_2 = E$, we can infer that your lower and upper conditional credences in A given E should not change when you learn E . But since we assume that learning E involves giving real-valued credence 1 to E , we know (by $(M_{**}=)$, $(M^{**}=)$, $(A_{**}=)$, and $(A^{**}=)$) that after learning E , your lower and upper unconditional credences in A equal your lower and upper credences in A given E .

1 in a disastrous way to many real-world inquiries.

2 To see the issue, note first that our analysis of *Two Spinners* wouldn't differ much if the
3 choice of spinner had been based on a heavily biased coin rather than a fair one. Wherever
4 you start out, your observations will cause your lower credence that you have the boring
5 spinner to approach 0 just as fast as your *credence* would have approached 0 if you had
6 initially been certain that the interesting spinner would produce exactly the outcomes you
7 observed. This is very fast! For example, even if you initially had credence 0.9999999 that
8 you had the boring spinner, any series of 30 observations will leave your lower credence
9 less than $\frac{1}{3}$. But this version of the case is quite similar to what we face whenever we try
10 to investigate the laws governing some perhaps-indeterministic kind of physical system,
11 such as the decay of free neutrons. If the concepts of lower and upper chance are in
12 good standing, it seems unreasonably dogmatic to begin our inquiry with credence *zero* in
13 hypotheses according to which the laws governing the relevant physical systems involve
14 imprecise chances analogous to those generated by the interesting spinner.³³ But so long
15 as we are open-minded enough to assign a positive upper prior credence to some such
16 hypothesis, GEN-PP and GENERALIZED CONDITIONALIZATION imply that any observations we
17 might make should cause our upper credences in it to rapidly approach 1, so that our
18 lower credences in all competing hypotheses must correspondingly approach 0.³⁴ Given
19 CONFIDENCE, this means that if we make sufficiently many observations of the systems in
20 question, then no matter what we see, we will be no more confident in any hypothesis
21 about how the chances work for such systems than we were initially. This would be a

³³Indeed, the hypothesis that free neutrons have lower chance 0 and upper chance 1 of decaying within any finite interval of time seems rather simple and elegant; some might find it more plausible *a priori* than the currently accepted hypothesis that their chance of decaying is approximately 0.5 per 611 seconds.

³⁴The phenomenon is not limited to hypotheses that ascribe lower and upper chances of 0 or 1. The result that a long enough sequence of observations will cause one's lower and upper credences that one has the boring spinner to approach 0 and 1 respectively still holds if the set of blessed points on the interesting spinner has inner measure x and outer measure y and the set of blessed points on the boring spinner has measure z , with $x < z < y$.

1 disaster for all scientific inquiry.³⁵

2 One precedent for the bullet-biting response to *Two Spinners* is the reaction of Joyce
3 (2010) to a similar argument—the inspiration for mine—against his thesis that credal
4 imprecision is sometimes rationally *compulsory*. Joyce considers a case where you are
5 given one of two coins chosen at random. You know the first coin is fair, but have “no
6 information” about the bias of the second coin. He expresses sympathy for the claim that
7 your representor should contain probability functions treating the second coin as having
8 every bias between 0 and 1. He moreover accepts a form of representor conditionalization
9 which implies that if you begin with such a representor, no sequence of observations
10 should increase your lower credence that you have the first coin. Joyce suggests that the
11 counterintuitiveness of this prescription can be defused by adopting a decision theory
12 on which even persistent, extreme imprecision in one’s credences need not show up in
13 any distinctive patterns of *action*. For example, one might allow a process of “pragmatic
14 sharpening” in which one proceeds *as if* one’s representor were smaller than it actually is.
15 IHH might, analogously, suggest that after your observations in *Two Spinners*, you should
16 *act as if* you had become more confident that you have the boring spinner, without in fact
17 becoming more confident.³⁶

18 However, it is hard to imagine what sort of normative decision theory could explain the
19 practical requirements required by this strategy. Since Joyce was only trying to “explain
20 away” the appeal of an initially plausible *permissibility* claim, his purposes would be served

³⁵As Hoek (2021: §2) points out, ordinary scientific reasoning seems to assume the existence of real-valued chances, so it might be hard for a view on which imprecise chances deserve nonzero prior credence to be *entirely* conservative with respect to standard scientific practice. But once we give up PERMISSIVE GEN-PP, I see no obstacle to the view that given our actual evidence, we should have negligible credence that there is any significant imprecision in the dynamical chances.

³⁶Such invocations of pragmatic sharpening will exacerbate the foundational worries about credal imprecision. Behaviourists certainly won’t like the idea that credal imprecision is compatible with being disposed to act just like someone with a real-valued credence. And even non-behaviourists may feel that pervasive pragmatic sharpening weakens their grip on the concept of credal imprecision.

1 by a view on which arbitrary pragmatic sharpenings are always permissible. By contrast,
2 the analogous strategy for “explaining away” an initially plausible ‘should’ claim like mine
3 will require a view on which some of the possible pragmatic sharpenings are *compulsory*,
4 while others (e.g., acting as if you always had a high credence that the interesting spinner
5 would generate the outcomes you observed) are impermissible. I see no basis for such a
6 discriminating requirement.

7 And even if the practical claim that you should act as if you had become more confident
8 could be integrated into a reasonable decision theory, it would still not be an adequate
9 substitute for the claim that you should in fact become more confident. Our real-world
10 scientific cases make this clear. Surely, for example, there are detailed observations that
11 should really leave us confident that neutron-decay involves certain fairly specific chances,
12 not just disposed to act as if we were confident. Science is about finding out what the
13 world is like, not just putting on a brave face!

14 **4 Replacing the Generalized Principal Principle**

15 I have argued that PERMISSIVE GEN-PP is false. Since anything rationally obligatory is
16 rationally permissible, it follows that GEN-PP is also false. But I don’t want to go so
17 far as to give up the original Principal Principle (PP), so I need to say something about
18 IHH’s argument that proponents of PP should also accept GEN-PP. Their thought is that
19 assuming imprecise chance is possible, there clearly needs to be *some* generalization of
20 PP that doesn’t just fall silent when applied to hypotheses that ascribe imprecise chances,
21 and approximates PP when there is only a little imprecision. For example, the hypothesis
22 that *A* has lower chance 0.49 and upper chance 0.51 should presumably constrain one’s
23 attitude towards *A* in similar ways to the hypothesis that it has real-valued chance 0.5.

24 I agree that some such generalization is called for. But I suggest that the following
25 weaker principle, which simply replaces the equalities in GEN-PP with inequalities, can
26 do the job:

WEAK GEN-PP If Cr_* and Cr^* are a rational pair of a lower and upper prior credence function, and E is admissible at t , then for any proposition A and $x \in [0, 1]$:

$$Cr_*(A | E \wedge Ch_{*t}(A) = x) \geq x$$

$$Cr^*(A | E \wedge Ch^*_t(A) = x) \leq x$$

1 so long as these are defined.

2 When you are certain of the lower and upper chances, WEAK GEN-PP requires your lower
3 and upper credences to be within the interval they span, but allows that they might be
4 closer together than the known lower and upper chances, or even equal. This makes it
5 useless for the purposes of arguing for the rational permissibility of imprecise credence.
6 Nevertheless, like GEN-PP, WEAK GEN-PP reduces to PP in the special case where $x = y$.

7 5 Symmetry

8 Besides the argument from PERMISSIVE GEN-PP I have been focusing on, IHH have an al-
9 ternative argument that could be used to support the permissibility of imprecise credences
10 in *Strange Spinner*, even if one accepted the impermissibility of the extreme imprecise cre-
11 dences mandated by GEN-PP. To run the alternative argument, we will need to elaborate
12 the story by having God also inform you that the set of blessed points is a *Vitali set*: one
13 that does not overlap the result of rotating it by any rational angle, and for which the
14 union of all such rational rotations is the whole circle.³⁷ The key premise is that it is per-
15 missible for your beliefs about the next spin to obey both *countable additivity* and *rotational*
16 *symmetry*. Countable additivity means that when you have a real-valued credence in each
17 of countably many pairwise inconsistent propositions, your credence in their disjunction

³⁷See IHH, §2. A Vitali set must have inner Lebesgue measure 0, but can have any outer Lebesgue measure in $[0, 1]$. Note that the complement of a Vitali set cannot be a Vitali set.

1 is the sum of your credences in them.³⁸ Rotational symmetry means that for every angle x ,
2 your lower credence that *the spinner will land on a blessed point* equals your lower credence
3 that *the spinner will land on a point x clockwise from a blessed point*. Conditional on the set of
4 blessed points being a Vitali set, you are certain there is exactly one *rational x* such that *the*
5 *spinner will land on a point x clockwise from a blessed point* is true. Given rotational symmetry,
6 all of these propositions must receive the same lower and upper (conditional) credence.
7 Since they are pairwise incompatible, this means they must all have lower credence 0.
8 But since their disjunction gets upper credence 1 and there are only countably many of
9 them, countable additivity implies that their upper credence must be positive. Of course,
10 if you assign each of them upper credence 1, you will be in the same state mandated by
11 GEN-PP, which I already argued to be impermissible. However, if you are willing to jet-
12 tison PERMISSIVE GEN-PP and can live with the arbitrariness involved, you could still obey
13 rotation-invariance and countable additivity by assigning each of them the same upper
14 credence strictly between 0 and 1.

15 But this less-extreme imprecise credential state doesn't really avoid the problem, since
16 our argument against the permissibility of the imprecise credences mandated by GEN-PP
17 can be adapted to target it. For any positive upper credence you might adopt, we can
18 consider a variant of *Two Spinners* where the measure of the set of blessed points on the
19 boring spinner is positive, but less than that number. An extension of the reasoning
20 in §2 will then require your lower credence that you have this new boring spinner to
21 approach 0, whatever observations you make. So given CONFIDENCE, we again have
22 the very implausible consequence that no possible observations could make you more
23 confident that you initially were.

24 This leaves us with a choice between rejecting (the compulsoriness of) countable ad-

³⁸In the imprecise setting, it is natural to derive countable additivity for credence from a requirement of countable *subadditivity* for upper credence, generalizing A^{**} : your upper credence in a countable disjunction can never exceed the sum of your upper credences in the disjuncts.

1 ditivity and rejecting (the permissibility of) rotational symmetry. As it happens, I op-
2 pose countable additivity for entirely independent reasons (see Arntzenius, Elga, and
3 Hawthorne 2004, Arntzenius and Dorr 2017).³⁹ But (as IHH note) we can modify the case
4 to avoid having to rely on countable additivity, by replacing our spinners with fancier
5 devices that choose points on the surface of a sphere rather than on the circumference
6 of a circle. By the Banach-Tarski theorem, even a *finitely* additive probability function
7 that assigns a probability to every set of points on a sphere cannot be invariant under all
8 rotations of the sphere. So in this variant of the case, we are forced to deny the rational
9 permissibility of having a rotation-invariant credential state.

10 But this does not seem so terrible: without PERMISSIVE GEN-PP to back it up, the theoret-
11 ical case for rotation-invariance does not look so strong. In the cases where considerations
12 of symmetry seem to carry the most epistemological weight, the propositions related by a
13 symmetry are also alike in respect of their *simplicity* (or naturalness), which already speaks
14 in favour of assigning them equal credence. For example, the judgment that one should
15 assign the same prior credence to the propositions that a coin will land heads more often
16 than tails and that it will land tails more often than heads seems intimately connected
17 with the fact that these two propositions are equally simple. But the proposition that the
18 interesting spinner will land on a blessed point is intuitively much simpler than, say, the
19 proposition that it will land on a point exactly $17\frac{9}{44}$ degrees clockwise from a blessed
20 point. (Correspondingly, *being a blessed point* is plausibly a more natural property than
21 *being exactly $17\frac{9}{44}$ degrees clockwise from a blessed point*.) A theory that mandates or permits

³⁹Rejecting countable additivity also takes care of another argument of IHH's which appeals to Ulam's theorem, according which there is no countably additive probability measure defined on the powerset of \aleph_1 that vanishes on singletons, and hence, if the continuum hypothesis is true, no such measure defined on the set of all propositions about where the spinner will land the first time it is spun. But even if one liked countable additivity and the continuum hypothesis, one could respond to this argument by denying that it is permissible even to *have* a lower and upper credence in every proposition about where the spinner will land. This seems defensible; indeed one might argue that it is not even possible for finite creatures like us to be in such an opinionated state.

1 treating these two propositions on a par is thus making a bold and controversial step
2 beyond the kinds of cases that initially suggested a role for the concept of symmetry in
3 the theory of rationality.

4 Could we shore up the case for rotation-invariance by switching to a pared-down
5 case that eliminates the spinner’s qualitative asymmetries (the differences with respect to
6 “blessedness” and the associated disposition to make the lamp come on)? The problem is
7 that if you’re just presented with a pointer attached to a featureless disc, it is unclear how
8 you could even *entertain*, or take any other attitude to, the proposition that the pointer will
9 end up in some specific non-measurable set of points. Once we remove the possibility of
10 exploiting God’s decision to confer a qualitative distinction on one such set, it is arguable
11 that you cannot assign any lower or upper credence to any such proposition.⁴⁰ And so
12 long as you do not, your doxastic state can still be rotationally invariant even if your lower
13 and upper credences never diverge.

14 **6 The aversion to arbitrariness**

15 As we noted at the outset, the idea of adopting maximally imprecise credences in *Strange*
16 *Spinner* is attractive, in part, because every alternative seems so *arbitrary*. In fact the appeal
17 of the argument from rotation-invariance seems to be based on arbitrariness-aversion:
18 if one’s credential state fails to be rotation-invariant, it is especially hard to imagine a
19 principled reason to favour it over any of its rotations. More generally, the idea that there
20 should be something principled to say in defence of every feature of our belief state has
21 been a major impetus behind the view that imprecise credence is sometimes mandatory.
22 Some such thought is arguably in the background when philosophers like Joyce (2010) and
23 Sturgeon (2010) maintain that the evidence we have to go on is so often so “incomplete,
24 imprecise or equivocal” (Joyce 2010: 283) that imprecise credence is the only rational

⁴⁰Proponents of the inner-outer analysis will not agree, since according to them every agent must have a lower and an upper credence in every proposition. Their view makes it easy to defend the permissibility of imprecise credence, but it is itself very controversial.

1 response to it. But even setting aside the arguments of the present paper, the demand for
2 a principled, non-arbitrary basis for each feature of one's doxastic state is independently
3 suspect, since it can so readily collapse into absurd forms of scepticism. Even proponents
4 of imprecise credence must grant that in many everyday cases, people manage to form
5 reasonable non-extreme lower or upper credences despite having no inkling of any first
6 principles from which one could derive that those are the right credences to have.⁴¹

7 Perhaps the demand for non-arbitrariness can be understood in a more relaxed way
8 that is content with some sort of reflective equilibrium, rather than a derivation from first
9 principles. This might still be thought to support extreme imprecision in *Strange Spinner*,
10 since it is hard to muster any non-extreme pre-theoretic intuitions about what your lower
11 and upper credences that the lamp will come on should be. But our central argument
12 has uncovered a different source of pre-theoretic judgments that can be drawn on during
13 the progress towards reflective equilibrium about this case, namely, judgments about
14 which sequences of observations would and would not make it reasonable to become
15 confident that one had this or that sort of boring spinner. We do seem to be in a position
16 to make some (albeit piecemeal) intuitive judgments about this. For example, I judged
17 that you should be *pretty confident* that you have the boring spinner after the given series
18 of 30 observations; I take this to imply that your lower credence should be well above $\frac{1}{2}$.
19 Given such judgments about how certain observations would bear on the hypothesis that
20 you have the interesting spinner, we can apply Bayes's theorem in reverse to make some
21 progress with the question what your credences conditional on that hypothesis should

⁴¹This would be denied by Sturgeon (2020), who maintains that ordinary people almost never have lower or upper credences on similar grounds to those on which many fans of imprecise credence maintain that ordinary people almost never have real-valued credences. Rinard (2013), similarly, thinks that most ascriptions of interval-valued credences should be rejected as involving a 'false precision'. But these arguments seem to turn on the kind of "no sharp boundaries" thought that cannot be endorsed by those of us who hold on to theorems of classical logic even in the face of the Sorites. (Indeed, Rinard's "false precision" argument seems pretty directly to require giving up the Law of Excluded Middle). The task of developing an analogue of (precise or imprecise) probability theory within non-classical logic raises challenges beyond the scope of this paper.

1 look like. Of course, these judgments aren't very specific and fall silent in many harder
2 cases. But in this respect, they are no different from our intuitive judgments about other
3 cases of induction. These suggest (for example) that it is reasonable to assign higher prior
4 credence to simpler hypotheses, but do not take us very far towards a theory about what
5 the relevant notion of simplicity amounts to, or how an a priori bias towards simplicity
6 should be implemented quantitatively. The arbitrariness worries raised by propositions
7 about unmeasurable regions thus do not, ultimately, look importantly different from those
8 that arise in connection with run-of-the-mill inductive reasoning.

9 **7 Conclusion**

10 I conclude that IHH's arguments for the rational permissibility of imprecise credence
11 rest on unconvincing premises. PERMISSIVE GEN-PP does not seem plausible once we trace
12 through its consequences for cases like *Two Spinners*. The premise that it is permissible to be
13 rotation-invariant is subject to similar objections, and is moreover not well-motivated apart
14 from PERMISSIVE GEN-PP. In considering these arguments, we have learnt something about
15 the force of the anti-arbitrariness considerations which drive the other main arguments
16 for imprecise credence in the literature. This helps clear the way for arguments against
17 the permissibility (e.g. White 2010, Elga 2010) or even possibility (e.g. Dorr, Nebel, and
18 Zuehl 2021) of imprecise credence.

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