Philos Stud
DOI 10.1007/s11098-011-9833-z

Grades of individuality. A pluralistic view of identity in quantum mechanics and in the sciences

Mauro Dorato · Matteo Morganti

Abstract This paper offers a critical assessment of the current state of the debate about the identity and individuality of material objects. Its main aim, in particular, is to show that, in a sense to be carefully specified, the opposition between the Leibnizian 'reductionist' tradition, based on discernibility, and the sort of 'primitivism' that denies that facts of identity and individuality must be analysable has become outdated. In particular, it is argued that—contrary to a widespread consensus—'naturalised' metaphysics supports both the acceptability of non-qualitatively grounded (both 'contextual' and intrinsic) identity and a pluralistic approach to individuality and individuation. A case study is offered that focuses on non-relativistic quantum mechanics, in the context of which primitivism about identity and individuality, rather than being regarded as unscientific, is on the contrary suggested to be preferable to the complicated forms of reductionism that have recently been proposed. More generally, by assuming a plausible form of anti-reductionism about scientific theories and domains, it is claimed that science can be regarded as compatible with, or even as suggesting, the existence of a series of equally plausible *grades of individuality*. The kind of individuality that prevails in a certain context and at a given level can be ascertained only on the basis of the specific scientific theory at hand.

Keywords Individuality · Discernibility · Identity · Reductionism · Primitivism · Contextualism · Quantum mechanics

M. Dorato · M. Morganti (✉)
Department of Philosophy, University of Rome 'RomaTre', Via Ostiense, 234, 00144 Rome, Italy
e-mail: mmorganti@uniroma3.it

Published online: 16 November 2011

1 Introduction

The aim of the present paper is to discuss philosophical perspectives on identity and individuality for material objects, and argue that—rather than being in an irreducible mutual opposition—the originally Scholastic *primitivism* (i.e., the view that individuality is intrinsic and irreducible) and the originally Leibnizian *reductionism* (i.e., the view that individuality reduces to uniqueness of properties) in fact give rise to a spectrum of positions. Such positions can (and in fact do) peacefully coexist, as each one of them may be (and in fact is) more appropriate than the others in a specific domain of application, and for specific ways of describing that domain. In arguing for this view, we devote special attention to non-relativistic quantum mechanics, with respect to which we offer reasons for believing, *contra* a widespread opinion, that it describes a domain of primitively individuated objects. More generally, we raise doubts about the view that some form of reductionism is obviously to be preferred from a naturalistic perspective that aims to supply metaphysical claims with a solid scientific basis. At the same time, we argue in favour of a 'pluralistic' approach to the issue of the identity and individuality of material objects that, we think, is more in keeping with science than any unyielding 'monistic' alternative (be it of the primitivist or of the reductionist kind).

We begin with a brief sketch of primitivism and reductionism, and of the key role played by the Principle of the Identity of the Indiscernibles in reductionist contexts (Sect. 1). In Sect. 2, we present progressively weaker versions of reductionism. In Sect. 3 we critically assess the current popularity of Leibnizian reductionism among philosophers of science, concluding that there is no real reason for scientifically informed philosophers to be Leibnizian reductionists. In Sect. 4 we argue in favour of primitivism in more detail, this time by questioning the philosophical basis for the *non-Leibnizian* sort of reductionism recently defended by Stachel, Ladyman and other 'structuralists'. As a specific case-study, in Sect. 5 we look at non-relativistic quantum mechanics. We suggest that, as a matter of fact, a careful consideration of recent arguments in favour of the 'weak' discernibility of quantum particles lends support to the claim that mere numerical difference (countability) is both epistemically *and ontologically* prior to the putative qualitative difference that 'neo-Leibnizian' strategies insist so much on. In Sect. 6, we conclude more generally in favour of a pluralistic and gradualistic view of individuality, moving from absolute discernibility grounded in monadic properties at one end of the spectrum, to a view of individuality as based on mere (non-contextual) numerical difference and countability at the other end. Such a pluralistic stance, we suggest, requires that we attribute to things the form of individuality that can be most straightforwardly extracted from the relevant scientific description. This means that: (a) In each case in which things are individuals, they are *primarily* individuals in one specific sense directly suggested by science; and (b) Given a plausible anti-reductionism about scientific theories and their 'levels' of application, there is no reason for thinking that individuality is given in the same form in all cases, as philosophers have instead tended to think.

2 Definitions of individuality, and the received opposition

To put it roughly, *individuality* consists in the possession of determinate self-identity and numerical distinctness from other things. It has often been argued that this fundamental aspect of things can be analysed in terms of some other, more 'down-to-earth' concept. The most prominent among such reductionist views is doubtlessly the view according to which the individuality of an entity supervenes on the entity's qualities. The idea is that *something is an individual if and only if its qualitative characteristics are not the same as those of any other entity*. Clearly, according to this approach, individuality is a *derivative* concept, and talk of individuality could in principle be entirely replaced with talk involving solely the qualitative features of things. The alternative to this is to regard individuality as primitive and non-reducible. The ensuing, indeed traditional, dichotomy is thus between:

a) The view that the world is, at root, entirely constituted by *qualitative facts* (i.e., facts other than those concerning identity and number), and individuality is consequently reducible to (qualitative) properties;
b) The view that the individuality of things is something over and above their qualitative aspects, so that there can be brute (primitive, ungrounded) metaphysical facts of self-identity and numerical distinctness.

In the terminology introduced by Adams (1979), the former approach takes the things' *suchnesses* as the only components of individuals, while the latter maintains that some form of *thisness* also exists and is the primitive source of individuality. In what follows, we will refer to option (a) as *reductionism* and to option (b) as *primitivism*.

In modern times, reductionism was clearly and forcefully upheld by Leibniz. Leibniz's reductionist perspective can be summarised as the view that individuality reduces to uniqueness of qualities, in such a way that the *Principle of the Identity of the Indiscernibles* (PII from now onward) holds:

$$\forall x \forall y((\forall \mathrm{P}(\mathrm{P}x \leftrightarrow \mathrm{P}y)) \rightarrow (x = y)).$$

Literally, the Principle says that if two entities have all the same *monadic* properties, then they are the same individual. This entails that each individual has a set of monadic properties unique to it, i.e., that individuality is the same as qualitative uniqueness.[1] It is clear that an assessment of the reductionist view essentially involves an inquiry into the validity and epistemic status of PII, which here—it is worth making this explicit at the outset—we will conduct from a metaphysical rather than merely formal/logical viewpoint.

[1] There is, of course, also an important connection with the work of Quine (1960, 1976). On the basis of ideas of Hilbert and Bernays, Quine showed that (provided that the vocabulary of non-analysed general terms is finite) the identity sign can be paraphrased away in any first-order language, and replaced with a conjunction of non-identity-involving formulas (in particular, conditionals of the form 'if Fx then Fy' for any x and y and any number of places in F). This must be mentioned, as authors that we will discuss later on worked in an explicitly Quinean setting.

3 The different readings of PII

Let us start with the rather well-known fact that if predicates involving identity are *included* in the scope of the relevant universal quantifier, PII turns out to be analytically true.[2]

It is a widespread (and, it would seem, well-motivated) opinion that, at least within the reductionist camp, PII cannot be used as a criterion of individuation if identity and difference are regarded as properties and, therefore, *presupposed* rather than *analysed* in terms of something else.[3] The question that needs to be addressed when assessing reductionism, therefore, is whether a *non-trivial* version of PII—in which identity is not presupposed in any way—can be defended as a valid criterion of individuation.

In answering this question, it is useful to distinguish between two different ways of interpreting PII. On a *metaphysical* reading, PII is intended to be *necessarily* true; that is, as a matter of metaphysical necessity, no two individuals can have all the same properties. On an *epistemic* reading, instead, the view is that—*as far as we know*—there are good reasons for believing that numerically distinct but indiscernible individuals do not actually exist. Importantly, the metaphysical reading is based on the idea that the necessary truth of PII can be established on non-empirical grounds, and that this fact has *consequences* for our knowledge of, and claims about, things in the world. The epistemic reading reverses the order of argumentation, and claims that it is experience that gives us reasons for using PII as a criterion for ascribing individuality to things.

Rather than discussing the various arguments for or against the metaphysical reading (all of which, in any case, appear to us far from conclusive), here we will simply take for granted that PII is *not* a necessary truth, and that it is its *epistemic* reading that deserves discussion, where "epistemic", crucially, presupposes a confrontation with our best scientific knowledge. We will consequently assess PII, and reductionism more in general, from a 'naturalistic' standpoint, one according to which our metaphysical claims should be supported by, and be compatible with, our best current scientific knowledge of the world.

However, before moving on, and as a first step towards the view that individuality 'comes in degrees', let us begin by reminding the reader that Leibniz committed himself to a *strong* version of PII (henceforth, ***PIIa***), one that *excludes* spatial location from the scope of the universal quantifier ranging over properties appearing in the principle, and only takes into account monadic intrinsic properties. Once Leibniz's peculiar theologico-metaphysical reasons for wanting such a restriction are dropped, however, a weaker form of PII, quantifying also over spatial locations (***PIIb***), presents itself as far more plausible. The most important consequence of the move from ***PIIa*** to ***PIIb*** is that the latter allows for otherwise qualitatively identical things to be made numerically distinct by the mere fact that

[2] For PII as an analytical truth, see Whitehead and Russell (1925, p. 57), Church (1956, p. 302) and Brody (1980, pp. 6–9).

[3] For a defence of this claim, see Black (1952, p. 155), Ayer (1954, p. 29), Katz (1983), and Rodriguez-Pereyra (2006).

they exist at different places. This weaker version of PII, that is, can be used to express the age-old idea of relying on space as a *principium individuationis*—an idea defended, among others, by Aquinas, Kant and Schopenhauer.[4] Obviously enough, any two objects that are distinct individuals according to ***PIIa*** are also distinct according to ***PIIb***, but not the other way around. Consequently, the 'grade of individuality' that ***PIIb*** ascribes to entities is proportionally weaker than that 'captured' by ***PIIa***.

But is such a weaker formulation of PII compelling? The *locus classicus* with respect to a critical assessment of ***PIIb*** is Black's (1952, p. 156) completely symmetric universe, only inhabited by two numerically distinct spheres having all the same monadic properties. In particular, it looks as though the spheres' spatial positions must be defined in *relational terms*, because—*by hypothesis—only* the two spheres exist "and nothing else".[5] It would seem that Black's thought experiment shows that there is at least one conceivable circumstance in which we cannot make recourse even to space as *principium individuationis* and, therefore, ***PIIb*** is violated: that *one* sphere is distinct from *the other* sphere seems to be a *primitive* fact, neither grounded in an intrinsic qualitative difference nor in a difference with respect to *location in space*. Ignoring the long-standing debate about the actual strength of Black's argument,[6] here we will instead notice that when presented with Black's universe, one has the feeling that something has been tacitly 'smuggled in' in an illegitimate way. This something, in particular, has to do with *the status of the spatial relation* holding among the identical spheres, which indeed seems to ground a relevant, purely qualitative state of affairs. Does this mean that the formulations of PII considered so far do not capture all possible qualitative facts about things, that is, all possible forms of individuality as discernibility?

Following certain Quinean reflections, a positive answer to this question has been recently given (Saunders 2006). Quine explained that what he calls *strong* and *moderate discriminability* are in fact not the only possibilities. It is also possible, says Quine (1976, p. 114), that two objects are *weakly discriminable*, a fact that occurs when they satisfy a formula containing a predicate satisfiable by two entities in any order, but not by one of them alone, such as, for instance, "…goes in the opposite direction to…". Black's spheres, says Saunders, clearly turn out to be weakly discernible (here, we will not follow Quine's idiosyncratic terminology), as an *irreflexive* spatial relation holds between them that determines that *each one of the*

[4] And by no means limited to historical figures in philosophy: for the influence of Schopenhauer's view of space on Einstein's thoughts on separability, for instance, see Howard (1997).

[5] It could be objected that Black doesn't explicitly rule out the existence of an absolute spatial background. This exegetical point, however, doesn't affect the strength of the counterexample: the location occupied by each sphere must in any case be described in absolutely general terms, turning out to be the same for the two spheres.

[6] Black's argument might be rejected as question-begging (as in, for example, Odegard (1964)) or as re-describable in reductionist terms (as in Hacking (1975)). But the former objection has no force here, as we are looking for a justification of PII in the first place. With respect to the latter, it must be noticed instead that a re-description may not always be available, and Hacking's strategy might in any case not be regarded as a legitimate reductionist response. An 'extreme' option is to follow O'Leary-Hawthorne (1995) in claiming that in Black's universe there is only one sphere at some distance from itself, but this really looks like a last resort for the defender of PII, especially from a naturalistic viewpoint.

spheres is at some distance from the other (but not from itself)—and, consequently, that there are *two* numerically distinct but qualitatively identical spheres.

The foregoing indicates that it is in fact possible to formulate a version of PII that sets even weaker requirements on individuality than both ***PIIa*** and ***PIIb***, thus individuating certain entities that both these forms of PII fail to individuate. Informally, what we will from now on refer to as ***PIIc*** claims that if any two entities share all the same monadic properties and partake in no irreflexive relation, then they are one and the same individual (and, consequently, that the participation in an irreflexive relation is sufficient for individuality).

However, ***PIIc*** appears to be much more controversial than the previous two versions of PII. This is due to the fact that, while ***PIIc*** is an unquestionable *logical* principle—recall that, as Quine showed, the identity relation is in fact *coextensive* with the conjunction of *all* the relevant non-identity-involving formulas whenever there is a finite number of unanalysed general terms—this does not *ipso facto* ground it as an indisputable *metaphysical* claim. For, there is no obvious correspondence between the predicates appearing in a language that we might decide to use for describing the world and the properties and relations that are actually exemplified in the world. And this is particularly true when it comes to relations, as these have a much more problematic metaphysical status than 'canonical', monadic properties. Incidentally, it is crucial here to appreciate this logic/metaphysics divide because, while we are happy to acknowledge that PII can be given independent logical motivations, here we are interested in its role within an aptly scientifically-informed metaphysics. Consequently, we feel authorised to bracket the discussion of the logical status of ***PIIc***, and focus instead on the ontological presuppositions it rests upon, and on its ontological consequences.

In this context, however, an immediate problem arises with the proposal of using weak discernibility to neutralise anti-reductionist counterexamples of the sort devised by Black. Namely, that such a proposal smells of circularity. In particular, it could be objected that no relation can be said to hold unless we have *two* relata to begin with, so that numerical distinctness must be *presupposed*. To this charge of circularity (irreflexive relations can discern only if we already have two individuals) it might be replied that relations need not always be derivative, i.e., dependent on the prior existence of their relata, and could at least in some cases *ground*, be prior to, the numerical distinctness of things. This is, of course, a contentious and possibly *ad hoc* move, as we don't seem to have independent evidence to assume the existence of such relations. But it must be admitted, on the other hand, that there is no clear evidence against this thesis either, but only indications coming from common sense. Not surprisingly, this issue has been and still is object of philosophical controversy.

Be that it is may, in order to give our opponent all the ground she needs, we won't exclude that the counterintuitive view of relations under discussion can be consistently upheld so as to avoid the circularity objection. For us, it will in fact be sufficient to show that since the reasons that are normally adduced for preferring reductionism to primitivism will not turn out to be compelling, the controversial status of relations existing prior to their relata (or, at any rate, metaphysically on a par with the latter) will give us an *additional* reason for doubting the force of weak discernibility as a weapon for the reductionist. Indeed, we will argue that at least in

some cases primitivism *should* in fact be preferred to reductionism, if only on mere grounds of simplicity and minimisation of metaphysical revision.

In more detail, in the sections to follow we will first of all argue that, contrary to what seems to be a widespread consensus, a proper naturalisation of metaphysics doesn't by any means entail that we should opt for a reductionist conception of individuality. The same holds, we will also argue, for a relatively new, non-Leibnizian, form of reductionism—endorsed by various structuralists—according to which identity is always contextually, albeit not necessarily qualitatively, determined. After that, we will look specifically at non-relativistic quantum mechanics and suggest that it is best interpreted as a theory of primitively individuated entities. At the same time, on the basis of a plausible form of anti-reductionism about scientific theories and their levels of application, we will argue that naturalists should nevertheless allow for a *plurality* of forms of individuality, i.e., for the possibility that entities in one scientific domain are individuated differently from entities in another scientific domain. What is fundamental, we will claim, is to look at the best available scientific description of the entities we are dealing with in a given context, and see which form of individuality can be most straightforwardly 'extracted' from that description in that context.

4 The alleged scientific basis of Leibnizian reductionism

As witnessed by the recent flourishing literature on identity and discernibility in the individual sciences (physics in particular), scientifically minded philosophers who aim to answer metaphysical questions are in the vast majority of cases sympathetic to reductionism about individuality. The main motivation for insisting on a reductionist view of individuality seems to be the desire to avoid mysterious metaphysical assumptions that are not empirically supported by well-corroborated science or, worse, that cannot in principle 'make a difference' at the observable level (where "observability" here is being understood in the broadest possible manner): haecceitates, bare particulars, etc.

In the quantum case, for example, it is exactly the endorsement of a form of naturalism that led many to regard the question whether particles are *discernible* as fundamental. For, that quantum particles may be regarded as individuals by attributing some form of 'transcendental individuality' to them is well-known at least since Post (1963) and French and Redhead (1988). Naturalists, however, while clearly interested in trying to preserve the idea that quantum mechanics describes a domain of individual objects (peculiar in some respects though these may be), have typically rejected this solution as evidently relying on non-scientifically respectable metaphysical posits. But is this line of reasoning truly unexceptionable?

The case of quantum mechanics will be discussed in more detail later. Here, we will focus on the more general inference from naturalism to reductionism. We believe that *three* important points must be made with respect to such inference, which show that it is much less straightforward and compelling than it may seem at first.

(1) As we see the issue, in most of the current literature on the topic there is an as simple as much as misleading conflation between *two* different ways of

determining what is supported by, or to be deemed meaningful on the basis of, science and what is not. The *first* equates what is acceptable with what is qualitative—that is, roughly, with properties like colour, mass, charge and the likes, that do not 'encode' any 'information' about the identity of any specific individual. The *second* defines what is naturalistically acceptable as *whatever* contributes to an as complete as possible description of things according to our best current science. While it is obvious that the former stance is much closer to the Leibnizian spirit, it seems to us that, as a matter of fact, it might sacrifice a lot of what qualifies as part and parcel of scientific theorising. So much so, we suggest, that at least in certain contexts, one should rather opt for the latter understanding of 'scientifically acceptable'. For one, notice that *indiscernible objects can make an empirical difference* in spite of their being indiscernible, *merely in virtue of the fact that they are numerically distinct*: that is, the qualitative uniqueness of material objects is *not* necessary for the empirical significance and the scientific meaningfulness of claims concerning (physical systems containing) those objects. For instance, a world with two exactly similar material objects exhibits twice the mass of a world with only one of them (see Hawley 2009). This only holds as long as properties are additive, of course, but it is clear that most of the relevant physical properties of material objects are indeed additive. This too often overlooked fact should already give some pause to the scientifically minded metaphysician who aims to accept only 'empirically grounded' metaphysical posits: there are in fact no obvious reasons for ruling out indiscernible objects as a matter of principle solely on the basis of empirical indistinguishability plus Ockham's razor!

(2) Secondly, the possibility that facts of numerical distinctness might be as fundamental as, or even *more fundamental* than, facts about qualities appear directly suggested by some scientific theories. We will argue in more detail later that the use of the term "more fundamental" is justified in non-relativistic quantum mechanics, where all formulations of physical problems consider the number of particles as a starting assumption, one which enters into the construction of the right kind of model for the physical problem at hand (i.e., a Hilbert space or a configuration space with the correct number of dimensions), independently of considerations related to the qualitative features of things. In other words, we will see that the presence of particle names or 'labels' in the quantum *formalism*, rather than being regarded as an accidental feature of quantum theory, can instead be legitimately given a direct physical significance (notice, incidentally, that we do not need to restrict this claim to anti-symmetric quantum states, but can extend it also to bosons).

(3) Our third point is that primitive intrinsic identities need *not* be taken to constitute 'mysterious metaphysical additions' to the qualities of things, and *may* simply coincide with fundamental, ungrounded facts about the existence of certain entities. In other words, there is no reason for thinking that primitive intrinsic individualities can only exist if they are based on full-blown 'properties' additional to the other properties of things. Historically, this was clearly stated already within the Scholastic tradition that many naturalistic

metaphysicians have been (too) quick to dismiss: while Duns Scotus did in fact regard haecceitates as ontological components of things, Ockham—in keeping with his general nominalistic attitude—insisted that individuality just corresponds to a fundamental 'way of being' of objects, i.e., to facts about those objects that neither allow for nor demand further analysis. It seems to us that the Ockhamian perspective is both internally consistent and perfectly compatible with a naturalistic methodology. To the reductionist who remains sceptical about this and wonders whether it is anything more than a 'terminological trick' we respond as follows: if the naturalist is *never* allowed to introduce something that is primitively what it is, how can the reductionist fully and satisfactorily develop his/her own theory? Properties too, be they universals or particularised instances, have well-defined identity conditions, in virtue of which they are the specific entities they are; and either these conditions are analysable, but then the threat of an infinite regress immediately arises, or they are primitive. The same seems to hold for *any* ontological construction.

5 Non-Leibnizian reductionism?

Indeed, self-proclaimed naturalists have already acknowledged the existence of valid counterexamples even to ***PIIc***. In referring to the PII in the field of mathematics, for instance, Ladyman (2007) considers two-node graphs with no edges (mathematical systems composed of two absolutely indiscernible objects) and concludes that, for cases like these, there is no reason to expect that some Leibnizian principle will turn out to apply.

By distinguishing sharply between mathematical and physical ontology, one could argue that this sort of considerations are simply irrelevant for those who are interested in the ontological nature of material, concrete objects. Apart from the fact that one could reject the existence of a sharp divide between mathematical and physical ontology, however, this conclusion would be hasty. For, it is certainly possible to refer to the abstract domain inquired into by mathematics as a *model* for how to conceive of the physical world, especially if there are reasons coming from philosophically inspired analyses of specific scientific theories that bring evidence to the claim that there are clear *analogies* between what can be said about mathematical objects on the one hand and physical objects on the other. In this sense, we take Ladyman's point to be exactly that reflections about mathematical ontology have at least heuristic value in the present context.[7]

Indeed, Ladyman convincingly relates the abovementioned graphs to quantum statistics and to space-time in General Relativity (GR)—where particles and space-time points, respectively, turn out to be permutation-invariant—going then on to draw a general philosophical conclusion.

[7] We are grateful to an anonymous referee for pressing us on this point concerning mathematical as opposed to physical ontology.

Let us look at this conclusion in more detail, considering in particular the relativistic case. If in GR the metric field is defined on the manifold functions as a globally defined individuating entity for the manifold points (Stachel 1993; Dorato and Pauri 2006), then diffeomorphically related models can be regarded as physically identical, as it should be in order to avoid undesirable violations of determinism implied by the hole argument (Earman and Norton 1987). Several authors, and Ladyman among these, take this to mean that the points of the manifold are not discernible in any way, and are only *contextually* individuated by the relations that characterise the metric field as it is defined on the manifold. More generally, Ladyman suggests that, given the available evidence and taking the analogy with graph theory seriously (something that, to repeat, we don't see a reason for not doing), there is only one move available to the reductionist in view of the (potential) counterexamples to PII just considered. That is, to give up one of the central tenets of his/her position—the qualitative analysability of individuality—but stick to the other key element of reductionism, namely the idea that *identity and individuality are contextually determined*.

Indeed, Ladyman explicitly claims that, in those cases in which the identity and individuality of an object cannot be grounded in qualitative differences, they *must* anyway be regarded as determined by the whole system to which the object in question belongs. In other words, one should postulate some sort of non-qualitative 'identity- and difference-making relations' that characterise the total system/structure.

This leads, then, straightforwardly to a '*non-Leibnizian' form of reductionism*, corresponding to what Stachel and Ladyman call 'contextualism'. Contextualism is, in particular, a form of reductionism because it invites one to analyse the individuality of objects in terms of something else, external to the objects themselves; but it is also non-Leibnizian because this something else is not (at least not necessarily) something qualitative. But why choose contextualism rather than primitivism? Recall the previous section's refutation of the traditional argument against primitivism, according to which primitive identities are unacceptable, non-scientifically grounded, metaphysical posits. To those arguments we now add the following remarks.

The contextualist inference from the examples coming from graph theory and GR to the claim that, *necessarily*, identity is determined contextually rests on another fundamental argument against primitivism, which goes as follows. Primitive intrinsic individuality entails *haecceitism*, i.e., differences between what distinct worlds say *de re* about certain individuals that do not correspond to overall qualitative differences among those worlds. But haecceitism is directly contradicted by contemporary science. In particular, the sort of permutation invariance that, as we have just seen, is pervasive across different scientific domains points to anti-haecceitism, as there are no two possible worlds described by the relevant theory that differ *merely* with respect to the identities of the things they contain. Hence, the naturalist metaphysician must in any case be a reductionist about individuality.

Now, is this a compelling argument for endorsing reductionism at least in its non-Leibnizian version? We think the answer to this question is negative, for two reasons.

First, *primitive intrinsic individuality need not entail haecceitistic differences*. What is true of distinct worlds is not univocally determined by the nature of the identity of each individual object inhabiting them, and intra-world and trans-world considerations in fact have an important degree of mutual *independence*. Indeed, metaphysical frameworks are available in which objects possess primitive intrinsic identities but this fact does not entail haecceitism.[8] A counterpart-theoretic treatment of possible worlds, for example, allows for primitive intra-world identities (i.e., primitive intrinsic individuality for objects) together with anti-haecceitism about modality. And it is interesting to notice that this is directly relevant with respect to the physical evidence Ladyman refers to. For instance, with reference to GR, Butterfield (1989) argued that, if points of the manifold have primitive intrinsic identities but these are not preserved if one exchanges them while preserving their overall relationships (which is what happens on the counterpart-theoretic account of trans-world identity), substantivalism is kept safe from the alleged dire consequences of the hole argument.

One may object that, if counterpart theory and other suspicious 'metaphysical tricks' are set aside, the assumption that entities possess primitive identities immediately leads one to acknowledge the possibility of haecceitistic differences. Here, however, one further distinction must be drawn: when one has to account for specific non-haecceitistic facts which apparently contradict the claim that things possess primitive intrinsic identities, there might be viable, *non-ad-hoc* explanations of the evidence that tell us *why* haecceitistic differences are not manifest—not empirically meaningful—*without at the same time involving a more general choice between haecceitism and anti-haecceitism*. Consider for example quantum statistics, in which exchanging indistinguishable particles does not give rise to new, statistically relevant states. In this case, contrary to contextualism as well as to the 'Received View' that the evidence points to the non-individuality of particles, it is *possible* to claim that particles possess primitive intrinsic identities but their state-dependent properties are *holistic in the sense that they* only belong to the whole and exclusively describe *correlations* between its parts (for more details, see Morganti 2009). This immediately explains the peculiar features of the quantum domain while leaving it open whether, *had* their state-dependent properties been monadic, quantum particles *would have* given rise to haecceitistic differences. Following this line of reasoning, it might even be suggested very generally that, for reasons having to do with the minimization of revision of our entrenched beliefs, a modification of our commonsense beliefs about other aspects of reality (in the present case the *properties* of composite systems) should always be preferred to one involving more established beliefs (in the present case, concerning the *identity and individuality* of objects). Whether this is convincing, and which explanation of the evidence is the most plausible and least costly, can of course be questioned. But, in any case, the mere existence of an alternative, plausible possibility suffices to show that the claimed implication between intrinsic primitive identity and 'empirically relevant' haecceitism doesn't hold.

[8] In other words, haecceitism and the view that individuals (may) possess haecceitates are distinct and largely independent theses.

Another important thing to notice is how relevant a consideration of contextualism is with respect to our earlier discussion of naturalism and putatively mysterious metaphysical posits. We have already argued that primitivists can agree wholeheartedly with reductionists that mysterious, real yet non-physical, entities should be avoided when providing a metaphysical account of reality. Here, we have just seen that contextualists present their thoroughly non-Leibnizian position as motivated by science, hence as eminently satisfactory from a naturalistic viewpoint. Logically, this can only mean that contextualists regard (contextual) non-qualitatively analysable facts of numerical identity and distinctness as not corresponding to mysterious metaphysical posits; and, thus, that they see naturalism as independent of Leibnizian reductionism. But then it follows that, *exactly in the same way as the relations of numerical difference emphasised by contextualists*, primitive intrinsic identities need *not* be taken to constitute 'metaphysical additions' to the qualities of things, and *may* simply coincide with fundamental, ungrounded facts about the existence of certain entities (one may even go so far as to claim that primitive intrinsic individuality is nothing but a limiting case of ungrounded *contextual* individuality where a system is composed of only one entity). In other words, *if* Ladyman/Stachel-style contextualism is a live option in the physical world in any context in which the relevant grade of individuality is, so to put it, 'very low' because not qualitatively grounded, in the very same contexts intrinsic, primitive individuality is also admissible, at least in what in the previous section we called its 'Ockhamian' form.

From these considerations and the ensuing possible metaphysical underdetermination, we infer that the necessity of contextualism has not been argued for convincingly, while primitivism gains further credibility. The analysis just carried out, however, does *not* simply point to the fact that the metaphysics of identity and individuation is to be deemed underdetermined by the evidence. Nor do we intend it as sufficient for regarding primitivism as valid across the board.

In the next section, we will consider one specific example, widely discussed in the recent literature, and provide reasons for regarding the entities in the relevant domain as primitively individuated. We will argue that the considerations brought to bear in the course of the case study have more general import and lend support to primitivism in that context. This, however, does not mean that one should generalize to other domains the specific conclusions about identity and individuality arrived at in the case study, since in other domains reductionism is (or, at least, may be) a better option. Our take-home lesson will be, in conclusion, that in most (if not all) cases it is reasonable to think that science can be supplied with a well-defined and compelling metaphysical interpretation; but a form of *pluralism* about identity and individuality is advisable which *includes* primitivism as one among several metaphysical stances that are acceptable from a naturalistic viewpoint.

Before continuing, however, one remaining question has to be addressed in this section. Could the naturalist introduce forms of PII that are able to capture non-qualitative, *but scientifically grounded*, facts of numerical identity and distinctness as those discussed above? This would mean to add what we might label ***PIId*** and ***PIIe*** to the various versions of PII already presented, the *former* capturing the relations of numerical difference grounding contextual non-qualitative individuality, the *latter* the intrinsic facts of numerical uniqueness grounding intrinsic,

non-qualitative individuality. We think that this attempt to extend the Leibnizian tradition (hence, reductionism) so as to allow for individuality facts only grounded in countability is not necessarily doomed to failure: for, the relevant properties are, as we argued, part of the scientific description of the domain in question, and it would be at least possible to interpret 'modern-day' reductionism about individuality as the view that *whatever* appears in the relevant scientific description should be quantified over in PII, and individuality be reconstructed accordingly.[9] However, we also believe that this perspective is in tension with the spirit of the Leibnizian/reductionist tradition, and in particular with the no-triviality requirement for PII discussed in Sect. 2. Because of this, we suggest, the proposed 'extension' of PII is in fact undesirable.

6 Non-relativistic quantum mechanics, weak discernibility and countability

As promised, we now move on to a discussion of one specific case study, concerning the identity and (non-)individuality of particles in non-relativistic quantum mechanics. In particular, we will look at the recent debate about the ontological status of quantum particles and argue that there are reasons for attributing primitive individuality to them, without thereby violating the spirit of a sensible form of naturalism about metaphysics.

Surely, our everyday experience and classical mechanics support the idea that distinct objects *must* differ with respect to their spatial location. However, in the quantum domain things stand otherwise. In a seminal paper, French and Redhead (1988) started from the (debatable but widespread) identification of quantum properties with the quantities denoted by the probabilities appearing in the formalism, and considered two-particle systems of identical particles. They concluded that, both for fermions and bosons, two particles of the same kind that partake in the same physical system have all the same properties, *including (potential) spatial location*.[10] However, French and Redhead only took into account ***PIIa*** and ***PIIb*** in their paper. Indeed, it is now generally agreed that ***PIIb*** (and, consequently, ***PIIa***) fails in quantum mechanics. What about ***PIIc***?

It would seem that the fact that quantum particles can share all their properties including potential spatial location (defined in terms of probabilities) makes ***PIIc*** fail too in this context. In a recent paper that we have already mentioned, however, Saunders (2006, p. 59) argued that fermions in the singlet state of spin are weakly discernible, because they are in an irreflexive relation expressed by the symmetric but irreflexive predicate '… has opposite ↑-spin component of spin to…'. Saunders' argument has been made more general and rigorous by Muller and Saunders (2008) and Muller and Seevinck (2009). The more general argument (from now on, 'MSS' argument')—which, quite importantly, doesn't employ probabilities to define the relevant properties—goes as follows:

[9] Remember our distinction above between two ways of understanding naturalism.

[10] French and Redhead's results have been later improved upon by Butterfield (1993) and Huggett (2003).

(1) Quantum particles are well-defined in number (COUNTABILITY);
(2) Relations can be metaphysically genuine and yet *fail* to be reducible to monadic properties of their relata;
(3) Physical discernibility must be grounded in *physically meaningful properties*;
(4) *By using COUNTABILITY*, relations can be constructed out of physically meaningful single-particle operators (hence, satisfying 3)) that hold in many-particle systems of identical particles and satisfy the requirement for weak discernibility;
(5) Since—in a way that should be deemed unproblematic in virtue of (2)—these relations are not reducible to monadic properties of their relata, weak discernibility is the maximum degree of discernibility that can be obtained in the relevant quantum systems;
(6) However, weakly discerning relations can be reconstructed for *all* quantum systems.

The conclusion of MSS' argument is thus that in quantum mechanics ***PIIc*** is *always* capable to discern, a fact that can be generalised to all particles and Hilbert spaces of all dimensions.[11] This is very important for our discussion, as it seems to provide a reason for regarding PII, and therefore reductionism, as at least contingently true in our world. In particular, while macroscopic, classical objects appear to invariably obey ***PIIa*** or at least ***PIIb***, it now seems that more basic constituents of reality never violate (at least) ***PIIc***—roughly for the same reason for which Black's spheres do not violate it. This would clearly suffice for supporting what we have called the epistemic reading of PII in Sect. 2.

Things are not so simple, however: MSS' argument raises a number of issues that are not easily sorted out and, we think, eventually lend support to the theses we are presenting in this paper.

First, Muller and Seevinck themselves explain that their proof leads one to regard two identical bosons in a factorisable, symmetric direct product state as *discernible* while, intuitively, such entities do *not* appear to be discernible. They then state that the discernibility of such bosons should be accepted in the same way in which one accepts other 'quantum mysteries', such as the possibility of Schrödinger's cat (Muller and Seevinck 2009; Remark 4). But of course, one may take the alternative route of *modus tollens* and use this consequence of their argument to question MSS' conclusion. One may even go so far as to reject the relations constructed by MSS and accuse the latter of a form of "naïve realism about operators" (Daumer et al. 1996). Indeed, MSS do repeatedly point out that the relations they regard as weakly discerning are derived from operators whose physical meaningfulness is not questioned by anybody; but does this mean that the relations themselves are unexceptionable?

Secondly, and relatedly, one may protest, with Dieks and Veerstegh (2008) and Ladyman and Bigaj (2010), that weak discernibility at least betrays the Quine-Leibniz reductionist spirit, in that it doesn't correspond to the possibility of *actually* telling particles apart from each other through physical means and/or of establishing the genuineness of the relevant relations via 'symmetry-breaking'. Here too, the

[11] In particular, Muller and Saunders use only spin degrees of freedom (specifically, total spin relations) in finite Hilbert spaces; and more general commutator relations holding between distinct single-particle operators (e.g., position and momentum) in the case of infinite-dimensional Hilbert spaces.

argument is certainly not conclusive, but definitely relevant for our present purposes. On the one hand, MSS need not be impressed by the sort of operationalist arguments presented by Dieks and Veerstegh and Ladyman and Bigaj. In general, reductionism need not be based on the possibility of *actually discerning* through physical means. On the other hand, one may take this objection to MSS' argument to foster the above doubts about the status of the alleged discerning relations.

It is also worth pointing out that MSS draw a distinction between *individuals* (objects that are absolutely discernible on the basis of monadic and/or relational properties) and *relationals* (objects that are only weakly discernible) without adding anything explicit about the motivation for this differentiation. Even if, in all probability, this is just the result of a terminological choice based on logical considerations and established philosophical usage, such a differentiation might be intended as having *ontological* weight, i.e., as being due to the belief that weak discernibility is insufficient for 'full-blown individuality'. Obviously enough, this would lend another bit of support to the pluralistic and 'gradualistic' view on individuality that we are developing in this paper.

Of course, the foregoing is far from sufficient for a rejection of MSS' argument. However, that is not what we are looking for here. Rather, our point is that, once one puts the above considerations together with the infamous question, pointed at earlier, whether or not relations can 'come first' with respect to their relata (or, at least, 'together with' them), the overall scenario can be reasonably represented as follows. MSS develop a rigorous and clever, but also rather complicated and philosophically controversial, argument. And the idea that such an argument has a clear and objective metaphysical import is mainly, if not *exclusively*, motivated by the presupposition of the correctness of Leibnizian reductionism, or at least by the alleged unavailability of a credible alternative approach to individuality.[12] But if these assumptions turned out to be not compelling, the naturalist would have a very good motivation for looking for a (naturalistically acceptable) alternative that doesn't force one to (i) posit irreducible relations; (ii) be naïve or at least 'very liberal' with respect to which properties are physically genuine; (iii) accept counterintuitive conclusions about certain physical systems (i.e., the above mentioned systems of two identical bosons in a factorisable, symmetric direct product state); and, perhaps, even (iv) allow for entities (the above 'relationals') belonging to a new, *sui generis* ontological category.[13]

It should be clear by now what, in view of this, our suggestion is going to be: a hitherto ignored (at least by naturalists) alternative—based on primitivism—is available and allows for a great methodological gain in terms of simplicity, clarity and conservativeness with respect to entrenched metaphysical beliefs and schemes, while being at least equally satisfactory in terms of defining an ontological

[12] We do not attribute this presupposition to MSS themselves because they seem quite neutral on this. In fact, what they say seems best interpreted as motivated by the idea that the question whether physical objects can adequately be described using PII is an interesting one in itself.

[13] Notice, in this connection, that a reductionist viewpoint seems to be shared also by authors who disagree with MSS. For instance, Dieks and Versteegh (2008) conclude their critique of MSS by arguing that one should opt for a holistic interpretation of quantum systems of identical particles (i.e., one where the total system simply has no component particles) (compare Hawley's (2009) 'summing defence' of PII). This clearly suggests that Dieks and Versteegh too reject primitivism as unscientific.

interpretation that meets the criteria and constraints set by a naturalistic methodology. As a consequence of this, we conclude that primitivism should be preferred to Leibnizian reductionism at least in the non-relativistic quantum case.

Crucially, the fundamental basis for this assertion is offered by physics itself. It consists of the simple and uncontroversial fact that premise (4) above is essentially based on premise (1), that is, on COUNTABILITY. If we are right in suggesting that primitivism is not necessarily in conflict with naturalism about metaphysics, it immediately follows from this that there is a much simpler alternative to the complicated scheme put forward by MSS, with all the issues it raises: namely, *to regard COUNTABILITY not just as a merely formal fact about particle labels, but as metaphysically and physically significant in itself*, without searching for additional elements on the basis of which the countable entities could be regarded as discernible. In other words, one could maintain that the "presence" of *n* particles at the formal level has a direct ontological counterpart, so that it can be concluded that those particles are *n* individuals *independently of their qualities*. After all, if the fact that a given physical system is composed of a certain number of particles in a purely formal sense is fundamental for even starting to show that the entities composing the system are discernible, it seems perfectly legitimate to regard quantum particles as ('low-degree') individuals by moving from a purely formal to a non-formal reading of countability, without caring about qualitative properties and (in)discernibility.[14]

Summing up, the moral of our case study is as follows. Quantum particles can and should be regarded as primitively individuated, simply because they are countable at the level of the formalism (a fact used in the extant proofs of their weak discernibility) and the existing ontological alternatives do not offer any advantage when an accurate critical comparison on the basis of methodological and pragmatic criteria is carried out. Primitivism, as a matter of fact, provides us with the most straightforward and uncomplicated ontological interpretation of the theory. However, accepting the view that countability can be read in an ontological sense directly from the formal language of the theory does not amount to prejudging the issue in our favour without giving arguments, nor is it "naïve realism about the formalism". It simply amounts to preferring the simplest choice available for the interpretation of the relevant domain of material objects. (Finally, notice that our form of primitivism is *not* incompatible with the view that quantum particles—under certain (non-negligible) assumptions—turn out to be weakly discernible: in our picture, it is perfectly possible to claim that primitively individuated objects also possess a derivative, less fundamental, grade of individuality captured in this case by ***PIIc*** above).

7 Should we be primitivists in general?

We have argued that primitivism (when aptly formulated) does not imply a 'jump' to a metaphysical viewpoint that is in principle unacceptable for the naturalist, and

[14] Interestingly, Ladyman and Bigaj (2010, p. 135) express ideas similar to ours when they suggest that perhaps "anything that is the value of a first-order variable is an individual" , but do not emphasise the role played by countability assumptions in the arguments in favour of the weak discernibility of quantum particles.

that it is in fact preferable in at least one important scientific domain. As a matter of fact, we suggested, a simple criterion—according to which one should always define one's ontology on the basis of the simplest and most direct interpretation (compatible with a naturalist methodology) of the language of the relevant theory—makes primitivism preferable in the case of non-relativistic quantum mechanics. An important question to be asked now is whether this doesn't make primitivism valid generally, i.e., for individuals at all levels. In particular, one might think that since quantum mechanics describes the most fundamental 'building blocks' of reality, of which everything else is made of, then individuals at all less fundamental levels, i.e., the individuals described by the other special sciences will also automatically be individuated primitively: they will just be the individuals arising from the composition of (primitively individuated) particles x_1, x_2, …, x_n.

However, this only holds if one assumes a strong, or "avid" form of reductionism, i.e., a micro-reductionism *à la* Putnam-Oppenheim, according to which it is in principle possible to translate the language of higher-level sciences exclusively in the vocabulary of fundamental physics, in this case, non-relativistic quantum mechanics (of course, one also needs to assume that the latter theory has reached the "bottom" level of reality). As is well-known, though, this reductionist view is not very popular in contemporary philosophy of science and, we believe, rightly so. If anything, what one can reasonably hope to achieve given the present status of the debate is (refined) Nagel-type reductions endowed with an explanatory, not an ontological, connotation.[15] In other words, it seems fair to claim that theories describing different levels of reality might turn out to be in relevant mutual relationships, and there may even be a lot in the language of a higher-level theory that can be said without a significant loss of content in the language of a more fundamental theory. But by no means should this be interpreted as justifying any project of ontological reduction.

Now, if this is the case, it follows that ontological questions such as that of individuality *can and should be asked in a level- and context-dependent fashion*, that is, always with specific and explicit reference to the entities belonging to the domain of inquiry of a specific science and a specific theory. This, we claim, gives us good reasons to be 'naturalistic pluralists' about individuality, in the sense that the opposition between primitivism and reductionism can and should be overcome in favour of a more comprehensive and flexible view, allowing for different 'grades of individuality' to be evaluated and assessed on the basis of their applicability and usefulness in the various specific theoretical contexts.

At this point, given our emphasis on the language of the theory and on countability, one may insist that it is more or less obvious that objects are always countable at least at the formal level. This, however, is not so. The attribution of the various forms of individuality on the basis of our best available descriptions of reality requires, no doubt, a much more detailed treatment, and is a task for which further work is certainly needed. For the time being, it seems at any rate safe to

[15] For an interesting defence of a neo-Nagelian account of reduction, see for example Dizadji-Bahmani, Frigg and Hartmann (2010). Another account of reduction, in terms of asymptotic behaviour, is spelled out by Batterman (2002).

claim, in a rather general fashion, that for the vast majority of macroscopic entities Leibniz's metaphysical views seem to apply at least contingently—think, in particular, of biological entities (apart from the negligible possibility of perfect clones, and of exactly similar organisms in general)—and ***PIIa*** consequently holds for them. Once classical mechanics (supplemented by an assumption of impenetrability which, incidentally, is not an integral part of the theory) is adopted, ***PIIb*** becomes a natural criterion of individuation: particles may have absolutely all the same monadic intrinsic properties, but they always differ with respect to position. In the quantum world, however, the landscape changes. As things stand, we have argued, primitive, non-qualitative individuality appears preferable to ***PIIc*** in the non-relativistic case. Things, however, are likely to be different as one moves to quantum field theory, where we can have *superpositions of particle number* and, consequently, *countability cannot be expected to play the same role as in the case of non-relativistic quantum mechanics*. And yet different conclusions might be in order when it comes to even more fundamental physical theories such as quantum gravity, string theory, etc. The sort of pluralism that we have recommended takes all this into account, and makes room for different ontological perspectives to be applied at different levels and in different domains. In this context, not even the weakest form of individuality is taken for granted in all cases.

Having mentioned that in quantum field theory the lack of countability may suffice for not endorsing an ontology of individual objects, we are now in a position to add one further clarification. Of course, also in non-relativistic quantum mechanics one could hold that particles really are modes of excitation of the underlying quantum field: after all, that might just be the right ontology for the domain in question. This is fine, but doesn't contradict our claims. For, it is important to see how exactly the alternative ontology is arrived at. Several authors (e.g., Saunders (2006)) argued in favour of it on the basis the claim that bosons are not discernible in any way in the non-relativistic context; this, however, is a move that is put into doubt by the latest results about weak discernibility that we discussed earlier (and by our results about countability and primitivism, of course). If, instead, the conclusion that the right ontology is one of "modes of field excitations" is obtained via considerations that do not have to do with non-relativistic quantum mechanics,[16] our point about this latter theory and what its formalism suggests remains intact (at least given our pluralist stance).[17]

8 Conclusions

In this paper, we have attacked entrenched prejudices against primitivism about individuality, and the connected idea that a naturalistic approach to metaphysics

[16] For instance, to the effect that quantum field theory is in general more empirically adequate than quantum mechanics.

[17] We are grateful to an anonymous referee for pressing us on this point.

inevitably leads to a Leibnizian–Quinean stance with respect to identity and individuality. Moreover, we have critically assessed a contextualist view that, in fact, amounts to a form of non-Leibnizian reductionism. In a case study dealing with non-relativistic quantum mechanics, we applied these general premises to make explicit certain assumptions that are customarily made in a tacit form in the literature and prove them unjustified. On this basis, we defended primitivism about the individuality of quantum particles. At a more general level, however, we recommended a more flexible sort of pluralism about identity and individuality, one that we regard as more in harmony with the actual relationship between the different sciences and their different domains of inquiry, and also between different theories and models in the same domain.

The proposed perspective enables one, among other things, to shed light on existing oppositions and conflicting views about the nature of individuals and the conditions of individuation of things. For instance, many authors, including historical figures such as, for example, Schrödinger, thought that quantum particles were *not* individuals. But this was, it seems to us, because they held the view of individuality captured by ***PIIa*** and ***PIIb*** as *generally and absolutely valid*. A similar presupposition, we believe, also underlies the current discussion about the ontology of quantum mechanics, which we looked at in some detail. However, if, as we have suggested, individuality is not a monolithic concept, then 'how much individuality' an object has can be meaningfully asked, and (objective and well-defined! We don't think there is any room for conventionalism or relativism here) answers can and should be sought by having recourse to our best knowledge of the relevant field. This, however, without expecting, as it happened so far, one form of individuality and one correct criterion of individuation to be valid across all fields of knowledge.

The most important work to be done in the future is thus, as mentioned above, to test the whole set of available forms of individuality against the background of specific scientific theories. We think that—especially once Leibnizian or at any rate reductionist prejudices are set aside—this approach can be expected to produce a vast array of novel philosophical results.

References

Adams, R. M. (1979). Primitive thisness and primitive identity. *Journal of Philosophy, 76*, 5–25.

Ayer, A. J. (1954). *The identity of the indiscernibles, in philosophical essays*. London: MacMillan.

Batterman, R. W. (2002). *The devil in the details*. Oxford: Oxford University Press.

Brody, B. (1980). *Identity and essence*. Princeton: Princeton University Press.

Black, M. (1952). The identity of indiscernibles. *Mind*, *61*, 153–164.

Butterfield, J. (1989). The hole truth. *British Journal for the Philosophy of Science, 40*, 1–28.

Butterfield, J. (1993). Interpretation and identity in quantum theory. *Studies in history and Philosophy of Science, 24*, 443–476.

Church, A. (1956). *Introduction to mathematical logic* (Vol. I). Princeton: Princeton University Press.

Daumer, M., Dürr, D., Goldstein, S., & Zanghì, N. (1996). Naïve realism about operators. *Erkenntnis, 45*, 379–397.

Dieks, D., & Versteegh, M. (2008). Identical quantum particles and weak discernibility. *Foundations of Physics, 38*, 923–934.

Dizadji-Bahmani, F., Frigg, R., & Hartmann, S. (2010). Who's afraid of nagelian reduction? *Erkenntnis, 73*, 393–412.

Dorato, M., & Pauri, M. (2006). Holism and structuralism in classical and quantum general relativity. In D. Rickles, S. French, & J. Saatsi (Eds.), *Structural foundations of quantum gravity* (pp. 121–151). Oxford: Oxford University Press.

Earman, J., & Norton, J. D. (1987). What price space-time substantivalism. *British Journal for the Philosophy of Science, 38*, 515–525.

French, S., & Redhead, M. (1988). Quantum mechanics and the identity of the indiscernibles. *British Journal for the Philosophy of Science, 39*, 233–246.

Hacking, I. (1975). The identity of indiscernibles. *Journal of Philosophy, 72*, 249–256.

Hawley, K. (2009). Identity and indiscernibility. *Mind, 118*, 101–119.

Howard, D. (1997). A peek behind the Veil of Maya: Einstein, Schopenhauer, and the historical background of the conception of space as a ground for the individuation of physical systems. In J. Earman & J. Norton (Eds.), *The cosmos of science: Essays of exploration, Pittsburgh–Konstanz series in the philosophy and history of science* (Vol. 6, pp. 87–150). Pittsburgh and Universitätsverlag, Konstanz: University of Pittsburgh Press.

Huggett, N. (2003). Quarticles and the identity of the indiscernibles. In K. Brading & E. Castellani (Eds.), *Symmetries in physics: Philosophical reflections* (pp. 239–249). Cambridge: Cambridge University Press.

Katz, B. D. (1983). The identity of the indiscernibles revisited. *Philosophical Studies, 44*, 37–44.

Ladyman, J. (2007). Scientific structuralism: On the identity and diversity of objects in a structure. *Proceedings of the Aristotelian Society, Supplementary Volume, 81*, 23–43.

Ladyman, J., & Bigaj, T. (2010). The principle of the identity of indiscernibles and quantum mechanics. *Philosophy of Science, 77*, 117–136.

Morganti, M. (2009). Inherent properties and statistics with individual particles in quantum mechanics. *Studies in the History and Philosophy of Modern Physics, 40*, 223–231.

Muller, F. A., & Saunders, S. (2008). Discerning fermions. *British Journal for the Philosophy of Science, 59*, 499–548.

Muller, F. A., & Seevinck, M. P. (2009). Discerning elementary particles. *Philosophy of Science., 76*, 179–200.

O'Leary-Hawthorne, J. (1995). The bundle theory of substance and the identity of the indiscernibles. *Analysis, 55*, 191–196.

Odegard, D. (1964). Indiscernibles. *Philosophical Quarterly, 14*, 204–213.

Post, H., (1963): *Individuality in physics*, in *The Listener*, 70, 534-537, reprinted in 1973 in *Vedanta for East and West*, *32*, 14–22.

Quine, W.v. O. (1960). *Word and object*. Cambridge: MIT Press.

Quine, W.v. O. (1976). Grades of discriminability. *Journal of Philosophy, 73*, 113–116.

Rodriguez-Pereyra, G. (2006). How not to trivialise the identity of the indiscernibles. In P. F. Strawson & A. Chakrabarti (Eds.), *Universals, concepts and qualities: New essays on the meaning of predicates* (pp. 205–224). London: Ashgate.

Saunders, S. (2006). Are quantum particles objects? *Analysis, 66*, 52–63.

Stachel, J. (1993). The meaning of general covariance. In J. Earman, A. Janis, G. Massey, & N. Rescher (Eds.), *Philosophical problems of the internal and external worlds: Essays on the philosophy of Adolf Grünbaum* (pp. 129–160). Pittsburgh and Universitätsverlag, Konstanz: University of Pittsburgh Press.

Whitehead, A. N., & Russell, B. (1925). *Principia mathematica* (Vol. I). Cambridge: Cambridge University Press.