Holism and Structuralism in Classical and Quantum GR - (Forthcoming in *Structuralism and Quantum Gravity*, S.French (ed.), OUP)

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Abstract

The main aim of our paper is to show that interpretative issues belonging to classical General Relativity (GR) might be preliminary to a deeper understanding of conceptual problems stemming from on-going attempts at constructing a quantum theory of gravity. Among such interpretative issues, we focus on the meaning of general covariance and the related question of the identity of points, by basing our investigation on the Hamiltonian formulation of GR. In particular, we argue that the adoption of a peculiar gauge-fixing within the canonical reduction of ADM metric gravity may yield a new solution to the debate between substantivalists and relationists, by suggesting a *tertium quid* between these two age-old positions. Such a third position enables us to evaluate the controversial relationship between entity realism and structural realism in a well-defined case study. After having indicated the possible developments of this approach in Quantum Gravity, we discuss the structuralist and holistic features of the class of spacetime models that are used in the above mentioned canonical reduction.

1 Introduction: two strands of philosophy of physics that ought to be brought together

In recent philosophy of science, there have been *two* interesting areas of research that, independently of each other, have tried to overcome what was beginning to be perceived as a sterile opposition between two contrasting philosophical stances.

The first area of research involves the age-old opposition between the so-called spacetime *substantivalism*, according to which spacetime exists over and above the physical processes occurring in it, and *relationism*, according to which spatio-temporal relations are derivative and supervenient on physical relations obtaining among events and physical objects. The plausible claim that substantivalism and relationism, as they were understood before the advent of relativity or even before the electromagnetic view of nature, simply do not fit in well within the main features of the general theory of relativity, is reinforcing the need of advancing a *tertium quid* between these two positions, which tries in some sense to overcome the debate by incorporating some claims of both sides (Dorato 2000).

On the second front, discussions fuelled by the historical work of Thomas Kuhn, have generated a contrast between those who believe that assuming the approximate truth of scientific theories is the best explanation for the predictive and explanatory success of science - the scientific realists - and those who insist that the history of science is so replete with the corpses of abandoned entities (the flogiston, the caloric, the ether, etc.) that one should believe only in the humanly-observable consequences of our best scientific theories - the instrumentalists. As an attempt to overcome this opposition and save the history of science from a complete incommensurability between successive theories, John Worrall (Worrall , 1989) has recently recuperated some forgotten lesson left to us by Poincaré, by pointing out that *structural realism* (i.e., belief in the relational content denoted by our mathematically-expressed laws of nature) is "the best of all possible worlds". While giving some content to the view that there is (structural) continuity in the history of science, and therefore justifying a claim typically endorsed by the realists, structural realism *á la* Worrall was also meant as a warning against believing in non directly observable physical entities. Discussing the example already put forth by Poincaré, Worrall remarked that while Fresnel's equations were later incorporated by Maxwell's synthesis, the ether-based models used by him to mathematically describe light have later been abandoned.

In this paper, we aim to bring together these two strands of philosophical research by claiming that a certain form of *structural spacetime real-*

ism (a view that we refer to as "point-structuralism") may offer the desired *tertium-quid* solution to the debate between substantivalists and relationists by simply overcoming it. As we will see, such a solution emerges naturally from a certain approach to the Hamiltonian formulation of the general theory of relativity (GR), which is important not just to shed light on the above debate but also to clarify - with the help of a well-defined case study - some philosophical problems that are currently affecting the literature on structural realism in general. Most importantly, taking a stance on the meaning of general covariance within classical GR seems to us a precondition also to develop a satisfactory quantum theory of gravity [1].

Given these aims, our paper is organized as follows. In the next section **(2)**, we try to clarify the relationships among the various forms of scientific realism that are currently discussed in the philosophical literature. Together with the question of clarifying the nature of a *physical* (*versus* a purely *mathematical*) structure, we believe that these issues are a precondition to understand the impact of *structural* realism on the issue of the *identity of point-events* in classical GR. After a brief review of the hole argument in **3.1**, in **3.2** we show how a peculiar gauge-fixing in the canonical reduction of ADM metric gravity, based upon a new use of the so-called Bergmann-Komar "intrinsic pseudo-coordinates", can help us to formulate a new structural view of certain models of classical, general relativistic spacetime. In **4**, we indicate some possible developments of this view on the status of "points" in Quantum Gravity. In **5** and **5.1** we draw some philosophical conclusions from the preceding discussion by showing how our *point structuralism* represents an overcoming of both traditional substantialist and relationist views of the spacetime of GR. Such a *point structuralism*, however, does *not* dissolve physical entities into mathematical structures, as it entails a typical "entity-realist" attitude toward both the metric field and its "point-events", as well as a theory-realist attitude toward Einstein's field equations.

2 Many "Realisms" or one?

With the progressive sophistication of our philosophical understanding of science, the issue of scientific realism seems to have undergone a process of complication that is not unlike the growth of a living cell or the development of an embryo. As evidence for this claim, note that nowadays there are at least *four* different ways of characterizing scientific realism, namely *theory realism*, *entity realism*, and more recently, *structural scientific realism*, where

[1] See also Belot and Earman (1999).

the latter characterization, in its turn, has originated a division between the so-called *epistemic* structural realists and the *ontic* structural realists.

In synthesis, a theory realist defends the claim that the theories of a mature science and its laws are true in the limits of the approximation of a physical model or, in short, *approximatively true* (whatever "approximately" may mean in this context, a difficult problem that here we will not address). Entity realists claim that entities that are not *directly* observable with the naked eye (quarks, electrons, atoms, molecules, etc.) exist in a mind-independent fashion. Structural realists claim, with Poincaré, that while real objects will always be hidden from our eyes, "the true relations between these objects are the only reality we can attain": "...les rapports veritable entre ces objects sont la seule réalité que nous puissions atteindre" (Poincaré 1905, p. 162).

One may wonder whether these various forms of realism are logically independent of each other, as many philosophers have claimed. We believe that they are not. For example, it is not at all clear whether it is really possible, *pace* Hacking (1983), to defend any form of entity realism without also endorsing some form of theory realism [2].

Analogously, it is highly controversial whether one may have structural realism without also embarking *theory realism* or *entity realism* of some form. For a necessarily brief defence of the implication from structural realism to theory realism, consider the following remark. If: (i) the only reality we can know (as the epistemic structural realist has it) are the relations instantiated by existing but unknowable entities described by mathematically expressed laws (Worrall (1989), see also Morganti, forthcoming); and, (ii) the relations expressed by the equations of mathematical physics represent the *only* element of continuity across scientific revolutions; then clearly one must assume, together with Poincaré, that at least such equations are *true* ("les équations différentielle sont toujours vraie": Poincaré (1905) *ibid.*). To the extent that realism about laws entails theory realism, Poincaré's and Worrall's position seem to presuppose a form of realism about theories, an implication that in the philosophical literature has gone strangely unnoticed.

For a defence of the implication from structural realism to entity realism, suppose with the *ontic* structural realists that the relations referred to by mathematically formulated laws are knowable just because they exhausts what exists, so that entity realism is false. Alternatively, suppose with the *epistemic* structural realism that entity realism is epistemically unwarranted. In both cases, how can we endorse the existence of relations

[2] For a forceful defense of the view that unless we trust theoretical laws, we cannot choose between alternative explanations of the data in terms of rival models of theoretical entities, see Massimi (forthcoming)

without also admitting the existence of something that such relations relate (their *relata*), namely something carrying intrinsic, non-relational properties?[3]. Chakravartty (1998) and Cao (2003b), for instance, agree with the epistemic realist that our *knowledge* about unobservable entities is essentially structural, but refuse to dissolve physical entities into mathematical structure, thereby classifying themselves as entity realists, and endorsing the view that *structural realism entails entity realism.*[4] This is also the view we want to defend by considering the case study of the ontological status of points-events in classical GR: point-events are structurally individuated by the metric field, but (i) the metric field exists as an extended entity and (ii) the law governing its behavior must be regarded as approximately true.

2.1 What, exactly, is a physical structure?

It should be clear at this point that a decisive progress on the issues concerning structural realism presuppose a clarification of the following, crucial question: "what, exactly, is a physical structure ?" As we will see, such a question is also crucial to address the problem of the nature of point-events in classical GR. Much seem to depend on how we want to understand a structure in *physical* terms, since for our purpose the definition of a *mathematical* structure can be taken as sufficiently clear, at least if the latter is regarded as *a system of differential equations* plus some abstract object purporting to describe a physical system.

The problem is that it seems very difficult even to define a *physical* structure without bringing in its constituents, and thereby granting them existence. We will take this difficulty as a preliminary argument in favor of the implication that we want to defend, namely that structural realism implies entity realism, let alone theory realism.

For instance, if we preliminarily regard a structure as "a stable system of relations among a set of constituents", i.e., a class of entities (see Cao, 2003a, pp. 6-7; Cao, 2003b, pp.111) – where "entities" is deliberately left sufficiently vague in order to cover cases in which the members of the above class lack individuality as it is the case for quantum particles – we immediately take an important stance in the above debate. By adopting this definition in fact, we are already presupposing the independent existence of entities [the

[3]This worry has been expressed also by Redhead in private conversation. See French and Ladyman (2003, p. 41).

[4]Conversely, it is much less controversial to agree on the fact that a theory realist must be committed to structural realism about scientific laws, as well as to the existence of unobservable entities, since *holding a theory as approximately true implies believing in the referential power of its assumptions about unobservable entities.*

constituents], thereby ruling out of the game *a priori* the ontic structural realism defended by French and Ladyman (2003)). Analogously, the so-called "partial structure approach" (Bueno et.al , 2003a), according to which a structure is a set of individuals *together* with a family of partial relations defined over the set, seems to run an analogous risk, because the definition of a partial structure includes a set of individuals.

In particular, what is unclear to us is whether it makes sense to consider a physical, "holistic structure as [*ontologically*] prior to its constituents", as Cao has it (Cao , 2003b, p. 111)), by simply arguing that its constituents, "as placeholders, derive their meaning or even their existence from their function and place in the structure." While the thesis of *meaning* holism may be uncontroversial but clearly irrelevant in the present ontological discussion, one must ask how a placeholder can have any ontological function in an evolving network of relationships without possessing at least some intrinsic *non-relational properties*. While we can imagine that placeholders with different intrinsic properties can contribute the *same* function in the holistic network, so that the structural, relational properties empirically underdetermines the intrinsic properties of the placeholders, it seems that no placeholder can even *have* a function without possessing *some* intrinsic properties[5]

In a word, we believe that in order to clarify the meaning of "structure" in the philosophy of physics in general and in the philosophy of space and time in particular, it is essential to revert to the original meaning that "structuralism" had in linguistics or anthropology. In such contexts, structuralism originally referred to a sort of *holistic thesis about the identity of the members of a set of stable relations*, and was not conceived, as today sometimes is, as an attempt to eliminate the constituents. The crucial question to ask then becomes: given a certain physical theory, to what extent do the relational properties of a set of constituents contribute to fix their identity?

In the next section, we will try to answer this question by showing how the structural and holistic identity of spacetime points in GR does not force us to abandon the typical entity-realist's attitude toward the metric field *and its point-events*. Such an attitude is compatible with the fact that the points of *a bare manifold*, lacking intrinsic identity, are deprived, to put it with Einstein, of "the last remnant of physical objectivity".

[5]A property is intrinsic or non-relational if and only if its attribution does not presupposes the existence of any other entity. For instance, "being a father" is clearly extrinsic or relational, while "being square" is intrinsic.

3 A case study: the holistic and structural nature of general-relativistic spacetime in a class of models of GR

3.1 The Hole Argument and its consequences

In the recent years, the debate on spacetime substantivalism in GR has been revived by a seminal paper by John Stachel (1980), followed by Earman and Norton's philosophical argument against manifold spacetime substantivalism (1987). Both papers addressed Einstein's famous Hole Argument ("Loch Betrachtung") of 1913-1915 (Einstein (1914, 1916)), which was soon to be regarded by virtually all participants to the debate[6] as being intimately tied with the nature of space and time, at least as they are represented by the mathematical models of GR.

In a nutshell, a mathematical model of GR is specified by a four-dimensional mathematical manifold $\mathcal{M}_4$ and by a metrical tensor field g, where the latter dually represents *both* the chrono-geometrical structure of spacetime *and* the potential for the inertial-gravitational field. Non-gravitational physical fields, when they are present, are also described by dynamical tensor fields, which appear as sources of the Einstein equations.

The above-emphasized dual role of the metric field has recently generated a conceptual debate, that can be summarized by the following question: *which is the best candidate to interpret the role of space and time in GR, the manifold or the (manifold plus) the metric?* Those opting for the bare manifold $\mathcal{M}_4$ (like Earman and Norton) correctly point out that g cannot be understood as interpreting the role of the "empty spacetime" of the traditional debate: by embodying the potential of the gravitational field, g is to be regarded as a (special) type of "physical field". Those opting - much more reasonably, in our opinion - for "the manifold plus the metric field" (Maudlin (1990), Stachel (1993)) also correctly point out that the metric provides the chrono-geometrical structure as well as, most significantly, the causal structure of spacetime. To the extent that one can see good arguments for *both* options - or even if, as we believe, the second option is the only plausible one - such an "ambiguous" role of the metric seems to provide one of the main arguments to claim that the early-modern debate between substantivalists and relationists is now "outdated", because in GR it does not admit of a clear formulation (Rynasiewicz 1996).

Before agreeing on this skeptical remark, however, it is appropriate to

[6]For example: Butterfield (1989); Earman (1989); Maudlin (1990); Norton (1987); Norton (1992, 1993).

go over the Hole Argument one more time, in order to show how it should be really tackled and what implications our proposed solution has on the above debate. Let us assume that $\mathcal{M}_4$ contains a *hole* $\mathcal{H}$: that is, an open region where all the non-gravitational fields vanish. On $\mathcal{M}_4$ we can define an *active* diffeomorphism D_A (see, for example, Wald (1984)) that re-maps the points inside $\mathcal{H}$, but blends smoothly into the identity map outside $\mathcal{H}$ and on the boundary. By construction, for any point $x \in \mathcal{H}$ we have (in the abstract tensor notation) $g'(D_A x) = g(x)$, but of course $g'(x) \neq g(x)$ (in the same notation). The crucial fact to keep in mind at this point is that the Einstein equations are generally covariant: this means that if g is one of their solutions, so is the *drag-along* field $g' \equiv D_A^* g$.

What is the correct interpretation of the new field g'? Clearly, the transformation entails an *active redistribution of the metric over the points of the manifold in* $\mathcal{H}$, so the crucial question is whether and how the points of the manifold are primarily *individuated*. Now, *if* we think of the points of $\mathcal{H}$ as *intrinsically individuated physical events*, where "intrinsic" means that their identity is independent of the metric - a claim that is associated with *manifold substantivalism* - then g and g' must be regarded as *physically distinct* solutions of the Einstein equations (after all, $g'(x) \neq g(x)$ at the *same* point x). This is a devastating conclusion for the causality, or better, the *determinateness*[7] of the theory, because it implies that, even after we completely specify a physical solution for the gravitational and non-gravitational fields outside the hole - in particular, on a Cauchy surface for the initial value problem - we are still unable to predict uniquely the physical solution within the hole. Clearly, if general relativity has to make any sense as a *physical* theory, there must be a way out of this foundational quandary, *independently of any philosophical consideration.*

According to Earman and Norton (1987), the way out of the hole argument lies in abandoning manifold substantivalism: they claim that if diffeomorphically-related metric fields were to represent different physically possible worlds, then GR would turn into an *indeterministic* theory. And since the issue of whether determinism holds or not at the *physical* level cannot be decided by opting for a *metaphysical* doctrine like manifold substantivalism, they conclude that one should go for spacetime relationism.

Now, *if* relationism in GR were entailed by the claim that diffeomorphically related mathematical models don't represent physically distinct solutions, most physicists would count themselves as relationists. After all,

[7] We prefer to avoid the term *determinism*, because we believe that its metaphysical flavor tends to overstate the issue at stake. This is especially true if *determinism* is taken in opposition to *indeterminism*, which is not mere absence of *determinism*.

the assumption that *an entire equivalence class of diffeomorphically related mathematical solutions represents only one physical solution* is regarded as the most common technical way out of the strictures of the Hole Argument (in the philosophical literature such an assumption is known, after Earman and Norton (1987), as *Leibniz equivalence*). However, we believe that it is not at all clear whether Leibniz equivalence really grinds corn for the relationist's mill, since the spacetime substantivalist can always ask: (1) why on earth should we identify *physical* spacetime with the bare manifold *deprived of the metric field*? (2) Why should we assume that the points of the mathematical manifold have an intrinsic *physical* identity independently of the metric field?

In order to lay our cards on table with respect to these (rethorical) questions, we start from the latter in order to note an unfortunate ambiguity in the use of the term "spacetime points": sometimes it refers to elements of the mathematical structure that is the first conceptual "layer" of the spacetime model (the manifold), sometimes it refers to the points interpreted as *physical* events. To remedy this situation, we stipulate to use the term *point-events* to refer to physical events and simply *points* to refer to elements of the mathematical manifold. In this respect we just want to add that in the mathematical literature about topological spaces, it is implicitly assumed that their elements are already distinguished. Otherwise, one could not even state the Hausdorff condition, let alone define mappings, homeomorphisms, or active diffeomorphisms. It is well known, however, that the points of a *homogeneous* space (as the manifold would be prior to the introduction of the metric) cannot have any intrinsic *individuality*. As Hermann Weyl (1946) put it:

> There is no distinguishing objective property by which one could tell apart one point from all others in a homogeneous space: at this level, fixation of a point is possible only by a *demonstrative act* as indicated by terms like "this" and "there."

Quite aside from the phenomenological stance implicit in Weyl's words, there is only one way to individuate points *at the mathematical level* that we are considering, namely by (arbitrary) *coordinatization*. By using coordinates, we transfer the individuality of n-tuples of real numbers to the elements of the topological set.

As to the first question above, we will have to limit ourselves to the following remarks: although the metric tensor field, *qua* physical field, cannot be regarded as the traditional *empty container* of other physical fields, we believe that it has *ontological priority* over all other fields. This preeminence has various reasons (Pauri, 1996), but the most important is that the metric field tells all other fields how to move causally. In agreement also with the general-relativistic practice of not counting the gravitational energy induced by the metric as a component of the total energy, we believe that physical spacetime should be identified with the manifold endowed with its metric, thereby leaving the task of representing matter to the stress-energy tensor.

In consonance with this choice, Stachel[8] has provided a very enlightening analysis of the conceptual consequences of *modern* Leibniz equivalence. Stachel stresses that asserting that g and $D_A^* g$ represent *one and the same gravitational field* implies that *the mathematical individuation of the points of the differentiable manifold by their coordinates has no physical content until a metric tensor is specified*. Stachel stresses that if g and $D_A^* g$ must represent the same gravitational field, they cannot be physically distinguished in any way. Consequently, when we act on g with D_A^* to create the *drag-along* field $D_A^* g$, no element of physical significance can be left behind: in particular, *nothing* that could identify a point x of the manifold as the *same* point of spacetime for both g and $D_A^* g$. Instead, when x is mapped onto $x' = D_A x$, it *brings over its identity*, as specified by $g'(x') = g(x)$.

These remarks led Stachel to the important conclusion that *vis á vis* the physical point-events, the metric plays in fact the role of *individuating field*. More than that, even the topology of the underlying manifold cannot be introduced independently of the specific form of the metric tensor, a circumstance that makes Earman and Norton's choice of interpreting the mere topological and differentiable manifold as *spacetime* (let alone *substantival spacetime*) even more implausible. More precisely, Stachel suggested that this individuating role should be implemented by four invariant functionals of the metric, already considered by Komar (1955).

However, he did not follow up on such a suggestion, something that we will do in the next section, with the aim of further clarifying the nature of the physical point-events. We believe in fact that their status as the intrinsic elements of physical spacetime needs further analysis,[9] especially in view of the questions of structural realism and spacetime substantivalism that we raised before.

3.2 The dynamical individuation of point-events

3.2.1 *Pure gravitational field without matter*

It is well known that only some of the ten components of the metric are physically essential: it seems then plausible to suppose that only this subset can act as individuating field, and that the remaining components play a

[8]See Stachel (1980; 1986; 1993.)

[9]For a similar viewpoint, see Friedman (2001).

different role.

Bergmann and Komar (1960) and Bergmann (1960; 1962; 1977) introduced the notion of *intrinsic invariant pseudo-coordinates* already in 1960. These authors noted that for a vacuum solution of the Einstein equations, there are exactly four functionally independent *scalars* that can be written using the lowest possible derivatives of the metric[10]. These are the four Weyl scalars (the eigenvalues of the Weyl tensor), here written in Petrov's compressed notation,

$$\begin{aligned} w_1 &= \mathrm{Tr}\,(gWgW), \\ w_2 &= \mathrm{Tr}\,(gW\epsilon W), \\ w_3 &= \mathrm{Tr}\,(gWgWgW), \\ w_4 &= \mathrm{Tr}\,(gWgW\epsilon W), \end{aligned} \tag{1}$$

where g is the *four*-metric, W is the Weyl tensor, and ϵ is the Levi–Civita totally antisymmetric tensor.

Bergmann and Komar then propose to build a set of *intrinsic pseudo-coordinates* for the point–events of spacetime as four suitable functions of the w_T,

$$\hat{I}^{[A]} = \hat{I}^{[A]}[w_T[g(x), \partial g(x)]], \quad A = 0, 1, 2, 3. \tag{2}$$

Indeed, under the *non-restrictive* hypothesis that *no* spacetime symmetries are present - in an analysis of the physical individuation of points, we must consider *generic* solutions of the Einstein equations rather than the null-measure set of solutions with symmetries - the $\hat{I}^{[A]}$ can be used to *label* the point-events of spacetime, at least locally. Since they are scalars, the $\hat{I}^{[A]}$ are invariant under passive diffeomorphisms (therefore they do not define a coordinate chart in the usual sense), and by construction they are also constant under the *drag-along* of tensor fields induced by active diffeomorphisms.

At this stage, however, it is far from clear how to explicitly use these intrinsic coordinates to solve the puzzles raised by the Hole Argument, *especially in view of its connection with the Cauchy problem.* For it is essential to realize that the *Hole Argument is inextricably entangled with the initial-value problem of general relativity*, although, strangely enough, it has never been explicitly and systematically discussed in this context. The main reason for this neglect is plausibly given by the fact that most authors have

[10]The fact that there are just *four* independent invariants for the vacuum gravitational field is not a coincidence. On the contrary, it is crucial for the purpose of point individuation and for the gauge-fixing procedure that we will propose. After all, recall that in general spacetimes with matter there are 14 invariants of this kind!

implicitly adopted the Lagrangian approach (or the *manifold way*), in which the initial-value problem turns out to be intractable because of the non-hyperbolic nature of Einstein's equations. This is also the main reason why we are obliged to turn to the Hamiltonian methods[11].

Three circumstances make the recourse to the Hamiltonian formalism especially propitious.

1. It is only within the Hamiltonian approach that we can separate the *gauge variables* - which carry the descriptive arbitrariness of the theory - from the *Dirac observables*, which are gauge invariant quantities and are subject to hyperbolic (and therefore "*determinate*" or "*causal*" in the customary sense) evolution equations.

2. In the context of the Hamiltonian formalism, we can resort to the Bergmann and Komar's theory of "general coordinate group symmetries" (1972) to clarify the significance of active diffeomorphisms as *on-shell dynamical symmetries* of the Einstein equations.

3. With respect to our main purpose of trying to understand the nature of point-events in classical GR, it is only within the ADM Hamiltonian formulation of GR that we can introduce a *peculiar gauge-fixing* that can be invoked for their *physical (dynamical) individuation*[12].

3.2.2 *Pure gravitational field: the ADM slicing of spacetime and the canonical reduction*

The ADM (Arnowitt, Deser and Misner, 1962) Hamiltonian approach starts with a slicing of the 4-dimensional manifold $\mathcal{M}_4$ into constant-time hypersurfaces Σ_τ, indexed by the *parameter time* τ, each equipped with coordinates σ^a (a = 1,2,3) and a three-metric ${}^3g$ (in components ${}^3g_{ab}$). In order to obtain the 4-geometry, we start at a point on Σ_τ, and displace it infinitesimally in a direction that is normal to Σ_τ. The resulting change in τ can be written as $d\tau$ = $Nd\tau$, where N is the so-called *lapse function.* In a generic coordinate system, such a displacement will also shift the spatial coordinates: $\sigma^a(\tau + d\tau) = \sigma^a(\tau) + N^a d\tau$, where N^a is the *shift vector.* Then the interval between (τ, σ^a)

[11]It is not by chance that the modern treatment of the initial value problem within the Lagrangian configurational approach (Friedrich and Rendall , 2000) must in fact mimic the Hamiltonian methods.

[12]The individuation procedure outlined here is based on the technical results obtained by Lusanna and Pauri (Lusanna and Pauri, 2003), see also Pauri and Vallisneri, (2002), hereafter quoted as LP and PV, respectively.

and $(\tau + d\tau, \sigma^a + d\sigma^a)$ results: $ds^2 = N^2 d\tau^2 - {}^3g_{ab}(d\sigma^a + N^a d\tau)(d\sigma^b + N^b d\tau)$. The *configurational* variables N, N^a, ${}^3g_{ab}$ together their 10 conjugate momenta, index a 20-dimensional phase space[13]. Expressed (modulo surface terms) in terms of the ADM variables, the Einstein-Hilbert action is a function of N, N^a, ${}^3g_{ab}$ and its first time derivative, or equivalently of N, N^a, ${}^3g_{ab}$ and the extrinsic curvature ${}^3K_{ab}$ of the hyper-surface Σ_τ, considered as an embedded manifold.

Since the original Einstein's equations are not hyperbolic, it turns out that the canonical momenta are not all functionally independent, but satisfy four conditions known as *primary* constraints. Other four, *secondary* constraints arise when we require that the primary constraints be preserved through evolution (the secondary constraints are called the *superhamiltonian* $\mathcal{H}_0 \approx 0$, and the *supermomentum* $\mathcal{H}_a \approx 0$, $(a = 1, 2, 3)$ constraints, respectively). The eight constraints are given as functions of the canonical variables that vanish on the constraint surface[14]. The existence of such constraints imply that that not all the points of the 20-dimensional phase space represent physically meaningful states: rather, we are restricted to the *constraint surface* where all the constraints are satisfied, i.e., to a 12-dimensional (20 - 8) *surface* which, on the other hand, does not possess the geometrical structure of a true phase space. When used as generators of canonical transformations, the eight constraints map points on the constraint surface to points on the same surface; these transformations are known as *gauge transformations*.

To obtain the correct dynamics for the constrained system, we need to modify the Hamiltonian variational principle to enforce the constraints; we do this by adding the *primary* constraint functions to the Hamiltonian, after multiplying them by arbitrary functions (the *Lagrange–Dirac multipliers*). If, following Dirac, we make the reasonable demand that the evolution of all *physical variables* be unique - otherwise we would have *real* physical variables that are indeterminate and therefore neither *observable* nor *measurable* - then the points of the constraint surface lying on the same *gauge orbit*, i.e. linked by gauge transformations, *must describe the same physical state*[15]. Conversely, only the functions in phase space that are invariant with respect to gauge transformations can describe physical quantities.

To eliminate this ambiguity and create a one-to-one mapping between

[13] Of course, all these *variables* are in fact *fields*.

[14] Technically, these functions are said to be *weakly* zero. Conversely, any *weakly* vanishing function is a linear combination of the *weakly* vanishing functions that define the constraint surface.

[15] Actually in GR, there are further and subtler complications concerning the geometric significance of the whole set of such transformations and the existence of geometrically-inequivalent states (see LP and PV).

points in the phase space and physical states, we must impose further constraints, known as *gauge conditions* or *gauge-fixings.* The gauge-fixings can be implemented by arbitrary functions of the canonical variables, except that they must define a *reduced phase space* that intersects each gauge orbit exactly once (*orbit conditions*). The number of independent gauge-fixing must be equal to the number of independent constraints (i.e. 8 in our case). The canonical reduction proceeds by a cascade procedure: the gauge-fixings to the *super-hamiltonian* and *super-momentum* come first (call it Γ_4); then the requirement of their time constancy fixes the gauges with respect to the primary constraints. Finally the requirement of time constancy for these latter gauge-fixings determines the Lagrange multipliers. Therefore, the first level of gauge-fixing gives rise to a *complete* gauge-fixing, say Γ_8, and is sufficient to remove all the gauge arbitrariness.

The Γ_8 procedure reduces the original 20-dimensional phase space to a *reduced phase-space* Ω_4 having 4 degrees of freedom per point (12 - 8 gauge-fixings). Abstractly, the reduced phase-space is the quotient of the constraint surface by the 8-dimensional group of gauge transformations and represents the space of variation of the true degrees of freedom of the theory. Ω_4 inherits a symplectic structure (*Dirac brackets*) from the original Poisson brackets and is a *true phase-space* coordinatized by four *Dirac observables* (two configurational and two momentum variables): call such field observables q^r, p_s (r,s = 1,2). These observables carry the physical content of the theory in that they represent the *intrinsic degrees of freedom of the gravitational field* (recall that at this stage we are dealing with a pure gravitational field). Concretely, for any *complete gauge fixing* Γ_8, we get a Γ_8-dependent *copy* of the abstract Ω_4 as a coordinatized realization of it in terms of Dirac observables. Though the Dirac observables are gauge-invariant, their functional form in terms of the original canonical variables depends upon the gauge, so that such observables - a priori - are neither tensors nor invariant under D_P. Yet, *off shell*, barring sophisticated mathematical complications, *any two copies of* Ω_4 *are diffeomorphic images of one-another.* After the canonical reduction is performed, the theory is completely determined: each physical state corresponds to one and only one set of canonical variables that satisfies the constraints and the gauge conditions.

It is important to understand qualitatively the geometric meaning of the eight infinitesimal *off-shell* Hamiltonian gauge transformations and thereby the geometric significance of the related gauge-fixings. i) The transformations generated by the four *primary* constraints modify the *lapse* and *shift* functions which, in turn, determine how densely the space-like hyper-surfaces Σ_τ are distributed in spacetime and also the *gravito-magnetism* conventions; ii) the transformations generated by the three *super-momentum* constraints induce a transition on Σ_τ from a given 3-coordinate system to another; iii) the transformation generated by the *super-hamiltonian* constraint induces a transition from a given *a-priori* "form" of the 3+1 splitting of M^4 to another one, by operating deformations of the space-like hyper-surfaces in the normal direction. The manifest effect of the related gauge-fixings emerges only at the end of the canonical reduction and after the solution of the Einstein-Hamilton equations has been worked out (i.e., *on shell*), since the role of the gauge-fixings is essentially that of choosing the *functional form* in which all the gauge variables depend upon the *Dirac observables*. Therefore, it is only after the initial conditions for the *Dirac observables* have been arbitrarily selected on a Cauchy surface that the whole four-dimensional chrono-geometry, (including - unlike the special relativistic case - all the relativistic "conventions") is *dynamically determined*.

Two important points must be stressed.

First, before the gauge-fixings are implemented, in order to carry out the canonical reduction *explicitly*, we have to perform a basic canonical transformation, the so-called Shanmugadhasan transformation, bringing from the original canonical variables to a new basis including the Dirac observables in a canonical subset in such a way that they have zero P.B. with all the other variables[16]. Now, the Shanmugadhasan transformation is highly *non-local* in the metric and curvature variables: even though, at the end, for any τ, the Dirac observables are *fields* indexed by the coordinate point σ^a, they are in fact *highly non-local functionals of the metric and the curvature over the whole surface* Σ_τ. We can write, *symbolically*:

$$\begin{aligned} q^r(\tau,\vec{\sigma}) &= {\cal F}_{[\Sigma_\tau]}{}^r[(\tau,\vec{\sigma})|\ {}^3g_{ab}(\tau,\vec{\sigma}), {}^3\pi^{cd}(\tau,\vec{\sigma})] \\ p_s(\tau,\vec{\sigma}) &= {\cal G}_{[\Sigma_\tau]_s}[(\tau,\vec{\sigma})|\ {}^3g_{ab}(\tau,\vec{\sigma}), {}^3\pi^{cd}(\tau,\vec{\sigma})], \quad r,s=1,2. \end{aligned} \tag{3}$$

Second: since the original canonical Hamiltonian in terms of the ADM variables is zero, it happens to be written solely in terms of the eight constraints and Lagrangian multipliers. This means, however, that this Hamiltonian generates purely harmless gauge transformations connecting different admissible spacetime 3+1 splittings, so that *it cannot engender any real temporal change* (in this connection see Earman (2002); Belot and Earman (1999, 2001)). The crucial point, however, is that, *in the case of the globally-hyperbolic non-compact spacetimes*, defined by suitable boundary conditions

[16]In practice, this transformation requires the solution of the super-hamiltonian constraint. Even if so far this result has proved elusive, the relevant properties of the new basis can nevertheless be worked out (De Pietri et al. (2002)).

and asymptotically flat at spatial infinity, just as those we are dealing with in this work[17], *internal mathematical consistency* entails that the generator of temporal evolution is the so-called *weak ADM energy*, which is obtained by adding the so-called De-Witt boundary surface term to the canonical Hamiltonian[18]. Indeed, this quantity *does generate real temporal modifications of the canonical variables*. Thus, the final Einstein-Hamilton-Dirac equations for the Dirac observables are

$$\dot{q}^r = \{q^r, H_{\rm ADM}\}^*, \quad \dot{p}_s = \{p_s, H_{\rm ADM}\}^*, \quad r, s = 1, 2, \tag{4}$$

where $H_{\rm ADM}$ is intended as the restriction of the *ADM weak energy* to Ω_4 and where the $\{\cdot,\cdot\}^*$ are the Dirac brackets.

3.2.3 *Pure gravitational field: the metrical fingerprint*

We can now turn to briefly illustrate the process of dynamical individuation of point-events. First of all we exploit a technical result by Bergmann and Komar (Bergmann and Komar, 1960), namely the fact that the four Weyl scalar invariants (1), *once re-expressed in terms of the ADM variables*, turn out to be independent of the *lapse function* N and the *shift vector* N^a. This means that the *intrinsic pseudo-coordinates* are in fact functionals of the variables ${}^3g_{ab}$ and ${}^3K_{ab}$ only . Then we write

$$\hat{I}^{[A]}[w_T(g, \partial g)] \equiv \hat{Z}^{[A]}[w_T({}^3g, {}^3\pi)], \quad A = 0, 1, 2, 3; \tag{5}$$

and select a *completely arbitrary* coordinate system $\sigma^A \equiv [\tau, \sigma^a]$ adapted to the Σ_τ surfaces. Finally we apply the peculiar gauge fixing Γ_4 defined by

$$\chi^A \equiv \sigma^A - \hat{Z}^{[A]}[w_T[({}^3g(\sigma^B), {}^3\pi(\sigma^D)]] \approx 0, \quad A = 0, 1, 2, 3, \tag{6}$$

to the *super-hamiltonian* (A = 0) and the *super-momentum* (A = 1,2,3) constraints. This is indeed a good gauge-fixing provided that the functions $\hat{Z}^{[A]}$ are chosen to satisfy the fundamental *orbit conditions* $\{\hat{Z}^{[A]}, \mathcal{H}_B\} \neq 0, \quad (A, B = 0, 1, 2, 3)$, which ensure the independence of the χ^A and carry information about the Lorentz signature. At the end of the gauge-fixing procedure Γ_8, the effect is that the values (i.e. the *evolution* throughout the mathematical spacetime $\mathcal{M}_4$) of the *Dirac observables*, whose dependence on space (and on parameter time) is indexed by the chosen coordinates

[17] These spacetimes are known as Christodoulou-Klainermann spacetimes, (Christodoulou and Klainerman , 1993).

[18] The ADM energy is a Noether constant of motion representing the *total mass* of the "universe", just one among the ten asymptotic Poincaré "charges". The mathematical background of this result can be found in Lusanna (2001) and references therein.

σ^A, *reproduces* precisely the σ^A as the Bergmann–Komar *intrinsic pseudo-coordinates, in the chosen gauge* Γ_8:

$$\sigma^A = \hat{Z}^{[A]}[w_T(q^r(\sigma^B),\ p_s(\sigma^C)|\Gamma_8)], \quad A = 0, 1, 2, 3; \tag{7}$$

where the notation $w_T(q, p|\Gamma_8)$ represents the functional form that the Weyl scalars w_T assume in the chosen gauge.

In the language of constraint theory, after the canonical reduction is performed - and *only for the solutions of the equations of motion* - (7) becomes a *strong* relation[19]. *Such a strong relation is in fact an identity with respect to the σ^A, and amounts to a "definition" of the coordinates σ^A as four scalars providing a physical individuation of any point–event, in the gauge-fixed coordinate system, in terms of the true dynamical gravitational degrees of freedom.* The price that we have paid for this achievement is of course that we have broken general covariance!

At first, this result may sound surprising: *qua* diffeomorphism-invariant quantities, the intrinsic pseudo- coordinates can be forced within a coordinate system corresponding to any experimental arrangement. From the Hamiltonian viewpoint, however, they are necessarily gauge-dependent functionals[20].

Note that the virtue of this elaborate construction does *not* depend on the selection of a set of physically preferred coordinates, because by modifying the functions $I^{[A]}$ of (2) we have the possibility of implementing *any* coordinate transformation. So diffeomorphism-invariance reappears under a different suit: we find exactly the same functional freedom of D_P in the functional freedom of the choice of the *pseudo-coordinates* $Z^{[A]}$ (i.e., of the gauge fixing Γ_4). Thus, it turns out that, *on shell*, at the Hamiltonian as well as the Lagrangian level, *gauge fixing is clearly synonymous with the selection of manifold coordinates* (recall the geometric meaning of the *off shell* gauge transformations). Yet, we can now claim that *any* coordinatization of the manifold can be seen as embodying the physical individuation of points, because it can be implemented - locally at least - as the Komar–Bergmann intrinsic pseudo-coordinates after we choose the correct $Z^{[A]}$ and we select the proper gauge.

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[19]This means that the relation is expressed by functions which not only vanish on the constraint surface but also have all vanishing derivatives in directions normal to the constraint surface.

[20]It is not known so far whether the 16 canonical variables of the Shanmugadhasan basis - which include the *Dirac observables* - could be replaced by 16 diffeomorphism-invariant quantities - which, in particular - would include *tensorial Dirac observables.* This is an important question which however could be answered in the positive if a *main conjecture* advanced in LP would turn out to be true. In that case we would have a *fully objective* (tensor-covariant) dynamical individuation of point-events.

The effect of the active diffeomorphisms D_A of $\mathcal{M}_4$ has been that of disclosing the Leibniz equivalence underlying the Hole Argument within the Lagrangian description. By now it should be clear that the Hole Argument has nothing to do with an alleged *indeterminism* of GR as a dynamical theory. Once we choose a complete gauge-fixing Γ_8 and we assign the initial values for such observables on a Cauchy surface Σ_{τ_0}, no such indeterminism appears to affect the Hamiltonian-Dirac observables .

This result, however, entails that one crucial point has still to be clarified. On the one hand, at the Lagrangian level, passive diffeomorphisms D_P are devoid of any physical significance, while active diffeomorphisms D_A differing from the identity within the Hole relate distinct Leibniz-equivalent solutions that have the same initial Cauchy conditions, a fact that allegedly could have had physical implications. On the other hand, at the Hamiltonian level, different solutions to the (deterministic) Einstein-Hamilton equations for the Dirac observables, corresponding to the same initial conditions on a Σ_{τ_0} Cauchy surface, are related by harmless gauge-transformations which are indeed *passive* by definition. Moreover, we have stated that a complete gauge-fixing is equivalent to the choice of a *spatiotemporally extended physical laboratory* [21]. It is clear that for a full understanding of the role played by *active* diffeomorphisms in the Hole Argument, it is necessary that they be also interpretable in some way as the *manifold-way* counterparts of suitable Hamiltonian gauge transformations. Here, we can only limit ourselves to state that this is actually possible by resorting to a nearly forgotten paper by Bergmann and Komar's (1972) about the general coordinate-group symmetries of Einstein equations. In fact, it turns out (see LP and PV) that active diffeomorphisms can be viewed as passive transformations only on the conjunction of the spacetime manifold and the function space of the metric fields. This entails that *solutions of Einstein's equations that within the Hole differ by an active transformation on manifold points, at the Hamiltonian level - the proper one for a correct treatment of the initial value problem - are simply solutions differing by a harmless gauge transformation.*

In a word, since outside and inside the Hole the gauge must be completely

[21]Recall that canonical reduction, which creates the distinction between gauge-dependent quantities and Dirac observables, is made *off shell*, i.e., *before* solving the equations of motion. On the other hand, the metric and the extrinsic curvature (and thereby also the complete definition of the Σ_τ embedding) are not completely defined until the Einstein-Hamilton equations are solved and the contribution of the Dirac observables calculated. Given the geometrical meaning of the gauge fixings, at this stage Γ_8 includes a choice of the conventions about *global simultaneity* and *gravito-magnetism*, together with the implicit definition of two *global congruences of time-like observers* and *an atlas of coordinate charts on the spacetime manifold*: in particular, *within the Hole* (see LP).

fixed *before* solving the initial-value problem and therefore find the solution of the field equation, *it makes little sense to apply active diffeomorphisms to an already generated solution to obtain an allegedly "different" spacetime.* Conversely, it should be possible to generate these "different" solutions by appropriate choices of the initial gauge fixing (the functions $\hat{Z}^{[A]}$).

3.2.4 *Gravitational field cum matter and the spacetime holistic texture*

In conclusion, what is relevant to our discussion is that *there is a peculiar class of gauge-fixings*, (6), that is instrumental both to the solution of the Cauchy problem and to the *physical individuation of spacetime point-events.* We propose to call this gauge the *intrinsic individuating gauge.* As we have seen above, each of the point–events of spacetime is endowed with its own physical individuation (the right metrical fingerprint!) as the value of the four scalar functionals of the *Dirac observables* (just four!), which describe the dynamical degrees of freedom of the gravitational field. It is important to stress that, due to the independence of the *pseudo-coordinates* from the *lapse* and *shift* functions, these degrees of freedom *are inextricably entangled with the structure of the whole 3-metric and 3-curvature in a way that is strongly gauge dependent* but *does not involve geometrical elements external to the Cauchy surface.* This result appears as an instantiation of three-dimensional *holism.* Yet, since (7) is four-dimensional and includes the temporal gauge (fixed by the scalar $Z^{[0]}$), as soon as the Einstein-Hamilton equations are solved and the evolution in τ of the *Dirac observables* fully determined, a peculiar instantiation of four-dimensional *stratified holism* is recovered. At this point we could even say that the existence of physical *point-events* in our models of general relativity appears to be synonymous with the existence of the Dirac observables for the gravitational field, and advance the *ontological* claim that - physically - *Einstein's vacuum spacetime is literally identifiable with the autonomous degrees of freedom of such structural field,* while the specific (gauge-dependent) functional form of the *intrinsic pseudo-coordinates* map specifically such coordinates into the manifold's points. The intrinsic gravitational degrees of freedom are - as it were - *fully absorbed in the individuation of point-events.* Thus, in this way, *point-events also keep a special kind of intrinsic properties*[22].

On the other hand, with respect to the physical interpretation of all the variables implied in the Hamiltonian approach to GTR, it can be argued (see LP) that while the Dirac observables essentially describe *generalized*

[22]Of course, once Einstein's equations have been solved, the metric tensor and all of its derived quantities, in particular the light-cones structure, can be re-expressed in terms of *Dirac observables* in a gauge-fixed functional form.

tidal effects of the gravitational field, the gauge variables, considered *off shell*, embody *generalized inertial effects* connected to the definition of the *laboratory* in which measurements take place, i.e. in which the gravitational phenomena manifestly *appear*.

Let us now briefly look at the most general case of ADM models of GR with *matter fields*, taking proper notice of the fact that we are still working with globally hyperbolic pseudo-Riemannian 4-manifolds $\mathcal{M}_4$ which are asymptotically flat at spatial infinity. The introduction of matter has the effect of modifying the Riemann and Weyl tensors, namely the curvature of the 4-dimensional substratum, and to allow a measure of the gravitational field in a geometric way (for instance through effects like the geodesic deviation equation). In the presence of matter, we have *Dirac observables for the gravitational field* and *Dirac observables for the matter fields*. As it is to be expected, however, even the functional form of gravitational observables *is modified* (relative to the vacuum case) by the presence of matter. Since the *gravitational* Dirac observables will still provide the individuating fields for point-events according to the conceptual procedure presented in this paper, *matter will come to influence the very physical individuation of spacetime point-events*. Yet, *the ontological conclusions reached above are not altered at all*.

Finally, even in the case with matter, time evolution is still ruled by the *w*eak ADM energy rather than by the simple canonical Hamiltonian. Therefore, the temporal variation corresponds to a *real change* and not merely to a harmless gauge transformation as in other models of GR. These latter include, for instance, the spatially compact spacetime without boundary (or simply closed models) which are exploited by Earman in his *Thoroughly Modern McTaggart* (2002). We acknowledge that the validity of our results is restricted to the class of models of GR we worked with. Yet, we are interested in a instantiation of a question of principle, and we want to argue that there is a basic class of models of GR embodying both a *real notion of temporal change* and a *new structuralistic and holistic view of spacetime.*

3.2.5 *The empirical side: the closure of the epistemic circuit*

As shown in PL and LP, in the absence of a dynamical theory of measurement, the epistemic circuit of GR can be approximately closed via an experimental three-steps procedure that, starting from concrete radar measurements and using test-objects, ends up in a complete and empirically coherent *intrinsic individuating gauge-fixing.*

4 Developing hints for the Quantum Gravity Program

Let us close our analysis with some hints for the quantum gravity program that are suggested by the above results. As is well-known, there are today two inequivalent approaches: i) the perturbative background-dependent *string* formulation, on a Fock space containing elementary particles; ii) the non-perturbative background-independent *loop* quantum gravity formulation, based on the non-Fock so-called *polymer* Hilbert space. The latter approach still fails to accommodate elementary particles, although Ashtekar has advanced some suggestions to define a *coarse-grained structure* as a bridge between standard *coherent states* in Fock space and some *shadow states* of the discrete quantum geometry associated to *polymer* Hilbert space.

Now, let us point out that (7) is a numerical identity that has an *in-built non-commutative structure*, deriving from the Dirac–Poisson structure on its right-hand side. The individuation procedure we have proposed transfers, as it were, the non-commutative Poisson-Dirac structure of the Dirac observables onto the individuated point-events, even if, of course, the coordinates on the l.h.s. of the identity (7) are c-numbers quantities. One could guess that such a feature might deserve some attention in view of quantization, for instance by maintaining that the identity (7) could still play some role at the quantum level. We will assume here for the sake of argument that the *main conjecture* advanced by Lusanna and Pauri (see footnote 20) is verified, so that all the quantities we consider are manifestly covariant.

Let us first lay down some qualitative premises concerning the status of Minkowski spacetime in relativistic quantum field theory (RQFT): call it *micro-spacetime* (see Pauri (2000)). Such a status is indeed quite peculiar. Since it is introduced into the theory through the group-theoretical requirement of the relativistic invariance of the *statistical* results of measurements with respect to the choice of *macroscopic reference frames*, the micro spacetime is therefore *anchored* to the macroscopic, medium-seized objects that asymptotically define the experimental conditions in the laboratory.[23] Thus, the spatiotemporal properties of the *micro Minkowski manifold*, including its basic causal structure, are, as it were, projected onto it *from outside*.

In classical field theories spacetime points play the role of individuals and we have seen how point-events can be individuated dynamically in a richer

[23] It is just in this asymptotic sense that a physical meaning is attributed to the classical spatiotemporal *coordinates* upon which the quantum fields' operators depend as *parameters*.

and holistic way. No such possibility, however, is consistently left open in a non-metaphorical way in RQFT. From this point of view, Minkowski's *micro-spacetime* in RQFT is in a worse position than classical general relativistic spacetime: it lacks the existence of Riemannian *intrinsic pseudo-coordinates*, as well as of all the non-dynamical (better, operational and pragmatic) additional macroscopic elements that are used for the individuation of its points, like rigid rods and clocks in rigid and non-accelerated motion, or various combinations of *genidentical* world-lines of free test particles, light rays, clocks, and other test devices.

Summarizing, *Minkowski's micro-spacetime* seems to be essentially functioning like an instrumental but *external translator* of the symbolic structure of quantum theory into the *causal* language of the macroscopic, irreversible traces constituting the experimental findings within *macro-spacetime*. Such an *external translator* should be regarded as an epistemic precondition for the formulation of RQFT in the sense of Bohr, independently of one's attitude towards the measurement problem in quantum mechanics.

Thus, barring macroscopic Schrödinger's cat-like states of the would-be quantum spacetime, any conceivable formulation of a quantum theory of gravity would have to respect, at the *operational* level, the *epistemic priority* of a classical spatiotemporal continuum. In fact, the possibility of referring *directly* to "the quantum structure of spacetime" faces at the very least a serious conceptual difficulty, concerning the *localization* of the gravitational field: what does it mean to talk about the *values* of the gravitational field *at a point* if the metric field itself is subject to quantum fluctuations? How could we identify point-events? In this case, we could no longer tell whether the separation between two points is space-like, null or time-like since quantum fluctuation of the metric could exchange past and future.

Accordingly, in order to give physical and operational meaning to the spatiotemporal language, we would need some sort of instrumental background, mathematically represented by a manifold structure, which, at the quantum level, should play more or less the role of a Wittgensteinian staircase. It is likely, therefore, that in order to attribute meaning to the individuality of points at some spatiotemporal scale - so as to build the basic structure of standard quantum theory - one should split, as it were, the individuation procedure of point-events from the true quantum properties, i.e., from the fluctuations of the gravitational field and the micro-causal structure. Our canonical analysis tends to prefigure a new approach to quantization, having in view a Fock space formulation which, unlike the loop quantum gravity, could even lead to a background-independent incorporation of the standard model of elementary particles (provided the Cauchy surfaces admit Fourier transforms). For a quantization program respecting relativistic causality, two options seem available:

1) Our individuation procedure suggests to quantize *only* the *gravitational* Dirac observables (assumed now as scalars in force of the *main conjecture* (LP)) of each Hamiltonian gauge, as well as all the *matter* Dirac observables, and than exploit the *weak ADM energy* of that gauge as Hamiltonian for the functional Schrödinger equation (of course there might be ordering problems). This quantization would yield as many Hilbert spaces as gauge-fixings, which would likely be grouped in unitary equivalence classes (we leave aside the question of what could be the meaning of inequivalent classes, were there any). In each Hilbert space the Dirac quantum operators would be distribution-valued quantum fields on a *mathematical micro spacetime* parametrized by the 4-coordinates $(\tau,\ \vec{\sigma})$ associated to the chosen gauge. Strictly speaking, due to the non-commutativity of the operators Z^A associated to the classical constraint $\sigma^A - \hat{Z}^A \approx 0$ defining that gauge, there would be *no spacetime manifold of point-events* to be mathematically identified by one coordinate chart over the *micro-spacetime* but only a *gauge-dependent non-commutative structure*, which is likely to lack any underlying topological structure. However, for each Hilbert space, a *coarse-grained* spacetime of point-events $(\bar{\Sigma})^A(\tau,\vec{\sigma})$, superimposed on the mathematical manifold $\mathcal{M}_4$, might be associated to each solution of the functional Schrödinger equation, *via* the expectation values of the operators $\hat{Z}^A$:

$$(\bar{\Sigma})^A(\tau,\vec{\sigma}) \equiv \langle\Psi\Big|\hat{Z}^A_\Gamma[\mathbf{Q}^r(\tau,\vec{\sigma}),\mathbf{P}_a(\tau,\vec{\sigma})]\Big|\Psi\rangle, \quad A=0,1,2,3; \quad r,s=1,2; \tag{8}$$

where $\mathbf{Q}^r(\tau,\vec{\sigma})$ and $\mathbf{P}_s(\tau,\vec{\sigma})$ are now Dirac *scalar field operators.*

Let us stress that, by means of (7), the *non-locality* of the *classical* individuation of point-events would be directly transferred to the basis of the ordinary, *quantum* non-locality. Also, one could evaluate in principle the expectation values of the operators corresponding to the lapse and shift functions of that gauge. Since we are considering a quantization of the 3-geometry (like in loop quantum gravity), evaluating the expectation values of the quantum 3-metric and the quantum lapse and shift functions could permit to reconstruct a coarse-grained foliation with coarse-grained so-called WSW hyper-surfaces[24].

2) In order to avoid inequivalent Hilbert spaces, we could quantize *before* adding any gauge-fixing (i.e. independently of the choice of the 4-coordinates and the physical individuation of point-events). For example, using the following rule of quantization, which complies with relativistic causality: in a

[24]This foliation is called the Wigner-Sen-Witten foliation due to its properties at spatial infinity (see Lusanna (2001)).

given canonical basis of the *main conjecture*, quantize the two pairs of (scalar) gravitational Dirac observables and matter Dirac observables, but leave the 8 gauge variables as *c-number classical fields*. Like in Schrödinger's theory with a time-dependent Hamiltonian, the momenta conjugate to the gauge variables would be represented by functional derivatives. Assuming that in the chosen canonical basis, 7 among the eight constraints be gauge momenta, we would thereby get 7 Schrödinger equations. Then, as suggested in (LP), both the *super-Hamiltonian* and the *weak ADM energy* would become operators and, if an ordering existed such that the 8 "quantum constraints" satisfied a closed algebra of the form $[\hat{\phi}_\alpha, \hat{\phi}_\beta] = \hat{C}_{\alpha\beta\gamma}\, \hat{\phi}_\gamma$ and $[\hat{E}_{ADM}, \hat{\phi}_\alpha] = \hat{B}_{\alpha\beta}\, \hat{\phi}_\beta$, $(\alpha, \beta, \gamma = 1, ..., 8)$, (with the quantum structure functions $\hat{C}_{\alpha\beta\gamma}, \hat{B}_{\alpha\beta}$ tending to the classical counterparts for $\hbar \mapsto 0$), we might quantize by imposing 9 integrable coupled functional Schrödinger equations, with the associated usual scalar product $\langle\Psi\Big|\Psi\rangle$ being independent of τ and of the gauge variables.

Again, we would have a *mathematical micro spacetime* and a *coarse-grained spacetime of "point-events"*. At this point, by going to *coherent states*, we could try to recover classical gravitational fields. The 3-geometry (volumes, areas, lengths) would be quantized, perhaps in a way that agrees with the results of loop quantum gravity.

It is important to stress that, according to both suggestions, *only the Dirac observables would be quantized*. The upshot is that fluctuations in the gravitational field (better, in the Dirac observables) would entail *fluctuations of the point texture* that lends itself to the basic spacetime scheme of standard RQFT: such fluctuating texture, however, could be recovered as a coarse-grained structure. This would induce fluctuations in the coarse-grained metric relations, and thereby in the causal structure, both of which would tend to disappear in a semi-classical approximation. Such a situation should be conceptually tolerable, and even philosophically appealing, especially if compared with the impossibility of defining a causal structure within all of the attempts grounded upon a quantization of the full 4-geometry. In this connection, it would be interesting to see whether the fluctuations of the point-events metrical texture could have any relevance to the macro-objectification issue of quantum theory (see Károlyházy et.al (1985), and Penrose (1985)).

Finally, in spacetimes with matter, this procedure would entail *quantizing the generalized tidal effects and the action-at-a-distance potentials between matter elements, but not the inertial aspects of the gravitational field*. As we have seen, the latter aspects are connected with gauge variables whose variations reproduce all the possible viewpoints of local accelerated time-like observers. Quantizing also the gauge variables would be tantamount to quantizing the metric *together* with the *passive observers* and their *reference frames*, a fact that is empirically meaningless[25].

5 Structural spacetime realism

The discussion in the previous sections is substantially grounded upon the fact that GTR is a gauge theory. Henneaux and Teitelboim (1992) gave a very general definition of gauge theories:

> These are theories in which the physical system being dealt with is described by more variables than there are physically independent degrees of freedom. The physically meaningful degrees of freedom then re-emerge as being those invariant under a transformation connecting the variables (gauge transformation). Thus, one introduces extra variables to make the description more transparent, and brings in at the same time a gauge symmetry to extract the physically relevant content.

The relevant fact is that, while from the point of view of the constrained Hamiltonian *mathematical* formalism general relativity is a gauge theory like any other (e.g., electromagnetism and Yang-Mills theories), from the *physical* point of view it is radically different. For, in addition to creating the distinction between what is observable and what is not, the gauge freedom of GR is unavoidably entangled with the constitution of the very *stage*, space–time, where the *play* of physics is enacted: a stage, however, which also takes an active part in the play. In other words, the gauge mechanism has the dual role of making the dynamics unique (as in all gauge theories), and of fixing the spatio-temporal, dynamical background. It is only after a complete gauge-fixing (i.e. after the individuation of a *well defined* physical laboratory), and after having found the solution of Einstein's equations, that the mathematical manifold $\mathcal{M}_4$ gets a *physical individuation*.

Unlike theories such as electromagnetism (or even Yang-Mills), in GR we cannot rely from the beginning on empirically validated, gauge-invariant dynamical equations for the *local* fields. In order to get equations for *local* fields we must pay the price of Einstein's general covariance which, by ruling out any background structure at the outset, conceals at the same time the intrinsic properties of point-events. With reference to the definition of Henneaux and Teitelboim, we could say, therefore, that the introduction of extra

[25] Of course, such observers have nothing to do with *dynamical measuring objects*, which should be realized in terms of the Dirac observables of matter.

variables does make indeed the mathematical description of general relativity more transparent, but it also makes its physical interpretation more obscure and intriguing, at least *prima facie*. Actually, our analysis discloses a deeper distinction of philosophical import. For it highlights a peculiar ontological and functional split of the metric tensor that can be briefly described as follows. On the one hand, the Dirac observables *holistically* specify the *ontic* structure of spacetime. On the other, we have seen that the gauge variables specify, as it were, the in-built *epistemic* component of the metric structure. Actually, in completing the structural properties of the general-relativistic spacetime, they play a multiple role: first of all, their fixing is necessary to solve Einstein's equations and to reconstruct the four-dimensional chrono-geometry emerging from the *Dirac observables*: they are essential to get a manifestly covariant and *local* metric field as a ten dimensional tensor (the *transparency* of Henneaux and Teitelboim); but their fixing is also necessary to allow empirical access to the theory through the definition of a spatio-temporal *laboratory*.

The isolation of the superfluous structure hidden behind Leibniz equivalence, which surfaces in the physical individuation of point-events, renders even more glaring the ontological diversity and prominence of the gravitational field with respect to all other fields, as well as the difficulty of reconciling the nature of the gravitational field with the standard approach of theories based on a *background spacetime* (to wit, string theory and perturbative quantum gravity in general). Any attempt at linearizing such theories unavoidably leads to looking at gravity from the perspective of a spin-2 theory in which the graviton stands on the same ontological level of other quanta: in the standard approach of background-dependent theories of gravity, photons, gluons and gravitons all live on the stage on an equal footing.

From the point of view gained in this paper, however, *non-linear gravitons* are at the same time both the stage and the actors within the causal play of photons, gluons, as well as of other "material characters" like electrons and quarks.

We can, therefore, say that general covariance represents the horizon of *a priori* possibilities for the physical constitution of the spacetime, possibilities that must be actualized within any given solution of the dynamical equations.

We believe in conclusion that these results cast some light over the *intrinsic structure* of the general relativistic spacetime that had disappeared behind Leibniz equivalence. While Leibniz could exploit the principle of sufficient reason since for him space was *uniform*, in GR the upshot is that space (spacetime) is not *uniform* at all and shows a rich *structure*. In a way, in the context of GR, Leibniz equivalence ends up hiding the very nature of spacetime, instead of disclosing it.

5.1 The nature of point-events and the overcoming of the substantivalism/relationism debate

In 1972, Bergmann and Komar wrote (1972):

> [...] in general relativity the identity of a world point is not preserved under the theory's widest invariance group. This assertion forms the basis for the conjecture that some physical theory of the future may teach us how to dispense with world points as the ultimate constituents of spacetime altogether.

Indeed, would it be possible to build a fundamental theory that is grounded in the reduced phase space parametrized by the Dirac observables? This would be an abstract and highly non-local theory of classical gravitation but, transparency aside, it would lack all the epistemic machinery (the gauge freedom) which is indispensable for the application of the theory. Therefore, we see that, even in the context of classical gravitational theory, the spatio-temporal continuum is an epistemic precondition playing a role which is not too dissimilar from that enacted by *Minkowsky micro-spacetime* in RQFT. We find here much more than a clear instantiation of the relationship between canonical structure and locality that pervades contemporary theoretical physics throughout.

Can this basic freedom in the choice of the *local realizations* be equated with a "taking away from space and time the last remnant of physical objectivity," as Einstein suggested? We believe that, discounting Einstein's "spatial obsession" with *realism as locality (and separability)*, a significant kind of spatio-temporal objectivity survives. It is true that - if the *main conjecture* of LP is not verified - the *functional form* of the Dirac observables in terms of the spatio-temporal coordinates depends upon the particular choice of the latter (or, equivalently, of the gauge); yet, there is anyway no a-priori *physical* individuation of the manifold points independently of the metric field, so we cannot say that the individuation procedures corresponding to different gauges individuate *different* point-events. Given the conventional nature of the primary mathematical individuation of manifold points through n-tuples of real numbers, we could say instead that the *real point-events* are *constituted* by the non-local values of gravitational degrees of freedom, while the underlying point structure of the mathematical manifold may be changed at will. A *really different* physical individuation should only be attributed to different initial conditions for the *Dirac observables*, (i.e., to a different "universe").

Taking into account our results as a whole, we want to spend a few words about their implications for the traditional debate on the absolutist/relationist dichotomy as well as for the issues surrounding structural realism in general.

First of all, let us recall that, in remarkable diversity with respect to the traditional historical presentation of Newton's absolutism *vis á vis* Leibniz's relationism, Newton had a much deeper understanding of the nature of space and time. In a well-known passage of *De Gravitatione* (see Hall and Hall (1962)), he expounds what could be defined a *structuralist view* of space and time. He writes:

> Perhaps now it is maybe expected that I should define extension as substance or accident or else nothing at all. But by no means, for it has its own manner of existence which fits neither substance nor accidents [...] the parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other *qua* individuals. The parts of duration and space are only understood to be the same as they really are because of their mutual order and positions (*propter solum ordinem et positiones inter se*); nor do they have any other *principle of individuation* besides this order and position which consequently cannot be altered.

We have just disclosed the fact that the points of general-relativistic spacetimes, quite unlike the points of the homogeneous Newtonian space, are endowed with a remarkably rich *non-point-like* and *holistic* structure furnished by the metric field. Therefore, the general-relativistic metric field itself or, better, its independent degrees of freedom, have the capacity of characterizing the "mutual order and positions" of points *dynamically*, and in fact much more than this, since such mutual order is altered by the presence of matter.

In conclusion, we agree with Earman and Norton that the Hole Argument is a decisive blow against strict *manifold substantivalism*. However, the isolation of the intrinsic structure hidden behind Leibniz equivalence - leading to our *point-structuralism* - does not support the standard relationist view either. As a matter of fact, by referring to Earman's third criterion (R_3) for relationism (see (Earman , 1989, p. 14)): "No irreducible, monadic, spatiotemporal properties, like 'is located at spacetime point p' appears in a correct analysis of the spatiotemporal idiom", we observe as follows: if by 'spacetime point' we mean our *physically individuated point-events* instead of the naked manifold's point, then - because of the autonomous existence of the intrinsic degrees of freedom of the gravitational field (an essential ingredient of GR) - the above mentioned spatiotemporal property should be admitted in our spatiotemporal idiom.

Indeed, a new kind of *holistic and structuralist* conception of spacetime emerges from our analysis, including elements common to the tradition of both *substantivalism* (spacetime has an autonomous existence independently of other bodies or matter fields) *and relationism* (the physical meaning of spacetime depends upon the relations between bodies or, in modern language, the specific reality of spacetime depends (also) upon the (matter) fields it contains). Indeed, even though the metric field does *not* embody the traditional notion of *substance* (rather than being wholly present, it has "temporal parts"), it *exists* and plays a role for the individuation of point-events. On the other hand, each point-event itself, though holistically individuated by the metric field, has - to paraphrase Newton - "its own manner of existence", since it "is" the "values" of the intrinsic degrees of freedom of the gravitational field. Finally, in presence of matter, such values become dependent also on the values of the Dirac observables of matter fields.

These remarks show how the structural texture of spacetime in classical GR does not force us to abandon the typical entity realist attitude toward both the metric field and its points. As our case study seems to indicate, we must reject the ontic structural realist claim that (the metrical) relations can exist without their relata (the points). At the same time, we can distance ourself from the epistemic structural realist's prudence in denying existence to entities (in our case, point-events): despite their holistic texture, the identity of point-events is sufficiently well characterized by the distinct values of the Dirac observable they exemplify. In a word, we can use structural realism to defend both (i) a moderate form of theory realism about the approximate truth of Einstein's field equations (within the limits fixed by their domain of application) and (ii) a full blown realism about spacetime in GR. As far as QG is concerned, the fluctuations of the Dirac observables do not eliminate the structuralist and holistic nature of the coarse-grained texture of "quantum spacetime". However, it would be difficult to claim that some kind of intrinsic individuality survives for "point-events".

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References

Arnowitt, R., Deser, S., and Misner, C. W. (1962), "The dynamics of general relativity", in *Gravitation: an introduction to current research*, L. Witten (ed.) (Wiley, Nework), pp. 227–265.

Belot, G. and Earman, J. (1999), "From Metaphysics to Physics", in *From Physics to Philosophy*, J.Butterfield and C.Pagonis, (eds.), Cambridge University Press, Cambridge, pp. 167–186; (2001), "Pre-Socratic Quantum Gravity", in *Physics Meets Philosophy at the Planck Scale. Contemporary Theories in Quantum Gravity*, C.Callender (ed.), (Cambridge University Press, Cambridge), p. 213–255.

Bergmann, P. G. (1960), "Observables in General Relativity", *Rev. Mod. Phys.* **33**, 510–514.

Bergmann, P. G. (1962), "The General Theory of Relativity", in *Handbuch der Physik*, vol. IV, *Principles of Electrodynamics and Relativity*, S. Flugge (ed.) (Springer-Verlag, Berlin), pp. 247–272.

Bergmann, P. G. (1977), "Geometry and Observables", in *Foundations of Space-Time Theories, Minnesota Studies in the Philosophy of Science*, Vol. VIII, J. S. Earman, C. N. Glymour, and J. Stachel (eds.) (University of Minnesota, Minneapolis), pp. 275–280.

Bergmann, P. G., and Komar, A. (1960), "Poisson Brackets between Locally Defined Observables in General Relativity", *Phys. Rev. Lett.* **4**, 432–433.

Bergmann, P. G., and Komar, A. (1972), "The Coordinate Group Symmetries of General Relativity", *Int. J. Theor. Phys.* **5**, 15–28.

Bueno O., French S., Ladyman J. (2002), "On Representing the Relationship between the Mathematical and the Empirical", *Philosophy of Science*, **69**, pp. 452–473.

Butterfield, J. (1989), "The Hole Truth", *British Journal for the Philosophy of Science* **40**, 1–28.

Cao T. (2003a)," Structural Realism and The Interpretation of Quantum Field Theory", *Synthese*, **136**, pp. 3–24.

Cao T. (2003b), "Can we Dissolve Physical Entities into Mathematical Structures?", *Synthese*, **136**, pp. 51–71.

Chakravartty, A. (1998), "Semirealism", *Studies in History and Philosophy of Science*, **29**, pp. 391–408.

Christodoulou, D. and Kleinerman, S. (1999), *The Global Nonlinear Stability of the Minkowski Space*, (Princeton University Press, Princeton).

De Pietri R., Lusanna L., Martucci L. and Russo S., *Dirac's Observables for the Rest-Frame Instant Form of Tetrad Gravity in a Completely Fixed 3-Orthogonal Gauge*, Gen.Rel.Grav. **34**, 877–922.

Dorato, M. (2000), "Substantivalism, Relationism, and Structural Spacetime Realism", *Foundations of Physics* **30**, pp. 1605–1628.

Earman J. (1989), *World Enough and Space-Time*, (The Mit Press, Cambridge Mass).

Earman, J. (2002), "Thoroughly Modern McTaggart or what McTaggart would have said if He had read the General Theory of Relativity", *Philosophers' Imprint* **2**, No.3, (http://www.philosophersimprint.org/002003/).

Earman, J., and Norton, J. (1987), "What Price Spacetime Substantivalism? The Hole Story", *British Journal for the Philosophy of Science* **38**, pp. 515–525.

Einstein, A. (1914) "Die formale Grundlage der allgemeinen Relativitätstheorie", in *Preuss. Akad. der Wiss. Sitz.*, pp. 1030–1085.

Einstein, A. (1916), "Die Grundlage der allgemeinen Relativitätstheorie", *Annalen der Physik* **49**, pp. 769–822; translation by W. Perrett and G. B. Jeffrey (1952), "The Foundation of the General Theory of Relativity", in *The Principle of Relativity* (Dover, New York), pp. 117–118.

French S. and Ladyman J. (2003), "Remodelling Structural Realism", *Synthese*, **136**, p. 31–56.

Friedman, M. (2001), "Geometry as a Branch of Physics: Background and Context for Einstein's 'Geometry and Experience'", in *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics to Honor Howard Stein on His* 70^{th} *Birthday*, D. Malament (ed.) (Open Court, Chicago).

Friedrich H. and Rendall A. (2000), "The Cauchy Problem for Einstein Equations", in *Einstein's Field Equations and their Physical Interpretation*, B.G.Schmidt ed., (Springer, Berlin, 2000), (gr-qc/0002074).

Hacking I. (1983), *Representing and Intervening*, (Cambridge University Press, Cambridge).

Hall, A. R., and Hall, M. B. (eds.) (1962), *De Gravitatione et Aequipondio Fluidorum, Unpublished Scientific Papers of Isaac Newton. A Selection from the Portsmouth Collection in the University Library* (Cambridge University, Cambridge).

Henneaux, M. and Teitelboim, C. (1992), *Quantization of Gauge Systems* (Princeton University Press, Princeton).

Károlyházy, F., Frenkel, A., and Lukács, B. (1985) "On the possible role of gravity in the reduction of the wave function" in *Quantum Concepts in Space and Time*, R.Penrose and C.J.Isham (eds.), (Clarendon Press, Oxford), pp. 109–128; Penrose, R. (1985) "Gravity and state vector reduction", *ibid.*, pp. 129–146.

Komar, A. (1955), "Degenerate Scalar Invariants and the Groups of Motion of a Riemann Space", *Proc. Natl. Acad. Sci.* **41**, pp. 758–762.

Komar, A. (1958), "Construction of a Complete Set of Independent Observables in the General Theory of Relativity", *Phys. Rev.* **111**, 1182–1187.

Lusanna, L. (2000), "Towards a Unified Description of the Four Interactions in Terms of Dirac–Bergmann Observables", in *Quantum Field Theory*, A. N. Mitra (ed.) (Indian National Science Academy), pp. 490–518.

Lusanna, L. (2001), "The Rest-Frame Instant Form of Metric Gravity", *Gen. Rel. Grav.* **33**, pp. 1579–1696.

Lusanna L., and Pauri, M. (2003), "General Covariance and the Objectivity of Space-Time Point-Events: The Physical Role of Gravitational and Gauge Degrees of Freedom in General Relativity", gr-qc/0301040.

Massimi M. (forthcoming), "Non-defensible middle ground for experimental realism: why we are justified to believe in coloured quarks",*Philosophy of Science*.

Maudlin, T. (1990), "Substances and Spacetimes: What Aristotle Would Have Said to Einstein", *Studies in the History and Philosophy of Science* **21**, pp. 531–61.

Morganti M. (forthcoming), "On the Preferability of Epistemic Structural Realism", *Synthese*.

Norton, J. (1987), “Einstein, the Hole Argument and the Reality of Space”, in *Measurement, Realism and Objectivity*, J. Forge (ed.) (Reidel, Dordrecht), pp. 153–18.

Norton, J. (1992), “The Physical Content of General Covariance”, in *Studies in the History of General Relativity*, Einstein Studies, vol. 3, J. Eisenstaedt, and A. Kox (eds.) (Birkhäuser, Boston), pp. 281–315.

Norton, J. (1993), “General Covariance and the Foundations of General Relativity: Eight Decades of Dispute”, *Rep. Prog. Phys.* **56**, pp. 791–858.

Pauri, M. (1996), “Realtà e Oggettività”, in *L’Oggettività nella Conoscenza Scientifica* (F. Angeli, Brescia), pp. 79–112.

Pauri, M., (2000), "Leibniz, Kant and the *Quantum* - A Provocative Point of View about Observation, Space-Time, and the Mind-Body Issue", in *The Reality of the Unobservable*, E.Agazzi and M.Pauri (eds.) Boston Studies in the Philosophy of Science, n.215, (Kluwer Academic Publishers, Dordrecht), pp. 257–282.

Pauri, M. and Vallisneri, M., (2002), "Ephemeral point-events: is there a last remnant of physical objectivity ?", *Dialogos*, **79**, pp. 263–303.

Poincaré H. (1905), *La science et l’hypothèse*, (Flammarion, Paris).

Rynasiewicz, R. (1996), “Absolute versus Relational Space-Time: An Outmoded Debate?”, *Journal of Philosophy* **43**, pp. 279–306.

Shanmugadhasan, S. (1973), “Canonical Formalism for Degenerate Lagrangians”, *Journ. Math. Phys.* **14**, pp. 677–687.

Stachel, J. (1980), “Einstein’s Search for General Covariance, 1912–1915”, paper read at the Ninth International Conference on General Relativity and Gravitation, Jena; published in *Einstein and the History of General Relativity: Einstein Studies*, vol. 1, D. Howard and J. Stachel (eds.) (Birkhäuser, Boston 1986), pp. 63–100.

Stachel, J. (1986), “What can a Physicist Learn from the Discovery of General Relativity?”, *Proceedings of the Fourth Marcel Grossmann Meeting on Recent Developments in General Relativity, Rome, 1–21 June, 1985*, R. Ruffini (ed.) (North-Holland, Amsterdam).

Stachel, J. (1993), “The Meaning of General Covariance – The Hole Story”, in *Philosophical Problems of the Internal and External Worlds, Essays on* *the Philosophy of Adolf Gruenbaum*, J. Earman, I. Janis, G. J. Massey and N. Rescher (eds.) (University of Pittsburgh Press, Pittsburgh), pp. 129–160.

Wald, R. M. (1984), *General Relativity*, (University of Chicago, Chicago), pp. 438–439.

Weyl, H. (1946), "Groups, Klein's Erlangen Program. Quantities", ch. I, sec. 4 of *The Classical Groups, Their Invariants and Representations*, 2nd ed., (Princeton University, Princeton), pp. 13–23.

Worrall J. (1989), "Structural Realism: the best of all possible worlds?", *Dialectica*, **43**, pp. 99–124.