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SCAN-9411346



DTP/94/45

## Symmetric Versions of Explicit Wavefunction Collapse Models

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*November, 1994.*

**ABSTRACT** Two versions of the GRW “hitting” model for explicit wavefunction collapse, which are consistent with preserving the symmetry of the wavefunction, are considered. The predictions of the models for excitation of bound systems are calculated and compared with experiment and with the predictions of other similar models. It is shown that our preferred model strongly supports the idea that collapse, if it occurs, has gravitational origin.

### 1. Introduction

In the original explicit collapse model of Ghirardi, Rimini and Weber (GRW) [1], the collapse was caused by a discrete, random, hitting process. Later, using methods previously developed by Pearle [2], continuous versions of the model were constructed.

A problem with the pioneering GRW model was that it did not respect the symmetry in the case of identical particles. This problem was solved for the continuous collapse models [3], but there does not appear to have been any complete discussion of how to treat identical particles in a hitting model. The first purpose of this paper is to fill this gap.

A further motivation comes from the work of Pearle and Squires [4] where it is shown that the original model, if applied to the quark structure of hadrons, gives an unacceptably large rate for nucleon decay. The continuous versions, which respect the symmetry, have two features which enable them to evade this difficulty. One is that there is an extra factor, related to the density of particles, in the collapse rate

for a pointer, and the other is that if the collapse is made to happen symmetrically over all particles, and if it has a rate proportional to the mass (as might be expected if it is caused in some way by gravity [5]), then in lowest order the excitation rate of bound systems is zero. Here we want to investigate further the origin of these features, and to see whether similar things arise in discrete models.

In the next section we briefly review the original GRW “hitting” model, paying particular attention to the extent to which it is unique. Then, in the next two sections, we consider two alternative methods for making the model consistent with maintaining the symmetry of the wavefunction for identical particles. In section 5 we introduce a hitting process that is dependent on the masses of the particles concerned. The consequences of the various models for collapse of a “pointer”, and for the excitation of a bound state are then calculated. Finally, in section 8, we give the conclusions of this work.

## 2. The GRW model

Consider an  $N$ -particle system with wavefunction given by  $\Psi(\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_N) \equiv \Psi(\mathbf{q})$ . At random times this is hit so that it changes instantaneously according to

$$\Psi \rightarrow \Psi_{\mathbf{x},j}^H = \frac{f(\mathbf{x} - \mathbf{q}_j)\Psi}{R_j(\mathbf{x})}, \quad (2.1)$$

where the function  $f$  is localised around the zero value of its argument. Following GRW we choose this function to be a normalised gaussian

$$f(\mathbf{z}) = \left(\frac{\beta}{\pi}\right)^{\frac{3}{4}} \exp\left(-\frac{\beta \mathbf{z}^2}{2}\right), \quad (2.2)$$

where the parameter  $\beta$  gives the order of magnitude of the radius of the collapsed wavefunction:

$$a = \sqrt{\frac{1}{\beta}}. \quad (2.3)$$

The function  $R$  is chosen so that the hitting preserves the normalisation of the wavefunction. Hence

$$|R_j(\mathbf{x})|^2 = \int d^3\mathbf{q}_1 \dots d^3\mathbf{q}_N |f(\mathbf{x} - \mathbf{q}_j)|^2 |\psi|^2. \quad (2.4)$$

It follows from these equations that the  $R$  functions are normalised, i.e., that

$$\int |R_j(\mathbf{x})|^2 d^3\mathbf{x} = 1. \quad (2.5)$$

Now we suppose that the probability of hitting particle  $j$  in time  $dt$  is given by  $\lambda_j dt$ , and that the probability distribution of the hitting positions  $\mathbf{x}$  is given by the function  $P_j(\mathbf{x})$ . Then the density matrix at time  $t + dt$  is given by

$$\rho(t + dt) = \left(1 - \sum \lambda_j dt\right) \left(\rho(t) - \frac{i}{\hbar}[H, \rho(t)]dt\right) + \sum \lambda_j \rho_j^H dt, \quad (2.6)$$

where  $\rho_j^H$  is the density matrix after particle  $j$  has been hit. From eq. (2.1) this is given by

$$\langle \mathbf{q}' | \rho_j^H | \mathbf{q} \rangle = \int d^3\mathbf{x} \frac{P_j(\mathbf{x}) f(\mathbf{x} - \mathbf{q}_j) \Psi(\mathbf{q}')^* f(\mathbf{x} - \mathbf{q}_j) \Psi(\mathbf{q})}{|R_j(\mathbf{x})|^2}. \quad (2.7)$$

Eq. (2.6) leads to

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] - \sum \lambda_j (\rho - \rho_j^H). \quad (2.8)$$

The model is now completely determined when we impose the requirement that *position* averages are conserved, i.e., that the diagonal elements of the density matrix are unchanged by the hitting. Hence, from eq. (2.7), we need

$$\sum_j \lambda_j \int d^3\mathbf{x} \frac{P_j(\mathbf{x}) (f(\mathbf{x} - \mathbf{q}_j))^2}{|R_j(\mathbf{x})|^2} = \sum_j \lambda_j. \quad (2.9)$$

Since this equation must be true for all  $\mathbf{q}$ , it follows that  $P$  is proportional to  $|R|^2$ , and so, because eq. (2.5) shows that  $|R|^2$  is already normalised as a probability, that

$$P_j(\mathbf{x}) = |R_j(\mathbf{x})|^2. \quad (2.10)$$

## 3. A Hitting Process for Identical Particles

We now suppose that all the  $N$  particles are identical, and that the uncollapsed wavefunction,  $\Psi$ , is correctly symmetrised (or antisymmetrised). Then the obvious generalisation of eq. (2.1) that preserves this property is

$$\Psi \rightarrow \Psi_{\mathbf{x}}^H = \frac{F(\mathbf{x}, \{\mathbf{q}_i\})\Psi}{R(\mathbf{x})}, \quad (3.1)$$

where

$$F(\mathbf{x}, \{\mathbf{q}_i\}) = \frac{C(\{\mathbf{q}_i\})}{N^{\frac{1}{2}}} \sum_j \left(\frac{\beta}{\pi}\right)^{\frac{3}{4}} \exp\left(-\frac{\beta}{2}(\mathbf{q}_j - \mathbf{x})^2\right). \quad (3.2)$$

The factor  $R$  is again included to preserve the normalisation of  $\psi^H$ . The necessity of the factor  $C$  will be clear below.

It should be emphasised at this point that later we will take  $N = N_{\mathbf{u}}$ , where  $N_{\mathbf{u}}$  is the number of particles in the universe. This means that a given hit can occur anywhere in the universe. We denote the probability of such a hit in time  $dt$  by  $\lambda_{\mathbf{u}} dt$ , where clearly  $\lambda_{\mathbf{u}}$  will be very large.

The evolution of the density matrix is now given by

$$\rho(t + dt) = (1 - \lambda_{\mathbf{u}} dt) \left(\rho(t) - \frac{i}{\hbar}[H, \rho(t)]dt\right) + \lambda_{\mathbf{u}} \rho^H dt, \quad (3.3)$$

which leads to

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] - \lambda_u(\rho - \rho^H). \quad (3.4)$$

In these equations  $\rho^H$ , the density matrix after a hit, is given by

$$\langle \mathbf{q}' | \rho^H | \mathbf{q} \rangle = \int d^3 \mathbf{x} \frac{P(\mathbf{x}) F(\mathbf{x}, \{\mathbf{q}_j'\}) \Psi(\mathbf{q}')^* F(\mathbf{x}, \{\mathbf{q}_j\}) \Psi(\mathbf{q})}{|R(\mathbf{x})|^2}. \quad (3.5)$$

The requirement that the probability predictions of quantum theory are preserved again means that the diagonal elements of the density matrix must be unchanged by the collapse. This implies

$$\int d^3 \mathbf{x} \frac{P(\mathbf{x}) (F(\mathbf{x}, \{\mathbf{q}_j\}))^2}{|R(\mathbf{x})|^2} = 1. \quad (3.6)$$

Substituting the form for  $F$  from eq. (3.2) and using  $[(\mathbf{q}_j - \mathbf{x})^2 + (\mathbf{q}_k - \mathbf{x})^2] = [2(\mathbf{x} - \frac{1}{2}(\mathbf{q}_j + \mathbf{q}_k))^2 + \frac{1}{2}(\mathbf{q}_j - \mathbf{q}_k)^2]$  we find

$$\frac{C^2}{N} \left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} \int d^3 \mathbf{x} \frac{P(\mathbf{x})}{|R(\mathbf{x})|^2} \sum_{j,k} \exp\left(-\beta \left[\mathbf{x} - \frac{1}{2}(\mathbf{q}_j + \mathbf{q}_k)\right]^2\right) \exp\left(-\frac{\beta}{4}(\mathbf{q}_j - \mathbf{q}_k)^2\right) = 1. \quad (3.7)$$

The last equation shows why  $C$  must be a function of the  $\mathbf{q}$  variables. If we require that it only depends on differences between the  $\mathbf{q}$ 's (which seems to be a reasonable physical requirement), then we must again choose

$$P(\mathbf{x}) = |R(\mathbf{x})|^2, \quad (3.8)$$

analogous to eq. (2.10). Then eq. (3.7) becomes

$$\frac{C(\{\mathbf{q}_i\})}{N^{\frac{1}{2}}} = \left[ \sum_{i,j} \exp\left(-\frac{\beta}{4}(\mathbf{q}_i - \mathbf{q}_j)^2\right) \right]^{-\frac{1}{2}}. \quad (3.9)$$

#### 4. An alternative hitting model which preserves the symmetry

The extra complication in the hit wavefunction of the previous section arises from the cross terms in, for example, eq. (3.7). We can, however, eliminate these terms if we replace eqs. (3.1) and (3.2) by

$$\Psi \rightarrow \Psi_{\mathbf{x}}^H = \frac{G(\mathbf{x}, \{\mathbf{q}_i\}) \Psi}{R(\mathbf{x})}, \quad (4.1)$$

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where

$$G(\mathbf{x}, \{\mathbf{q}_i\}) = \frac{1}{N^{\frac{1}{2}}} \left(\frac{\beta}{\pi}\right)^{\frac{3}{4}} \left[ \sum_j \exp(-\beta(\mathbf{q}_j - \mathbf{x})^2) \right]^{\frac{1}{2}}. \quad (4.2)$$

Here there is no need for any extra factor analogous to the  $C$  in the previous section.

If we have a system ( $s$ ) isolated from the rest of the universe ( $u$ ), then we can write

$$\left[ \sum_j \exp(-\beta(\mathbf{x} - \mathbf{q}_j)^2) \right]^{\frac{1}{2}} = \left[ \sum_{j \in u-s} \exp(-\beta(\mathbf{x} - \mathbf{q}_j)^2) \right]^{\frac{1}{2}} + \left[ \sum_{j \in s} \exp(-\beta(\mathbf{x} - \mathbf{q}_j)^2) \right]^{\frac{1}{2}}, \quad (4.3)$$

since one of these terms will be zero for all values of  $\mathbf{x}$ . If we assume that  $\mathbf{q}_i = \mathbf{q}_i'$  for  $i \notin s$ , then we can write the hit density matrix,  $\int d^3 \mathbf{x} P(\mathbf{x}) |\Psi_{\mathbf{x}}^H \rangle \langle \Psi_{\mathbf{x}}^H|$ , as

$$\langle \mathbf{q}' | \rho^H | \mathbf{q} \rangle = \left(\frac{1}{N_u}\right) \left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} \int d^3 \mathbf{x} \left[ \sum_{j \in s} \exp(-\beta(\mathbf{x} - \mathbf{q}_j)^2) \right]^{\frac{1}{2}} \left[ \sum_{j \in s} \exp(-\beta(\mathbf{x} - \mathbf{q}_j')^2) \right]^{\frac{1}{2}} + \frac{1}{N_u} \left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} \int d^3 \mathbf{x} \left[ \sum_{j \in u-s} \exp(-\beta(\mathbf{x} - \mathbf{q}_j)^2) \right] \langle \mathbf{q}' | \rho | \mathbf{q} \rangle. \quad (4.4)$$

The second term in this expression may of course be evaluated exactly, giving a value of  $\frac{N_u - N_s}{N_u}$ , where  $N_u$  is the number of particles in the universe and  $N_s$  is the number of particles in the system.

#### 5. A mass dependent symmetrical hitting model

As we shall see below there are reasons for believing that the collapse model should treat all "matter" particles (fermions) in a symmetrical way, apart from a dependence on the mass. In order to permit this dependence, we will replace eq. (3.2) by

$$F(\mathbf{x}, \{\mathbf{q}_i\}) = \frac{C(\mathbf{q})}{N^{\frac{1}{2}}} \sum_j \alpha_j \left(\frac{\beta}{\pi}\right)^{\frac{3}{4}} \exp\left(-\frac{\beta}{2}(\mathbf{q}_j - \mathbf{x})^2\right), \quad (5.1)$$

where  $\alpha_j$  is the "coupling" of the  $j^{\text{th}}$  particle, and the sums now go over all particles. The simplest mass dependence, which would be expected for a gravitational effect, comes from taking  $\alpha_j = \frac{m_j}{m_0}$ . Later we will consider atomic systems where we have electrons that couple with strength  $\alpha$  and nucleons with strength  $1 - \alpha$ . With eq (5.1) as the hitting function, eq. (3.9) becomes replaced by

$$\frac{C}{N^{\frac{1}{2}}} = \left[ \sum_{i,j} \alpha_i \alpha_j \exp\left(-\frac{\beta}{4}(\mathbf{q}_i - \mathbf{q}_j)^2\right) \right]^{-\frac{1}{2}}. \quad (5.2)$$

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Similarly we shall replace eq. (4.2) by

$$G(\mathbf{x}, \{q_i\}) = \frac{1}{N^{\frac{1}{2}}} \left( \frac{\beta}{\pi} \right)^{\frac{3}{4}} \left[ \sum_j \alpha_j \exp(-\beta(\mathbf{q}_j - \mathbf{x})^2) \right]^{\frac{1}{2}}. \quad (5.3)$$

## 6. Pointer Collapse

We now wish to compare the rate of pointer collapse in the various models. In particular we shall compare the original GRW model, the continuous localisation model (CSL), with mass dependent collapse (see ref.[4]), the symmetric hitting model of section 3, modified as in section 5, (SHM1) and the symmetric hitting model of section 4, again modified as in section 5 (SHM2).

We assume that the pointer is in a state that is a superposition of two macroscopically distinguishable states, and that it is isolated from the rest of the universe. Thus we write:

$$\Psi = \left( \frac{1}{2} \right)^{\frac{1}{2}} (\psi_1 + \psi_2) \chi_{universe}. \quad (6.1)$$

We now consider the collapse rate in various models.

### 1. GRW.

Each particle is hit independently of all of the others. Therefore the rate of pointer collapse is given by

$$\lambda_p = N_p \lambda, \quad (6.2)$$

where  $N_p$  is the number of particles in the pointer.

### 2. CSL

We quote the result obtained in [3],

$$\lambda_p = (N_p D_N a^3) \lambda, \quad (6.3)$$

where  $D_N$  is the particle-number density of the pointer. Here we see the extra factor, noted in §1, which makes the pointer collapse faster.

If we consider the model modified as in §5, then we must sum over all particles with the  $\alpha$  factors included. In particular, if we have electrons with coupling  $\alpha$  and nucleons with coupling  $1 - \alpha$ , then for a pointer made of one element (Z,A), this becomes

$$\lambda_p = (N_p D_N a^3 [\alpha Z + (1 - \alpha) A]^2) \lambda, \quad (6.4)$$

where  $N_p$  is now the number of atoms in the pointer, and  $D_N$  is the density of atoms in the pointer.

### 3. SHM1

As the pointer is isolated from the rest of the universe, we may rewrite eq. (3.9) in the form

$$\frac{C(\mathbf{q})}{N^{\frac{1}{2}}} = \left[ \sum_{j,k \in u-p} \exp\left(-\frac{\beta}{4}(\mathbf{q}_j - \mathbf{q}_k)^2\right) + \sum_{j,k \in p} \exp\left(-\frac{\beta}{4}(\mathbf{q}_j - \mathbf{q}_k)^2\right) \right]^{-\frac{1}{2}}, \quad (6.5)$$

where the first sum is over all the universe apart from the pointer and the second sum is over the pointer. Clearly we can approximate this as:

$$\frac{C(\mathbf{q})}{N^{\frac{1}{2}}} = \left( \frac{1}{\Sigma_u} \right)^{\frac{1}{2}} \left[ 1 - \frac{1}{2\Sigma_u} \sum_{j,k \in p} \exp\left(-\frac{\beta}{4}(\mathbf{q}_j - \mathbf{q}_k)^2\right) \right], \quad (6.6)$$

where

$$\Sigma_u = \sum_{j,k \in u} \exp\left(-\frac{\beta}{4}(\mathbf{q}_j - \mathbf{q}_k)^2\right). \quad (6.7)$$

This is a large, and approximately constant, number.

From eq. (3.5) we then have

$$\begin{aligned} \langle \mathbf{q}' | \rho^H | \mathbf{q} \rangle &= \frac{1}{\Sigma_u} \left[ 1 - \frac{1}{2\Sigma_u} \sum_{j,k \in s} \Phi(\mathbf{q}_j - \mathbf{q}_k) \right] \left[ 1 - \frac{1}{2\Sigma_u} \sum_{j,k \in s} \Phi(\mathbf{q}_{j'} - \mathbf{q}_{k'}) \right] \\ &\times \left( \Sigma_u + \sum_{j,k \in s} \Phi(\mathbf{q}_j - \mathbf{q}_{k'}) \right) \langle \mathbf{q}' | \rho | \mathbf{q} \rangle, \end{aligned} \quad (6.8)$$

where  $\Phi(\mathbf{z}) = \exp\left(-\frac{\beta}{4}z^2\right)$ .

Taking into account that  $\Sigma_u$  is very large, and using eq. (3.4), we obtain for the equation giving the evolution of the density matrix:

$$\begin{aligned} \frac{\partial \langle \mathbf{q}' | \rho | \mathbf{q} \rangle}{\partial t} &= -\frac{i}{\hbar} \langle \mathbf{q}' | [H, \rho] | \mathbf{q} \rangle - \frac{\lambda}{2} \langle \mathbf{q}' | \rho | \mathbf{q} \rangle \\ &\times \sum_{j,k \in s} [\Phi(\mathbf{q}_j - \mathbf{q}_k) + \Phi(\mathbf{q}_{j'} - \mathbf{q}_{k'}) - 2\Phi(\mathbf{q}_j - \mathbf{q}_{k'})]. \end{aligned} \quad (6.9)$$

where we have written

$$\lambda = \frac{\lambda_u}{\Sigma_u}. \quad (6.10)$$

The above equation for the evolution of the density matrix is identical to that obtained in CSL, and hence the rates for pointer collapse in SHM1 will be those of eq. (6.3) and (6.4).

It is worth noting that the SHM1 model does allow the possibility of a novel type of superluminal signalling. This is because the effective collapse rate, proportional to  $\lambda$ , depends on the quantity  $\Sigma_u$ , which can be altered by changing the density anywhere in the universe (at least in a finite universe). If such signalling is considered unacceptable, then we would have to reject this model.

### 4. SHM2

As the pointer is in a superposition of two macroscopically distinguishable states, the first term in eq. (4.4) is essentially zero, since one of the factors will vanish for each value of  $\mathbf{x}$ . Using eq. (3.4), and ignoring the Hamiltonian term, we obtain

$$\frac{\partial \langle \mathbf{q}' | \rho | \mathbf{q} \rangle}{\partial t} = -\lambda N_s \langle \mathbf{q}' | \rho | \mathbf{q} \rangle, \quad (6.11)$$

where we have written

$$\lambda = \frac{\lambda_u}{N_u}, \quad (6.12)$$

analogous to eq. (6.10). Therefore

$$\lambda_p = N_p \lambda, \quad (6.13)$$

With the modification of §5, we obtain a similar expression;

$$\lambda_p = \sum_p \alpha_j \lambda, \quad (6.14)$$

where now we have defined

$$\lambda = \frac{\lambda_u}{\sum_u \alpha_j}. \quad (6.15)$$

In particular, for a pointer made of atoms (Z,A) with the coupling  $\alpha$  for electrons and  $1 - \alpha$  for nucleons, then

$$\lambda_p = (N_p[Z\alpha + (1 - \alpha)A])\lambda. \quad (6.16)$$

We can see that these values for the rate of collapse of a pointer have the same form as those in the GRW model (i.e. linear in the number of particles).

## 7. Bound State Excitation

In the previous section we were concerned with the effect which we want to obtain from our model. It thus gives a lower limit to the collapse rate. Now we consider an effect which has not been observed, and which therefore gives an upper limit.

We suppose that the particles belong to an isolated bound state, with spatial separation much less than  $a$ , as for example in an atom. Initially the system is in its ground state  $\psi_0$ . We consider the rate of excitation to an orthogonal excited state  $\phi$ , in the various models. Since the  $\mathbf{q}_j$  are small, we expand to lowest order in  $\beta q_j^2$ .

### 1. GRW

The expression for the rate of excitation is given by

$$R(\phi) = \frac{\lambda\beta}{2} \sum_{k=1}^{N_s} |\langle \phi | \mathbf{q}_k | \psi_0 \rangle|^2. \quad (7.1)$$

This has the order of magnitude  $\frac{\lambda}{2} \left(\frac{a_0}{a}\right)^2$ , where  $a_0$  is the radius of the bound state.

### 2. CSL

For the continuous case, the corresponding expression to eq. (7.1) is

$$R(\phi) = \frac{\lambda\beta}{2} |\langle \phi | \sum_{k=1}^{N_s} \alpha_k \mathbf{q}_k | \psi_0 \rangle|^2, \quad (7.2)$$

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where the sum is over all the particles in the bound state. If the  $\alpha_k$  are all equal, the effect is dominated by the electrons, and the excitation rate is similar to that given above for GRW.

A very different situation arises if we choose  $\alpha_i = \frac{m_i}{m_0}$ , in particular for atoms,  $\alpha_e = \frac{m_e}{m_e + m_p}$  and  $\alpha_p = 1 - \alpha_e$ . Then

$$R(\phi) = \left(\frac{M_s}{m_0}\right)^2 \frac{\lambda\beta}{2} |\langle \phi | \mathbf{Q} | \psi_0 \rangle|^2, \quad (7.3)$$

where  $\mathbf{Q}$  is the centre of mass operator and  $M_s$  is the mass of the bound atom. This last expression is equal to zero, as the centre of mass operator cannot excite the internal atomic states. Thus there is no excitation to lowest order in the mass-dependent case. The next order gives rise to an excitation rate of order  $\lambda \left(\frac{a_0}{a}\right)^4$ .

### 3. SHM1

As we noted above, the equation for the evolution of the off-diagonal elements of the density matrix is identical to that obtained in CSL, and hence the expressions for the rate of excitation of a bound state will be as in eqs. (7.2) and (7.3) above.

### 4. SHM2

To calculate the rate of excitation, we use eq. (4.4). Whereas the second term can be evaluated exactly, we need to make an approximation to proceed further with the first term. With  $\Sigma_s = \sum_{j \in s} \exp(-\beta(\mathbf{x} - \mathbf{q}_j)^2)$ , we may write

$$\Sigma_s^{\frac{1}{2}} = \exp\left(-\frac{\beta}{2}x^2\right) \left[ \sum_{j \in s} \exp(\beta(2\mathbf{q}_j \cdot \mathbf{x} - q_j^2)) \right]^{\frac{1}{2}}. \quad (7.4)$$

The gaussian  $\exp\left(-\frac{\beta}{2}x^2\right)$  ensures that only terms with  $x < a$  will be significant. Since we are concerned with a system with spatial separation much less than  $a$ , i.e.  $q \ll a$ , we have  $(xq) \ll a$  and hence  $\beta(xq) \ll 1$  so we can expand the exponentials:

$$\Sigma_s^{\frac{1}{2}} = \exp\left(-\frac{\beta}{2}x^2\right) \left[ \sum_{j \in s} (1 + \beta(2\mathbf{q}_j \cdot \mathbf{x} - q_j^2) + 2\beta^2(\mathbf{x} \cdot \mathbf{q}_j)^2) \right]^{\frac{1}{2}}. \quad (7.5)$$

We are only interested in terms that give a contribution to first order in  $q^2$ . We similarly expand the square root to give:

$$\Sigma_s^{\frac{1}{2}} = N_s^{\frac{1}{2}} \exp\left(-\frac{\beta}{2}x^2\right) \left[ 1 + \beta \mathbf{Q}_s \cdot \mathbf{x} - \frac{1}{N_s} \sum_{j \in s} \left( \frac{\beta}{2} q_j^2 - \beta^2(\mathbf{x} \cdot \mathbf{q}_j)^2 \right) - \frac{\beta^2}{2}(\mathbf{x} \cdot \mathbf{Q}_s)^2 \right]. \quad (7.6)$$

This can be simplified by making use of

$$\left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} \int d^3\mathbf{x} (\mathbf{x} \cdot \mathbf{q}_j)^2 \exp(-\beta x^2) = \frac{1}{2\beta} q_j^2. \quad (7.7)$$

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Eq (4.4) then becomes

$$\begin{aligned} \frac{\langle \mathbf{q}' | \rho^H | \mathbf{q} \rangle}{\langle \mathbf{q}' | \rho | \mathbf{q} \rangle} &= \frac{N_u - N_s}{N_u} + \frac{N_s}{N_u} \left( \frac{\beta}{\pi} \right)^{\frac{3}{2}} \int d^3 \mathbf{x} \exp(-\beta x^2) \left( 1 - \frac{\beta^2}{2} [\mathbf{x} \cdot (\mathbf{Q} - \mathbf{Q}')]^2 \right) \\ &= 1 - \left( \frac{N_s}{N_u} \right) \frac{\beta}{4} (\mathbf{Q} - \mathbf{Q}')^2. \end{aligned} \quad (7.8)$$

We can substitute this expression back into eq. (3.4) to get

$$\frac{\partial \langle \mathbf{q}' | \rho | \mathbf{q} \rangle}{\partial t} = -\frac{i}{\hbar} \langle \mathbf{q}' | [H, \rho] | \mathbf{q} \rangle - \frac{N_s \lambda \beta}{4} (\mathbf{Q} - \mathbf{Q}')^2 \langle \mathbf{q}' | \rho | \mathbf{q} \rangle. \quad (7.9)$$

To calculate the rate of excitation,  $R(\phi) = \frac{\partial \langle \phi | \rho | \phi \rangle}{\partial t}$ , we must multiply both sides of eq (7.10) by  $\langle \phi | \mathbf{q}' \rangle \langle \mathbf{q} | \phi \rangle$ , and integrate over  $\mathbf{q}$  and  $\mathbf{q}'$ , leading to:

$$R(\phi) = \frac{\lambda \beta}{2N_s} \left| \langle \phi | \sum_{k=1}^{N_s} \alpha_k \mathbf{q}_k | \psi_0 \rangle \right|^2, \quad (7.10)$$

where we have included the factors  $\alpha_j$  of §5. Apart from the factor  $N_s$ , which is 2 for hydrogen, for example, this is the same as the rate in CSL (or SHM1).

If we now denote the ratio of the excitation rates to the pointer collapse rates by  $\Gamma_{CSL}$  and  $\Gamma_{SHM2}$ , for the two models, then we find that

$$\frac{\Gamma_{CSL}}{\Gamma_{SHM2}} = \frac{N_s}{D_n \alpha^3 [\alpha Z + (1 - \alpha)A]}, \quad (7.11)$$

which will be very small, of order  $10^{-8}$  when we consider a carbon pointer and take  $a = 10^{-5}$  cm. This is the reason why the constraints on the parameters of SHM2 are much tighter than those of CSL.

Of course eq. (7.11) does not hold in the special case where the  $\alpha_j$ 's are proportional to the mass because then, as noted in eq. (7.3), the lowest order contribution to  $R(\phi)$  is zero.

For completeness, we now consider the next order term in this special case.

The calculation for CSL and SHM1 is straightforward, leading to

$$\begin{aligned} R(\phi) &= \frac{\lambda \beta^2}{16} \int d^3 \mathbf{q} d^3 \mathbf{q}' \left[ \sum_{j,k} \frac{m_j m_k}{m_0^2} (q_j^2 q_k'^2 + 2(\mathbf{q}_j \cdot \mathbf{q}_k')^2) \right] \\ &\quad \times \langle \phi | \mathbf{q}' \rangle \langle \mathbf{q}' | \psi_0 \rangle \langle \psi_0 | \mathbf{q} \rangle \langle \mathbf{q} | \phi \rangle. \end{aligned} \quad (7.12)$$

For the case of SHM1, we have already expanded  $\Sigma_s^{\frac{1}{2}}$  as far as we require. Only terms which contain both primed and unprimed coordinates will give a contribution. Further, we may neglect any occurrence of the centre of mass, as it does not excite the internal states. The next order will give terms that must be multiplied by  $\beta \mathbf{Q} \cdot \mathbf{x}$

to give rise to  $\beta^2$  terms after integration, and hence may be neglected. The mass-dependence introduces a factor  $\frac{m_i}{m_0}$  into the sums in eq. (7.5), and  $N_s \rightarrow \frac{M_s}{m_0}$ . This means the relevant part of  $\Sigma_s^{\frac{1}{2}}$  can be written

$$\Sigma_s^{\frac{1}{2}} = \left( \frac{M_s}{m_0} \right) \exp \left( -\frac{\beta}{2} x^2 \right) \left[ 1 - \frac{m_0}{M_s} \sum_{j \in s} \left( \frac{\beta}{2} q_j^2 - \beta^2 (\mathbf{x} \cdot \mathbf{q}_j)^2 \right) \right]. \quad (7.13)$$

We can make further use of eq. (7.7) to simplify the expression obtained, together with

$$\left( \frac{\beta}{\pi} \right)^{\frac{3}{2}} \int d^3 \mathbf{x} (\mathbf{x} \cdot \mathbf{q}_j)^2 (\mathbf{x} \cdot \mathbf{q}_k')^2 \exp(-\beta x^2) = \frac{1}{4\beta^2} (q_j^2 q_k'^2 + 2(\mathbf{q}_j \cdot \mathbf{q}_k')^2), \quad (7.14)$$

leading to

$$\begin{aligned} R(\phi) &= \frac{\lambda \beta^2}{2} \left( \frac{m_0}{M_s} \right) \int d^3 \mathbf{q} d^3 \mathbf{q}' \sum_{j,k} \left( \frac{m_j m_k}{m_0^2} (\mathbf{q}_j \cdot \mathbf{q}_k')^2 \right) \\ &\quad \times \langle \phi | \mathbf{q}' \rangle \langle \mathbf{q}' | \psi_0 \rangle \langle \psi_0 | \mathbf{q} \rangle \langle \mathbf{q} | \phi \rangle. \end{aligned} \quad (7.15)$$

To get a numerical result, we need to substitute explicit forms for the wavefunctions into eqs. (7.12) and (7.15). For simplicity we will take the system to be a solitary hydrogen atom, and consider the collapse happening to the proton and electron. We calculate the rate of excitation to the lowest excited states, the 2S and 2P levels. The  $m=0$  state of the 2P level is not excited here, as it is to first order in GRW.

	GRW	CSL/SHM1	SHM2
To 2S level	0	$0.231 \lambda \beta^2 \left( \frac{m_e}{m_0} \right)^2 a_0^4$	$0.370 \lambda \beta^2 \frac{m_0}{M} \left( \frac{m_e}{m_0} \right)^2 a_0^4$
To 2P( $m=0$ ) level	$0.277 \lambda \beta \left( \frac{m_e}{m_0} \right) a_0^2$	0	0
To 2P( $m=\pm 1$ ) level	0	$0.267 \lambda \beta^2 \left( \frac{m_e}{m_0} \right)^2 a_0^4$	$1.070 \lambda \beta^2 \frac{m_0}{M} \left( \frac{m_e}{m_0} \right)^2 a_0^4$

If we take  $m_0 = m_e + m_p$  as suggested before, then the extra factor  $\frac{m_0}{M}$  in SHM2 is unity, and the excitation rates have a dependence on  $\lambda \beta^2 \left( \frac{m_e}{M} \right)^2 a_0^4$ , for both CSL/SHM1 and SHM2.

## 8. Conclusions

Provided we can naively extend the GRW model to quarks then the original form appears to be ruled out by the experimental data on nucleon stability [4] (by at least 6 orders of magnitude). It is probably also ruled out by the stability of Germanium atoms [6].

The CSL model acting independently on electrons or protons is just on the margin of being possible with regard to the Germanium data [6]- being saved by the extra density factor in pointer decay. The CSL model in the special case where the  $\alpha$ 's are proportional to the mass is certainly not ruled out by any data, the combination of the density factor and the need to go to higher order to obtain particle excitation means that such excitation is unlikely ever to be observed.

As we have seen the first form of symmetric hitting model has predictions essentially identical to those of CSL.

For SHM2, we follow Collett et al.[6], and consider a pointer consisting of a sphere with diameter  $\approx 4 \times 10^{-5}$ cm. We require the collapse time,  $\tau = \frac{1}{\lambda}$  (see [6] for discussion) to be less than 0.01 sec. The theoretical value is given in eq. (6.16) for the case when the separation  $l$  is greater than  $a$ ,

$$\tau \approx \frac{T}{N[\alpha Z + (1 - \alpha)A]}, \quad l \geq a, \quad (8.1)$$

where  $T = \frac{1}{\lambda}$ .

We must also consider the case where the separation is smaller than  $a$ . Then we can write

$$\exp(-\beta(\mathbf{x} - \mathbf{q}_k)^2) = \exp(-\beta(\mathbf{x} - \mathbf{q}_k)^2) \exp(-\beta l^2) \exp(2\beta(\mathbf{Q}' - \mathbf{Q}) \cdot (\mathbf{x} - \mathbf{q}_k)). \quad (8.2)$$

As  $|\mathbf{Q}' - \mathbf{Q}| = l \ll a$ , we can neglect the final factor in this expression. Using this expression in eq. (4.4) gives

$$\langle \mathbf{q}' | \rho^H | \mathbf{q} \rangle = \left( 1 - \frac{N_s}{N_u} \left( \exp\left(-\frac{l^2}{2a^2}\right) - 1 \right) \right) \langle \mathbf{q}' | \rho | \mathbf{q} \rangle. \quad (8.3)$$

Modifying this expression to include the factor  $\alpha$  of §5, and using eq. (3.4) gives

$$\tau \approx \frac{2T\alpha^2}{Nl^2[\alpha Z + (1 - \alpha)A]}, \quad a > 4 \times 10^{-5}, l \ll a. \quad (8.4)$$

Supposing that the sphere is made out of carbon,  $N \approx 3.8 \times 10^9$  atoms,  $Z = 6$  and  $A = 12$ , the two constraints eqs. (8.1) and (8.4) become

$$T \leq 2.3 \times 10^8 (2 - \alpha), \quad (8.5)$$

$$T\alpha^2 \leq 0.1(2 - \alpha), \quad a > 4 \times 10^{-5} = l. \quad (8.6)$$

We also have an experimental constraint from the Germanium data [6],

$$T\alpha^2 \geq 1.1 \times 10^{11} \left( 1 + \frac{m_e}{m_p} \right)^2 \left( \alpha - \frac{m_e}{m_e + m_p} \right)^2 (74 - 42\alpha)^{-1}. \quad (8.7)$$

The inequalities (8.6) and (8.7) impose very strong limits on the value of  $\alpha$ . In fact,

$$\left| \frac{\alpha}{m/(m+M)} - 1 \right| \leq 0.03. \quad (8.8)$$

Given this result, it would be surprising were  $\alpha$  not equal to  $\frac{m}{m+M}$ , i.e. mass-proportional coupling. In other words in this model, which appears to be the most natural hitting model consistent with preserving wavefunction symmetry, the experimental requirements force a mass dependence suggestive of the effect being ultimately connected with gravity.

With this particular coupling, the first order excitation vanishes, and we have a new order of magnitude constraint from the second order

$$T\alpha^4 \geq 10^{-16}. \quad (8.9)$$

The limits from eqs. (8.5), (8.6) and (8.9) are also shown in Fig. (1), which may be compared with the corresponding diagram for CSL (see [6]). This figure also shows that the GRW value  $T = 10^{16}$  sec, is forbidden in the SHM2 model by some 8 orders of magnitude.

#### Acknowledgements

We are grateful to Philip Pearle for discussion of this work. C.D. thanks the SERC for a maintenance grant.

#### REFERENCES

1. G.C.Ghirardi, A.Rimini and T.Weber, Phys. Rev., **D34** (1986) 470
2. P.Pearle, Int. J. of Theor. Phys., **18** (1979) 489
3. P.Pearle, Phys. Rev., **A39** (1989) 2277; G.C.Ghirardi, P.Pearle and A.Rimini, Phys. Rev., **A42** (1990) 78
4. P.Pearle and E.J.Squires, Phys. Rev. Letters, **73** (1994) 1
5. F.Karolyhazy, Nuovo Cimento **42A**, (1966) 390; A.Frenkel, Found. Phys., **20** (1990) 159; R.Penrose, in *Quantum Concepts in Space and Time*, edited by R.Penrose and C.Isham (Oxford Univ. Press, Oxford, 1986); L.Diosi, Phys. Rev., **A40** (1989) 1165; E.J.Squires, *The Mystery of the Quantum World* (Adam Hilger, Bristol, 1986), pp. 111 and 156; and University of Durham Report No. 89/15.
6. B.Collett, P.Pearle, F.Avignone and S.Nussinov, Hamilton College Preprint, *Constraint on Collapse Models by Limit on Spontaneous X-Ray Emission in Ge*, submitted to Phys. Rev. A.

**Figure Caption**

Figure 1. Graph of the boundaries imposed by the "theoretical constraints", eqs. (8.5) and (8.6), in the log T(sec)-log a(cm) plane, for the case  $\alpha = \frac{m_e}{m_e+m_p}$ . The allowed region lies below each boundary. Also shown is the Germanium experimental constraint boundary, eq (8.9), with the allowed region lying above the boundary.

