

HEGEL'S PHILOSOPHY OF LOGIC AS A RECKLESS PROLONGATION OF CUSANUS' LOGICAL DISCOVERIES

Abstract. I take advantage of two recent results: 1) the recognition of an alternative theoretical organization to the deductive-axiomatic one; it is characterized by a sequence of four logical steps belonging to intuitionist logic; 2) the recognition of the logical content of Cusanus' philosophical works; also this content pertains to intuitionist logic, which Cusanus anticipated by even identifying some its logical laws. Many Cusanus' books present the alternative theoretical organization; whose yet he did not apply in a clear way its last two steps, the *ad absurdum* arguing and the application of the principle of sufficient reason. For this reason in the last books his thinking transcended to a metaphysical level. Hegel's philosophy of logic shares Cusanus' philosophical effort for exiting out classical logic. He seems to have assumed Cusanus' anticipation of the new theoretical organization; all goes as if Hegel wanted to improve Cusanus' poor method, by introducing a process of additions of two negations which however does not agree with the intuitionist logic. Moreover, Hegel presupposes, like Cusanus in his last years, the free application of the principle of sufficient reason to the entire subject of his philosophy of logic without awareness of the logical constraints to which is subject a correct application of this principle; hence, he introduced a metaphysical arguing which according to modern mathematical logic has no consistent content. Also Hegel's supposed prolongation of Cusanus' logical discoveries was an essentially metaphysical view.

Keywords: Cusanus, intuitionist logic, doubly negated propositions, *ad absurdum* reasoning, principle of sufficient reason, Hegel's philosophy of logic.

1. Introduction: Hegel well knew previous Cusanus' logical search

In the literature on the relationship Hegel-Cusanus there exists an agreement on three points: 1) contrarily to a common attitude of past scholars of Nicolas Cusanus (1401-1464), his thinking cannot be interpreted through Hegel's philosophy (Gabriel 1970); Hegel suggested a different theory which is not compatible with Cusanus'; 2) Hegel's writings lack of any quotation of Cusanus' antecedent philosophical work; 3) but his logical thinking is in a great consonance with Cusanus'. Indeed, in times dominated by classical logic both wanted to exit out it in order to build an alternative kind of logic. On his hand, Cusanus wanted to found a different theology from both the affirmative one and the negative one and hence from Aristotelian logic; this is the same logic that Hegel denied as static and abstract in order to build a logic of historical and dynamical nature, in sum in order to perform "a total makeover" of past logic (Hegel 1813, "Introduction"; see also the "Introduction" to the II ed.). Both attempts for a new logic started from the reflection on the contradiction principle; they reduced its role to a lateral one, for giving birth to new logical principles, first of all the principle of "the contradiction of the opposites", which apparently clashes with the basic principles of classical logic.

Hence, no surprise if many recent studies gave evidence that through the books that Hegel surely read and meditated he well knew Cusanus and was even familiar with his ideas. A scholar wrote:

No direct connection has ever been established between Nicholas of Cusa and Hegel. It has been accepted that Hegel did not know of him. In fact, there is clear evidence that Hegel knew of Cusanus, and *in detail*. Not only did Bruno twice specifically refer to him, and with the highest praise. Buhle, whose history Hegel read, discussed Cusanus at length, citing all of his most important works.

In the first volume of his *Lectures on the Philosophy of Religion* Hegel quoted Eckhart from one of his sermons - "The eye with which God sees me is the eye with which I see him." In volume III of his *Lectures on the History of Philosophy* Hegel discussed Bohme - 'the first German philosopher' the reading of whose works was 'wondrous,' over eleven pages, giving five pages in the same volume to Bruno, yet he never even mentioned Cusanus - a figure in the German mystical tradition between Eckhart and Bohme.

But the 'divine' Cusanus *who Bruno named*, 'the inventor of geometry's most beautiful secrets,' is Bruno's guide in *Cause, Principle and Unity*, in which Bruno referred to key aspects of Cusanus'

philosophy. Bruno again named and referred to Cusanus as 'divine' in *The Ash Wednesday Supper*, citing his single most important treatise, *De Docta Ignorantia*.

Particularly, in the same volume of his *Lectures on the History of Philosophy*, Hegel wrote simply of Bruno 'The tidiest information about him is to be found in Buhle's history of philosophy. In that extremely interesting work (not only because of the sections on Cusanus but because in it Buhle discusses the impact of Neoplatonism, Kabbalism and Hermeticism in general on German philosophy) Buhle wrote on Cusanus in detail, naming his most significant treatises.

Why did Hegel never even name Cusanus, in any of his writing - a man who was far more philosophical, and in the 'Hegelian manner,' than either Eckhart and particularly Boehme, of whom Hegel also wrote that his articulation was 'unmistakably barbarous' and that he 'grasps the antitheses in the harshest' crudest fashion"?'

My contention in my thesis will be that Hegel never named Cusanus, not only because he was so indebted to one who was *known* to be a Christian mystic/Neoplatonist (I have identified more than thirty points in the philosophy of Cusanus which occur in that of Hegel), but because to do so would immediately open to question the nature of Hegel's vaunted *concepts* (several of which, I will contend, came directly *from* Cusanus), the *apparent* intellectual rigour of his philosophy, and particularly, the *meaning* of his 'reason' - his claims to and for it.

In denying through his utter silence the significance of Cusanus and the foresight of his genius, Hegel was emblematic not only of the German intellectuals of his time [...] but of Western culture as a whole - suppressing key elements of our history, of what has made us, so that we may perceive ourselves, as Hegel did, the bearers and masters of 'Reason.' (Stanfield 2020)

Abstract. In this extract from my thesis I argue that Hegel not only knew of Cusanus but was familiar with his ideas and that he never even named him because of the implications of that acknowledgement indebtedness to Cusanus, the certainty of his own Neoplatonism and for his ambition. Hegel was by no means the only philosopher in his time and later to fail to acknowledge those who significantly influenced them (particularly Neoplatonic and mystical writers). This, in effect, denial of the significance of Neoplatonism and mysticism is representative of the pandering of Western academic philosophers to patriarchal and supremacist notions of what comprises 'reason'. [Instead,] It was the fruits of Neoplatonism (particularly via Cusanus and then Descartes) that enabled Western philosophy to be broken from scholasticism and advanced...

Nothing could more exemplify the dishonesty that permeates modern Western philosophy, a dishonesty motivated by a careerist pandering to the requirements of the dominant ideology, than the relationship between Neoplatonism and the Philosophy of the German idealists, particularly Hegel. [...] Hegel showed clear features of the type of thought found in the Platonism of late antique philosophers like Plotinus and Proclus (Beierwaltes 2004; Vieillard-Baron 1979)... The neoplatonistic thought of Plotinus and Proclus had been a recurring feature of German religious and philosophical thought since the late middle ages, having appeared in influential thinkers like Meister Eckhart and Nicholas of Cusa and later, Leibniz and Jacob Böhme. (Stanfield 2017; emphasis added by A.D.)

In the following three sections I will present the preliminaries for interpreting the supposed connection between Cusanus and Hegel, which will be illustrated by going beyond the lack of any Hegel's reference to this antecedent philosopher. These preliminaries have been established by some recent papers (Drago 2012; Drago 2017a). In sect. 2 I list a series of doubly negated propositions which occur as fundamental propositions within some scientific theories. They belong to intuitionist logic; as a consequence, these theories are organized according to a model of theory different from both (apodictic) Aristotelian, deductive one and its improvement into Hilbert's axiomatic one; their theoretical organizations are problem-based because each of them is aimed to discover a new scientific method by solving a given basic problem. In sect. 3 the four logical steps of a problem-based theory are listed and illustrated. Sect. 4 applies this kind of logic and this new theoretical model to Cusanus' philosophical works. Some books written by Cusanus in his last years (1460-1464) play a decisive role for manifesting a logical attitude that anticipated even some laws of intuitionist logic; but he was unaware of the general logical system and in particular of the correct way of arguing through *ad absurdum* arguments; so that at the end of his life he searched new improvements of his thinking in a purely metaphysical way. Sect. 5 compares the best

advancements of Cusanus' philosophy with Hegel's philosophy of logic. I suggest that all went as if Hegel wanted to improve Cusanus' logical advancements, mainly the use of doubly negated propositions by inventing a syllogism-like process of reasoning (addition of two negations) which instead misunderstands the relations among the three versions (affirmative, negative, doubly negated) of a same proposition in intuitionist logic. As a fact, Hegel's dialectic is a deviant beginning of a non-Aristotelian logic which later will be formally recognized in the intuitionist one. Hence, Hegel developed a metaphysical thinking without a logical consistency.

2. Doubly negated propositions in scientific theories

In ancient Greece the art of reasoning was in a first time a pluralist one. Ancient Greek word for truth: un-veiling. The ancient Greek word for truth was “ἀλήθεια (alétheia)” which is composed by a privative ἀ and λήθω (to be hidden); hence, un-veiling, i.e. a double negation without an equivalent affirmative word, since it means a process, not a fixed idea. In this sense ‘Alétheia’ is in opposition to the Latin word ‘Veritas’, i.e. the present common meaning of truth. The original meaning of ‘Alétheia’ was then changed (maybe by Plato) into an abstract, eternal entity which to believe in (in Italian language wedding ring is called “vera”, the feminine of “vero”, true). Afterwards, Aristotle improved classical logic till up to perform mechanical implications, as it occurs in syllogistic. Along a period of two millennia Aristotle's formalization of logic persisted without relevant changes, apart the birth of a modal logic which however never achieved a self-consistency. During this period logic was mainly considered as an instrument, not as a theory deserving attention by itself. Owing to its precious formal mechanisms, classical logic was assumed as the most productive one. Therefore, it was so greatly developed that in next centuries Western culture considered classical logic in an exclusive way (this was e.g. Kant's prejudice).

Yet, scientific theories a different kind of fundamental propositions; they have the specific features that are doubly negated propositions, whose corresponding affirmative ones lack of evidence (DNPs)¹; hence for each of them the classical law of double negation (commonly said as “Two negations affirm”) fails, just as in the non-classical kinds of logic, first of all the intuitionist logic². Let us consider the following list of this kind of propositions.

By means of the principle "All substances that we could not decompose in any way are for us elements" Lavoisier has given rise to the chemical theory (1789, “Discours préliminaire”, p. 7). Manifestly, previous proposition is a methodological principle giving to a theory a stable viewpoint to its investigations. Later, Mendeleev eventually acquired a proper way of completing the theory through his periodic table, just by reasoning through DNPs, in particular the word “equivalent” (= not different) for representing the relations between two elements under a chemical aspect or a physical one.

In economy, the basic principle is the following one: “No free [= without money] lunch”.

The traditional theory of mechanics begins with the principle of inertia; this principle was enunciated by Lazare Carnot in a different manner from Newton's: “Once a body is at rest, alone does not change its state; if it is in motion, alone does not change speed or direction.” (L. Carnot 1803, p. 49) By cancelling the two negations of L. Carnot's proposition we exactly obtain Newton's affirmative, yet idealistic word "perseveres", whereas L. Carnot's DNP is an operative proposition,

¹ Here we meet a prejudice pertaining to the Anglo-Saxon culture: the use of double negations is a characteristic feature of primitive languages and all such expressions constitute an un-frankly speaking (Horn 2002, pp. 79 ff.; Horn 2010). This prejudice leads to cancel the double negations because all they have to be considered equivalent to the corresponding affirmative expressions.

² In the following I will underline the two negative words inside a DNP in order to make easier to the reader the recognition of the nature of the proposition. The double negation cannot oppose to the affirmation because it represents an open situation; e.g. the proposition: “Acquitted for insufficiency of evidence of guilty” does not means a correct behavior of the defendant nor his misconduct, but only the incapacity of the Court to sharply decide the author of the crime.

because its words may be verified by an experimental physicist³. Moreover, Carnot' formulation of mechanics is aimed to discover the "in-variants" of the impact of bodies. (L. Carnot 1783)

Sadi Carnot's thermodynamics may be summarized by the proposition: "It is not true that heat is not equal to work" that of course does not mean "Heat is equal to work". Its basic principle was: "No change of temperature inside the bodies employed for obtaining the motrice power of heat occurs without a change in the volume". (S. Carnot 1924, p. 23)

In Lobachevsky booklet introducing the first non-Euclidean geometry, after fifteen preliminary propositions, the proposition no. 16 put the problem of how many are the parallel lines; "in the uncertainty" (Lobachevsky 1840), i.e. "... in the ignorance whether the parallel lines are not reduced to only one..." he decides to explore the case of two parallel lines. First, he suggests a new, more general definition of parallelism; then he states two theorems corroborating his definition and then he proves four theorems whose general conclusion is the following proposition: "The second assumption [of two parallel lines] can likewise be admitted without leading to any contradiction in the results..." This proposition is a DNP since it is not equivalent to: "The second assumption is consistent", i.e. "There exist two parallel lines"; which actually Lobachevsky never stated. The doubly negated proposition represents a surmise allowing to correctly reasoning for proving the existence of two parallel lines.

According to Andrey Kolmogorov the basic principle of intuitionist logic is: "Any [affirmative] proposition that is not without content should refer to one or more completely determined states of affairs accessible to our experience." (Kolmogorov 1932, p. 332) This proposition is not equivalent to the following one "All intuitionist affirmative propositions are supported by full evidence", because we cannot control all these propositions.

In sum, there exist logically different principles from the affirmative ones; in logical terms they are DNPs.

Previous investigations on the occurrences of DNPs within original scientific texts suggested the following main rules for recognizing them.

First of all, three cases of doubly negated propositions have to be discarded because they are not DNPs:

a) a doubly negated proposition which is a merely rhetoric one, because a verification of its correspondence with reality makes apparent that it concerns an objective fact; in such a case the proposition belongs to classical logic; e.g. "This move does not lead to you out of the room".

b) a doubly negated proposition which is equivalent to a negated proposition whose negation is emphasised, i.e. one negation of the former proposition reinforces in a psychological way the other one; e.g. the already quoted propositions "I cannot go no further"; "I cannot get no satisfaction".

c) a doubly negated proposition where a negation explains the other one, e.g. "I have no answer from this deaf (wo)man" (actually this proposition is a shortened version of two propositions: "I have no answer from this (wo)man because (s)he is deaf").

Instead, a doubly negated proposition is a genuine DNP when it includes:

- 1) A single word may be composed by two negations; e.g. in-nocent (\neq righteous).
- 2) The word "only", being equivalent to "nothing else", works as a DNP.
- 3) Modal words "must", "possible", "necessary", etc. work as DNPs, owing to the S4 translation between modal logic and intuitionist logic. (Hughes, Cresswell 1996, pp. 224ff.). These words will be dotted underlined.

3. From the DNPs the discovery of an alternative model of theoretical organization to the axiomatic one. The four logical steps of problem-based organization of a theory

³ In addition notice that here we are talking about "a" well established body, not, as Newton says, "all" bodies, whose experimental evidence is inattainable.

In the past, owing to the inaccuracy in circumscribing its contents, a DNP was considered as a blind alley for understanding the reality, or at most as the characteristic feature of a mysticism (see e.g. a common interpretation of Cusanus' writings). *From all in the above instead one draws that a DNP constitutes the mark of the non-classical rationality; and it is the base of an alternative way of reasoning.*

Along two millennia a clear presentation of a scientific theory required a systematic organization of a deductive kind starting from few axioms. But this kind of organization cannot include a DNP. Indeed, a DNP cannot play the role of an axiom, because its content cannot be stated with certainty (it corresponds to an open set in Stone representation of intuitionist logic; it states a(n imprecise) bound to our inaccurate thinking). In logical terms, being the content of $\neg\neg A$ different from the content of A , no affirmative proposition can be derived from it. On the other hand, an axiom, being expressed by an affirmative proposition which is equivalent to its doubly negated proposition, cannot produce by logical implication a DNP, which is not equivalent to its corresponding affirmative proposition. Hence, the classical consequences of an affirmative axiom are propositions of only classical, never a DNP pertaining to a non-classical logic. In sum, even the occurrence in a theory of a single DNP prevents a deductive organization of this theory.

So no theoretical organization through DNPs? Most scholars seem to have this opinion about what they call the "negative" thinking. Instead, a comparative analysis on the texts of the above-mentioned theories shows the well-defined characteristic features of a common model of theoretical organization.

- i) Each of these theories presupposes no more than the common knowledge about the field at issue.
- ii) It states a problem which is unsolvable through usual tools. Lavoisier's chemistry: what are the elements of the matter? Lazare Carnot's mechanics: what are the invariants in an impact of bodies? S. Carnot's thermodynamics: what is the maximum efficiency of heat/work transformations? Lobachevsky's geometry: How many parallel lines exist? Kolmogorov's above-mentioned paper formalizing intuitionist logic: how formalize in logical terms a calculus of mathematical problems?
- iii) It is aimed at inventing a new method for solving the previously given problem.
- iv) In order to solve the given problem the author of the theory argues by linking together the DNPs through words as "otherwise we obtain an absurd result" an *ad absurdum* argument (AAA)⁴. This DNP may work as a premise for a next AAA; hence, many AAAs may constitute a chain of AAAs.
- v) The final AAA concludes a DNP suggesting a universal predicate, $\neg\neg UT$, which represents a possible resolution for all cases of the starting problem and the related problems. Hence, the conclusion of the final AAA suggests a surmise; but being a DNP, it cannot be accurately tested with reality.
- v) At this point, an author of such a theory as a matter of fact performs the step called PSR, i.e. he translates the above predicate $\neg\neg UT$ into the corresponding affirmative predicate UT ⁵. One can suppose that the author thinks to have already collected enough evidence to be justified in promoting his conclusion $\neg\neg UT$ to the corresponding affirmative proposition UT , although this change is not allowed by non-classical logic, which previously he adhered to.

⁴ This AAA is of a weak kind, i.e. its final proposition is a DNP. Notice that a common opinion holds that an *ad absurdum* argument concludes an affirmative proposition; yet this conclusion is possible only if one applies to the conclusion of an AAA, i.e. a DNP, the double negation law, which however pertains to only classical logic.

⁵ The logical step which in the above was denoted as PSR is explicitly manifested by three eminent scientists. Galilei (1958, p. 191), Lobachevsky (1840, prop. 22) and Einstein (1905b, p. 891).

vi) Now he draws from this affirmative proposition all possible derivations, to be subsequently tested against reality for a verification of the built theory. (Drago 2012) Hence, the application of PSR translates the conclusion of the AAAs into an affirmation whose derivations are tested with reality. If the answers that he obtains are positive, what was a hypothetical reasoning is changed into a theory of reality. I conclude that there exists evidence of a scientific nature of the logical step PSR to the conclusion of (the sequence of DNPs and AAAs of) a PO theory.

vii) Let us examine this step, PSR, in formal terms of the square of opposition. It corresponds to a change of the main thesis A (“S is P”) from its non-classical version (“not (S is not P)”) to the classical one. Through Dummett’s table of all implications between a couple of classical and intuitionist predicate (Dummett 1977, p. 29) it is apparent that previous translation is enough for changing the entire intuitionist predicate logic into the classical one (indeed, the similar change of thesis E is obtained by merely negating thesis A; the change of thesis I is obtained by doubly negating thesis A).

This step PSR apparently represents an application of Leibniz’ principle of sufficient reason; its antecedent is itself a universal DNP (“Nothing is without reason”) and the consequent is the corresponding affirmative proposition (“Everything has a reason”). Through the step PSR an author of a problem-based theory performs on a specific predicate of this theory a logical translation which formally is the same translation performed by the application of this general principle, i.e. the logical translation occurring from the antecedent of Leibniz’ principle of sufficient reason into its consequent:

$$\neg\exists x \neg P(x) \Rightarrow \forall x P(x).$$

Notice that the translation performed by this principle merely constitutes the inverse translation of the so-called ‘negative translation’ (i.e. a suitable addition of two negations to each classical proposition (Troelstra and van Dalen 1988, pp. 56 ff.). Yet, whereas the latter one can be always performed under some simple rules on the way to add the two negations, the application of PSR is valid, as suggested by Markov, under two requirements on the doubly negated predicate on which it is applied: it has to be 1) derived from an AAA and 2) decidable (Markov 1961, p. 5; Drago 2012) for the simple reason that for passing from a hypothetical world to a real world we have to be supported by reality criteria.

Let us remark that the essential steps of a PO theory are the previous four: ii), iii), iv), and vi).

4. The results of Cusanus’s searching for a suitable name of God

At glance Cusanus’ illustrations of his subjects seem an inextricable link of concepts devoid of content. A scholar went to write that Cusanus presents “une jonglerie de mots” (“a word juggling”) resulting into “tours de passe passe” (turns of inconsistent connections; Duhem 1913, p. 262). However, a more accurate reading of his writings shows that Cusanus dramatized an exhausting search looking for the best name of God. From the *coincidentia oppositorum* of *Maximum* and *Minimum* in the book of 1440 (*De Docta Ignorantia*⁶), in 1460 he went to the name *Posse=est* (*Power=is*), shortened in *Possest*. Here his thinking relied on modal logic, determined by the word *posse*. Two years later he went to discover the (intuitionist) name *Non Aliud* (Not-Other) giving the title of a new book: *De Non Aliud*. It is remarkable that in this new book, by reasoning about theological subjects Cusanus anticipated some laws of intuitionist logic (of terms). He stated that not-other is not the same (Cusanus 1462, chap. 5, p. 1117, no. 19; p. 1164, no. 123); this distinction between the double negation word and the positive word represents the basic law of intuitionist logic. He stated also that “the negation does not exist before the not-other.” (ibidem, chap. 4, p. 1113, no.11); this proposition affirms the validity of the doubly negated term whereas the negative term and the positive term are only partially true. Notice also his following proposition: “Negation does not oppose to affirmation”. These intuitionist logical laws were stated for the first

⁶ All Cusanus’ books will be referred in English translation from Jasper Hopkins’ site: <https://jasper-hopkins.info/>.

time by Cusanus; and *vice versa*, modern logic (essentially, the intuitionist one) helps to legitimate and clarify Cusanus' basic ideas⁷.

The name Not-Other raised his enthusiasm, because it allowed him to make great intellectual advancements; first of all, a concise verbal formula designating the Tri-Unity: *Non aliud est non aliud quam non aliud* (Not-Other is Not-Other than Not-Other); by suitably reiterating three times the word "Not-Other" this formula may represent the three distinct Persons of God; at the same time the Unity of God is represented by (the three times) use of a same name.

In correspondence to the results of his research of new names of God his numerous books have been severed in distinct periods (Drago 2017a, p. 159). The following table represents this periodization which is more accurate than previous Kurt Flasch's (2001), which was obtained by a longtime reiterated reading and reflecting on Cusanus' texts.

Table 1. PERIODIZATIONS OF CUSANUS' PHILOSOPHICAL WORKS

Cusanus' Works	Flasch 1998	Drago 2017a
<i>DDI 1440</i>	2° A system based on the <i>coincidentia oppositorum</i>	2° He thinks through both the negative and the <i>coincidentia oppositorum</i> ; e.g. God as the coincidence of <i>Maximum</i> and <i>Minimum</i>
<i>De Coniecturis 1444</i>	3° The <i>coniectura</i> as self-awareness of the <i>intellectus</i>	3° By a <i>coniectura</i> ⁸ the <i>mens</i> obtains the <i>coincidentia oppositorum</i> and applies it to the reality as well as the divine.
<i>Idiota</i> (1450) and the following works	4° applying <i>coniecturae</i> to the reality	
<i>Possest 1460</i>	5° Pure Power	4° God as <i>Posse=est</i> : Modal logic; Principle of Sufficient Reason
<i>De Non Aliud 1462</i>		5° God as <u>Not-Other</u> ; Intuitionist logic; Heuristic theory
<i>De Venatione Sapientiae 1463</i> , <i>Compendium 1463</i> , <i>Apice Theoriae 1664</i>		6° God as <i>Posse</i> ; elaboration in Modal logic; Deductive-like theory

⁷ First Ernst Cassirer suggested that Cusanus has introduced a new kind of logic (Cassirer 1927).

⁸ He stated that "... a surmise (*coniectura*) is a positive assertion that partakes—with a degree of otherness—of truth as it is." (Cusanus 1441-1442, I, 11, no. 57, p. 190) The word "partakes" summarizes a DNP because it is equivalent to: "It is not the case that it is not...". However, Cusanus did not perceive this proposition as a DNP.

(The qualifications “heuristic theory” and “deductive-like theory” refer to the respective final summaries of *De non Aliud* and *De Apice Theoriae* and correspond to the two kinds of theoretical organization, respectively PO and AO).

5. How Cusanus may have paved the way to Hegel’s philosophy of logic

Here I suggest a more accurate periodization of Cusanus’ books of the last period of his life (1460-1464). According to Flasch’s periodization the contents of all these books are directed as their natural conclusion, to the content of the last work (*De Apice theoriae*, 1464, the year of his death) exactly as Cusanus presented it: the same title of this last book announces to have achieved the ultimate result of his reflections. Instead Drago’s periodization has detailed this period by distinguishing Cusanus’ book based on modal logic (*Possest*) from his book based on intuitionist logic (*Non Aliud*). But he did not give reason of the occurrence of last four books. Apart *De Ludo Globi* (1463) (which illustrates through an odd kind of game Cusanus’ conception of mechanics’ dynamics and by analogy the soul), they are *De Venatione Sapientiae*, summarizing ten strategic paths for “hunting” the wisdom (of approaching God), *Compendium*, summarizing the paths of his previous researches and for first time introducing the word *Posse* for naming God, and *De Apice Theoriae* (1464) which is presented by him as the point of arrival of the research of his entire life (Cusanus 1464, p. 1424, 4).

I suggest that the last two important books represent recoil of Cusanus’ reflection from the method described by *De non Aliud*. The latter book was exalting in suggesting great novelties, but, beyond his first results, in the end it suggested a very different method of arguing from Cusanus’ usual one; in other terms, the word not-other was impracticable by him for arguing further on both God and Tri-Unity.

Actually, he had already looked for discovering a better way of reasoning. In *De Aequalitate* (1459), although it was clear to him that Aristotle’s syllogistic concerns classical logic, i.e. the kind of logic from which he wanted to exit out, he devoted space to analyze this traditional way of formally reasoning. He attempted to suggest some novelties; but at last he did not obtain nothing of relevant for his aim of reasoning in a new way with respect to Aristotle’s one.

We know that according to a PO theory Cusanus’ reflection on *De Non Aliud* ought to reason by composing through DNPs *ad absurdum* arguments; which really his book presents, but in an occasional circumstances and without noticing their importance. Since he had failed to discover a new way of reasoning, in the end the double negation of the exalting name Not-Other seemed to him a *cul de sac* of his research.

Rather, the previous method of modal logic in *Possest* appeared to him more productive than the latter one; since modal logic is more apt to argue through intuitive, linguistic turns of phrase according to his reasoning through primitive rules⁹; through modal words he could space on apparently unlimited horizons. For this reason in his last works Cusanus came back to arguing through modalities, as in the book *Possest*. Indeed, in *De Apice Theoriae*, although he suggested a number of AAAs, many of the following form “Without...., nothing...”, he persisted in arguing in an intuitive way on modal words, i.e. by combining the terms through daring connections.

In addition, according to the PO of his theory Cusanus had to conclude by applying PSR. He actually expressed the PSR in *De Non Aliud*, a first time in chap. IX: “it is not the case that anything is created unreasonably”; and then in proposition 20 of its summary: “Nothing is in vain”. In the former case he recognizes it as the Ground of the world, i.e. as a basic principle of reason. But he did not attribute to it a theoretical role, neither explicitly or implicitly, in his theories.

In particular, Cusanus does not apply the PSR to his main result of *De Non Aliud*, the word *non aliud*, because it would become *idem* and his intuitionist logic would be denied. Instead, he had applied PSR in previous book *Possest*. We know that the name *Posse=est* represents an application

⁹ An inspection of his way of arguing shows that his crucial words are the following ones: without greater thing, *maximum* and *minimum*, antecedent and consequent, cause, generate, depend, “idempotence” (e.g. all of all, form of the form), etc.

of PSR to the modal name *Posse*, which as a modality is equivalent to a doubly negated term (“It is not true that cannot...”) and hence the result of the application of PSR (“it can”) may be the term *est*¹⁰. The entire book depends from this application of PSR.

But he ignored that in order to avoid that the reasoning enters into a metaphysical realm the correct application of PSR to a predicate requires that it fulfills two constraints. Surely, he did not care to examine if his predicates are decidable; in fact, they are not. Therefore, in *Possest* he inadvertently entered into a metaphysical realm which did not distinguish the reality from the idealistic concepts and beings. Through his free application of PSR Cusanus went to, according his words in *De Visione Dei* (1453), overcome the wall constituted by the *coincidence of oppositorum*, i.e. he entered into the Paradise.

The word *Posse* is a modal word, but Cusanus’ attribution to God gives to it a metaphysical substance, so much to be considered by a believer in God as an objective word. In such a sense in last Cusanus’ work, *De Apice Theoriae*, the word *posse* has two meanings: the original meaning of the modal word and an objectively demonstrated capacity to do, as it is attributed by a believer to God. In such a way this term alone, *posse*, joins in itself the two meanings which otherwise would be joined by only an application of PRS as he had did in previous book *Possest*. At last, the book is an entire metaphysical vision produced by the exalting last discovery of his life (*De Apice Theoriae*, 5, pp. 1424-1425 and 14, pp. 1429).

This transcendent plan of arguing now concerns first of the *quidditas* of everything, in particular God; hence, he does no more distinguishes between real world and transcendent world; the reality has joined metaphysics; and *vice versa*. In this vision God as *Posse* subsumes in Himself the entire world as His direct expression or prolongation. It seems that Cusanus perceived that this conclusion contradicts all what he had written in previous books, i.e. God is unattainable, because he introduces (*De Apice Theoriae*, 10-11, pp. 1427-1428) a correction, by distinguishing between “to have the power to see” and “to have the power to understand”; where only the last capacity of human beings is limited. However, each qualification of a subject or being is now of a qualitative nature, not of a quantitative nature.

Clearly the viewpoint of Cusanus’ arguing is different in correspondence with the kind of logic applied by Cusanus and the kind of application of PSR. The book *De non Aliud* presents the viewpoint of a person searching the best name of God (even the first part of the complete title of the book states this viewpoint: “*Directio speculantis seu De non Aliud*”, i.e. “Address to a searching philosopher...”). Instead, Cusanus’ application of the PSR to the word *Posse*, giving *est* without caring of the two constraints (the doubly negated term has to result from an *ad absurdum* argument and decidable) suggests to him a metaphysical viewpoint looking for the true nature of God, a nature which transcends all personal viewpoints. In other words, *Possest* presents a transcendent viewpoint; no surprise if Cusanus performed without problems his investigation on this metaphysical plan, because he ignored to have missed a logical consistency.

Two years later he temporarily abandoned this viewpoint when discovered the intuitionist name “Not-Other”, which more adequately represented his personal research. But in *Compendium* (1463) and in *De Apice Theoriae* (1464) Cusanus renounced to the personal viewpoint which appeared to him no more improvable beyond his previous use of the word not-other and chose to pursue his metaphysical view. He shortened the name *Possest* into the name *Posse* (or *Posse ipsum*; which, without changing the logical nature of the name *Posse*, merely emphasizes it) and elaborated it in a metaphysical way. At last, his trespassing in both *Compendium* and *De Apice Theoriae* to metaphysics, his viewpoint is clearly the same transcendent viewpoint obtained by naming through the application of the PSR God as *Posse=est*.

In the history of philosophy this Cusanus’ result constitutes the birth of an absolute Spirit, although in a Christian version, since he identified Him with “God trine and one”, as he writes in

¹⁰ By following classical logic Scholastic school well knew that “*Ab esse ad posse valet consequentia*”; but not viceversa: “*Ab posse ad esse non valet consequentia*”.

the summary of *De Apice Theoriae*, the beginning of the last proposition. This conclusion of the book may seem a paying off debt which he as cardinal of Catholic Church had to profess according to the Christian faith; instead it represents the failure in exploiting the word *Posse* for characterizing the Tri-Unity more than he had did in *De Non Aliud*; hence, it represents also the failure of his inexhaustible hunting the wise knowledge of the divine beings notwithstanding he had changed his viewpoint in a manifestly metaphysical one.

6. Hegel's reckless prolongation of Cusanus' thinking

Scholars proved that Hegel knew Cusanus' books or at least their essential contents.

Like Cusanus' logic, the main characteristic features of Hegel's philosophy of logic are the following ones:

1) it is based on terms; that is, it is ancient logic with respect to modern mathematical logic which started in the second half of the 19th Century (Boole, Frege) on the analysis of the propositions and their connections. 2) It reasons through relations of merely compatibility of the terms composing a discourse. 3) It represents an attempt of exiting out the Aristotelian (i.e. classical) logic for introducing a new one. 4) Both attempts for a new logic started from the reflection on the contradiction principle; they reduced its role to a lateral one, for giving birth to new logical principles, first of all the principle of "the contradiction of the opposites", which apparently clashes with the basic principles of classical logic.

Hegel seems prolong Cusanus' search on the main more important logical problems of the latter: 1) how to reason out of Aristotelian logic through the use of doubly negated terms (or propositions); 2) how to characterize reason's approaching and even achieving a transcendent world.

Let us start with the first problem. The main difference of Hegel's logic from the old logic is his claiming to argue through a dialectics which proceeds through the application of a sequence of two negations to a positive term. Since for Hegel this term represents the being in itself, it belongs to classical logic. One may remark that also the negation of this term is again a logical operation of classical logic. Hence, the entire novelty of Hegel's logic concerns only the second negation. Actually, Hegel characterizes this application, giving rise to the synthesis of both the positive term and its negation, as an *Aufhebung*. He illustrated this logical step in philosophical terms, but to his posterity the logical interpretation of this process remained unclear.

First of all, I suggest that Hegel's search for a new logic clearly constitutes a PO theory rather than a deductive theory. Moreover, in order to an *Aufhebung* really constitute a very difference of Hegel's logic from the classical logic it has to represent a logical translation from the classical one to a kind of logic which considers the doubly negation of a term as essentially different from the positive term; that means the intuitionist logic. In addition, we know that in a PO theory this translation corresponds to an application of the PSR. It is therefore the PSR that translates Hegel's doubly negated term of the original positive term (or proposition) into the synthesis.

Notice that the philosophical meaning of the PSR is an act of faith in the rationality of the reality; i.e. it is the belief that our correct reasoning correspond to the intimate logic of reality. No surprise then if Hegel systematically applying his *Aufhebung* process, actually the PSR, states with emphasis that "What is rational is real" and *vice versa* "What is real is rational".

In such a way Hegel may claim that his logic is not simply a science of the form of our thoughts; it is also a science of actual *content* as well, and as such has an ontological dimension; clearly, all that pertains to metaphysics only. He thought to be successful in his aim because his kind of Logic, as a whole, has as first Principle that his "rational" though can develop by using *only* the resources available to thought itself, without meeting any constraint.

Notice that Hegel ignores that in a PO context even the positive term should be considered as belonging to intuitionist logic, i.e. as only a partial truth which is not in opposition to both its negation and its double negation (as Cusanus had well explained it). In other terms, intuitionist

negation is different from the classical negation. So, inside a PO context the application of a negation to an affirmative term in order to have the negative term gives no arguing advancement; it simply wanders into the indefinite world of the partial truths. It is only the doubly negated term that gives something of firm, stable, clear meaning. In this sense, Hegel was not wrong in considering the synthesis as an *Aufhebung* of previous positive and negative terms and moreover in exalting this step as a crucial one; but since his previous listing the positive terms and the negative terms is not a valid premise for trespassing to the synthesis; this trespassing it is not a logical translation. In conclusion he wrote nothing concrete about his dialectical process.

We can refer all said in the above to the most celebrated instance of his dialectical process, through which he separated his philosophy from Kant dialectics of the notions of Being and Nothing. When Hegel states that given “Being”, its negation is “Nothingness” and at last the synthesis of this process of successive negations is “Becoming”, he does not determine the last logical step; the word “Becoming” represents a nice find, ma no one can assure that this word is the only one possible for closing the process; the final word that he suggested (“Becoming”) is merely one among the many word which are not incompatible with the previous doubly negated term. As a matter of fact, Hegel’s final logical step is not an application of PSR, as a PO context could suggest, of the doubly negated term “Not Not Being”, but an almost free elaboration of it, leading to an ill-characterized term.

A similar conclusion about the under determination of Hegel’s typical process is obtained by considering it, in the reverse order. By starting from “Becoming”, we clearly see that this term is only in an approximate way both the negation of “Nothingness” and the doubly negated term of the first positive previous term, “Being”.

We know that an application of the PSR allows escape into an uncontrolled metaphysics unless the two Markov’s constraints are fulfilled. Hegel’s elaborations of the doubly negated term did not prove *ad absurdum* theorems or decidabilities. Hegel had no idea of these constraints and therefore freely applies his PSR, which led to transcend in an uncontrolled way the reality of the basic term¹¹.

I conclude that the distance of Hegel’s logical process from the unique way of interpreting it through modern logic, i.e. an intuitionist logical process, is very great. His inaccurate definition of an *Aufhebung* process and his implicit application of PSR in a free way opened a great room to not only a metaphysical thinking but also misunderstandings and deviations. No surprise if more than two centuries of studies of Hegel’s logic were unsuccessful to establish a common agreement on a minimal content of his logic.

How all in the above may be linked to Cusanus’ thinking? Through Cusanus’ notion *Posse*. Hegel seems to have developed in an a-religious way the identification between substance and spirit suggested by Cusanus’ *posse*; the substance of a being is become spirit and even God is become (absolute) Spirit. In addition, Hegel seems to have thought to have solved Cusanus’ problem of discovering the way of reasoning inside the new logic: he suggested his dialectics as the wanted new logical tool.

But under the light of intuitionist logic both his innovations result to be ineffective. About these innovations holds true the old motto: “What was new (dialectics) was wrong, and what was correct (according to Cusanus: Absolute Spirit) was not new”.

In sum, implicitly Cusanus followed a clear kind of logic, in the book *De Non Aliud*; through previous book *Possest* and the last books he closely approached the reality to metaphysics; instead never a precise kind of logic underlie Hegel’s thought; moreover Hegel’s free use of PSR always identified in a metaphysical way reality and thinking, so much that he conceived a “super PSR” concerning more than the relationship of the reasoning with the static reality of the world; this “super PSR” constitutes a strict relationship between the temporal evolution of human reality with a

¹¹ He also ignored also that not only in classical logic but also in intuitionist logic three negations give one negation; so that its suggestion of a chain of negation from a given affirmative term actually stops after the second one, because a third negation give back the negation of the starting term.

reason which however cannot belong to a single human being, rather a collective being which of course is the Absolute Spirit. He believed that the philosophy of history is to comprehend "what is and what was"; wherein, it is "truer, while adhered to reality" (Hegel, 1956, p. 292). In this way his theory claimed to have achieved completeness beyond any previous philosophy; it have interpreted the historical dynamics of human world, like in theoretical physics the paradigmatic Newton's theory culminates into a dynamics.

Conclusions

In the above, by applying (modal and) intuitionist logic I suggested an answer to the mystery of the origin of Hegel's thought on logic. All went as if Hegel wanted to solve the two main problems of Cusanus' reflection by developing the contents of Cusanus' last books, in particular *De Non Aliud* and *De Apice Theoriae*.

This answer suggests the many reasons of the two centuries of unsuccessful studies aimed at finding out both the origin and the internal logic of Hegel's novelty. First, the guilty silence of Hegel on his philosophical sources; this silence constituted a great hindrance to discover his background. Second, the possible term *a quo* of Hegel's philosophy of logic, i.e. Cusanus' works, was longtime ignored by Hegel's scholars because these books were very difficult to be interpreted in a consistent way. Third, the common qualification of cardinal Cusanus' books, mainly the books of the last years of his life as mystical discourses on one hand made very arduous to disentangle Cusanus intellectual path which was an essentially logical one (only at the end of last century a consistent and almost complete understanding of his philosophical production was achieved; cfr. Flasch 2001) and on the other hand made unlikely a connection to Hegel's a-religious thought. Fourth, Cusanus' anticipation of intuitionist logic was unthinkable by Cusanus' scholars, as well as Hegel's scholars, all informed by only classical logic. Fifth, the common prejudice of both AO as the only possible theoretical organization and classical logic as the only valid kind of logic prevented the recognition of the model of a PO theory in both Cusanus' and Hegel's productions, as well as their essential use of intuitionist logic; the overcoming this prejudice was the crucial step for deeply interpreting Cusanus' books. Sixth, last but not least, the scholars conceived PSR as unavoidably leading to uncontrolled thinking; yet, they did not notice Markov's suggestion of two constraints, which gives to PSR a realistic efficiency in performing a fundamental logical operation, i.e. the translation from intuitionist logic to classical logic.

So many difficulties surpassed the capacity of the naïve logical search performed by a multitude of scholars of Hegel's philosophy of logic, in particular the followers of clever Marx' suggestion: "With him [Hegel] it [dialectic] is standing on its head [metaphysics]. It must be turned right side up again [the reality], if you would discover the rational kernel within the [Hegel's] mystical shell." (Marx 1906, p. 25)

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