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Dossier
Methods and Cognitive Modelling in the History and Philosophy of Science–&–Education

Suggestion for Teaching Science as a Pluralist Enterprise

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Abstract.
The change in the organization of science education over the past fifty years is quickly recalled. Being its cultural bound the lack of a conception of the foundation of science, the multiple innovations have resulted as temporary improvements without a clear direction, apart from the technocratic goal of an automation of learning processes. The discovery of two dichotomies as the foundations of science suggests a pluralist conception of science, and hence the need to entirely renew science education, in particular by introducing the notions of incommensurability and radical variations in meaning of the basic notions. This change was already anticipated by several proposals within each subject-matter.

Keywords:
Two Dichotomies; Four Models of Scientific Theory; Incommensurability; Radical Variation of Meaning; Suggestion for a New Didactic of Pluralist Nature

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The Current Situation of the Scientific Education

Notwithstanding a radical change occurred in society in the last half of century, and a radical change in the scholastic educational process, at the present a teacher still enjoys ample space for developing a gratifying work within the school. First of all, a teacher still has a visual relationship with the students, even if this relationship is polluted by youth fashions, consumerism, drugs and communication devices; through this visual relationship a teacher remains the only one person that creates convictions, not opinions. In addition, he can find the collaboration of other colleagues (also at world level), associations of teachers and some

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other innovative schools; finally, with respect to a teacher of fifty years ago, he has at his disposal an enormous number of teaching aids as well as cultural resources.

However, present science teaching is informed by the ideal of the unity of science, although the variety of the foundations of science cannot be hidden to high school pupils: Chemistry is usually under evaluated as a low-level theory; Thermodynamics' notions of friction and cycle oppose to Mechanics' abstract notions, which are commonly presented as the most assuredly scientific notions; then, modern physics requires a so deep change of scientific mentality with respect to classical Physics to result as inconclusive (even in many physicists' minds); in addition theoretical Biology does not accept anything from a three centuries-long development of theoretical Physics. Last but not least, no appraisal of the historical development of science along four centuries can be transmitted to the pupils, because no appraisal is shared by scientists' community, apart from the vulgar conception of a unceasing accumulation of formidable achievements. In order to cover this obscure cultural situation, the popularization of science presents to the pupils surprising scientific notions, brilliant discoveries, marvelous advancements, without any relationship with the hard scientific method (whatsoever this method may be, after the inconclusive debate among science philosophers).

Since the present, highly informative science teaching presenting no evidence for the claimed unity of science, and modern Physics teaching, stressing many scientific notions which are at odds with the intuitive ones, corroborate a separation between the two cultures, i.e. the humanistic one and the scientific one. Unfortunately, a conciliation of these cultures cannot be obtained from a study of five centuries long pondering by the philosophers of science on this subject; indeed, inside the classroom philosophy teaching pursues a separate own target, i.e. the autonomous rise and fall of Kantian synthesis, or the subsequent Hegel's system. For this lack of a valid cultural synthesis, a pupil is attracted by ephemeral philosophies of the present stage of the scientific development as they would be the last word on the subject.

A Quick History of the Paradigms in Science Teaching

In this ill-certain cultural framework, the critical question is whether a teacher is fully aware of the work he is doing and in particular whether he commands the subject of his teaching so that to have the awareness of the culture that he transmits to his pupils. Unfortunately, this consciousness is not there yet; neither the current education at undergraduate level stimulates a teacher to perform an autonomous process of learning on the subject.

Not only, as Kuhn says, (Kuhn 1969) the history of science has gone through paradigms (Ptolemaic, Newtonian, etc.), but also science teaching has gone through different paradigms. A quick reflection identifies four didactic paradigms, valid at least for Physics teaching. In the following, in order to facilitate the grasping of their content each of them is marked by a specific word:[2]

1st Paradigm (<1958) science is "Truth"; experimental results are absolute truths, which are codified by textbooks in a systematic way and authoritatively presented by teachers. These truths are independent of society, men and human faculties; hence, a science teacher

[2] The year 1958 is chosen as the borderline of two science teaching attitudes since in such a date the launch of the first satellite to orbit the Earth was a coup for the Soviet Union, a Communist country and America's principal international rival in the Cold War. The Soviets' history-making accomplishment created concern that the Soviets had beaten Americans into space and may be after a short time in the whole challenge. In 1958, Congress approved $1 billion for the National Defense Education Act, or NDEA, the first of more than a dozen programs at all levels of American education, meant to help US students compete with the Soviets. Since few high school pupils usually chose scientific subjects the entire scientific education was radically changed in order to present more appealing and pleasant subjects. (Hare 1999)

transmits to the pupils truths that are independent of their persons and their interpersonal relationships inside the classroom.

2nd *Paradigm* (1958-1980) Physics teaching is facilitated by reducing Physics to its backbone (plus some innovations); it is referred mainly to experiments since they characterize science with respect to any other kind of knowledge; mathematics is added as a further constitutive element. Moreover, by overcoming the old Physics didactics, which ignored the student as a person, some hard problems within the process of Physics learning are recognized; however, it is believed that they are surmountable by introducing the kind of pedagogy which radically changes the pedagogical tradition, i.e. the "active pedagogy"; this one facilitates the learning process by, above all, suggesting appropriate motivations for studying Physics; and then by inserting all kinds of attractions (fascinating experiments, colored figures, photos, cartoons, amusement mathematics, ideals of a great scientist career). Also the organization of the education process inside the classroom undergoes a complete revolution; teaching process changes from being centered on scientific truths, collected by textbooks, to being centered on pupils, whose learning process – more than performed by studying a textbook – is supported and facilitated by both attractive experiments and an assist by the teacher, who is informed by his specific guide about the best teaching method.

However, no critical view on this kind of the scientific knowledge was introduced; hence, two attitudes to learning this kind of indubitable knowledge were possible within a pupil; either a disaffection for a knowledge which is detached from the common life or an a priori consensus to an attractive teaching. With respect to this divide, the active pedagogy had the merit to carry a minority of students from the former to the latter attitude, but the learning process of a large majority of Physics students remained determined by activities and influences experienced outside the school.

3rd *Paradigm* (1980 – present) In order to be most effective, the Physics teaching has looked for a pedagogy that also it is scientific in nature: therefore the new teaching introduced computer-aided teaching and first of all "the cognitive sciences", those sciences promising to indicate how the human mind understands and learns; it is hoped that these sciences will suggest a safe teaching formula, in view of an automation of as most as possible education processes. In synthesis, science teaching changed into a paradigm centered on cognitive processes.[3]

4th *Paradigm* (2000 – present) the paradigm is centered on the "web" (and more in general, mass media), where the pupil is a mere knot and the teacher a navigation assistant. The learning process constitutes an introduction to a community network. People within a classroom is changed into an almost informal grouping of persons, who are very different in ages and levels of culture and moreover lacking any rule which is intrinsic to the cultural process the persons are producing each one for himself.

Physics Teachers Have Moved to Teach through Physical Notions

After sixty years of innovations, science teachers are not still satisfied by the received suggestions for solving the problems of their Physics teaching. From the time of the second paradigm onwards, Physics teachers have begun to be aware that their teaching is objectively difficult because the main physical theories cannot be reduced to elementary experiments plus some pre-calculus computations; in this way, Physics teaching seems reduced to a practicality. The basic question is: Beyond objective Physics, what other aspects of Physics should be taught? In other words, to have abandoned the (positivistic) Truth of the science left a cultural void which can be correctly filled by only an autonomous

[3] This historical development is parallel to the historical development of the factory work organization (Drago 1977).

philosophical view of the scientific knowledge, but in the lack of specific suggestions, how to avoid an indoctrination? None of these trends may gratify a Physics teacher wanting to teach true scientific knowledge. In addition, he has begun to be aware the undergraduate studies did not suggest any tool to deal with these problems.

In sum, the paradigm-changes resulted to be weakly effective in improving pupils' knowledge and facilitating their education; even less to answer teachers' problems.

Then spontaneously Physics teachers tried to treat their subject–matter similarly to how the other teachers, especially the humanist ones, treat their subjects; they ventured to work on the concepts of Physics (subjective science). Inside science, a single physical quantity (e.g., space, time, force, mass, etc.) is usually well defined through both its process of measurement and its mathematical formula (objective physics). However, in the history of theoretical Physics the concept of e.g. "space" has received the following very different physical meanings: absolute space, a container, reference system, force field, Hilbert space, space-time; the same holds for the notion of mass which in special Relativity changed in mass-energy; for not saying nothing about the chameleonic notion of energy. Hence a Physics teacher is led to conceive in a unitarian way so many theoretical meanings of a single physical quantity. But this unifying conceptual process leads him to add philosophical meanings. In fact, each concept, when detached from its objective determinations for being referred also to philosophical aspects (such as the relationships of its several physical determinations) may present a multitude of meanings, not only in Physics' language but also in natural language. This fact gives rise to a great cultural difficulty; at the present, nobody can control the language of these widened concepts, because it belongs to the philosophy of knowledge; which in the last two centuries met a great crisis.

Rather, few decades ago some teachers were successful to suggest an innovative Physics teaching by changing the roles of physical notions inside a theory, e.g. Mechanics. Karlsruhe School of Physics (Hermann 1995, Hermann 2000) has changed the basic notions of theoretical Physics; e.g., it considers as fundamental momentum instead of force, flows instead of quantities. Thus, its new Physics education gives a new conceptual framework of theoretical Physics. By this change of the roles played by basic notions, it overcame the bound of all the previous paradigms: it has operated on the foundations of the physical theories; so that it suggests a different epistemological conception of theoretical Physics. However, this novelty did not result more productive than the traditional conception.

This fact proves that the main stumbling block of the improvement of Physics teaching is constituted by the lack of a comprehension of the foundations of science, being not known the foundations of Physics as first.

The Foundations of Science: The Two Dichotomies

I obtained a characterization of the foundations of science as constituted by more than one notion (e.g., space, force, etc.), or several notions (e.g., all notions produced by ruler and compass; the Lagrangian *T-V*), or a theory (e.g., Euclidean Geometry, Newton's Mechanics, etc.) or a semi-philosophical notion (e.g., set, chaos, determinism, etc.), or a single dichotomy (e.g., either classical Physics or modern Physics), but by two mutually independent dichotomies. These dichotomies are supported by ten evidences, i.e. a Leibniz's suggestion (Drago 1994a), an interpretation of Koyré's and Kuhn's historiographies, (Drago 2017a) a historical analysis of the classical physical theories (Drago 1996) and also classical Chemistry (Drago 2010), an interpretation of both special relativity and the birth of Quantum Mechanics (Drago 2013), plus an interpretation of the histories of Logic, Mathematics (Drago 2012; Drago 2017b; Drago 2017c) and Biology (Drago 2015).

These foundations are constituted by two well-known philosophical dichotomies, respectively the dichotomy on the kind of infinity – either actual infinity (AI), or potential infinity (PI) – and the dichotomy on the kind of the organization of a theory – either the deductive one from few axioms (AO), or the organization aimed at finding out a new scientific method for solving a given crucial problem (PO).[4] To them correspond two formally defined dichotomies, one on the kind of Mathematics – either classical one or constructive one (Bishop 1967) – and the other dichotomy on the kind of Logic – either classical one, or intuitionist one (Heyting 1960).

		Philosophical aspect	Scientific aspect
THE FOUNDATIONS OF SCIENCE	Infinity	Actual Infinity	Classical Mathematics
		Potential Infinity	Constructive Mathematics
	Organization	Organization through few Axioms	Classical Logic
		Organization aimed at solving a Problem	Intuitionist Logic

Tab. 1: The foundations of science

The four possible pairs of choices on the two dichotomies define four models of a scientific theory (MSTs). In theoretical Physics each MST may be baptized by means of its most representative theory: Newtonian (AI&AO) Descartesian (PI&AO), Lagrangian (AI&PO), Carnotian (PI&PO).

Notice that each MTS has its own representative theory; hence, the four centuries long creative theorizing by physicists on the several aspects of the reality exhaustively explored all the foundational possibilities. The collective mind of physicists was very powerful.

Being impossible within science a deeper cultural difference than that between the two kinds of infinity (more accurately, the two kinds of Mathematics) or that between the two kinds of organization (more accurately, the two main kinds of Logic), two theories founded on different choices are mutually incommensurable. This phenomenon manifests itself through the radical variations in meaning of the common notions (RVMs).[5] Being mutually different in an essential way in both the basic choices and in the basic notions, the four MSTs cannot be unified. As a consequence, a theoretical physicist founding a new theory has to choose among the four MSTs the more suitable MST to his purpose, since his work, aimed at constructing a theory, has to conform itself to a particular MST. An MTS that is culturally dominant over others is called a paradigm.

I stressed the mutual incommensurability of Newton's formulation of Mechanics and L. Carnot's formulation (Drago 2004a). They suggest an instance of the RVMs of their common notions; e.g. within the former formulation, the notion of time is continuous, while within the latter formulation is discrete. Within the former, the notion of space is absolute and infinite while within the latter is a zone of space; within the former the force plays the role of basic notion, while within the latter the true notion is *ma* and force is its shortening symbol; etc.). This divergence is puzzling for the supporters of the unity of science; also because a convergence appears instead between the notions of two theories concerning very different fields of phenomena, i.e. L. Carnot's Mechanics and S. Carnot's Thermodynamics; these theories share some concepts (e.g., work) without RVM (Drago

[4] The relevance of this dichotomy for Physics teaching was stressed for the first time by (Ferguson-Hessler, de Jong 1987).
[5] (Drago 1996) Both notions of incommensurability and radical variation of meaning have been independently introduced by T. S. Kuhn (Kuhn 1969) and P. Feyerabend (Feyerabend 1976).

2007, Drago 2017a, 114). Instead, by conceiving the foundations of science through the two dichotomies the explanation of these relationships is plain; the basic choices of the former pair of theories are different (PI&PO vs. AI&AO) and hence some their basic notions are different; while the choices of the latter pair are the same (PI&PO) and hence their basic notions may have the same meanings.

A Corroboration Coming from the Creative Experience of both Physics Teaching and Chemistry Teaching

The problem of the foundations of modern science began at the very moment of its birth; Galilei has recognized the two dichotomies,[6] but he did not choose on them. Then, the research on this subject has risen up to the most abstract speculations of philosophy, but without decisive results. Instead, the humble (with respect to scientific glory), but long practice of high school Physics teachers, who are attentive to transmit to pupils the entire Physics, has recognized, by intuition, four theories as the most representative of theoretical Physics; remarkably, they well represent the four MTS's mentioned in the previous section. The following table shows this correspondence (Drago 2004b).

	OA	OP	
IA	Newton's Mechanics	Electricity and Magnetism	*trajectory, line of force*
IP	Geometrical Optics	Thermodynamics	*distance, process*
	absolute space, reference system	*field, system*	

Tab. 2: The four theories taught by high school physics teachers
according to the four models of scientific theories

Notice that:

1) Both Mechanics and Electricity & Magnetism make use of infinitesimal operations (AI), while geometrical Optics and Thermodynamics do not (PI).

2) the theories of Mechanics and Optics are deduced from principles; hence, they choose AO; the elementary introduction of Electricity and Magnetism at the high school level implicitly poses the problem of their connection; hence, it chooses PO. The placement of Thermodynamics may seem erroneous because this theory is currently presented as derived from principles; but these principles is disputable for two reasons: his first principle is idealistic (heat would be entirely converted into work!) and it, oddly enough, was added twenty-five years after the birth of the second principle (which suggests how to solve the problem of the heat conversions into as much as possible work); in fact, at its historical origin Thermodynamics was OP, being aimed to solve the problem of the maximum efficiency in the conversions of heat into work.[7]

[6] Galilei posed the problem of the kind of the organization of a scientific theory when he wrote the *Dialogue* (1638), which concerned not only the organizations of both celestial and terrestrial worlds, but the organization of a theory too; in fact, he illustrated his theory in a dialogical form, as in Plato's books, not so much in a deductive way, as Aristotle taught that one has to organize a theory. Galilei posed also the problem of the foundational role played by the notion of infinity inside a physical theory at least on the first journey of the *Discourse* (1640); here he correctly posed the alternative between the actual and the potential infinity (Drago 2017d).

[7] Most textbooks, as first PSSC, substitute "Statistical Mechanics" for as most as possible of Thermodynamics. Actually, what high school teaching calls "Statistical Mechanics" is no more than the Kinetic theory of gases. This theory interprets some thermodynamics laws through mechanical

3) Because of the different choices, each pair of the four theories is mutually incommensurable; inside the Table, an arrow represents the incommensurability between Mechanics and Thermodynamics; it is the most apparent one because it occurred inside their histories and moreover the comparison of their conceptual frameworks manifests it.

4) There exist RVM's in the basic notions shared by Mechanics and Thermodynamics: space, time, body (either a point-mass or a system); some of them are of the kind all-or-nothing; e.g., the notions located at the side of the various theories are mutually exclusive (while they are suitable for the pairs of theories at the corresponding row or column).

5) Classical Chemistry shares the same choices of Thermodynamics since its Mathematics is even more elementary (PI) and its organization is based on the problem (PO) of how many elements there exist inside the matter.

The undergraduate teaching of Chemistry presents a similar scheme of four kinds of chemical theories (Bauer and Drago 2005).[8]

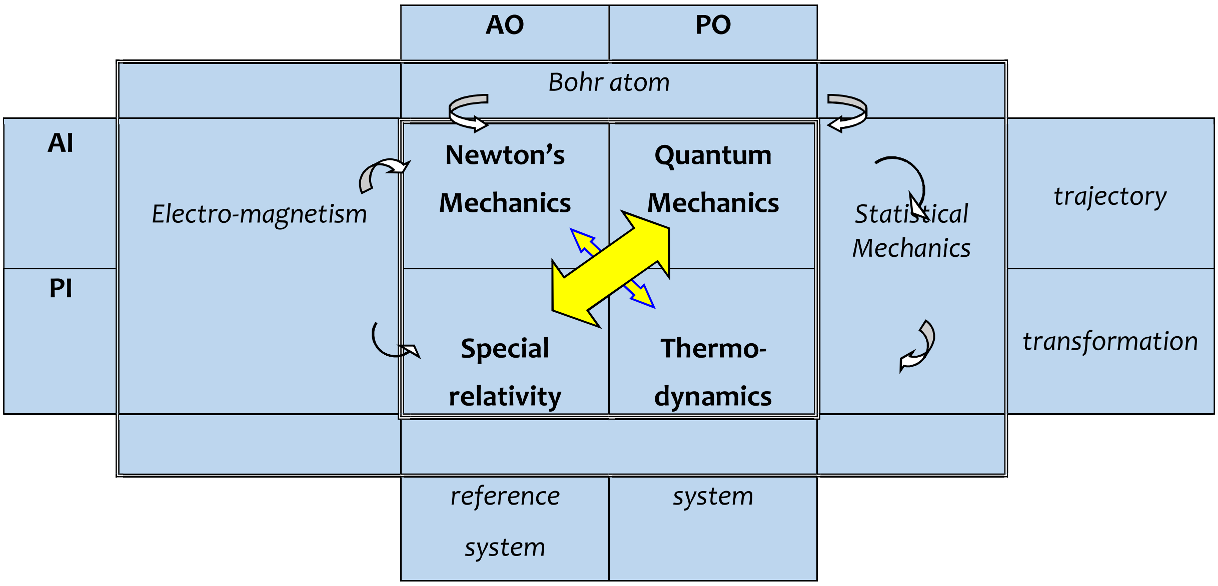
	AI	**PI**	
AO	Quantum Chemistry	Physical Chemistry	*state, entropy*
PO	Chemical Kinetics	Classical Chemistry	*chemical reaction*
	continuous time, electron	*before / after time, equilibrium*	

Tab. 3: The undergraduate didactics of chemistry

Also the undergraduate teaching of Physics presents four representative theories, but according to a more complex scheme. Owing to the theoretical change from classical to modern Physics, some of the previous theories are different: Special relativity (usually taught according to an AO) supersedes geometrical Optics, and Quantum Mechanics (clearly a PO theory within undergraduate didactics), Electricity and Magnetism. Undergraduate teaching includes some more theories, which remarkably play the roles of bridge theories (in each case two curved arrows indicate the two theories connected). The most apparent incommensurability phenomenon is not so much the classical one – between Newton's Mechanics and Thermodynamics –, but that between Quantum Mechanics and Special Relativity; it was the stumbling block of 20th Century theoretical Physics.

laws; but that does not concern Newton's Mechanics, which was committed to the model of the perfectly hard body, whose impact does not conserve energy owing to its fixed shape; moreover, this theory wrongly modeled a gas as composed by fixed mass-points interacting through gravitational forces. Rather, Daniel Bernoulli started a kinetic theory of gases as a continuation of Leibniz's laws of the elastic impact of bodies; and in 1858 eventually Maxwell recuperated (in a geometrical way) these laws for founding the modern theory. Therefore, this theory was implicitly derived from Lazare Carnot's Mechanics. (Drago 2017e). Hence, high school didactics makes violence to both history and theoretical physics when it presents the "Statistical Mechanics" as a prolongation of Newton's Mechanics.

[8] High school Chemistry teaching deals with little more than classical Chemistry, or rather, by following a technicistic attitude, it mixes the concepts of many chemical theories idealistically, without worrying for the rigor; for example, it often anticipates the concept of atom of Quantum Chemistry in order to study the periodic table of classical Chemistry.



Tab. 4: The undergraduate didactics of physics

In Biology teaching the deep fracture between Darwinism and Mendelism, or rather between functionalism and structuralism (Mayr 1972; Gould 2002), parallels the mutual opposition between the Newtonian MST and the Carnotian MST; it is represented by the opposition of the same two pairs of choices (Drago 2015).

In conclusion, it is an extraordinary fact that high school teachers of scientific subject-matters have discovered a deep foundational structure inside a subject where even the philosophers of science were unable to obtain a result of the common agreement. However, the merit of this collective intuition of these teachers is diminished by the fact that they usually adhere to the traditional vision of a unity of all four physical theories; actually, this unity has existed only in the eighteenth century, when Mechanics monopolized theoretical Physics; but after that time, it was forever gone.

The Incommensurability as the Bound of Physics Teaching: The Radical Variations in the Foundations of the Different Formulations of Mechanics

Let us analyze more in detail the implications of accepting the above pluralism. We see that Table 2 summarizes the main problems met by a Physics teacher, beyond all RVM's of the basic notions, as e.g., force, trajectory, transformation, system, etc.

1) the main incommensurability implies that the theories present a radical difference in the kind of Mathematics. Thermodynamics lacks of Geometry, which instead plays an essential role in the two theories of the higher row (Geometry plays no more a foundational role within Electricity and Magnetism since it is subordinate to the physical notion of force).

2) The discrete time of Thermodynamics opposes to the continuum of all variables of the other theories.

3) The cyclical way of reasoning of both the periodic table of elements within Chemistry and Sadi Carnot's theorem within Thermodynamics, opposes to the mathematical reasoning of the other theories.

4) the use of doubly negated propositions (as the *impossibility* of a *perpetual* motion and the *non-decreasing* variation of entropy within Thermodynamics) opposes to the certitude of the deductive development of Newton's mechanics.

The RVM's concerning Physics teaching may be stressed by analyzing the different versions of the inertia principle, the door to the modern science.

Let us recall Newton's version: "Every body perseveres in its state of being at rest or of moving uniformly straight forward except insofar as it is compelled to change its state by forces impressed". Lazare Carnot suggested a different version of the same principle: "Once a body is at rest, it cannot move by itself; and once it is in motion, it cannot change by itself its velocity nor its direction" (L. Carnot 1803, 49).

A first difference is that Newton refers to "every body" in every time and in every place; instead, Carnot bounds the content of his proposition to indicate a limited set of situations, in which it can be stated that a single body under examination is at rest or it is in motion. He refers to them by means of a deliberately inaccurate premise: "Once"; thanks to it, Carnot's version avoids the problem which is implicit within Newton's statement, namely how recognize by means of experimental tools a perfectly uniform and perfectly straight motion, which would take place "until" a unspecified time occurs (Hanson 1965); that is, Carnot's version does not require the verification of the absence of forces ($F = 0$) on the entire (possibly infinite) trajectory of the body, nor it mentions such notion of force (which for Carnot, as for D'Alembert too, is the merely shortened symbol of *ma*).

Carnot was well aware of this novelty. He maintained that it is not possible to judge in a certain way "if a motion is absolute [as Newton says it], or if there is a motion or a dragging force", and that it took "A lot of effort to correct this error".

A further problem comes from Newton's claim to establish exactly when a body is at rest or it is in the state of motion; that is, to decide whether both $v = 0$ exactly (not $v < \epsilon$!) and $v \neq 0$. In general, each of these checks is impossible through experimental means which always give approximate values (apart from specific situations); the same occurs in constructive Mathematics which is based on the approximations of real numbers, only in specific cases it obtains accurately determined real numbers. Owing to this impossibility, Carnot deliberately made use of the expression "once", which refers to particular (experimental and / or theoretical) circumstances in which we know how to decide whether a body is at rest or in uniform motion along a straight line; in other words, this evaluation remains our judgment of an empirical and occasional nature, rather than a general one. In fact, in the history of Mechanics the principle of inertia has been applied when the physicists were capable to approximately indicate a state of either uniform rectilinear motion or rest – with corrections increasingly sophisticated – with respect to the initial inertial reference system of the ground, considered as immobile. At this point we understand that the situation described by Newton's statement, the absence of forces idealizes the real situation; he thought both limits of F as exactly 0 and v as an exact constant of an exactly uniform value, plus a reference system idealized as absolute, i.e. absolute space.

Furthermore, the premise of Carnot's statement assumes that we have already operationally defined the state of uniform rectilinear motion of the body under examination. In this particular situation, the body in motion may well be used for operationally defining both the inertial reference system and the instrument for measuring time, i.e. a clock whose notion is based on a uniform motion. Thus, Carnot's version allows building, without any vicious circle, the three constitutive notions of the conceptual scheme of dynamics, i.e. "the physical content of the principle", "a reference system", "a clock". Instead, within Newton's version (of course deprived by the two metaphysical notions, i.e. absolute space and absolute time) each of the above three notions cannot be defined without referring to the other notions; that is, they constitute a definitory circle, which is commonly accepted inasmuch as it is located at the limit of idealized experiences.

By considering a motion which occurs inside a finite space and is measurable within experimental errors, Carnot restricts the discourse to what is observable experimentally. Understood in this way, the inertia principle indicates a method of investigation: if the observed body changes its motion, it is necessary to look to other bodies in order to find out those which act on it. This search introduces to an interaction dynamics.

Thus, Carnot suggested a version of the inertia principle that it is very adequate for what has been actually done and also to what currently is done by experimental physicists; instead, Newton's claiming to have established some universally valid properties turns out to be a tautology. Under Newton's cultural influence, also present didactics does it, at the cost to mislead and contradicting the experimental nature of theoretical Physics.

Let us now notice that Carnot's version of inertia principle, as all methodological principles, is a doubly negated proposition: "It can*not change*" (where the word "*to change*" has to be considered negative because it requires a physicist's explanation why the body exited out the state of rest). Now let us pause for a moment on the theoretical importance of a doubly negated proposition, as used by L. Carnot's version. It marks a great distance between the two kinds of organization of the two formulations of Mechanics under examination. It represents the way of conceiving the whole theory which this proposition belongs to. In fact, if a doubly negated proposition (of non-classical logic) is translated (by removing the two negatives) in the corresponding affirmative sentence (of classical logic), the content of the latter is unverifiable; it may belong to a physical theory as long as the theory allows idealizations, as an AO theory does in some principles, which, by summarizing a variety of phenomena through few propositions, idealizes the experimental facts; in such a case, also the consequences of these idealized principles may be idealized propositions with respect to reality, provided that they are eventually compatible with the experimental data. This gives a degree of theoretical freedom which is apparent for example inside Newtonian Mechanics, which makes use with impunity of the idealized notions of point-mass, perfectly rigid body, the continuum of matter.

However, we know that there are chemical, physical and mathematical PO theories that have been organized without idealized principles. First, classical Chemistry (whose basic problem is to know how many elements there exist inside the matter) and, originally, Thermodynamics (which wants to solve the problem of the efficiency of heat conversion into work), as well as other physical and mathematical theories, such as Lobachevsky's non-Euclidean Geometry, aimed at solving the problem of the number of the parallels lines. The fundamental problem of such a PO theory is exactly expressed through a doubly negated proposition for which it is not valid the logical law ¬¬A = A; e.g., in Sadi Carnot's Thermodynamics the problem is expressed by the proposition: "It is *not* true that the heat is *not* work". Within a PO the proposition ¬¬A cannot be idealized as A since the problem motivating the PO theory would be cancelled. E.g. if one states that heat is the same as work, then no longer there is a problem. Then the formal characteristic of an OP theory is the presence of doubly negated propositions; the first one of them expresses the problem of the theory; it and their consequences cannot be replaced by equivalent positive propositions because there are no operational means to prove them. So it is clear that while inside a deductive theory, AO, one deduces from the top of the first principles a pyramid of theorems, succeeding each other, to infinity; inside a PO theory the theoretical development is essentially of a cyclical kind, in the sense that, after having presented the problem as ¬¬A; the theory develops for discovering a general method that eventually closes the circle of the argumentations by finding out how much more content is possible of that proposition A that at the beginning could not be made true by means of a simple intellectual operation of suppressing the two negations.

In fact, by removing the two negations of L. Carnot's version one obtains an abstract word lacking physical meaning: "perseveres", or "continues" which are animistic in nature. Actually, they are the words of Newton's version! This version is precisely the affirmative, idealistic proposition which corresponds to L. Carnot's doubly negated proposition. Thus, L. Carnot's double negation is not a rhetorical figure without importance for theoretical physics, because within Physics this double negation lacks an affirmative equivalent, as instead, all rhetorical figures of speech do.

All in the above manifests the relevance of the choice AO/PO of Physics teaching.

Newton's version postulates an infinite inertial straight motion, which is not different from a perpetual motion (Thomson and Tait 1879). Lazare Carnot denies this kind of infinite motion because in the commentary to the seven hypotheses of his Mechanics he remarks that it is impossible because always there exist limitations due to other bodies, as well as unavoidable loss of energy; hence, the fundamental doubly negated principle: "Perpetual [=*without* an end] motion is *impossible*" (L. Carnot 1803, Preface) holds true.[9]

This principle can be operationally translated by stating that there is no finite series of physical operations (i.e., a machine) that allows us to produce an endless motion. Then this principle corresponds to a translation into Physics of the basic principle of constructivism; according to which a mathematical notion exists only if we can calculate it; hence, both definitions require operations, which are performed, in the case of the physical theory of Carnot, by means of experimental apparatus or a machine; and by calculations in the case of mathematical constructivism. Having established this connection, it is natural that Carnot's version of inertia principle is expressed through constructive Mathematics. Also Carnot's language ("once") mimics the language of constructive Mathematics in order to express a problem which is undecidable in general, which yet is soluble in special cases. For example, in constructive Mathematics it is a undecidable problem to state for which point one function $f(x)$ has an extreme, but in the case the function is a polynomial of degree two in the variable x, then we can establish the point $x°$ such that the derivative of $f(x)$ is zero by calculating the discriminant equated to 0 (in constructive Mathematics the last problem is decidable because there exists an algorithm for calculating a square root of the discriminant). Similarly, Lazare Carnot's proposition takes into account those special cases in which we are capable, in an appropriate way with respect to our experimental tools, to effectively establish if $v = 0$ or $v = \text{const}$.

Let us add that the first formulation of infinitesimal analysis was suggested by both Cavalieri and Torricelli through the new method of the "indivisibles". They considered as numbers the final elements of infinite series, notwithstanding Eudoxos has considered this passage as an improper one. During this historical passage from the traditional, constructive Mathematics to the Mathematics including the notion of AI, both have proposed a specific version of the inertia principle (Drago 1997). More than Cavalieri, Torricelli considered as legitimate the passage to the limit of a given sequence to its "final element"; this passage according to him was "the privilege of a geometer"; actually, it corresponds to the use of the least upper bound, which is performed by a more powerful Mathematics than the constructive one. Torricelli considered also the corresponding physical situations. In particular, he discussed whether the supports of the two plates of a scale with equal arms are parallel or not. He correctly stated that they are not parallel since on the Earth all vertical lines meet at its center, but he suggested that the "true" situation is the ideal one that is obtained when the scale was moved more and more from the Earth; then in the "final"

[9] Notice that there exist no operative means to test its corresponding affirmative proposition: "Every body in motion stops", because one cannot establishes the terminal time or the terminal point of Earth's motion, or even of a pulsed billiard ball; one would have to know in anticipation the friction experienced by the mobile along the entire, but a priori unknown, path.

condition the supports would be exactly parallel lines.[10] Similarly, Torricelli suggested inertia principle (albeit through the following incidental period, occurring in the course of a demonstration of a property of a parabolic motion): "It is clear that, without the attraction of gravity, the mobile would proceed to rectilinear and equitable motion along the direction line AB" (Torricelli 1644, 156) Twelve years after him Descartes enounced his version, which was reiterated by Newton (wrongly attributing it to Galilei).

A fourth version of inertia principle has been recognized by Enriques. His "inertia generalized principle" suggests an infinitesimal act of motion (AI), in order to offer a solution to the universal problem (PO) of the motion of a material point in whatsoever reference system.[11] Since the principle works as a methodological principle for studying a complete motion in all cases, it introduces to a PO theory. Hence it corresponds to the choices AI&PO, whereas Newton's one to AI&AO, Torricelli PI&AO,[12] Carnot's to PI&PO.[13]

The Conservative and Misleading Role Played by High School Mathematics Teaching

The above foundational scheme of four theories does not apply to present Mathematics teaching. The reason is not the lack of deep internal divisions among the mathematical theories, but a different attitude of the great majority of mathematicians from that of physicists. Whereas the latter ones had to bravely start anew their science after the crisis of the first years of the 20th Century, the former ones, since the time they have perceived problems inside the foundations of Mathematics (i.e. since the birth of non-Euclidean geometries and much more in the first years of 20th Century) tried to merely reform them in a conservative sense, i.e., by searching to preserve the unity of all theories of Mathematics, as it held true before the birth of calculus. I will quickly sketch this story by making use of the notion of a paradigm.

Since the ancient times, Euclidean geometry played the role of a paradigm for the entire science, and in particular the entire Mathematics. In the 18th Century a new theory, i.e. infinitesimal analysis, threatened (e.g. through Lagrange) to dethrone Euclidean Geometry from this role. But at the end of this Century some experimental theories, Chemistry and Thermodynamics, born without infinitesimal analysis and even without the assured basis of the previous theories (e.g. Optics and Newton's Mechanics), i.e. Geometry. In the first half of 19th Century new kinds of geometries born; hence, Euclidean Geometry could no longer play the role of foundational theory for the entire Mathematics. However, at the end of this Century, Cantor claimed that his Set theory enjoyed this role. Yet, he was unsuccessful owing to both the discovery of antinomies (e.g., Russell's) and its insufficient covering of all important subjects of Mathematics. At the beginnings of 20th Century Hilbert suggested as the foundations of the entire science – in particular, the entire Mathematics – a unique, uniform methodology for assuring their certainty; he wanted to formally axiomatize each

[10] Recall that even Galilei had presented a similar experience; a falling ball from a certain height of an incline (on the left hand) which at the bottom goes up (on the right hand) on another incline till up the same height; the inclination of the latter incline may be varied; in the extreme case of angle 0 – i.e. the incline is the extreme situation of a horizontal plane – then the ball should proceed indefinitely. Galilei proposed this case only as a mental experience, which he maintained as impossible in reality.

[11] (Enriques 1906, 424). This principle is extensively quoted and then commented by (Drago 2007, 140.)

[12] The choice on the infinity is PI in a subjective term, since both Cavalieri and Torricelli thought that their inclusion of the final elements was basically justified by the geometrical intuition of these points.

[13] The incompatibility among four formulations of Mechanics has been perceived also by Darrigol (2007): the distance action Mechanics, contact action Mechanics, Mechanics of continuous media, and impact Mechanics. They may be respectively attributed to Newton, Lagrange, Cauchy and L. Carnot. They can be interpreted as the formulations corresponding to respectively the pairs of choices AI&AO, AI&PO, PI&AO, PI&PO (Drago 2012).

scientific theory, first of all, the basic theories of Mathematics. Yet, after few decades Goedel's theorems stopped his program, by showing that even Arithmetic is not assuredly deductively true.

However, most mathematicians did not renounce to their hope for an ultimate unity of their science. At the same time of the previous result, Bourbaki renounced to found Mathematics on either a single theory or one methodology and even deliberately dismissed the problem of the foundations; he introduced a new conception of the entire Mathematics in a technical sense only, i.e. by promoting as basic some "structures" which are useful for coordinating research within the several branches of Mathematics, accumulated along the previous two centuries. Yet, also this attempt was unsuccessful because some parts of Mathematics are ill-covered by Bourbaki's structures (Drago 2017c).

In addition, since the year 1905 Brouwer has introduced a kind of Mathematics relying upon PI. He presented this Mathematics in exclusive terms, so much to reject any other kind of Mathematics. For this reason, inside Mathematics occurred a harsh debate on its foundation, without any reconciliation before the 1960s; however, this reconciliation has represented more an accommodation than a comprehension of the basic reasons of the divergent viewpoints (Martin-Loef 2007). In retrospect, mathematicians supported a prejudice, i.e. the unitarian nature of Mathematics, which assures a unique truth to each scientific proposition. All in the above suggests that Brouwer implicitly introduced the dichotomy PI/AI.

In addition, 20th Century studies have suggested that classical logic no longer can be considered as the unique logic; there exist many well-formalized non-classical logics, which are to be considered of the same importance (if not more important) than the former one according to the specific situations to which they are applied.[14] As a consequence, Mathematics is subjected to the dichotomy on the kind of logic, i.e. either classical logic or intuitionist logic (as the most representative logic among the non-classical ones), or equivalently, the dichotomy AO/PO.

In conclusion, neither external events (the birth of new physical theories) nor interior events (the births of both non-Euclidean geometries and Galois' algebraic theory), nor the great crisis of the first half of the 20th Century led mathematicians to reform according to a pluralist attitude the foundations of their science. Instead, present Mathematics is fractured not so much in many theories which are mutually different, but according to the above-mentioned dichotomies.

Ironically, during this long period of a crisis concerning foundations, high school Mathematics teaching could apply no innovation attempting to solve the crisis. Set theory was plagued by antinomies since its beginnings. In 1899 Euclidean geometry received a modern version by Hilbert. However, in his longtime study of Mathematics never a pupil is exposed to axiomatic Geometry since it is too difficult (24 axioms!). Recent attempt of attributing to Bourbaki's version of Set theory the same extraordinary role played by Euclidean geometry along past centuries[15] failed, because this theory does not cover in easy terms a variety of indispensable mathematical subjects.

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[14] In addition, we must take note of the crisis of the axiomatic attitude owing to not only Goedel's theorems but also the failure, against Hilbert's expectations, of this method within Physics, since one cannot suggest the correspondence axioms of the theory with reality. Moreover, the recent "reverse Mathematics" has shown that only three axioms are enough to produce almost all undergraduate Mathematics! (Simpson 1999) All of this was outside of the reach of Hilbert, Cantor and Bourbakists.

[15] Whereas after the Sputnik the Federal government of US suggested to change Mathematics teaching through the MSP, in the 1960s it was the OECE (European Organism of Economic Cooperation) and then e.g., in Italy the Ministry of the Education that have imposed the "advancement" which was dominant among mathematicians, that of Bourbaki, through a scholastic version of it (Drago and Forni 1978).

Also owing to the failures of the attempts (by Cantor, Hilbert, Bourbaki, etc.) to improve the foundations of Mathematics, Mathematics teaching remained subordinated to the conservative attitude of Mathematicians (a unique Mathematics assuring "certainty"!) which has influenced the whole science. At the present time, being the foundations of all mathematical theories (Arithmetic too) under examination, Mathematics teaching has preserved the old choice to attribute a paradigmatic role to Euclidean geometry according to a minimally modernized version, stressing its deductive organization (AO), to be considered par excellence as the kind of organization of a theory – actually, a mild support to the misfortunately failed Hilbert's program.[16] More in general, this teaching remains committed to Platonist philosophy of ideal and absolute truths (according to the myth of Plato's cave).

More precisely, Mathematics teaching has been confined within one MST, that of the conservative choices for OA (mild Hilbert program) and IA (sympathetically to both Set theory and Bourbaki), without recognizing any alternative.

Owing to the fascination for the theory of Euclidean Geometry, the program of Mathematics teaching adds to Geometry only some techniques of disconnected theories (e.g. algebraic equations, real numbers, analytical geometry, trigonometry, calculus, projective geometry, Cantor's Set theory); notice that no one of them is presented as a theory. Hence, no global view of the different theories inside Mathematics is transmitted to a pupil, leaving him to think that after the aristocratic era no new mathematical theory born. Of course, this decision greatly reduces the cultural value of Mathematics (Drago 2008).

As a result, high school Mathematics teaching, far from enlightening the basic language of all scientific theories, has degenerated to a teaching of a set of mathematical tools, to be applied soon or later in an instrumental way to Nature. This technicist kind of teaching destroys the great advantage given to the scientific teaching by the wise recognition of the four fundamental theories, as the basic ones, by both Physics and Chemistry teachers. Rather, the present technicist attitude of Mathematics teaching influences the teaching of each scientific theory according to an instrumental attitude for a personal empowerment sharing a mythical mankind's progress.

Suggestions for a Pluralist Scientific Teaching

In the present time, the knots are coming to the comb; two centuries of failed attempts to re-found Mathematics can no longer be ignored. Indeed, we see that within Mathematics a conservative cultural attitude can no longer be proposed. The time is come to overcome the search for only one Mathematical theory on which may be based the entire body of the scientific theories, or at least all mathematical theories. Not only the attitude of physicists to found on only calculus both rational Mechanics and mathematical Physics has been denied by the birth of discrete quanta (anticipated by the elementary Mathematics of both Chemistry and Thermodynamics); not only theoretical physicists since the 1960s have preferred group algebra to the differential equations, but the studies on both Mathematics and Logic foundations have recognized the two internal dichotomies. In particular inside social life the Mathematics without actual infinite has become essential because of the relevance of computers (they choice is for PI since their physical operations cannot materialize AI); and, in addition, they make use of several non-classical logics.

[16] Of course, metaphysical notions (e.g. that of a point) have been suppressed and Euclid's version of the parallel postulate is superseded by that suggested by Playfair in 1846; it includes unicity of the parallel line, which is considered as including its points at infinity ("In a plane, given a line and a point not on it, at most one line *parallel* to the given line can be drawn through the point."). Of course, the last characteristic features are idealistic in nature, so that Euclidean Geometry through this postulate is based on AI.

It is also time for a great renewal of scientific teaching, long delayed by the suggestions of new pedagogical paradigms that however changed only non-fundamental aspects.

Currently, the insufficiency of the conservative attitude inside Mathematics teaching is experienced by all teachers who have to introduce the computer into the classroom: this novelty suggests an upsetting practice which is an alternative to traditional Mathematics. This theory adds evidence, beyond the evidence coming from both Chemistry and Thermodynamics, for the validity of a PO (Computer science basic problem is what is a computation). Also, at present time, this novelty in science teaching is exorcised by de-evaluating it as a mere application. But this will not last, also because this theory is one of the most profound ones, not a simple application of Mathematics (Drago 2016).

All in the above suggests that the time is come to enlarge the pluralist attitude of Physics teaching to the teaching of the entire science. From this new attitude, many indications for educational innovations may be drawn. The first indication is to explain the present situation of science teaching to students, with respect to the foundations of science. But surely that does not mean the suggestion of a lesson of the philosophy of science; rather it constitutes the conclusion of an innovative didactics that highlights the foundations of alternative science to the dominant one.

In order to this aim I suggest some innovations.

1° Start Physics teaching from the most operative and the most (according to both Mach and Einstein) foundational theory, Thermodynamics, and then introduce Mechanics as a limit theory – since frictionless –, of the former one (Drago 1994b).

2° Teach the two dichotomies by comparing Newton's Mechanics and Thermodynamics (Drago 1996). Being classical Chemistry a Carnotian theory, i.e. PI and PO, (Bauer and Drago 2005) add a comparison between Newtonian Mechanics and classical Chemistry.

3° Critically analyze some basic notions suffering RVM's (e.g. space, time, inertia, force, mass).

4° Introduce the four principles of inertia according to the four MTS.

5° Reveal and stress the choice of the four theories of Physics teaching and then reveal the pluralism in science.

6° Revise traditional Mathematics teaching by not only criticizing its idealistic notions of real number, limit and set, but also introduce four foundational theories in correspondence to the four MST's; for instance: Euclidean geometry (PI through the ruler and compass, AO), limits and the extension of integer numbers to real ones (AI&PO), Boolean algebra (PI&PO), Cantor's set theory (AI&AO) (Drago 2008). [17]

7° Introduce Logic teaching through the elementary distinction between the two kinds of logic as given by the failure or not of the double negation law. The main doubly negated propositions occurring inside the original texts of the theories which make the history of PO inside Mathematics, Physics, Chemistry and Biology constitute a basis upon which a teacher can plainly introduce pupils to the two kinds of logic.

8° Add to the usual school's information about computers a quick theory of Computer Science through both a presentation of Turing Machine and elementary proofs of both universal Halting theorem and Universal machine theorem; then characterize the choices of this theory as those of the Carnotian MST (Drago 2010).

9° Illustrate the historical emergence process of each MST and also the occurrences of paradigms (as along two centuries the Newtonian MST, dominating all others physical theories, was).

[17] Calculus may be easily presented through its version of Cavalieri's mathematics (Drago and Vella 2006).

10° Ponder on the history of Western philosophy of knowledge in order to recognize the defective steps of the main philosophers; recognize that not Kant's ideas, but Leibniz's suggestion of two labyrinths, anticipating the two philosophical dichotomies, constituted an introduction to the scientific pluralism.[18]

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[18] There is no space for illustrating one more suggestion concerning the detailed model of PO (which may be illustrated on the Lobachevsky's text suggesting the first non-Euclidean Geometry; i.e. analyze the *ad absurdum* proof as it is presented by both Sadi Carnot theorem and the abovementioned theorems of Computer Science; each one concluding a doubly negated predicate; the author feels himself justified to translate it (through Leibniz's principle of sufficient reason) into an affirmative hypothesis, which then is tested with experiments. Leibniz's pluralist attitude is confirmed by his further suggestion of two are the logico-philosophical principles of human reason, i.e. non-contradiction principle (concerning AO theory) and the principle of sufficient reason (playing the role of concluding a PO theory) (Drago 2012).

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