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Research Article

Forecasting Crude Oil Consumption in China Using a Grey Prediction Model with an Optimal Fractional-Order Accumulating Operator

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Crude oil, which is an important part of energy consumption, can drive or hinder economic development based on its production and consumption. Reasonable predictions of crude oil consumption in China are meaningful. In this paper, we study the grey-extended SIGM model, which is directly estimated with differential equations. This model has high simulation and prediction accuracies and is one of the important models in grey theory. However, to achieve the desired modeling effect, the raw data must conform to a class ratio check. Unfortunately, the characteristics of the Chinese crude oil consumption data are not suitable for SIGM modeling. Therefore, in this paper, we use a least squares estimation to study the parametric operation properties of the SIGM model, and the gamma function is used to extend the integer order accumulation sequence to the fractional-order accumulation generation sequence. The first-order SIGM model is extended to the fractional-order FSIGM model. According to the particle swarm optimization (PSO) mechanism and the properties of the gamma function of the fractional-order cumulative generation operator, the optimal fractional-order particle swarm optimization algorithm of the FSIGM model is obtained. Finally, the data concerning China's crude oil consumption from 2002 to 2014 are used as experimental data. The results are better than those of the classical grey GM, DGM, and NDGM models as well as those of the grey-extended SIGM model. At the same time, according to the FSIGM model, this paper predicts China's crude oil consumption for 2015–2020.

1. Introduction

Energy is an important material basis for global economic growth and human social development. As an important component of energy consumption, the production and consumption of crude oil can drive or hinder economic development. At present, China is facing rapid economic growth, changes in consumer spending structures, and an economic development with an increasing dependence on crude oil resources [1, 2]. Crude oil supply and demand imbalances are becoming increasingly prominent. Low utilization of crude oil, irrational consumption structures, serious pollution, and other issues can restrict the development of China's economy. With China's industrialization, its urbanization, energy, and environmental constraints will increase. The settling of the contrast between the energy and economic development is related to the sustainable development of China's economy and society.

Crude oil demand forecasting is an important part of the development of crude oil development strategies and the scientific, reasonable, and accurate analysis of China's crude oil demand, which is needed not only to protect China's energy security and effectively prevent the bottlenecking of crude oil supplies but also for the realization of China's economic health. Sustainable and rapid development will have important impacts on these processes. China's rapidly growing energy consumption and its structural changes continue to challenge China's energy supply security. Therefore, effective methods of addressing the demand for crude oil are expected to become the basis for the policy formulation of China's energy supply security and will directly affect the stability of social production and national energy security in addition to helping the Chinese government establish an independent demand forecasting mechanism for crude oil and the energy sector to achieve an effective market transformation.

There are many ways to forecast crude oil demands, including the autoregressive moving average (ARMA) model [3], autoregressive conditional heteroscedasticity (ARCH) model [4], generalized ARCH (GARCH) model [5], and other time series methods as well as via artificial neural networks [6], fuzzy theory predictions [7, 8], and grey system methods [9, 10]. Liu et al. [11] used a time series approach to forecast the US West Texas Lightweight (WTI) crude oil prices based on crude oil demands. Liang et al. [12] predicted China's crude oil price using wavelet decomposition. Zhang [13] used the quadratic moving average method to predict the annual consumption of the next five years of oil consumption. Guo et al. [14] used soft computing and hard computing to forecast China's crude oil demand. Azadeh et al. [15] analyzed the oil consumption of Canada, United States, Japan, and Australia from 1990 to 2005 using fuzzy-regression data envelopment. Azadeh et al. [16] predicted the crude oil prices using a fuzzy-regression algorithm. Park and Yoo [17] studied the dynamics of oil consumption and economic growth in Malaysia.

The grey model is simple and adaptable, can handle mutations of parameters, and does not require many data points for predictive updates. The forecasting model GM (1,1) [18] has been widely used in many fields, such as those of transportation, medicine, industry, agriculture, and military [19–21], since its introduction. Researchers have expanded a variety of new models, such as DGM (1,1), NDGM (1,1), and GM (1, N) [22–28], from the classic GM (1,1) model. Concurrently, the grey prediction model has been studied in detail, including its background value, modeling mechanism, combinatorial model, and model optimization [29–33]. Grey forecasting models have been successfully applied for crude oil demand forecasting: Huang et al. [34] have used the grey prediction model to predict global crude oil consumption. Xu [35] used the grey model to forecast China's crude oil consumption. Mu Hailin et al. also used the grey model to predict China's crude oil consumption.

The SIGM model [10] is an extended version of the classical GM (1,1) model. The SIGM model can optimize the model parameters, which are directly estimated from the differential equation, making its simulations and predictions more accurate. However, the parameters in the literature [10] are too cumbersome to estimate, so this paper uses the least squares estimation method to simplify the parameter estimations of the SIGM model and to obtain the corresponding formula. At the same time, the modeling data of the SIGM model is a first-order cumulative generation sequence. To achieve the desired modeling results, the raw data must conform to the class ratio test, but the data characteristics of China's crude oil consumption do not meet the class ratio test. Therefore, this paper will promote the use of the SIGM model, which uses the gamma function to extend the integer order cumulative generation operator into the fractional-order cumulative generation operator, to extend the first-order cumulative generation sequence to the fractional-order cumulative sequence and to establish the FSIGM model of the fractional-order operator. At the same time, by using the mechanism of the particle swarm optimization (PSO) and the properties of the gamma function of the fractional-order generation operator, the optimal fractional particle swarm optimization algorithm of the FSIGM model is obtained, and the optimal fractional order is obtained using different data. Finally, the data describing the consumption of the crude oil in China from 2002 to 2014 are analyzed. The results show that the newly proposed FSIGM model has an improved accuracy and prediction accuracy over those of the original SIGM; however, its simulation accuracy is much higher than the classic GM, DGM, and NDGM models. The accuracy of the prediction is not much different from that of the GM and FSIGM models, but the simulation accuracy is obviously better than the DGM and NDGM models.

The sections of this paper are organized as follows: In Section 2, the basic concepts and properties of the GM (1,1) and SIGM models are introduced. In Section 3, the fractional-order SIGM model is proposed and its important properties are analyzed. Based on the mechanisms of the particle swarm optimization, the particle swarm optimization algorithm is obtained. In Section 4, the crude oil consumption in China from 2002 to 2014 is used for empirical analysis. The simulation results and prediction results of the FSIGM model are compared with the classical grey model GM, DGM, and NDGM models and the grey-extended SIGMD model. In Section 5, conclusions are drawn.

2. Preliminaries

This section mainly introduces the definition and basic properties of the GM (1,1) model and the definition of the SIGM model. The least squares estimation is used to estimate the parameters of the SIGM model, which is simpler than the method used in the literature [10].

2.1. GM (1,1) Model. Assume that the sequence:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\right) \tag{1}$$

is an original data sequence, and the sequence:

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\right) \tag{2}$$

is the accumulated generation sequence of X (0), where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \dots, n.$

$Z^{(1)}$ is the mean sequence of $X^{(1)}$.

$$Z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n)\right), \tag{3}$$

where $z^{(1)}(k) = 0.5(x^{(1)}(k) + x^{(1)}(k-1)), k = 2, 3, \dots, n.$

Definition 1. Assume that the sequence $X^{(0)}$, $X^{(1)}$, and $Z^{(1)}$ is shown as (1), (2), and (3), then

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{4}$$

is a first-order equation with a variable grey system prediction model, which is referred to as GM (1,1) model [18]. Its parameter estimation:

$$\begin{pmatrix} a \\ b \end{pmatrix} = \left(B^{\mathrm{T}}B\right)^{-1}B^{\mathrm{T}}Y, \tag{5}$$

where

$$B = \begin{pmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{pmatrix},\quad Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}. \tag{6}$$

The intrinsic reduction value of the GM (1,1) model is

$$\hat{x}^{(1)}(k+1) = \left(x^{(1)}(1) + \frac{b}{a}\right)e^{-ak} + \frac{b}{a}. \tag{7}$$

2.2. SIGM Model

Definition 2 (see [10]). For $X^{(0)}$, $X^{(1)}$, and $Z^{(1)}$ given by (1), (2), and (3), and c is a constant, then the following equation:

$$x^{(0)}(k) + az^{(1)}(k) = kb + c \tag{8}$$

is the expanded form of GM (1,1) model.

By definition, we can get the following.

Property 1. The parameter vector of SIGM model is $\hat{a} = [a, b, c]^{\mathrm{T}}$, using least squares estimation.

$$\hat{a} = \left(A^{\mathrm{T}}A\right)^{-1}A^{\mathrm{T}}X, \tag{9}$$

where A, X are

$$A = \begin{bmatrix} -z^{(1)}(2) & 2 & 1 \\ -z^{(1)}(3) & 3 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & n & 1 \end{bmatrix},\quad X = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}. \tag{10}$$

Definition 3. The equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = bk + c \tag{11}$$

is the whitening equation of FSIGM model $x^{(r-1)}(k) + az^{(r)}(k) = bk + c$.

Thus, we can get the following theorem.

Theorem 1. *Assume that B, Y, and $\hat{a}$ are given by Definition 1 and Property 1, and*

(1) The time response function of the whitening (12) is

$$\hat{x}^{(1)}(t) = \left[x^{(1)}(1) + \frac{b}{a^2} - \frac{c}{a}\right]e^{-at} + \frac{b}{a}t - \frac{b}{a^2} + \frac{c}{a}. \tag{12}$$

(2) The time response function of the whitening (28) is

$$\hat{x}^{(1)}(k) = \left[x^{(1)}(1) + \frac{b}{a^2} - \frac{c}{a}\right]e^{-a(k-1)} + \frac{b}{a}k - \frac{b}{a^2} + \frac{c}{a}. \tag{13}$$

(3) Restore value is

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{14}$$

Proof 1. From (11):

$$\begin{aligned} x^{(1)}(t) &= e^{-\int adt}\left[\int(bt+c)e^{\int adt}dt + C\right] \\ &= Ce^{-\int adt} + e^{-\int adt}\int(bt+c)e^{\int adt}dt \\ &= Ce^{-at} + \frac{b}{a}e^{-at}\left(te^{at} - \frac{1}{a}e^{at} + e^{-at}\frac{c}{a}e^{at}\right) \\ &= Ce^{-at} + \frac{b}{a}t - \frac{b}{a^2} + \frac{c}{a}. \end{aligned} \tag{15}$$

When $t = 0$, there is $C = x^{(1)}(1) + (b/a^2) - c/a$. Thus, we can get (12), then from Definition 2, we can get (13) and (14).

3. The FSIGM Model

In this section, we propose a new FSIGM model based on fractional-order accumulation generation, which uses the gamma function [36] to represent the parameter estimation of the fractional-order cumulative generation sequence and finds the optimal order using the adaptive particle swarm optimization [37] method.

3.1. Fractional Extension Operator. In Section 2.1, we have assumed that $X^{(1)}$ is 1-AGO; the r-order cumulative generation sequence is defined below.

Definition 4. Let $X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \dots, x^{(r)}(3))$ by (1) be r-AGO, where

$$x^{(r)}(k) = \sum_{i=1}^{k} x^{(r-1)}(i) = \sum_{i=1}^{k}\sum_{j=1}^{i} x^{(r-2)}(j), \quad r \in R^{+}, k = 1, 2, \dots, n. \tag{16}$$

Equation (16) can be expressed as

$$x^{(r)}(k) = \sum_{i=1}^{k} \frac{(k-i+1)(k-i+2)\cdots(k-i+r-1)}{(r-1)!} x^{(0)}(i), \quad r \in Z^{+}, k = 1, 2, \dots, n. \tag{17}$$

When $r \in N$, $X^{(r)}$ is called as integer order accumulation sequence; when $r \in R^{+}$, $X^{(r)}$ is called as fractional-order accumulation generation sequence.

In order to express the r-order cumulative generation sequence with the gamma function, the definition and nature of the gamma function are given below.

Definition 5. $n \in R$ and $n \notin \{0, -1, -2, -3, \dots\}$; $\Gamma(n)$ is the gamma function of the real number n defined as

$$\Gamma(n) = \int_{0}^{\infty} e^{-t} t^{n-1} dt. \tag{18}$$

Through the integral points, we can deduce the properties of the gamma function as follows:

Property 2. $\Gamma(n+1) = n\Gamma(n)$, when $n \in N$, $\Gamma(n+1) = \int_{0}^{\infty} e^{-t} t^{n} dt = n!$.

Through Definition 5 and Property 2, (17) can be expressed as

$$x^{(r)}(k) = \sum_{i=1}^{k} \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} x^{(0)}(i), \quad r \in Z^{+}, k = 1, 2, \dots, n. \tag{19}$$

Particularly, when $r \in Z^{+}$, $x^{(r)}(k)$ expanded coefficient is

$$a_k = \frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)} = \frac{(r+k-i-1)!}{(k-i)!(r-1)!}. \tag{20}$$

The grey reducing generation corresponds to the grey accumulating generation, which can be viewed as a process of grey release; it is the grey cumulative generation sequence to restore. Therefore, the grey accumulating generation operator and the grey reducing generation operator must satisfy the reciprocity.

Definition 6 (see [36]). For $X^{(0)}$ given by (1), an r-order reducing generation operator (RGO) sequence $X^{(-r)} = \{x^{(-r)}(1), x^{(-r)}(2), \dots, x^{(-r)}(n)\}$, $r \in R^{+}$ can be generated by r-RGO as follows:

$$x^{(-r)}(k) = \sum_{i=1}^{k-1} \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} x^{(0)}(k-i). \tag{21}$$

$X^{(-r)}$ is called as fractional reducing generation operator r-RGO $(r \in R^{+})$.

3.2. The FSIGM Model. This section mainly introduces the fractional-order SIGM model, which is the FSIGM model, and studies its important properties. First, define the FSIGM model.

Definition 7. Let $X^{(0)}$ be the original sequence, from Definition 1, and $X^{(r)}$ is the r-order accumulation generation sequence of $X^{(0)}$, which is given by Definition 4.

$$x^{(r-1)}(k) + az^{(r)}(k) = bk + c \tag{22}$$

is called as FSIGM model, where $x^{(r)}(k)$ is given by (19) and

$$\begin{aligned} x^{(r-1)}(k) &= x^{(r)}(k) - x^{(r)}(k-1), z^{(r)}(k) \\ &= \frac{1}{2}\left(x^{(r)}(k) + x^{(r)}(k-1)\right). \end{aligned} \tag{23}$$

Specifically, when $r = 1$, (19) becomes $x^{(0)}(k) + az^{(1)}(k) = kb + c$; it is the original form of the SIGM model.

According to the definition of the model FSIGM model, we can get the following properties.

Property 3. The parameter vector of the FSIGM model $\hat{a} = [a, b, c]^{T}$, using least squares estimation:

$$\hat{a} = \left(B^{\mathrm{T}}B\right)^{-1} B^{\mathrm{T}} Y, \tag{24}$$

where B, Y are

$$B = \begin{bmatrix} -z^{(r)}(2) & 2 & 1 \\ -z^{(r)}(3) & 3 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(r)}(n) & n & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, \tag{25}$$

then

$$x^{(r-1)}(k)=x^{(r)}(k)-x^{(r)}(k-1)=\sum_{i=1}^{k}\frac{\Gamma(r+k-i)}{\Gamma(k-i+1)\Gamma(r)}x^{(0)}(i)-\sum_{i=1}^{k-1}\frac{\Gamma(r+k-i-1)}{\Gamma(k-i)\Gamma(r)}x^{(0)}(i),\quad k=1,2,\ldots,n,$$
$$z^{(r)}(k)=\frac{1}{2}\left(x^{(r)}(k)+x^{(r)}(k-1)\right)=\frac{\sum_{i=1}^{k}(\Gamma(r+k-i)/\Gamma(k-i+1)\Gamma(r))x^{(0)}(i)+\sum_{i=1}^{k-1}(\Gamma(r+k-i)/\Gamma(k-i+1)\Gamma(r))x^{(0)}(i)}{2}. \tag{26}$$

Property 4. The matrix B, Y in Property 3 and Property 4 can be represented by the gamma function as follows:

$$B=\begin{bmatrix}-z^{(r)}(2) & 2 & 1\\ -z^{(r)}(3) & 3 & 1\\ \vdots & \vdots & \vdots\\ -z^{(r)}(n) & n & 1\end{bmatrix}=\begin{bmatrix}-\frac{x^{(r)}(1)+x^{(r)}(2)}{2} & 2 & 1\\ -\frac{x^{(r)}(2)+x^{(r)}(3)}{2} & 3 & 1\\ \vdots & \vdots & \vdots\\ -\frac{x^{(r)}(n-1)+x^{(r)}(n)}{2} & n & 1\end{bmatrix}=\begin{bmatrix}-\frac{1}{2}\sum_{i=1}^{2}\frac{\Gamma(r+2-i)}{\Gamma(2-i+1)\Gamma(r)}x^{(0)}(i)+\sum_{i=1}^{1}\frac{\Gamma(r+2-i)}{\Gamma(2-i+1)\Gamma(r)}x^{(0)}(i) & 2 & 1\\ -\frac{1}{2}\sum_{i=1}^{3}\frac{\Gamma(r+3-i)}{\Gamma(k-i+1)\Gamma(r)}x^{(0)}(i)+\sum_{i=1}^{2}\frac{\Gamma(r+3-i)}{\Gamma(3-i+1)\Gamma(r)}x^{(0)}(i) & 3 & 1\\ \vdots & \vdots & \vdots\\ -\frac{1}{2}\sum_{i=1}^{n}\frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i)+\sum_{i=1}^{n-1}\frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i) & n & 1\end{bmatrix}$$
$$=\begin{bmatrix}-\frac{1}{2}\left[(r+1)x^{(0)}(1)+x^{(0)}(2)\right. & 2 & 1\\ -\frac{1}{2}\left[\frac{r(r+3)}{2}x^{(0)}(1)+(r+1)x^{(0)}(2)+x^{(0)}(3)\right. & 3 & 1\\ \vdots & \vdots & \vdots\\ -\frac{1}{2}\sum_{i=1}^{n}\frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i)+\sum_{i=1}^{n-1}\frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i) & n & 1\end{bmatrix},$$
$$Y=\begin{bmatrix}\sum_{i=1}^{2}\frac{\Gamma(r+2-i)}{\Gamma(2-i+1)\Gamma(r)}x^{(0)}(i)-\sum_{i=1}^{1}\frac{\Gamma(r+2-i-1)}{\Gamma(2-i)\Gamma(r)}x^{(0)}(i)\\ \sum_{i=1}^{3}\frac{\Gamma(r+3-i)}{\Gamma(k-3+1)\Gamma(r)}x^{(0)}(i)-\sum_{i=1}^{2}\frac{\Gamma(r+3-i-1)}{\Gamma(3-i)\Gamma(r)}x^{(0)}(i)\\ \vdots\\ \sum_{i=1}^{n}\frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i)-\sum_{i=1}^{k-1}\frac{\Gamma(r+n-i-1)}{\Gamma(n-i)\Gamma(r)}x^{(0)}(i)\end{bmatrix}=\begin{bmatrix}(r-1)x^{(0)}(1)+x^{(0)}(2)\\ \frac{r(r-1)}{2}x^{(0)}(1)+(r-1)x^{(0)}(2)+x^{(0)}(3)\\ \vdots\\ \sum_{i=1}^{n}\frac{\Gamma(r+n-i)}{\Gamma(n-i+1)\Gamma(r)}x^{(0)}(i)-\sum_{i=1}^{k-1}\frac{\Gamma(r+n-i-1)}{\Gamma(n-i)\Gamma(r)}x^{(0)}(i)\end{bmatrix}. \tag{27}$$

Definition 8.

$$\frac{dx^{(r)}}{dt}+ax^{(r)}=bk+c \tag{28}$$

is the whitening equation of FSIGM model $x^{(r-1)}(k)+az^{(r)}(k)=bk+c$. The following theorem:

Theorem 2. *B, Y, and $\hat{a}$ are given by Definition 7 and Definition 5, then*

(1) The time response function of the whitening (28) is

$$\hat{x}^{(r)}(t)=\left[x^{(r)}(1)+\frac{b}{a^2}-\frac{c}{a}\right]e^{-at}+\frac{b}{a}t-\frac{b}{a^2}+\frac{c}{a}. \tag{29}$$

(2) The time response function of the whitening (28) is

$$\hat{x}^{(r)}(k) = \left[x^{(r)}(1) + \frac{b}{a^2} - \frac{c}{a}\right] e^{-a(k-1)} + \frac{b}{a}k - \frac{b}{a^2} + \frac{c}{a}. \tag{30}$$

(3) Restore value is

$$\hat{x}^{(0)}(k) = \hat{x}^{(r)(-r)}(k) = \sum_{i=0}^{k-1} (-1)^i \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \hat{x}^{(r)}(k-i), \tag{31}$$

where $k = 2, 3, \ldots, n, \hat{x}^{(0)}(1) = x^{(0)}(1)$.

Proof 2. The FSIGM and SIGM models have the same structures, such that the SIGM model is a special case of FSIGM. The difference between the two models is that the FSIGM model uses the r-order cumulative sequence $X^{(r)}$ of the original sequence $X^{(0)}$ as its modeling sequence, and the SIGM model uses the first-order accumulation sequence $X^{(1)}$ of the original sequence $X^{(0)}$ as the modeling sequence, so the conclusion is true.

3.3. Optimization of the FSIGM Model. Particle swarm optimization (PSO) is a type of global optimization evolution algorithm and was proposed by Kennedy and Eberhart in 1995 [36]. The concept of the PSO algorithm is simple, needing adjustments of a small number of parameters, and is also easy to program. The method has been widely used in function optimization, neural network training, and other fields.

From Theorem 2, the restored value $\hat{x}^{(0)}(k)$ can be calculated. Next, the mean absolute percentage error (MAPE) is defined.

$$\text{MAPE} = \frac{1}{n-1} \sum_{k=2}^{n} \frac{\left|x^{(0)}(k) - \hat{x}^{(0)}(k)\right|}{x^{(0)}(k)} \times 100\%, \tag{32}$$

where in $x^{(0)}(k)$ represents the raw data and $\hat{x}^{(0)}(k)$ represents a simulation value or a predicted value.

We want to obtain the optimal order r, which minimizes the MAPE between $x^{(0)}(k)$ and $\hat{x}^{(0)}(k)$, by solving the following optimization problem:

$$\min \quad f(r) = \frac{1}{n-1} \sum_{k=2}^{n} \frac{\left|x^{(0)}(k) - \hat{x}^{(0)}(k)\right|}{x^{(0)}(k)}, \quad r \in R^+. \tag{33}$$

The PSO algorithm based on adaptive mutation of population fitness variance [37] is used to optimize the order, such that (33) is used as the fitness of the particle. The order of the minimum mean relative error can then be obtained. The adaptive mutation particle swarm optimization algorithm of the optimal sequence is as follows:

Step 1. Randomly initialize the position and velocity of the particle swarm, taking $p\text{Best} = 1$, which is the mean of the FSIGM model.

Step 2. Set pBest in the particle to the current position; thus, gBest is set to the best particle position in the initial population.

Step 3. Calculate the average relative error of the fractional operator FSIGM model when $r = p\text{Best}$. The specific steps are as follows:

(1) Calculate the r-order cumulative generation sequence $X^{(r)}$ of the original sequence $X^{(0)}$, produce the mean generation sequence with consecutive neighbors of $X^{(r)}$, and calculate the first-order cumulative generation operator $X^{(r-1)}$ of $X^{(r)}$.

(2) Solve the parameter $\hat{a} = [a, b, c]^{\mathrm{T}}$ and then calculate the reduction value $\hat{x}^{(0)}(k)$ according to (31) to find the simulation value $\hat{X}^{(0)}$of $X^{(0)}$.

(3) Calculate the average relative error of (pBest) according to (32).

(4) Determine whether $|f(p\text{Best}) - f(g\text{Best})|$ is less than the given convergence value λ; if this condition is satisfied, then implement the ninth step; otherwise, implement the fourth step.

Step 4. For all particles of the particle group, do the following.

(1) Update the position and speed of the particle:

$$\begin{aligned} V = \varpi \times V + d_1 \times \text{rand} \times (p\text{Best} - \text{present}) + d_2 \\ \times \text{rand} \times (g\text{Best} - \text{present}), \end{aligned} \tag{34}$$

where present = present + V, $\varpi = \varpi_{\max} - \text{run}((\varpi_{\max} - \varpi_{\min})/\text{run max})$.

(2) If the particle fit is better than the fit of pBest, pBest can be set as the new position.

(3) If the particle fit is better than the fitness of gBest, g Best can be set as the new position.

Step 5. Calculate the population variance fit λ^2 and $f(p\text{Best})$

$$\begin{aligned} \lambda^2 &= \sum_{i=1}^{n} \left(\frac{f_i - f_{\text{avg}}}{f}\right), \\ f &= \begin{cases} \max\left\{|f_i - f_{\text{avg}}|\right\}, & \max\left\{|f_i - f_{\text{avg}}|\right\} > 1, \\ 1, & \text{others}. \end{cases} \end{aligned} \tag{35}$$

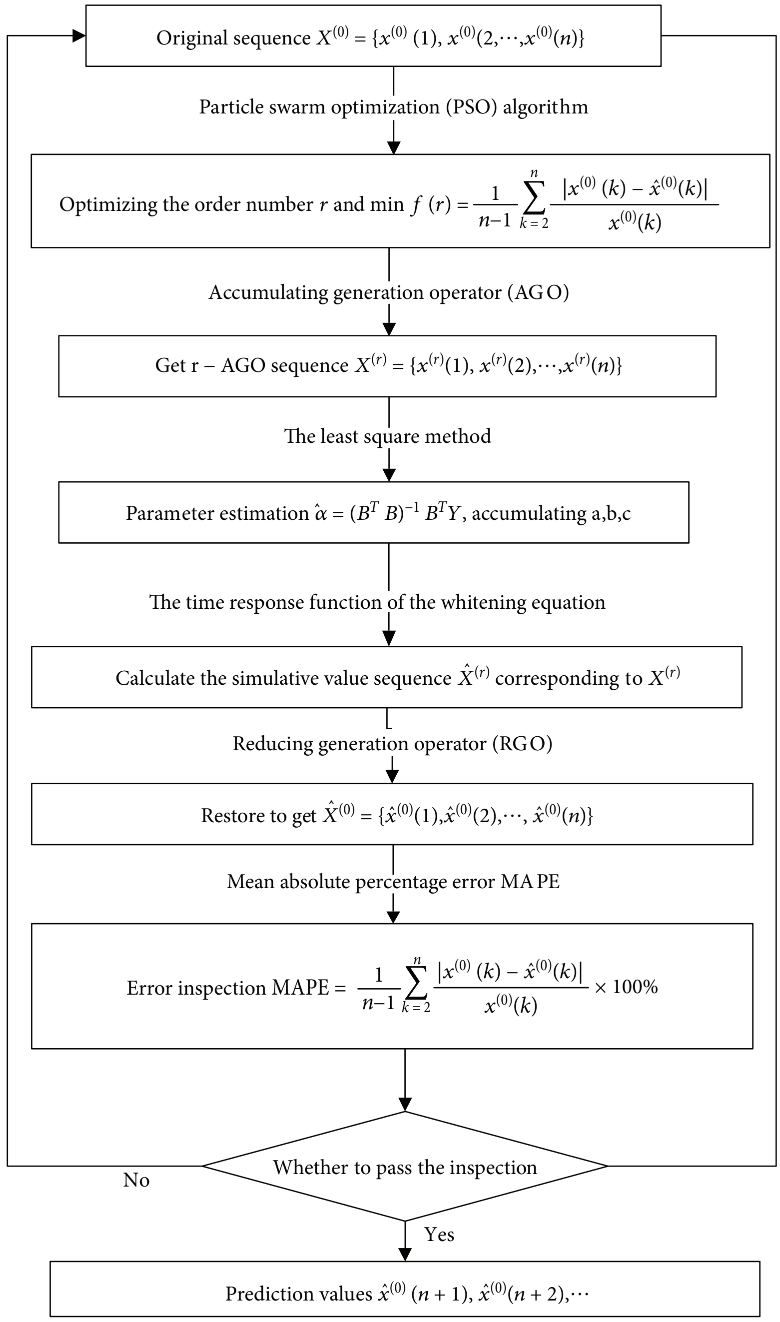


Figure 1: The flowchart of the FSIGM model.

Step 6. Calculate the probability of the variation p_m, where

$$p_m = \begin{cases} k, \lambda^2 < \lambda_d^2, & f(gBest) > f_d, \\ 0, & \text{others}. \end{cases} \quad (36)$$

Step 7. Generate a random number $\varepsilon \in [0, 1]$; if $\varepsilon < p$, perform a mutation operation according to (36); otherwise, perform the eighth step.

Step 8. Determine whether the algorithm convergence criteria are met; if these conditions are satisfied, perform the ninth step; otherwise, return to the third step.

Step 9. Output gBest such that r is the optimal value. Output $r = g$Best. At the same time, determine the predictive value of the FSIGM model and the average relative error MAPE.

It can be seen that the modeling steps of the FSIGM model are shown in Figure 1.

Table 1: China's crude consumption (units: 100 million tons) during 2002–2014.

Year	Crude consumption
2002	2.2544
2003	2.4922
2004	2.8749
2005	3.0086
2006	3.2245
2007	3.4032
2008	3.5498
2009	3.8129
2010	4.2875
2011	4.3966
2012	4.6679
2013	4.8652
2014	5.1547

4. Simulating and Forecasting the Demand of China's Crude Oil

This section mainly analyzes the current situation of crude oil consumption. FSIGM model of crude oil consumption data were analyzed. Based on the analysis results, it gives some policy suggestions.

4.1. Current Situation of China's Crude Oil Consumption. In recent years, with the sustained and rapid development of China's national economy, energy construction has also developed by leaps and bounds, such that the annual output of crude oil now ranks fifth in the world. However, with the demand for continued expansion, China has now become the world's third largest oil consumer, and China's output cannot fully meet the consumption demand. Since the 1990s, crude oil consumption has increased at an average annual rate of 5.77%. Crude oil self-sufficiency has become an important reason for the imbalance between the supply and demand of crude oil in China, which is only by strengthening the forecasts of crude oil demands. However, we can prevent future possible energy security problems.

China's crude oil demand forecast work is conducive to promoting China's crude oil industry with market-oriented reform and industry restructuring. At present, China's oil industry and its domestic market are undergoing profound historical changes. China's crude oil resources that will be available for exploitation are forecasted at 16 billion tons, with the remaining recoverable reserves of 2.38 billion tons. According to the China National Petroleum Corporation forecast analysis, the domestic crude oil self-sufficiency rate of 82% in 2000 will have been reduced to 60% in 2020.

4.2. Data Analysis. With the limited historical data available, the grey prediction model becomes suitable for use with this small sample size. Lists of statistics concerning China's crude oil consumption may be found via the following links: http://data.stats.gov.cn/easyquery.htm?cn=C01&zb=A070E&sj=2016. We use the actual data from 2002 to 2011 as a modeling sample (Table 1), using only 10 data points to meet the "small sample" features. Meanwhile, in order to verify the model's predictive performance, the real data from 2012 to 2014 will be used as the benchmark data for comparing model performances.

4.3. Simulation and Forecasting. According to the original data of the crude oil consumption shown in Table 1 for 2002 to 2011, the model FISGM has the smallest average relative error when the optimal fractional $r = 2.0175$ is obtained by the PSO algorithm. The model iterates according to the following steps:

Step 1. Data processing.

By (17), we can obtain the 2.0175-AGO sequence:

$$X^{(r)} = (2.2544\ 7.0429\ 14.7740\ 25.6051\ 39.7706\ 57.4667\ 78.8563\ 104.2173). \tag{37}$$

Step 2. Parameter estimation.

$$\widehat{a} = [a, b, c]^{\mathrm{T}} = \left(B^{\mathrm{T}} B\right)^{-1} B^{\mathrm{T}} Y = \begin{bmatrix} 1.0609 \\ 2.6509 \\ -0.6540 \end{bmatrix}. \tag{38}$$

Step 3. Construct the FSIGM model:

$$\begin{aligned} \widehat{x}^{(r)}(k) &= (-8.4845) \times 1.0609^{k-1} + 2.6509 \sum_{j=0}^{k-2} (k-j) \\ &\quad \times 1.0609^{j} + 10.7389, \quad k = 2, 3, \ldots, n. \end{aligned} \tag{39}$$

Step 4. Compute the simulated values $\widehat{X}^{(r)}$.

$$\widehat{X}^{(r)} = (2.2544\ 7.0396\ 14.7674\ 25.6170\ 39.7788\ 57.4544\ 78.8582\ 104.2173). \tag{40}$$

Step 5. Compute the simulated values $\widehat{X}^{(0)}$.

$$\widehat{x}^{(0)}(k) = \left(\widehat{x}^{(r)}\right)^{(-r)}(k) = \sum_{i=0}^{k-1} \frac{\Gamma(r+1)}{\Gamma(i+1)\Gamma(r-i+1)} \widehat{x}^{(r)}(k-i), \quad k = 2, 3, \ldots, n. \tag{41}$$

Step 6. Compute and compare the simulation/prediction errors.

The predicted values $\widehat{x}^{(0)}(k)$ of the amount of Chinese crude oil consumption and the mean absolute percentage error are taken from (33). To compare the simulation/prediction performances, FSIGM model and SIGM model are employed to simulate the amounts of crude oil consumed in China during 2012–2014, and their simulative and predictive errors are shown in Table 2.

As seen in Table 2, the simulation accuracy of the fractional-order FSIGM model is significantly improved

Table 2: The simulation/prediction values and MAPE of the models for China's crude oil consumption.

(a)

Year	Actual value	FSIGM Simulation value	SIGM Simulation value	GM Simulation value	DGM Simulation value	NDGM Simulation value
In-sample						
2002	2.2544	2.2544	2.2544	2.2544	2.2544	2.2544
2003	2.4922	2.4889	2.5374	2.6229	2.6235	2.5374
2004	2.8749	2.8749	2.7897	2.7941	2.7945	2.7897
2005	3.0086	3.0305	3.0204	2.9764	2.9766	3.0204
2006	3.2245	3.2019	3.2314	3.1706	3.1707	3.2313
2007	3.4032	3.3868	3.4243	3.3774	3.3774	3.4243
2008	3.5498	3.5848	3.6007	3.5978	3.5975	3.6007
2009	3.8129	3.7962	3.7620	3.8325	3.8320	3.7260
MAPE (in)		0.4949%	1.2526%	1.9179%	1.9145%	1.2525%

(b)

		Prediction value	Prediction value	Prediction value	Prediction value	Prediction value
Out-of-sample						
2010	4.2875	4.0215	3.9094	4.0826	4.0818	3.9095
2011	4.3966	4.2612	4.0443	4.3490	4.3479	4.0444
2012	4.6679	4.5160	4.1676	4.6327	4.6313	4.1678
2013	4.8652	4.7869	4.2804	4.9350	4.9332	4.2806
2014	5.1547	5.0746	4.3835	5.2569	5.2548	4.3837
MAPE (out)		3.1401%	10.9054%	2.0069%	9.1817%	10.9031%

compared with those of the classical grey GM, DGM, and NDGM models. The FSIGM model has a high prediction accuracy, which is much higher than those of the DGM and NDGM models. The simulations and predictions of the MAPE values of the FSIGM model and grey-extended SIGM model are lower than those of the SIGM model, giving a better overall effectiveness. When $r = 2.0175$, the simulated MAPE value of the FSIGM model is 0.4949 and the predicted value is 3.1401. These values increase when $r = 1$, such that the simulated MAPE value of the SIGM model is 1.2526 and the predicted MAPE value is 10.9054, which are significantly improved in the new model. To further see the obvious effects of the two models that were first shown in Table 2, the absolute simulation and prediction percentage errors of the above two models for China's crude oil consumption are illustrated in Figure 2.

Figure 2 shows that the simulations and predictions of the fractional-order FSIGM model at $r = 2.0175$ are much better than the simulations and predictions of the SIGM model at $r = 1$. At the same time, we can see that the precision of the FSIGM model is higher than that of the classical grey GM, DGM, and NDGM models. SIGM and NDGM overlap almost exactly. When $r = 2.0175$, the data coincide with the original data, and the predicted value is close to the original data. To further see the simulation and prediction effects of the two different models, the detailed graphs are shown in Figures 3(a)–3(e).

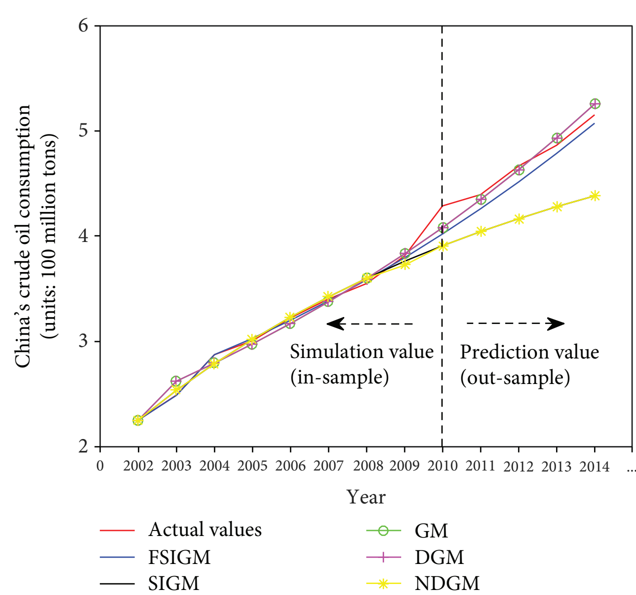
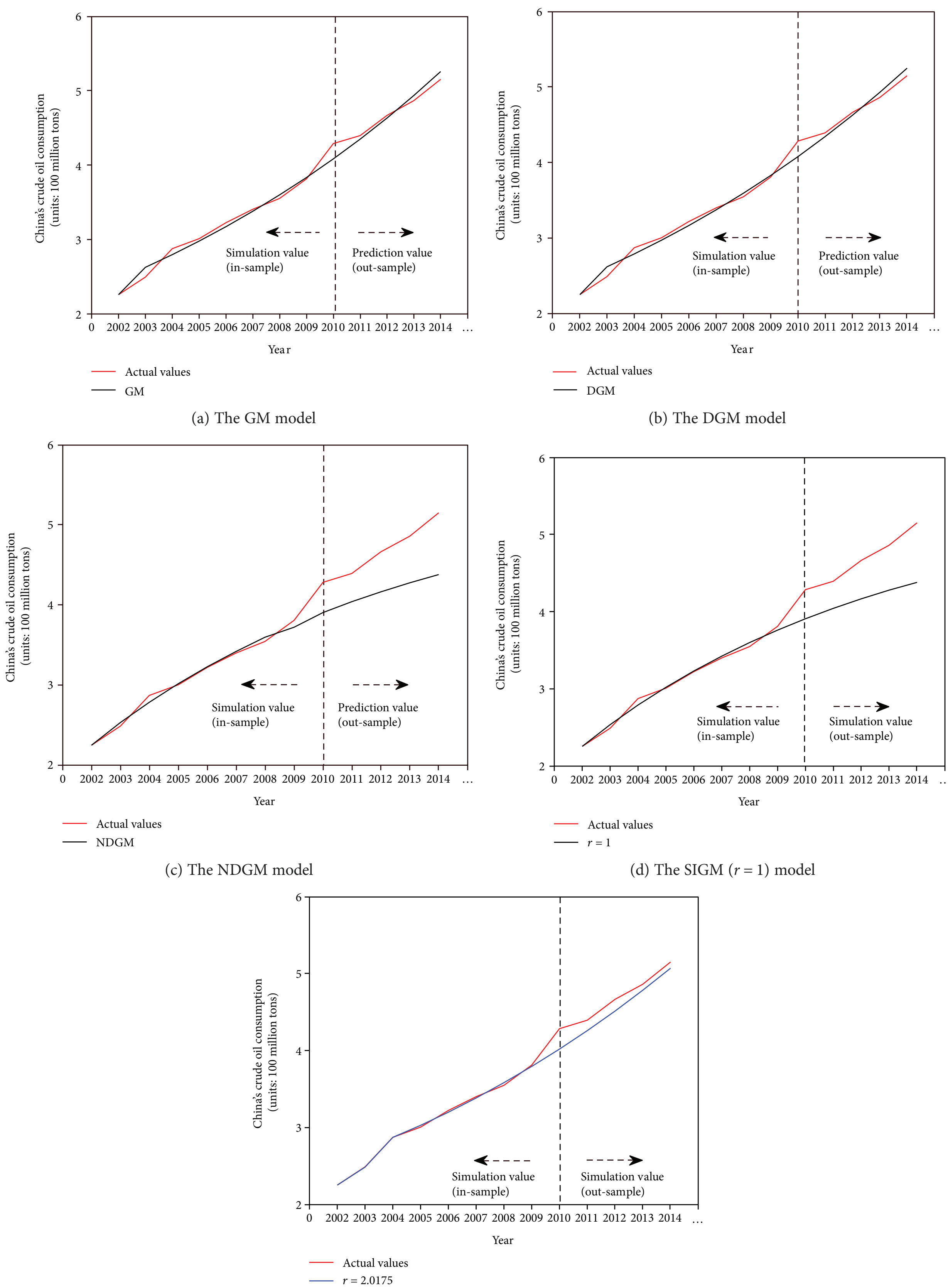


Figure 2: Simulation and prediction value of China's crude consumption with two models.

In Table 2 and Figures 2 and 3(e), the average errors of the simulations and predictions for FSIGM are shown to be 0.4949% and 3.1401%. It can be determined from therefore table of the precision grades of the grey



(a) The GM model

(b) The DGM model

(c) The NDGM model

(d) The SIGM ($r = 1$) model

(e) The FSIGM ($r = 2.0175$) model

FIGURE 3

Table 3: The predictive values of China's crude demand (units: 100 million tons) during 2015–2020.

Year	2015	2016	2017	2018	2019	2020
Crude demand	5.3802	5.7047	6.0492	6.4149	6.8031	7.2151

prediction models [38] that the precision grade of the FSIGM model for forecasting China's crude oil is between class I and class II and is proposed to hold for the short-term projections. China's crude oil demand during 2015–2020 in the FSIGM model can be employed to predict the median real demand, and this forecast can be implemented using the MATLAB program of FSIGM. The predictive results are shown in Table 3.

In Table 3, we can see that, in 2015–2020, China's crude oil demand will maintain a rapid growth rate. In this case, the Chinese government should implement any strategy or measures to maintain a balance between China's crude oil supply and its demand, which we will be discussed in detail in the next section.

4.4. Policy Suggestions. We can see from Table 3 that the FSIGM model forecast of China's 2015–2020 crude oil consumption can provide the likely demands of the next few years. On this basis, the government can determine domestic production according to the predicted supply and demand characteristics and take some preventive measures to maintain the balance of the supply and demand of crude oil. The specific measures include establishing a certain scale of national strategic crude oil reserve, which can reduce the impact of international crude oil market volatility on China's development. This will be one of the important strategic tasks of China's national economic development, which can prevent a disruption of the supply of crude oil. At the same time, international oil prices can remain stable or reach lower prices, reducing the holdings or ability to sell at higher prices. Thus, the proposed measure cannot only reduce foreign exchange spending but can also stabilize China's oil price fluctuations, stabilizing China's domestic economy.

5. Conclusion

Crude oil demand forecasting is an important part of setting crude oil development strategies. Scientific, reasonable, and accurate analyses of China's crude oil demand can protect China's energy security, providing an effective way to solve the bottlenecking problem of crude oil. Based on the features of the grey prediction model and the fractional extension operator, this paper proposes a new FSIGM model from the expansion of the SIGM expansion of the classic GM (1,1) model. The details are as follows:

(1) The least squares estimation method is used to simplify the SIGM model parameter calculation method from the literature [10].

(2) A new FSIGM model is proposed based on the fractional extension operator. The parameters of the model are calculated by using the least squares estimation method. The reduction value is obtained by using the differential equation. The fractional-order generation operator is expressed using the properties of the gamma function, and the representation of the gamma function of the FSIGM model is obtained.

(3) According to the characteristics of China's crude oil consumption data, the new model FSIGM is more flexible and intelligent, and based on the optimization mechanism of particle swarm optimization (PSO) and the properties of the gamma function of the fractional-order generation operator, the optimal particle swarm optimization algorithm of the FSIGM model is obtained. According to the original data, which is used to select the greatest fractional order, this method improves the adaptability of the model.

(4) According to the experimental analysis of China's oil consumption, the new FSIGM model is more accurate than the classical grey GM, DGM, and NDGM models, and the simulation and prediction accuracy of the grey-extended SIGM model is also higher. Based on this, China's crude oil consumption in 2015–2020 was forecasted. Based on the results of the forecast, establishing a certain scale of national strategic oil reserves could be an effective preemptive measure to take.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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