

On Bolzano's Alleged Explicativism

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ON BOLZANO'S ALLEGED EXPLICATIVISM

ABSTRACT. Bolzano was the first to establish an explicit distinction between the deductive methods that allow us to recognise the certainty of a given truth and those that provide its objective ground. His conception of the relation between what we, in this paper, call “subjective consequence”, i.e., the relation from *epistemic reason* to consequence and “objective consequence”, i.e., *grounding (Abfolge)* however allows for an interpretation according to which Bolzano advocates an “explicativist” conception of proof: proofs *par excellence* are those that reflect the objective order of grounding. In this paper, we expose the problems involved by such a conception and argue in favour of a more rigorous demarcation between the ontological and the epistemological concern in the elaboration of a theory of demonstration.

1.

There are two constraints on the notion of proof. The first constraint is *reliability*: if there is a proof of ϕ , then it must be the case that ϕ . The second constraint is *epistemic immanence*: if there is a proof of ϕ , then it must be possible to know it. One way to tie the two constraints together is to say that a proof ensures us that what it proves is true like litmus paper ensures us that the solution in which it is immersed is alkaline: a proof is what allows us to *detect* the truth of a theorem, which means, on the one hand, that there cannot be a proof of a theorem without this theorem's being true and, on the other hand, that it is on the basis of the existence of the proof that we recognize that the theorem is true. Following the “detection-view”, a proof is a relation between an agent A_1 and a proposition ϕ : when this relation is the case, it implies both A_1 's conviction *and* the truth of ϕ . The trouble with this view is that the word “imply” is, in this sentence, perfectly ambiguous. What implies A_1 's conviction that ϕ , namely, according to this view, the proof, is not what implies ϕ itself. In this example just as in many others, A_1 's conviction that ϕ follows from A_1 's grasping a consequence of ϕ which is more easily accessible than ϕ itself. But neither A_1 's grasping of this consequence, nor, of course, this consequence

as such can be seen to be what grounds ϕ . Since this is precisely what the detection-view maintains, it is absurd.

We owe to Bernard Bolzano (1781–1848) an argument that goes along these lines. Bolzano distinguishes two notions of proof, that is, two notions of consequence: one objective, what he calls *grounding* or relation from ground to consequence, one subjective, the relation from *epistemic reason* to consequence. In other terms, Bolzano makes a distinction between the objective grounding of a truth, and the subjective means that enable us to know it. A proposition is a subjective consequence (*subjektive Folge*) of other propositions if the recognition of the truth of the former depends on the recognition of the truth of the latter: they stand in a relation of epistemic dependence. On the other hand, a proposition is the objective consequence (*objective Folge*) of other propositions if the truth of the former presupposes the truth of the latter, that is, if they stand in a relation of alethic dependence. Bolzano himself explains the difference by appealing to the distinction he establishes in the *Wissenschaftslehre* between objective and subjective propositions – judgements and, if they are true, cognitions (*Erkenntnisse*) –, that is, the mental episodes in which objective propositions are “grasped” by the agent.¹ The subjective consequence relation differs from the objective one for the following reason: the objective consequence relation is a relation between true propositions *in themselves* while the subjective consequence relation is a relation between true propositions that is mediated by the recognitional capacities of the agent.

Following Bolzano’s canonical example,² the proposition that a thermometer stands higher in summer than in winter is an objective consequence of the proposition that it is warmer in summer than in winter. The latter, however, is a subjective consequence of the former, for it is on the basis of our recognising the truth of the propositions about the thermometer that we typically recognise the truth of the proposition about the temperature. Let us generalise on this example and let us suppose that ψ_1, \dots, ψ_n are objective consequences of ϕ : ϕ is then the subjective consequence of a set of propositions ψ_i , namely the propositions whose recognition as true leads to the recognition of ϕ as true. In short, the subjective consequence relation appears to be part of the converse of the objective consequence relation

$$(1) \quad SF \subset OF'$$

Of course, this account is still too imprecise, for it does not specify which are the objective consequences of ϕ that follow subjectively

from ϕ . Manifestly, the recognition of the truth of ϕ on the basis of the recognition of the truth of one of its objective consequences requires that we also recognise that the truth of the objective consequence (in our example, the thermometer's standing higher) necessarily supposes the truth of the propositions ϕ (the increase in temperature). Amongst all the propositions that are objective consequences of a given proposition ϕ , only those propositions of which we can both recognise that they are true and that they cannot be true unless ϕ also is have ϕ as a subjective consequence. The richness of this relation corresponds to the richness of the recognitional capacities of the agent

(2) If $X \subset X'$, then $SF_X \subset SF_{X'}$.

For an agent who is in a position to know that the temperature rose, not only because he notices that the thermometer stands higher, but because he is, say, aware of events such as the minute dilatation of a metal rod and can identify the cause of such phenomena, *all* objective consequences of the raise of temperature will be likely to indicate this raise. Following this explanation, a being endowed with asymptotically vast recognitional capacities would recognise the truth of a proposition on the basis of any of its objective consequences and would indeed gain the conviction of this truth as soon as he came across the first of these consequences with sufficient evidence. In other words, the wider the perceptive, inferential and discriminative powers of an agent, the greater the way in which he might convince himself of the truth of propositions is likely to vary. This unlegislated variation which goes along with the richness and diversity of the experiential and inferential possibilities drastically contrasts with the rigid and invariant objective grounding network between propositions: the range of patterns of epistemic consequence and conviction is never so remote from the fixity of objective groundings as when we are dealing with infinitely rich and prolix cognitive resources.

2.

The relation of grounding is an “ontological” relation whose realisation is perfectly independent of the question as to whether it has been contemplated or recognised and which would remain unchanged even if no one ever contemplated it. In other words, the fact that the recognition of the truth of a certain proposition may render intelligible the truth of its objective consequences is a mere by-product that

need not feature in the definition of the notion of objective consequence itself: epistemic factors such as “explicative value” must not be part of the account of objective consequence and, if there is such a thing, it is the prerogative of an eventual linguistic presentation of the objective network they determine. But what about the reality of these consequences?

Bolzano maintains that the ideal, strictly scientific order of a science should mirror the objective connection of truths, that is, it should conform to the order defined by the objective grounding-relation. Explicativists usually claim that a strictly scientific exposition in Bolzano’s sense achieves *more* than its rivals which are taken to merely establish the truth of the propositions to which they conclude: in addition to establishing their truth, expositions that conform to the objective grounding order are said to provide the ground of these truths. We contend that this interpretation is wrong, that the recognition of the truth of the propositions that are demonstrated is not a consequence we can expect from the “strictly scientific” exposition Bolzano advocated. This type of exposition does not do *more* than its rivals, but *something else*.

Most explicativists look at the issue from a standpoint which, for reasons to which we’ll return later, is eminently favourable to their position, namely that of mathematics. We’ll therefore approach the problem from a broader standpoint that embraces the principles of a strictly scientific exposition of empirical propositions. In order to elucidate his own distinction between subjective and objective consequence, Bolzano often resorts to the notorious division put forward by Aristotle in the *Posterior Analytics* between proofs that merely establish the reality or the *that* (the *οτι*) of the thing proven and proofs that impart its *why* (the *διοτι*). In the example used by Aristotle to illustrate the distinction,³ two ways of establishing astronomical truths are contrasted. We can establish the proximity of the planets by invoking the fact that they do not shine. We then infer to a mere fact since the proximity of planets is not grounded in their not shining. One may however, construe things so as to infer from the proximity of the planets to their not shining, in which case we infer from the ground to its consequence. This example aptly shows the misunderstanding on which explicativism thrives. Explicativists will hold that we get more out of the recognition of the “why” of a fact than from the mere recognition of that fact. When we recognise the “why” of a fact, we have both the fact and its ground: it would be impossible to be presented with the proof of a fact that would not be

the case. But this inference from *διότι* to *οτι* does not hold and Aristotle's example shows why: what is asserted is the reality of the proximity of the planets, but the fact whose ground is established is that they are not shining. There is therefore not one fact of which we both assert the reality and explain why it occurs. There are two facts, of one of which we assert the reality and the other of which we explain why it occurs. While the fact whose reality we establish is the ground of the other, its reality is not itself cognised through its ground, but precisely, through its consequence.

Explicativists claim that there is a type of exposition which can both persuade us of the reality of a fact and provide its ground: ϕ , and ϕ because ψ . They claim, moreover, that the recognition of the objective ground is itself the epistemic reason – the subjective ground – of our cognition of the consequent: ϕ , because ϕ because ψ . Of course, Bolzano's notion of objective consequence relation hardly forbids such an interpretation: the objective consequence relation holds only between true propositions, so that one who cognises that ϕ is an objective consequence of ψ is bound to recognise the truth of both ϕ and ψ . Nevertheless, Aristotle's example shows that this interpretation is inadequate. The judgement according to which the planets do not shine because they are in close proximity to one another is the conjunction of the judgement according to which the planets are close to one another, and of the judgement according to which heavenly bodies' being at a distance is a sufficient condition for their shining. Hence, either the "exposition" favoured by explicativists is the exposition of the conjunction of these two propositions and we see that the recognition of the fact is required for the recognition of the grounding-relation, or this "exposition" includes only the conditional proposition about the necessary connection between the two hypothetical events (if there is no shining, then there is proximity), in which case it is excluded that we derive the recognition of the fact from the latter. In short, either the bare recognition of the fact is required in order to establish its ground, or establishing the ground is insufficient in order to vouch for the consequence. In both cases, it is wrong to take the recognition of the *why* for a mode of cognition of the *that*: the second cognition only follows from the first if it has expressly been appended, that is, if it comes from another source. Explicativists hold that, in an exposition that follows the order of objective knowledge, the recognition of what implies the thing implies the recognition of the thing. This is a mere variant of the "detection-view", which is generally untenable.

3.

Aristotle identifies the additional condition under which the type of exposition to which the explicativists aspire could be realised. It would require, he says,⁴ that the objective ground be also the element that is best known. Only under this condition would the recognition of the why have as its consequence the recognition of the that: if it is known that the realisation of ϕ implies the realisation of ψ and if ϕ is cognised, then ψ is equally cognised, and it is cognised “through its ground”. This additional condition is obviously not satisfied, neither in Bolzano’s meteorological example nor in Aristotle’s astronomical one. In both cases, the consequence is better known than its ground, and the recognition of the ground therefore follows from the recognition of the consequence, not the other way around. As Aristotle writes, “in cases in which... the non explanatory term is more familiar, the fact is proved but the reason why is not.”⁵

Let us from now on call *cognition through grounds* (by contrast with cognition *of* the ground, that is, the mere cognition of the conditional connection between two propositions) a cognition that establishes the consequence of a proposition on the basis of its objective ground when the latter is best known. What are the features of such knowledge and under which conditions would it be possible?

Clearly, whenever such cognition is the case, the particular subjective consequence relation obtained is part of the objective consequence relation and not of its converse

$$(3) \quad SF^* \subset OF.$$

This peculiar subjective consequence relation ($\langle\langle \{\phi_i, \dots, \phi_n\} SF^* \phi \rangle\rangle$ means $\langle\langle \phi$ is known through its grounds $\phi_1, \dots, \phi_n \rangle\rangle$) raises two questions. There is, on the one hand the question whether SF^* , just like SF (the “ordinary” subjective consequence relation) depends on the agents’ recognitional capacities. The answer to this question is negative since SF^* is a mere linguistic presentation of the objective grounding relation which is, for its part, intrinsic and entirely determined. One detail aside – and we will get back to this detail later – this relation is thus unique. On the other hand, there is the question whether cognition through grounds is in general possible, that is, even in the case in which Aristotle’s condition is satisfied. We will attempt to show that it is not and that an explicativist interpretation of Bolzano is devoid of relevance.

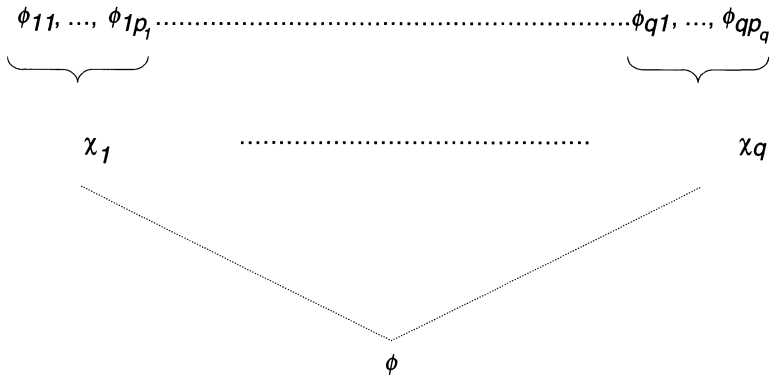
4.

As far as uniqueness is concerned, it is clear that cognition through grounds is not subject to the same variations as ordinary inferential knowledge (in Bolzano's meteorological example, we may infer to the increase in temperature on the basis of any of its perceived consequences, the range of which varies widely, as a function of the agents' discriminative powers). Here, the downstream variation, on the side of consequences, obviously does not occur. But it is eventually balanced out by an upstream variation on the side of the grounds of the cognised proposition. Aristotle discusses this point in the *Posterior Analytics* when he attempts to discard, as candidate to the title of cognitions through the grounds conclusions that are obtained by "hyperbolic" means (*καθ' υπερβολη*), that is, inferred from propositions exaggeratedly remote in the grounding-order. This is the case, Aristotle says, when we conclude that there are no flute players among the Scythians because they do not grow wines while it would be sufficient, in order to conclude to their absence, to invoke the fact that the Scythians are never inebriated enough to indulge in this frivolous distraction. On the contrary, cognition through the grounds must be construed in the sense of "immediate", and not "remote" ground. Thus we understand that walls do not breathe when we realise that they do not have lungs, not in recalling that they are not animals, this ground being too remote.⁶ Bolzano considers the objective grounding-relation as a derivative relation, namely as the transitive closure of a simpler relation that connects a set of propositions to those that are its immediate objective consequences: ϕ is an objective consequence of ϕ_1, \dots, ϕ_n exactly when there is a "chain" of immediate consequences that begins with ϕ_1, \dots, ϕ_n and finishes with ϕ . Under these conditions, there is, for each true proposition, a unique objective tree-structure that connects it in an ordered manner with the propositions that ground it and of which it is an objective consequence. To cognise a true proposition through its grounds thus consists in grasping the true propositions of which it is an immediate objective consequence. Since there are no individual fluctuations in the objective grounding-order, there are no individual fluctuations in the way in which we establish the ground of a cognition.

The only residual of indeterminacy comes from the fact that the linear linguistic exposition of the objective grounding tree-structure must cope with the absence of a complete order among the immediate

grounds of a determinate truth. Indeed, the tree-structure may have, for instance, the following form, where

$$\phi_{11} = \phi_1, \dots, \text{ and } \phi_{qp_q} = \phi_n.$$



In this example, χ_1 and χ_q are the immediate consequences of $\phi_{11}, \dots, \phi_{1p_1}$ and $\phi_{q1}, \dots, \phi_{qp_q}$, respectively, but neither of the two propositions precedes the other in the tree-structure. The “chain” of consequence-relations thus allows for permutations between propositions that are, so to say, on a par. Neither of the two propositions *Caius is at the forum* and *Scipio is at the forum*, to which the proposition *Caius and Scipio are at the Forum* owes its truth, can be considered to precede the other in the objective immediate grounding tree-structure.⁷ In other words, a proposition will be an immediate consequence of a non-structured set of propositions, and in this respect, the – necessarily diachronic – process of recognition of the grounding of true propositions may involve a minimal and venial form of interindividual variation.

5.

Explicativists endeavour to give an epistemic interpretation of Bolzano’s idea of an objective immediate grounding order among true propositions. According to them, the recognition of the truth of propositions according to this hierarchical structure presents a virtue which all other types of recognition lack since it alone provides the means to grasp true propositions as should be, that is, through the propositions to which they owe their truth. In order for this “canonical” grasping to be possible, four conditions must be fulfilled by the objective structure in question:

- (i) by virtue of the Aristotelian argument developed above, the propositions to which the proposition under consideration owes its truth must be at least as easily cognisable as the latter;
- (ii) the complete structure of relations that connects the proposition under consideration with those whose immediate objective consequence it is should be finite (“horizontal” finitude principle);
- (iii) the tree-structure of connections that connects, step by step, the proposition under consideration with the totality of its antecedents must be adequately grounded (“vertical” finitude principle);
- (iv) each objective consequence relation between the proposition under consideration and the propositions that are the source of its truth must be individually recognisable as such.

5.1.

The satisfaction of the first condition, as discussed above, is problematic and constitutes a serious objection to explicativism *in general*: explicativism is at best applicable in domains in which this requisite is satisfied.

5.2.

The horizontal finitude condition would not be satisfied in – at least – either of two cases. On the one hand, it would be violated if a proposition could be the immediate consequence of an infinite collection of propositions composed on the basis of diverse simple ideas. Complex ideas are, in Bolzano’s theory, composed recursively and hierarchically. The idea of an equilateral triangle, for instance, is complex. It contains other ideas as its parts (according to Bolzano’s canonical conception of the structure of ideas): *triangle*, *which*, *has*, and *equilaterality*; and if it is the case that the latter are themselves complex, it also contains – mediately – the parts of these parts, and the parts of the parts of these parts, etc.⁸ In order to safeguard horizontal finitude, Bolzano thus needs to exclude that the antecedent of a proposition contain an infinite number of propositions composed on the basis of an infinite number of ideas. Bolzano shows that, for a certain class of propositions, namely “purely conceptual” propositions (*reine Begriffsätze*), by contrast with empirical propositions, this situation cannot arise. His argument is three-fold.⁹

- (i) a purely conceptual proposition can only be the immediate consequence of propositions of the same nature (the epistemic

counterpart of this idea is that the truth of a purely conceptual proposition must be cognisable on the sole basis of conceptual reflection);

- (ii) the conceptual complexity of a proposition, that is, the number of concepts it contains, cannot exceed the conceptual complexity of the propositions that follow it immediately in the objective grounding tree-structure;
- (iii) the number of simple concepts is finite, and so are their modes of combination in a proposition.

By (i), the immediate antecedents of a purely conceptual proposition are all purely conceptual propositions which differentiate themselves from one another only by virtue of the simple concepts they contain and by the modes of combination of these concepts. By (ii), the number of concepts contained in the antecedent is inferior to the number, say n , of concepts which are contained in the proposition under consideration. By (iii), this proposition cannot have infinitely many distinct immediate antecedents since every one of them is completely determined by the choice of at most n concepts which are to be selected from a finite set and ordered according to finite modes of connection.

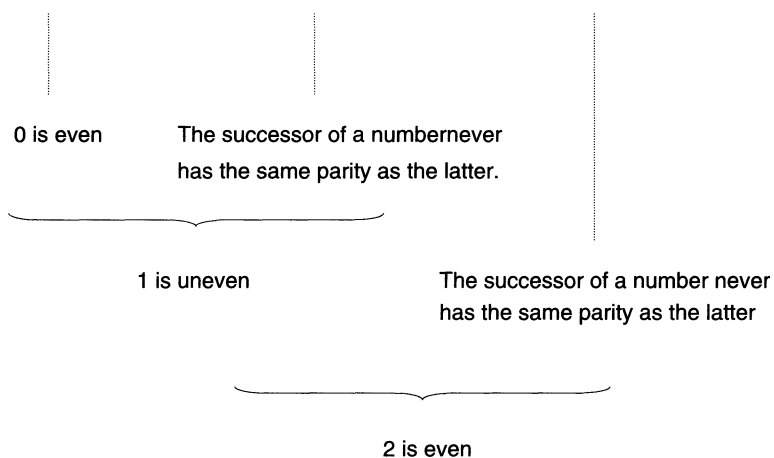
Condition (i) also prevents another form of horizontal infinitude. For were Bolzano to admit for universal propositions relative to an infinite domain of objects the constraint would, again, be violated. If we were to interpret the universal quantifier in terms of generalised conjunction, the proposition, for instance, that all equilateral triangles are equiangular, could owe its truth, immediately, to the collection of all propositions according to which *this* or *that* triangle is equilateral and equiangular, a collection of propositions which a finite being would hardly be in a position to recognise as true individually in their totality. Bolzano's conception of grounding does not allow this in the case of conceptual propositions. Now, the standard way to avoid this unassessable infinity consists in putting forward an analysis of propositions that does not allow to interpret the universal quantifier in terms of the infinite collection of individual equilateral triangles. One way to do this could consist in supposing that the object the propositions are about are in fact singular. Bolzano seems to suggest this when, at *WL* §57, he explains that the quantifier "all. . ." in the linguistic expression of a proposition is not a genuine component: it is considered to be "redundant" and is meant to remind us that the subject-idea must be considered in its widest

extension. Propositions of the traditional form 'All As are Bs' should be analysed as 'A has b': in the proposition *Equilateral triangles have equiangularity* we are talking about the equilateral triangle *überhaupt*.¹⁰ But this is not Bolzano's way to avoid the problem. For Bolzano is clear on the fact that the idea *equilateral triangle* is not a singular idea that denotes some general object but a general one – a *Gemeinvorstellung* – that denotes an indefinite number of objects (he is also unmistakable on the fact that the proposition is neither about this idea itself¹¹ nor about the unique collection of all equilateral triangles¹²). The Bolzanian argument against the idea that a "general" conceptual truth 'A has b' could be grounded in the infinite collection of propositions of the form 'This which has a has b' consists in pointing to the fact that since (1) 'this' invariably designates an intuition and (2) a proposition that contains an intuition is by definition an intuitive – or empirical – and not a conceptual truth, allowing for conceptual truths to be grounded in them would violate the "conceptuality" condition (i). This does not imply that the Bolzanian conception of universal quantification – which space does not allow us to discuss here – excludes that universally quantified empirical propositions range over an infinite number of objects. What this implies is, in a nutshell, that as far as conceptual truths within the objective-grounding order are concerned, the truth of general propositions – i.e., propositions whose subject-idea is a general concept – does not depend on their being a conjunctive set of more primitive singular truths about the individual objects since no such truth are in fact allowed in the structure. Rather, as Bolzano recurrently says, the truth of such propositions is grounded in the very nature of the concepts they involve. More on this in what follows.

5.3.

Are we also assured, in the domain of purely conceptual truths, that the regression from a proposition to the propositions of which it a consequence must end? It could be that the tree-structure contain infinite branches¹³ – just as is the case with some other propositions which are the immediate consequence of infinitely many others, such as empirical propositions which require a never ending collection of intuitions for their establishment (this is ϕ , this is *also* ϕ , etc...) – The previous argument, which rests on the finitude of the number of concepts and combination modes, would be unable to exclude this possibility. It only insures that a conceptual truth cannot have infinitely many *distinct*

immediate antecedents. But the finitude of the number of these immediate antecedents *tout court* is due to the fact that their collection is considered, ontologically, as a non structured set in which elements of identical types are identified rather than as a what substructuralist logicians call a “basket” and in which the number of occurrences is relevant. It is impossible to appeal to such a feature here since Bolzano-type tree-structure are structured vertically, but not horizontally. We can illustrate this with a segment of a derivation, built for the purpose, according to which two is an even number:



Nevertheless, the fact that the same proposition occurs many times in the grounding tree-structure of a purely conceptual proposition is perfectly natural and is not likely to generate what should precisely be avoided, namely the existence of an infinite branch in the structure. Identical propositions never appear on the same path and Bolzano’s principle according to which the objective consequence relation is irreflexive makes “cycles” of the sort impossible. In this respect, cognition through the grounds is insured to reach, regressively, “primitive” propositions that are not themselves consequences of other propositions and this is the result that was aimed for.

5.4.

Finally, it is generally assumed that the possibility of cognising the connection of immediate objective dependence between propositions is the natural counterpart to the indecomposability of these connections. The argument is that, if the objective dependence relation of ϕ with respect to ψ were irremediably opaque, it could mean that a

certain mediate proposition fails to be grasped and, consequently, that the dependence of ϕ with respect to ψ is not, properly speaking, immediate. In other terms, the required intelligibility is warranted by the minimal character of the immediate objective grounding relation: there cannot be any insuperable doubt about the nature of the connection between a proposition and each of those that ground its truth. If a ground fails to be grasped, it is because another one comes in between the latter and what is to be explained. The epistemic counterpart to the detailed objective structure of reality – i.e., determined up to the objectively indecomposable connections that constitute it – is the existence of minimal inferential steps whose rectitude cannot, for this reason, be doubted by any rational creature. Contrary to proofs which are solely meant to insure the truth of what they establish and in which it is sufficient that the rules of inference transmit regularly to their conclusion the eventual truth of their premises, a proof that records the immediate ground of this truth can only allow for transitions that are more restricted. In a realted context, Frege writes

Most of the time, one is satisfied if every step in the proof is evidently correct, and this is reasonable if one merely wants to persuade [others] of the truth of the propositions to be proven. But when what is at stake is to provide an insight in the nature of this evidence, this procedure does not suffice, and one must write down all intermediate steps in order to let the full light of consciousness fall on them. Mathematicians are usually concerned solely with the content of a proposition, and that it should be proven. What is new here (in the *Grundgesetze*) is not the content of the proposition, but the way in which the proof is set out, and the foundations on which it rests.¹⁴

6.

With purely conceptual and in particular mathematical propositions, explicativists thus find themselves on an eminently favourable soil to justify their preferred thesis: the detailed exposition of the grounding tree-structure of true propositions is such as to enable us to recognise the truth of these propositions while indicating the ground of their truth since: (1) the tree-structure shows in an indisputably intelligible manner their individual connection with the propositions to which they owe their truth; (2) these grounding propositions are at least as clear as their consequences and (3) the totality of these connections have the two finitude properties required in order to claim that they epistemically accessible.

We are not going to argue against what precedes and, for the sake of discussion, we will grant without further reservation the otherwise controversial premises on which the crucial properties we have just listed rest, such as the one concerning the finitude of the set of simple concepts. In fact, one of the “accessibility” arguments, the one concerning the “vertical” finitude of Bolzano-type tree-structures, can be strengthened in the following manner. In order to establish this finitude, we had recourse to the argument according to which the number of concepts that appear in the immediate antecedents (and thus, step by step, in the antecedents *tout court*) of a given conceptual proposition cannot exceed the number of concepts that appear in the proposition under consideration

If we imagine that all purely conceptual truths whose complexity does not exceed a certain limit (for instance all truths that do not contain more than 100 simple parts) are united in a collection, it follows from what precedes that each truth which appears in it and which is not primitive finds in these resources its complete ground, not only its immediate ground but also all its remote grounds.¹⁵

But after all, it is the set of simple concepts that appear in these antecedents, be they immediate or remote, which is itself included in the set of concepts that appear in the grounded proposition:

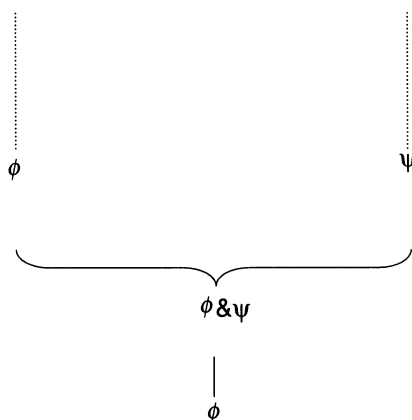
If a given proposition consists of mere concepts, such as, for instance, the proposition that virtue deserves respect or that two sides of a triangle taken together are bigger than the third, etc.; then the truth or falsity of the latter depends only on the properties of these concepts; and, at least in many cases, nothing else will be required in order to convince yourself of its truth, than that you examine attentively the concepts themselves of which it is composed. Thus, it will be possible for you to recognise the truth that virtue deserves respect from the mere fact that you have the concepts virtue, to deserve and respect. You cognise truths of this kind (purely conceptual truths) by virtue of the fact that you know the concepts of which they are composed. It is different with judgement which contain intuitions [...] For judgements of this species... their truth does not, indeed, depend solely on the ideas you have, but also on the properties of the external objects which they represent.¹⁶

Consequently, it is not only that the canonical knowledge of conceptual truths requires no intuition and thus does not involve any singular experience of this or that which we may have or may not have. It is also that the purely conceptual reflection demanded from the agents requires only that they possess a finite number of simple concepts. It is therefore the case according to explicativism that one who is able to state truths of these species finds himself *eo ipso* in possession of the conceptual resources required in order to appraise

truth. So what, given this flawless picture, could be the weakness of explicativism?

Bolzano's programme aims at substituting for "ordinary" proofs (*Gewissmachungen*) proofs that solely imply the conviction in the truth of the propositions they prove, by means of grounding proofs (*Begründungen*) that proceed in indicating, at each stage, the propositions to which the conclusion owes its truth immediately.¹⁷ Some notorious results have come out of this programme, such as the proof of the intermediate value theorem without recourse to geometric or cinematic concepts, which Bolzano deemed foreign to the proven proposition. Nevertheless, we know enough, ever since Gentzen,¹⁸ to appreciate that this programme, if it is practicable at all, presents shortcomings to which explicativists do not seem to pay sufficient attention.

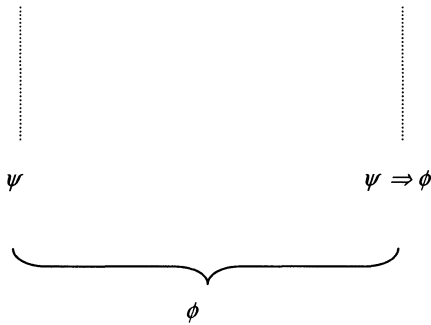
The transformation of an "ordinary" proof into a Bolzano-type proof consists, for the most part, in eliminating inferential configurations by means of which concepts foreign to the proved proposition are introduced. Thus, a configuration such as



will be replaced by a configuration reduced to "φ", since ψ and its antecedents are not qualified to appear among the propositions to which φ owes its truth. It can be shown that the order in which these replacements or deletions are made is indifferent: the process presents the "Church-Rosser" property according to which its result is a unique and well-defined Bolzano-type proof in whatever way we reach it.

It is indeed hard to conceive why the inference by means of which we would come to convince ourselves of the truth of φ should take the previous form: elementary considerations with respect to

cognitive economy would certainly lead us to stop right after the first occurrence of “ ϕ ” rather than to append a proof of ψ to the proof of ϕ which we already have before we actually conclude to ϕ again. But the situation is different in the case of *modus ponens*, that is, in the case of the configuration (“cut”)



There is no visible redundancy in this example. I may have a second hand knowledge of the truth of $\psi \Rightarrow \phi$, I may have it from my friend Wisdom, who, on his side, may or may not have had a canonical proof of it. Be it as it may, I do not *unwrap* his proof, I take it as a lemma, for I have enough to do on my part to establish ψ . I could have, it is true, not know Wisdom or I could not have inspired enough trust in him for him to let me know of his results. This belongs to the range of experiences which I may not have had, just like I may not have had the empirical experience of this or that. A being with minimal recognitional powers would not have know Wisdom and an agent equipped with less modest ones but determined not to benefit from them would have gained nothing from his advice. Less humble, or less of a puritan, I yielded to these commodities and I now have a proof that convinces me of the truth of ϕ , but which does not provide me with its ground.¹⁹

Veritatis amicus, sed veritatis causae magis amicus? Now, let us suppose that we give in to the explicativists' plea as to the benefit of transforming an impure proof into a demonstration that will provide us, he assures us, the additional advantage of knowing why his result is true. We are indeed insured that if we follow Gentzen's method for eliminating configurations of the above type (“cuts”) we will obtain a *bona fide* Bolzano-type proof in which all propositions are derived from their immediate grounds and where we encounter no concepts which are not contained in the result (the “subformula property”). But what does the resulting object look like, assuming that we be able

to construct it? If the length of my initial *Gewissmachung* is d , and if the complexity of Φ is n , then the length of the *thing* we get is given by

an exponential tower: $2^{2^{\dots^{2^n}}}$ } $_d$.

There may be explicativists who think that we lose nothing in trading for a proof *par excellence* the impure demonstration that was the motive of our rational conviction. Perhaps there are some among them to maintain that there is, in this respect, no noteworthy difference between an impure proof of two pages and a grounding proof, although the number of symbols involved in the latter exceeds that of nanoseconds since the Big-Bang;²⁰ that it is sufficient for the theory of knowledge that it watches out for theoretical impossibilities, and that if it were to take care of “medical” incapacities as well, philosophy would lose itself in trivialities. As far as we are concerned, we see in arguments of this type a patent ignorance of the difference between ontology and epistemology: ontological properties such as grounding are preserved by the passage from a relation to its transitive closure while epistemic properties, such as intelligibility, are not.

NOTES

¹ cf. for instance, Bolzano, 1837, §198 note, vol. 2, p. 341ff.

² *Ibid.*, § 162, vol.2, p. 341.

³ Aristotle, *Posterior Analytics*, I, 13 (in Jonathan Barnes (ed.), *Complete Works of Aristotle*, Vol. I, Princeton, 1984).

⁴ *Ibid.*, 78 a 27–28.

⁵ *Ibid.*, 78 b 11.

⁶ *Ibid.*, 78 b 12s.

⁷ Bolzano: 1837, §211, Vol. 2, p. 369.

⁸ *Ibid.*, §59, Vol. 1, p. 257s.

⁹ *Ibid.*, §221, Vol. 2, p. 384s.

¹⁰ cf. *Ibid.*, §57, Vol. 1, p. 247ff.

¹¹ In which case the subject would be a “symbolic idea”, i.e. a second order concept. Cf. Bolzano 1837, §90, Vol. 1, p. 426ff.

¹² Cf. *Ibid.*, Vol. 1, §83ff. where Bolzano discusses the concepts of collection, sum, set, series, unity, plurality, totality, infinity, etc.

¹³ For an example, cf. *Ibid.*, § 216, Vol. 2, p. 376s.

¹⁴ Frege, Gottlob, *Grundgesetze der Arithmetik*, Vol. 1, 1893, reprint. Hildesheim, Georg Olms, 1966, p. viii. Our translation.

¹⁵ Bolzano: 1837, §221, Vol. 2, p. 386.

¹⁶ *Ibid.*, § 42, Vol. 1, p. 181. cf. also §305.3, Vol. 2, p. 180.

¹⁷ Cf. *Ibid.*, §525, Vol. 4, p. 261.

¹⁸ For a comparison of Bolzano's ideas with Gentzen's theory of demonstration, see, for instance, K. Schröter, « Theorie des Logischen Schliessens », I et II, *Zeitschrift für mathematischen Logik und Grundlagen der Mathematik*, I-1955, p. 37-86 et IV-1958, p. 10-65, as well as J. Berg, *Bolzano's Logic*, Stockholm, Almqvist & Wiksell, 1962.

¹⁹ Bolzano does not ignore the fact that given pragmatic constraints impose themselves when carrying out demonstrations and he discusses this in detail (cf. Bolzano: 1837, §512, Vol. 4, p. 237s.). But even in pragmatic contexts, he advocates that one must, as often as possible, "bring the reader to a clear consciousness of the totality of the objective grounds on which the proof rests" (*Ibid.*, §517, Vol. 4, p. 248).

²⁰ Boolos, George: 1998 'Don't Eliminate Cut', in *Logic, Logic, and Logic*. Harvard U. Press, pp. 365-369.

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