Deleuze's Philosophical Lineage

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Albert Lautman

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Albert Lautman (1908–44) was a philosopher of mathematics working in the decades between the two world wars in the first half of the twentieth century. He postulated a conception of mathematics that is both formalist and structuralist in the Hilbertian sense. The reference to the axiomatic structuralism of Hilbert is foundational for Lautman, and it is because of this that his views on mathematical reality and on the philosophy of mathematics parted with the dominant tendencies of mathematical epistemology of his time. Lautman considered the role of philosophy, and of the philosopher, in relation to mathematics to be quite specific. He writes that: 'in the development of mathematics, a reality is affirmed that mathematical philosophy has as its function to recognize and to describe'.[1] He goes on to characterise this reality as an 'ideal reality' that 'governs' the development of mathematics. He maintains that 'what mathematics leaves for the philosopher to hope for, is a truth which would appear in the harmony of its edifices, and in this field as in all others, the search for the primitive concepts must yield place to a synthetic study of the whole'.[2]

One of the tasks, indeed the challenges, that Lautman set himself, but never carried through because of his early tragic demise – he was captured by the Nazis in 1944 and shot for being an active member of the resistance – was the task of deploying his mathematical philosophy in other domains. The commentator who, by taking up this challenge, shows the most assiduity in his engagement with Lautman is Gilles Deleuze. The mathematical work that is drawn upon and that plays a significant role in Deleuze's philosophical project is that of Lautman. Indeed, the speculative logic that Deleuze constructs as a part of his project of constructing a philosophy of difference is dialectical in the Lautmanian sense. The aim of this chapter is to give an account of this Lautmanian dialectic, of how it operates in Lautman's work, and to determine what, if anything, Deleuze does to this dialectic when it is incorporated into his project of constructing a philosophy of difference.

Lautman's axiomatic structuralism

What is quite clear in Lautman's work is that he was not concerned with specific foundational questions in mathematics, neither with those relating to its origins, its relationship to logic or to the problem of foundations. What he is interested in, rather, is shifting the ground of this very problematic by presenting an account of the nature of mathematical problematics in general.

Lautman had a wider and more precise schooling in both the French and German mathematics of the 1920s–30s than the majority of the mathematicians of his generation, who were often narrowly specialised.[3] Lautman, along with Cavaillès, was one of the introducers of the German axiomatic into a French context dominated at the time by the 'intuitionisms' of Poincaré, Borel, Baire and Lebesgue.[4] The two main ideas that are foregrounded in his primary theses in the philosophy of mathematics,[5] and which dominate the development of his subsequent work, are 'the concept of *mathematical structure* and the idea of the essential *unity* underlying the apparent multiplicity of diverse mathematical disciplines'.[6] It should be noted that, 'in 1935, the concept of structure' in mathematics 'had not yet been made completely explicit'.[7] Lautman's project is therefore novel. Lautman was inspired by the work of Hilbert on the axiomatic concept of mathematics to deploy the potential of an axiomatic structuralism in mathematics. The essential point that motivated this move was Lautman's conviction 'that a mathematical theory is predominantly occupied with the relations between the objects that it considers, more so than with the nature of those objects'.[8]

Lautman considers the idea that there is 'an independence of mathematical objects compared to the theories in which they are defined'[9] to be steeped in the analysis and geometry of the nineteenth century. He, by contrast, championed the modern algebra, and maintained that 'if classical mathematics was constructivist . . . modern algebra is on the contrary axiomatic'.[10] The introduction of the axiomatic method[11] into mathematics means that there is an 'essential dependence between the properties of a mathematical object and the axiomatic field to which it belongs'.[12] The isolation of 'elementary mathematical facts' that would function as building blocks is ruled out. Lautman can therefore claim that 'the problem of mathematical reality arises neither at the level of facts, nor at that of objects, but [rather] at that of theories'.[13] This of course is not to put mathematical facts *per se* into question. Lautman considered mathematics to be constituted like physics: 'the facts to be explained were throughout history the paradoxes that the progress of reflexion made understandable by a constant renewal of the meaning of the essential concepts'.[14] Rather than being isolatable elementary objects, mathematical facts, such as the 'irrational numbers, the infinitely small, continuous functions without derivatives, the transcendence of *e* and of ϖ, and the transfinite', 'were admitted by an incomprehensible necessity of fact before there was a deductive theory of them'.[15] He argues that mathematical and physical facts 'are organized thus under the unity of the concept which summarizes them'.[16]

Lautman's 'axiomatic structuralism' was the new mathematics that inspired the Bourbaki project which was influential in mathematics for several subsequent decades,[17] notably in the figure of Jean Dieudonné, who wrote the foreword to Lautman's collected works.[18] The structuralist point of view has been so influential on the development of mathematics since 1940 that it has become rather commonplace.[19] However, this was not yet the case when Lautman was writing.[20]

The first move that Lautman makes to develop his structural conception of mathematics is against the logical positivism of the Vienna Circle logicists. Lautman considered their effort 'to build mathematical concepts starting from a small number of concepts and from primitive logical propositions' to be in vain, because it 'loses sight of' what he refers to as 'the qualitative and integral character of the constituted theories'.[21] He argues that 'It is impossible to consider mathematical wholes as a result of the juxtaposition of elements defined independently of any overall consideration of the structure of the whole in which these elements are integrated'.[22] For Lautman, this impoverishment of logical positivism is the consequence of its conception of mathematics in propositional terms, as 'nothing more than a language indifferent to the content that it expresses'.[23]

Lautman also protests against the use made of Hilbert by the Vienna Circle logicists. Despite their claims to endorse the Hilbert programme,[24] Lautman is critical of the logicist interpretation of the term 'formalism', which he considers to be unrepresentative of Hilbert's thought.[25] While the logicists derive theorems in a formal system, such that the theorems are genetic or constitutive of the system, for Lautman, Hilbert is rather looking for theorems about formal systems, such as consistency or non-contradiction, completeness, decidability, etc.[26] Rather than confounding mathematical philosophy with the study of the different logical formalisms, Lautman considered it necessary to try to characterise mathematical reality 'from the point of view of its own structure'.[27] Lautman considered this to be a more accurate characterisation of Hilbert's meta-mathematical program, which, he argued, 'internalised the epistemological problem of foundations by transforming it into a purely mathematical problem'.[28]

Against the logicist interpretation of Hilbert's work Lautman argues that 'Hilbert substitutes for the method of genetic definitions that of axiomatic definitions, and far from wanting to rebuild the whole of mathematics starting from logic, introduced on the contrary, while passing from logic to arithmetic and from arithmetic to analysis, new variables and new axioms which each time broaden the domain of results.'[29] The (Hilbertian) axiomatic structural conception of mathematics that Lautman mobilises in his work is a nonconstructivist axiomatic, and he argues that 'Mathematics thus arises as successive syntheses where each stage is irreducible to the former.'[30] He continues by making the important point, again drawn from Hilbert, that 'a theory thus formalized is unable to bring with it the proof of its internal coherence; a meta-mathematics should be superimposed on it which takes the formalized mathematics as its object and studies it from the double point of view of non-contradiction and completeness'.[31] This double point of view distinguishes Lautman's concept of mathematics from the formalism of the logicists, which considered the study of mathematical reality to consist solely in the demonstration of the non-contradiction of the axioms which define it. The consequence of this 'duality of plans' that Hilbert establishes between 'formalized mathematics and the meta-mathematic study of this formalism' is that while the formalism is governed by 'the concepts of non-contradiction and completeness', these concepts are not themselves defined by this formalism. Hilbert expresses this governing role of meta-mathematical concepts over formalised mathematics when he writes that

> the demonstrable axioms and propositions, i.e. the formulas which are born from the play of these reciprocal actions (namely formal deduction and the addition of new axioms), are the images of thoughts that constitute the ordinary processes of mathematics developed up to now, but are not truths in the absolute sense. Truths in the absolute sense are rather the points of view . . . that my theory gives of the demonstration with regard to the resolvability and the non-contradiction of these systems of formulas.[32]

So, according to Lautman, the value of a mathematical theory is determined by 'the meta-mathematical properties that its structure incarnates'.[33]

While Lautman took a position against the version of logicism and formalism proposed by the Vienna Circle, he also distanced himself from the empirico-psychologising perspective of French mathematicians such as Léon Brunschvicg. Brunschvicg developed 'the idea that the objectivity of mathematics was the work of the intelligence in its effort to triumph over the resistance that the material on which it works opposes to it'.[34] Brunschvicg goes so far as to maintain that 'any effort of *a priori* deduction tends . . . to reverse the natural order of the mind in mathematical discovery'.[35] While Lautman follows Brunschvicg in distrusting all attempts 'to deduce the unity of mathematics starting from a small number of initial principles', including 'the reduction of mathematics to logic',[36] he doesn't endorse Brunschvicg's concept of mathematical philosophy 'as a pure psychology of creative invention'.[37] For Lautman, the task of characterising the mathematical real must be undertaken rather by 'mediating between' these two extreme positions. By extracting the minimal elements of each, the 'logical rigour' of the former and 'the movement of the intelligence' of the latter, Lautman proposes a third alternative characterisation of the mathematical real that is both axiomatic-structural and dynamic, where the fixity or temporal independence of the logical concepts and the dynamism of the temporal development of mathematical theories are combined.

The metaphysics of logic: a philosophy of mathematical genesis

In order to do this, Lautman distinguishes two periods in mathematical logic, the first he characterises as 'the naive period', which goes from 'the first work of Russell until 1929', which is the 'date of the meta-mathematical work of Herbrand and Gödel'. The latter marks the beginning of what Lautman calls 'the critical period'. He characterises the first period as 'that where formalism and intuitionism are opposed in discussions which prolong those that had been raised by Cantor's set theory'.[38] These involved the criticism of classical analysis and the foundational disputes which were largely characterised by the dispute over the legitimacy of the actual infinite. While the formalists, as partisans of the actual infinite, claim the right to identify a mathematical object 'as a result of its implicit definition by a system of non-contradictory axioms', the intuitionists, on the contrary, maintain that 'to affirm the possibility of an unrealizable operation', for example, 'with regard to an object whose construction would require an infinite number of steps, or to a theorem that is impossible to check' because it relies on impredicative definitions,[39] 'is to affirm something which is either stripped of sense, or false, or at least undemonstratable'.[40]

Lautman's interpretation of the unity of mathematics distinguishes him from the constructivist perspective of his French intuitionist contemporaries (including Brouwer) because Lautman considered the actual infinite to be legitimate in its algebraic-axiomatic presentation. And, contrary to the intuitionists and constructivists, he grants to mathematical logic all the consideration which it deserves. That is, he accepts the logical principle of the excluded middle.[41] However, he maintains that 'logic is not *a priori* compared to mathematics, but that for logic one needs a mathematics to exist'.[42] He considered the simple idea that the logicists of the 'naive period' had made of 'an absolute and univocal anteriority of logic in relation to mathematics' to be 'out-of-date'.[43]

For Lautman, the philosophy of mathematics is not reducible to a secondary epistemological commentary on problematic logical foundations, nor to historical or *a fortiori* psycho-sociological research, nor to reflections on marginal movements such as intuitionism.[44] It is, however, precisely in the research of the critical period relating to the non-contradiction of arithmetic that Lautman considers a new theory of the mathematical real to have been affirmed. One that is 'as different from the logicism of the formalist as from the constructivism of the intuitionist'.[45] Lautman claims that between the naive and critical periods there is an 'internal evolution of logic', and he sets himself the task of disengaging from this new mathematical real 'a philosophy of mathematical genesis, whose range goes far beyond the field of logic'.[46]

While Hilbert's meta-mathematics proposes to examine mathematical theories from the point of view of the logical concepts of non-contradiction and completeness, Lautman notes that 'this is only an ideal towards which research is directed, and one knows at what point this ideal actually seems difficult to attain'.[47] This is an implicit reference to Gödel's second incompleteness theorem, which demonstrates that any non-contradictory formal system cannot demonstrate its completeness by way of its own axioms. Lautman concludes from this that 'Meta-mathematics can thus consider the idea of certain perfect structures, possibly realizable by effective mathematical theories, and this independently of the fact of knowing if there are theories enjoying the properties in question.'[48] What we have with the critical conception of the mathematical real is 'the statement of a logical problem without at all having the mathematical means of resolving it'.[49] What this means for Lautman is that the critical period marks the appearance of innovation in mathematics, not only at the level of results, but also at that of the problematic.[50] Lautman proposes to characterise the problematic 'distinction between the position of a logical problem and its mathematical solution'[51] by means of an 'exposé' of what he calls 'the metaphysics of logic'.[52] This takes the form of 'an introduction to a general theory of the connections which unite the structural considerations' of the critical axiomatic-structural conception with the 'affirmations of existence' of a particular dynamic conception.[53]

The particular dynamic conception of mathematics that Lautman deploys is further characterised when he qualifies his conception of the essential nature of mathematical truth as follows: 'Any logical attempt which would claim to dominate *a priori* the development of mathematics thus ignores the essential nature of mathematical truth, because this is related to the creative activity of the mind, and takes part in its temporal nature.'[54] Lautman is careful here to point out that mathematical truth is only partially related to the creative activity of the mind of the mathematician. In order to distinguish his account of dynamism from Brunschvicg's, Lautman considers it 'necessary to grasp, beyond the temporal circumstances of a discovery, the ideal reality which is solely capable of giving its sense and value to the mathematical experience'.[55] The lynchpin of this distinction is that Lautman conceives 'this ideal reality as independent of the activity of the mind'. For Lautman, the activity of the mind of the mathematician 'only intervenes . . . once it is a matter of creating effective mathematics', that is, effective mathematical theories.[56] This ideal reality is constituted by what he refers to as 'abstract Ideas'. Lautman proposes to call the relation between the independent activity of the mind of the mathematician and the ideas of this ideal reality 'dialectical', and he refers to these ideas as 'dialectical ideas'.[57] Lautman's principal thesis is that mathematics participates in a dialectic that governs (*domines*) it in an abstract way. He argues that the ideas 'which appear to govern the movement of certain mathematical theories', and which are conceivable as independent of mathematics, 'are not however susceptible of direct study'.[58] He goes on to claim that it is these dialectical ideas that 'confer on mathematics its eminent philosophical value'.[59] This is why Lautman considers mathematics, and especially 'modern mathematics' (and here Lautman is referring to the post-critical developments in algebra, group theory and topology), to tell, in addition to the constructions in which the mathematician is interested, 'another more hidden story [that is] made for the philosopher'.[60] The gist of the story is that there is a 'dialectical action [that] is constantly at play in the background and it is towards its clarification' that Lautman directs his research.[61] Lautman characterises this dialectical action as follows: 'Partial results, comparisons stopped midway, attempts which still resemble gropings, are organized under the unity of the same theme, and in their movement allow a connection to be seen which takes shape between certain abstract ideas, that we propose to call dialectical.'[62] Lautman argues that the nature of the mathematical real, and indeed the nature of physical reality, 'its structure and the conditions of its genesis are recognizable only by returning to the Ideas'.[63]

Lautman's speculative logic

This account of Ideas does commit Lautman to a version of Platonism. It is, however, a Platonism that is quite distinct from what is usually called 'Platonism' in mathematics, which consists rather in the practice of summarily indicating with the name 'Platonism' any mathematical philosophy for which the existence of a mathematical object is held as assured. Lautman considers this to be only one 'superficial understanding of Platonism'.[64] Nor does he 'understand by Ideas the models of which mathematical objects would only be copies'.[65] Lautman is here opposed to the Platonism traditionally founded on a certain realm of Ideas, which interprets mathematical theories as copies, reproductions, translations, or simple transpositions of eternal ideal models or Forms. Instead he wants to 'remove the idea of an irreducible distance between the "eidos" and its representation to affirm the productive power of ideas which are incarnated in the theories'.[66] What Lautman wants to do is restore to Ideas what he considers to be 'the true Platonic meaning of the term', that is, the understanding of these abstract dialectical ideas as 'the structural schemata according to which effective theories are organized'.[67]

Lautman characterises these structural schemata as establishing specific connections between contrary concepts such as: local–global; intrinsic–extrinsic; essence–existence; continuous–discontinuous; and finite–infinite. Lautman provides many examples of these contrary concepts, including the introduction of analysis into arithmetic, of topology into the theory of functions, and the effect of the penetration of the structural and finitist methods of algebra into the field of analysis and the debates about the continuum.[68]

The nature of mathematical reality for Lautman is therefore such that 'mathematical theories . . . give body to a dialectical *ideal*'.[69] This dialectic is constituted 'by couples of opposites' and the Ideas or structural schemata of this dialectic are presented in each case 'as the problem of establishing connections between opposing concepts'.[70] Lautman makes a firm distinction between concepts and dialectical Ideas: the Ideas 'consider possible relations between dialectical concepts',[71] or conceptual couples,[72] and 'these connections are only determined within the fields where the dialectic is incarnated'.[73] What Lautman is proposing is a speculative logic that considerably broadens the field and range of the meta-mathematics that he adopts from Hilbert. While meta-mathematics examines mathematical theories from the point of view of the concepts of non-contradiction and completeness, Lautman argues that there are 'other logical concepts, also likely to eventually be connected to one another within a mathematical theory'.[74] These other logical concepts are the conceptual couples of the structural schemata,[75] and Lautman argues that, 'contrary to the preceding cases (of non-contradiction and completeness)', each of which is bivalent, 'the mathematical solutions to the problems' which these conceptual couples pose can comprise 'an infinity of degrees'.[76]

So, for Lautman, Ideas constitute, along with mathematical facts, objects and theories, a fourth point of view of the mathematical real. 'Far from being opposed, these four conceptions are naturally integrated with one another: the facts consist in the discovery of new objects, these objects organize themselves in theories and the movement of these theories incarnates the schema of connections of certain Ideas.'[77] For this reason, the mathematical real depends not only on the factual base of mathematical facts but also on dialectical ideas that govern the mathematical theories in which they are actualised. Lautman thus reconsiders meta-mathematics in metaphysical terms, and postulates the metaphysical regulation of mathematics. However he is not suggesting the application of metaphysics to mathematics. Mathematical philosophy such as Lautman conceives it 'does not consist . . . in finding a logical problem of traditional metaphysics within a mathematical theory'.[78] Rather it is from the mathematical constitution of problems that it is necessary to turn to the metaphysical, that is to the dialectic, in order to give an account of the ideas which govern the mathematical theories. Lautman maintains that the philosophical meaning of mathematical thought appears in the incorporation of a metaphysics (or dialectic), of which mathematics is the necessary consequence. 'We would like to have shown', he argues, 'that this bringing together of metaphysics and mathematics is not contingent but necessary'.[79] Lautman doesn't consider this to be 'a diminution for mathematics, on the contrary it confers on it an exemplary role'.[80] Lautman's work can therefore be characterised as metaphysical, which, in the history of modern epistemology, characterises it as 'simultaneously original and solitary'.[81]

Problematic Ideas and the concept of genesis

A key point for Lautman is that dialectical ideas 'only exist insofar as [they are] incarnated mathematically'.[82] Lautman insists on this point. He argues that 'the reality inherent in mathematical theories comes to it from the fact that it takes part in an ideal reality which is governing of the mathematics, *but which is only recognizable through it*'.[83] This is what distinguishes Lautman's conception from 'a naive subjective idealism'.[84]

The dialectical Ideas are therefore characterised by Lautman as constituting a problematic.[85] He argues that 'while the mathematical relations describe connections existing in fact between distinct mathematical objects, the Ideas of dialectical relations are not affirmative of an existing connection between any concepts whatsoever'.[86] They constitute rather a problematic, that is, they are 'posed problems . . . relative to the connections that are [only] likely to be supported by certain dialectical concepts'. As such, they are characterised by Lautman as 'transcendent (in the usual meaning of the term) in relation to mathematics'.[87] The effective mathematical theories are constituted in an effort to bring a response to the problem posed by these connections, and Lautman interprets 'the overall structure of these theories in terms of the immanence of the logical schemata to the sought after solution'.[88] That is, the conceptual couples of the logical schemata '*are not anterior to their realization within a theory*'. They lack what Lautman calls 'the extra-mathematical intuition of the urgency of a logical problem'. The fundamental consequence is that the constitution of new logical schemata and problematic Ideas '*depend on the progress of mathematics itself*'.[89] Mathematical philosophy such as Lautman conceives it consists in 'apprehending the structure of [a mathematical] theory globally in order to extract the logical problem which is both defined and resolved by the very existence of this theory'.[90] 'There is thus an intimate link', for Lautman, 'between the transcendence of the Ideas and the immanence of the logical structure of the solution of a dialectical problem within mathematics.' It is in direct relation to this link that Lautman characterises 'the concept of genesis'[91] that he considers to be operative in the relation between the dialectic and mathematics. However, 'the order implied by the concept of genesis is not the order of the logical reconstruction of mathematics' as undertaken by the logicists. For the latter, the genetic definitions 'of a theory give rise to all the propositions of the theory'; whereas for Lautman, although the dialectic is anterior to mathematics, it 'does not form part of mathematics, and its concepts are without relationship to the primitive concepts of a theory'.[92] Nor is the genesis conceived in the Platonic sense as 'the material creation of the concrete starting from the Idea', but rather as what Lautman describes as the genesis 'of concepts relative to the concrete at the centre of an analysis of the idea'.[93] Lautman defines the 'anteriority of the dialectic' as that of the 'question' in relation to the 'response': 'it is of the nature of the response to be an answer to a question already posed . . . even if the idea of the question comes to mind only after having seen the answer'.[94]

The dialectic therefore functions by extracting logical problems from mathematical theories. The apprehension of the conceptual couple, that is, the logical schema of the problematic Idea, only comes after having extracted the logical problem from the mathematical theory. This is the basis for Lautman's understaning of the genesis of concepts from the concrete that is operating in the dialectic. And, it is the logical problem itself, rather than the problematic Idea, that directly drives the development of mathematics. The problematic idea governs the extraction process that deploys the logical problem in the further development of new mathematical theories. So for Lautman, 'the philosopher has neither to extract the laws, nor to envisage a future evolution, his role only consists in becoming aware of the logical drama which is played out within the theories'.[95] This effort on the part of the philosopher to 'adequately comprehend dialectical Ideas' is itself 'creative of the system of more concrete concepts where the connections between the [concepts] are defined'.[96] The only '*a priori* element' that is able to be conceived 'is given in the experience of the urgency of the problems', which preceeds not only 'the discovery of their solutions',[97] but also the extraction of the logical problem from the mathematical theory under scrutiny.

The virtual in Lautman

The method that Lautman uses in his mathematical philosophy is 'descriptive analysis'. The particular mathematical theories that he deploys throughout his work constitute for him 'a given' in which he endeavours 'to extract the ideal reality in which this material participates'.[98] That is, Lautman starts with mathematical theories that are already in circulation. For example, he incorporates all the new work in algebraic topology of the German mathematicians Alexandroff, Hopf and Weyl, and connects it to the work of Elie Cartan in complex analysis and to that of André Weil in what was then the emerging field of algebraic geometry.[99] He is also one of the first to anticipate the philosophical interest in algebraic topology, a branch of mathematics that was then under full development. In relation to these mathematical theories Lautman argues that while

> it is necessary that mathematics exists as an example where the ideal structures of the dialectic can be realised, it is not necessary that the examples which correspond to a particular dialectical structure are of a particular kind; what generally happens on the contrary is that the organizing power of the same structure is affirmed in different theories; they present affinities of mathematical structure which testify to the common dialectical structure in which they take part.[100]

One of the examples developed by Lautman is the operation of the local–global conceptual couple in the theory of the approximate representation of functions.[101] The same conceptual couple is illustrated in geometry.[102] Distinct mathematical theories can therefore be structured by the same conceptual couple.[103] Lautman sees in the local–global conceptual couple the source of a dialectical movement in mathematics that produces new theories. He argues that 'one can grasp closely the mechanism of this operation where the analysis of Ideas is produced in effective creation, where the virtual is transformed into reality'.[104] In the case of the example of the local–global conceptual couple, the new mathematical theory that was effectively created was Poincaré's qualitative theory of differential equations, or the theory of automorphic functions.[105]

According to Lautman, the problematic nature of the connections between conceptual couples 'can arise apart from any mathematics, but the effectuation of these connections is immediately mathematical theory'.[106] As a consequence, he maintains that 'Mathematics thus plays with respect to the other domains of incarnation, physical reality, social reality, human reality, the role of model where the way that things come into existence is observed.'[107] This is an important point for Deleuze, one which shapes his strategy of engagement with a range of discourses throughout his work. Lautman's final word on mathematical logic is that it 'does not enjoy in this respect any special privilege; it is only one theory among others and the problems which it raises or which it solves are found almost identically elsewhere'.[108] Lautman claims that 'for the mathematician, it is in the choice of original definitions and judicious axioms that true invention resides. It is by the introduction of new concepts, much more than by transformations of symbols or blind handling of algorithms, that mathematics has progressed and will progress.'[109]

Deleuze and the calculus of problems

At the time, opinion amongst mathematicians and philosophers was largely unfavourable to Lautman. Mathematicians were at odds with what was for them his incomprehensible 'philosophical speculation' and its 'subtleties'.[110] While the philosophers reproached him for what they considered to be a certain inaccuracy in his use of the term 'dialectical':[111] was it Socratic, Kantian or Hegelian?[112] It was another 30 years before an adequate account of the dialectic proposed by Lautman was able to be given. This was offered by Deleuze in his major work *Difference and Repetition*. Despite Deleuze's work, the confusion over the nature of the dialectic in Lautman remains pretty much intact, with quite recent commentators such as Jean Petitot – a French mathematician and philosopher of mathematics who, contraray to Lautman's peers, considers Lautman to be one of the most inspiring philosophers of the twentieth century[113] – suggesting that the dialectic proposed by Lautman is a Hegelian one.[114] It is only in recent work on Deleuze's engagement with mathematics that the significance of Lautman to the development of Deleuze's philosophy, and of Deleuze to the recent reception of Lautman's work, is being recognised.[115] Even Petitot proclaims that 'with Ferdinand Gonseth and very recently Jean Largeault, Gilles Deleuze is one of the (too) rare philosophers to have recognised the importance of Lautman'.[116] Jean-Michel Salanskis acknowledges that it was Deleuze's *Difference and Repetition* that led him to read Lautman's work and to appreciate its significance to the subsequent developments in mathematics, in particular to the Bourbaki project.[117] And both Petitot and Salanskis draw attention to the 'visionary and profound character of Deleuze's presentation of the notion of structural multiplicity'[118] in *Difference and Repetition* (DR 182–4).

It is in the chapter of *Difference and Repetition* entitled 'Ideas and the Synthesis of Difference' that Deleuze mobilises mathematics to develop a 'calculus of problems' (TP 570 n. 61)[119] based on Lautman's work.

> Following Lautman's general theses, a problem has three aspects: its difference in kind from solutions, its transcendence in relation to the solutions that it engenders on the basis of its own determinant conditions; and its immanence in the solutions which cover it, the problem being the better resolved the more it is determined. Thus the ideal connections constitutive of the problematic (dialectical) Idea are incarnated in the real solutions which are constituted by mathematical theories and carried over into problems in the form of solutions. (DR 178–9)

Deleuze explicates this process by referring to the operation of certain conceptual couples in the field of contemporary mathematics: most notably the continuous and the discontinuous, the infinite and the finite, and the global and the local. The two mathematical theories Deleuze draws upon for this purpose are the differential calculus and the theory of dynamical systems, and Galois' theory of polynomial equations. For the purposes of this chapter I will only treat the first of these,[120] which is based on the idea that the singularities of vector fields determine the local trajectories of solution curves, or their 'topological behaviour'.[121] These singularities can be described in terms of a given mathematical problematic – for example, how to solve two divergent series in the same field – and in terms of the solutions, as the trajectories of the solution curves to the problem. What actually counts as a solution to a problem is determined by the specific characteristics of the problem itself, typically by the singularites of this problem and the way in which they are distributed in a system.[122] Deleuze understands the differential calculus essentially as a 'calculus of problems', and the theory of dynamical systems as the qualitative and topological theory of problems, which, when connected together, are determinative of the complex logic of differen*t*/*c*iation. (DR 209).[123] Deleuze develops the concept of a problematic idea from the differential calculus, and following Lautman considers the concept of genesis in mathematics to 'play the role of model . . . with respect to all other domains of incarnation'.[124] While Lautman explicated the philosophical logic of the actualisation of ideas within the framework of mathematics, Deleuze (along with Guattari) follows Lautman's suggestion and explicates the operation of this logic within the framework of a multiplicity of domains, including, for example, philosophy, science and art in *What is Philosophy?*, and the variety of domains which characterise the plateaus in *A Thousand Plateaus*. While for Lautman a mathematical problem is resolved by the development of a new mathematical theory, for Deleuze, it is the construction of a concept that offers a solution to a philosophical problem; even if this newly constructed concept is characteristic of, or modelled on, the new mathematical theory.

One of the differences between Lautman and Deleuze is that while Lautman locates the ideas in a specifically Platonic and idealist perspective, the ideas that Deleuze refers to are rather more Kantian than Platonic[125], and Lautman's idealism is displaced in Deleuze's work by an understanding of the Lautmanian idea as 'purely' problematic. There is no ideal reality associated with ideas in Deleuze but rather ideas are constituted by the purely problematic relation between conceptual couples. Deleuze defines the 'Idea' as 'a structure. A structure or an Idea is . . . a system of multiple, non-localisable connections between differential elements which is incarnated in real relations and actual terms' (DR 183). For Deleuze, it is the problematic nature of the relations between conceptual couples that incarnate problematic ideas and which govern the kinds of solutions that can be offered to them.

What Deleuze specifically draws from Lautman is a relational logic that designates a process of production, or genesis, which has the value of introducing a general theory of relations that unites the structural considerations of the differential calculus to the concept of 'the generation of quantities' (DR 175). The process of the genesis of mathematical theories that are offered as solutions to mathematical problems corresponds to the Deleuzian account of the construction of concepts as solutions to philosophical problems.

The mathematical problematics that Deleuze extracts from the history of mathematics, following Lautman's lead, are directly redeployed by Deleuze as philosophical problematics in relation to the history of philosophy. This is achieved by mapping the alternative lineages in the history of mathematics onto corresponding alternative lineages in the history of philosophy, that is, by isolating those points of convergence between the mathematical and philosophical problematics extracted from their respective histories. The redeployment of mathematical problematics as philosophical problematics is one of the strategies Deleuze employs in his engagement with the history of philosophy. Deleuze actually extracts philosophical problematics from the history of philosophy and then redeploys them either in relation to one another, or in relation to mathematical problematics, or in relation to problematics extracted from other discourses, to create new concepts, which Deleuze and Guattari consider to be the task of philosophy (WP 5).

Deleuze is therefore very much interested in particular kinds of mathematical problematics that can be extracted from the history of mathematics, and in the relationship that these problematics have to the discourse of philosophy. He can therefore be understood to redeploy not only the actual mathematical problematics that are extracted from the history of mathematics in relation to the history of philosophy, he also redeploys the logic of the generation of mathematical problematics, that is, the calculus of problems, in relation to the history of philosophy, in order to generate the philosophical problematics which are then redeployed in his project of constructing a philosophy of difference. It is in relation to the history of philosophy that Deleuze then determines the logic of the generation of philosophical problematics as the speculative logic characteristic of a philosophy of difference.

The speculative logic characteristic of a philosophy of difference

This speculative logic, the logic of the calculus of problems, is determined in relation to the discipline of mathematics and the mathematical problematics extracted from it. It is not simply a logic characteristic of the relation between the history of mathematics and its related mathematical problematics, or between axiomatics and problematics,[126] or between what Deleuze and Guattari characterise as Royal science and nomad science. It is rather a logic of the generation of each mathematical problematic itself, or of nomad science itself. Deleuze writes that:

> It is sufficient to understand that the genesis takes place in time not between one actual term, however small, and another actual term, but between the virtual and its actualization – in other words, it goes from the structure to its incarnation, from the conditions of a problem to the cases of solution, from the differential elements and their ideal connections to actual terms and diverse real relations which constitute at each moment the actuality of time. This is a genesis without dynamism. (DR 183)

It is this logic that Deleuze redeploys in relation to the history of philosophy as a logic of differen*t/c*iation in order to generate the philosophical problematics that he then uses to construct a philosophy of difference.

Lautman refers to this whole process as 'the metaphysics of logic',[127] and, in *Difference and Repetition*, Deleuze formulates a 'metaphysics of logic' that corresponds to the local point of view of the differential calculus. He endorses Lautman's broader project when he argues that 'we should speak of a dialectics of the calculus rather than a metaphysics' (DR 178), since, he continues, 'each engendered domain, in which dialectical Ideas of this or that order are incarnated, possesses its own calculus. . . . It is not mathematics which is applied to other domains but the dialectic which establishes . . . the direct differential calculus corresponding or appropriate to the domain under consideration' (DR 181). It is not the particular method of the differential calculus which is applied to the dialectical logic to support its development, but rather the dialectical logic which determines the direct differential calculus which corresponds or is appropriate to its own development.

There is therefore a convergence between the logic of the local point of view of the differential calculus and the logic of the theory of relations that is characteristic of Deleuze's philosophy of difference. The manner by means of which an idea is implicated in the mathematical theory which determines it, converges with, or serves as a function or mathematical model of, the manner by means of which a philosophical concept is implicated in the philosophical problematic which determines it. There are 'correspondences without resemblance' (DR 184) between them, insofar as both are determined according to the same speculative logic, that is, according to the logic of differen*t*/*c*iation. The philosophical implications of this convergence are developed by Deleuze in *Expressionism in Philosophy* in relation to his reading of Spinoza's theory of relations in the *Ethics*,[128] and in *Cinema 1* and *Cinema 2* in relation to his understanding of Bergson's intention 'to give multiplicities the metaphysics which their scientific treatment demands' (B 112).

The problematic Ideas that 'it is possible to recover within mathematical theories', and that are 'incarnated in the same movement of these theories',[129] are characterised by the relations between the conceptual couples. These Ideas, which are recast by Deleuze as philosophical concepts, are used to develop the logical schema of a theory of relations characteristic of a philosophy of difference. It is in the development of this project that Deleuze specifically draws upon Lautman's work to deploy a speculative logic that, in *Difference and Repetition*, is determined in relation to the history of the differential calculus as the logic of differen*t*/*c*iation; in *Expressionism in*

Philosophy is determined in relation to Spinoza's theory of relations as the logic of expression; and in the *Cinema* books, is determined in relation to the work of Bergson as a logic of multiplicities.

Lautman outlined a 'critical' programme in mathematics that was intended to displace the previous foundational discussions that were occupied with the criticism of classical analysis. Against the logicist claim that the development of mathematics is dominated *a priori* by logic, Lautman proposes a 'metaphysics of logic', and calls for the development of a 'philosophy of mathematical genesis'. Deleuze responds to this call. His Lautmanian preoccupation with mathematics is primarily focused on locating what Lautman characterises as 'logical Ideas', which are recast by Deleuze as philosophical concepts to develop the logical schema of a theory of relations characteristic of a philosophy of difference. Lautman's work on mathematics provides the blueprint for adequately determining the nature not only of Deleuze's engagement with mathematics, but also of Deleuze's metaphysics, the metaphysics of his speculative logic.

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Notes

1. Albert Lautman, *Essai sur l'unité des mathématiques et divers écrits* (Paris: Union générale d'éditions, 1977), p. 23.
2. Lautman, *Essai sur l'unité*, p. 24.
3. See Jean Dieudonné in Lautman, *Essai sur l'unité*, p. 15.
4. Jean Petitot, 'La dialectique de la vérité objective et de la valeur historique dans le rationalisme mathématique d'Albert Lautman', in *Sciences et Philosophie en France et en Italie entre les deux guerres*, edited by J. Petitot and L. Scarantino (Napoli: Vivarium, 2001), p. 83.
5. Albert Lautman, *Essai sur les notions de structure et d'existence en mathématiques. I. Les Schémas de structure. II. Les Schémas de genèse.* (Paris: Hermann, 1938); Albert Lautman, *Essai sur l'unité des sciences mathématiques dans leur développement actuel* (Paris: Hermann, 1938).
6. Dieudonné in Lautman, *Essai sur l'unité*, p. 16.
7. Dieudonné in Lautman, *Essai sur l'unité*, p. 16.
8. Dieudonné in Lautman, *Essai sur l'unité*, p. 16.
9. Lautman, *Essai sur l'unité*, p. 145.
10. Maurice Loi, 'Foreword', in Lautman, *Essai sur l'unité*, p. 13.
11. The axiomatic method is a way of developing mathematical theories by postulating certain primitive assumptions, or axioms, as the basis of the

theory, while the remaining propositions of the theory are obtained as logical consequences of these axioms.

12. Lautman, *Essai sur l'unité*, p. 146.
13. Lautman, *Essai sur l'unité*, p. 147.
14. Lautman, *Essai sur l'unité*, p. 25.
15. Lautman, *Essai sur l'unité*, p. 25.
16. Lautman, *Essai sur l'unité*, p. 136.
17. The Bourbaki project explicitly espoused a set-theoretic version of mathematical structuralism.
18. Dieudonné in Lautman, *Essai sur l'unité*, pp. 15–20.
19. According to mathematical structuralism, mathematical objects are defined by their positions in mathematical structures, and the subject matter that mathematics concerns itself with are structural relationships in abstraction from the intrinsic nature of the related objects. See Geoffrey Hellman, 'Structuralism', in Stewart Shapiro (ed.), *The Oxford Handbook of Philosophy of Mathematics and Logic* (Oxford: Oxford University Press, 2005), p. 256.
20. Dieudonné in Lautman, *Essai sur l'unité*, p. 16.
21. Lautman, *Essai sur l'unité*, p. 24.
22. Lautman, *Essai sur l'unité*, p. 38.
23. Lautman, *Essai sur l'unité*, p. 23. The logicist thesis was that the basic concepts of mathematics are definable by means of logical notions, and the key axioms of mathematics are deducible from logical principles alone.
24. The main aim of Hilbert's programme, which was first clearly formulated in 1922, was to establish the logical acceptability of the principles and modes of inference of modern mathematics by formalising each mathematical theory into a finite, complete set of axioms, and to provide a proof that these axioms were consistent. The point of Hilbert's approach was to make mathematical theories fully precise, so that it is possible to obtain precise results about properties of the theory. In 1931 Gödel showed that the programme as it stood was not possible. Revised efforts have since emerged as continuations of the programme that concentrate on relative results in relation to specific mathematical theories, rather than all mathematics. See José Ferreirós, 'The Crisis in the Foundations of Mathematics', in *The Princeton Companion to Mathematics*, edited by Timothy Gowers, June Barrow-Green and Imre Leader (Princeton: Princeton University Press, 2008), Ch. 2.6.3.2.
25. Lautman, *Essai sur l'unité*, p. 282.
26. See Jean Largeault, *Logique mathématique. Textes* (Paris: Armand Colin, 1972), pp. 215, 264.
27. Lautman, *Essai sur l'unité*, p. 9.
28. Petitot, 'La dialectique de la vérité', p. 98. The term 'meta-mathematics' is introduced by Hilbert in 'Uber das Unendliche', *Mathematische Annalen* 95 (1926), pp. 161–90.

29. Lautman, *Essai sur l'unité*, p. 26.
30. Lautman, *Essai sur l'unité*, p. 26.
31. Lautman, *Essai sur l'unité*, p. 26.
32. David Hilbert, *Gesammelte Abhandlungen* (New York: Chelsea Pub. Co., 1965), p. 180. Cited in Lautman, *Essai sur l'unité*, p. 30.
33. Lautman, *Essai sur l'unité*, p. 27.
34. Lautman, *Essai sur l'unité*, p. 25.
35. Lautman, *Essai sur l'unité*, p. 25. See Léon Brunschvicg, *Les Étapes de la philosophie mathématique* (Paris: A. Blanchard, 1993).
36. Lautman, *Essai sur l'unité*, p. 25.
37. Lautman, *Essai sur l'unité*, p. 25.
38. Lautman, *Essai sur l'unité*, p. 87.
39. A mathematical definition is impredicative if it depends on a certain set, **N**, being defined and introduced by appeal to a totality of sets which includes **N** itself. That is, the definition is self-referencing.
40. Lautman, *Essai sur l'unité*, p. 88.
41. The law of the excluded middle states that every proposition is either true or false. In propositional logic, the law is written 'P V ¬P' ('P or not-P').
42. Lautman, *Essai sur l'unité*, p. 48.
43. Loi in Lautman, *Essai sur l'unité*, p. 13.
44. Petitot, 'La dialectique de la vérité', p. 81.
45. Lautman, *Essai sur l'unité*, p. 89.
46. Lautman, *Essai sur l'unité*, p. 89.
47. Lautman, *Essai sur l'unité*, p. 27.
48. Lautman, *Essai sur l'unité*, p. 28.
49. Lautman, *Essai sur l'unité*, p. 28.
50. Lautman, *Essai sur l'unité*, p. 211.
51. Lautman, *Essai sur l'unité*, p. 28.
52. Lautman, *Essai sur l'unité*, p. 87.
53. Lautman, *Essai sur l'unité*, p. 87.
54. Lautman, *Essai sur l'unité*, p. 140.
55. Albert Lautman, *Nouvelles recherches sur la structure dialectique des mathématiques* (Actualités scientifiques et industrielles. Paris: Hermann, 1939), p. 630.
56. Lautman, *Nouvelles recherches*, p. 630.
57. Lautman, *Essai sur l'unité*, p. 28.
58. Lautman, *Essai sur l'unité*, p. 29.
59. Lautman, *Essai sur l'unité*, p. 29.
60. Lautman, *Essai sur l'unité*, p. 28.
61. Lautman, *Essai sur l'unité*, p. 28.
62. Lautman, *Essai sur l'unité*, p. 28.
63. Lautman, *Essai sur l'unité*, p. 147.
64. Lautman, *Essai sur l'unité*, p. 143.

65. Lautman, *Essai sur l'unité*, p. 204.
66. See Catherine Chevalley, 'Albert Lautman et le souci logique', *Revue d'Histoire des Sciences* 40:1 (1987), p. 61.
67. Lautman, *Essai sur l'unité*, p. 204. See also pp. 143–4, 302–4; Emmanuel Barot, 'L'objectivité mathématique selon Albert Lautman: entre Idées dialectiques et réalité physique'. *Cahiers François Viète* 6 (2003), p. 7 n. 2.
68. See Chevalley, 'Albert Lautman et le souci logique', p. 60.
69. Lautman, *Essai sur l'unité*, p. 253.
70. Lautman, *Essai sur l'unité*, p. 253.
71. Lautman, *Essai sur l'unité*, p. 210.
72. Which are also referred to and operate as 'dualities'. See Charles Alluni, 'Continental Genealogies: Mathematical Confrontations in Albert Lautman and Gaston Bachelard', in *Virtual Mathematics: The Logic of Difference*, edited by S. Duffy (Manchester: Clinamen Press, 2006), p. 78.
73. Lautman, *Essai sur l'unité*, p. 253.
74. Lautman, *Essai sur l'unité*, p. 28.
75. Which he therefore also refers to as 'logical schemata'. See Lautman, *Essai sur l'unité*, p. 142.
76. Lautman, *Essai sur l'unité*, p. 28.
77. Lautman, *Essai sur l'unité*, p. 135.
78. Lautman, *Essai sur l'unité*, p. 142.
79. Lautman, *Essai sur l'unité*, p. 203.
80. Lautman, *Essai sur l'unité*, p. 10. From Lautman's correspondence with Fréchet dated 1 February 1939.
81. See Chevalley, 'Albert Lautman et le souci logique', p. 50.
82. Lautman, *Essai sur l'unité*, p. 203.
83. Lautman, *Essai sur l'unité*, p. 290.
84. Petitot, 'La dialectique de la vérité', p. 86.
85. Lautman, *Essai sur l'unité*, p. 211.
86. Lautman, *Essai sur l'unité*, p. 210.
87. Lautman, *Essai sur l'unité*, p. 212.
88. Lautman, *Essai sur l'unité*, p. 212.
89. Lautman, *Essai sur l'unité*, p. 142.
90. Lautman, *Essai sur l'unité*, p. 143.
91. Lautman, *Essai sur l'unité*, p. 212.
92. Lautman, *Essai sur l'unité*, p. 210.
93. Lautman, *Essai sur l'unité*, p. 205.
94. Lautman, *Essai sur l'unité*, p. 210.
95. Lautman, *Essai sur l'unité*, p. 142.
96. Lautman, *Essai sur l'unité*, p. 205.
97. Lautman, *Essai sur l'unité*, p. 142.
98. Lautman, *Essai sur l'unité*, p. 40.

99. Barot, 'L'objectivité mathématique selon Albert Lautman', p. 22.
100. Lautman, *Essai sur l'unité*, p. 213.
101. Lautman, *Essai sur l'unité*, pp. 32, 45–7. The 'global conception of the analytic function that one finds with Cauchy and Riemann' (p. 32) is posed as a conceptual couple in relation to Weierstrass' approximation theorem, which is a local method of determining an analytic function in the neighbourhood of a complex point by a power series expansion, which, by a series of local operations, converges around this point (pp. 45–7).
102. The same conceptual couple is illustrated in geometry by the connections between 'topological surface properties and their local differential properties', that is, between the curvature of the former and the determination of second derivatives of the latter, both in the 'metric formulation' of geoemtry in the work of Hopf (Lautman, *Essai sur l'unité*, pp. 40–3) and 'in its topological formulation' in Weyl and Cartan's theory of closed groups (pp. 43–4).
103. See Barot, 'L'objectivité mathématique selon Albert Lautman', p. 10; Chevalley, 'Albert Lautman et le souci logique', pp. 63–4.
104. Lautman, *Essai sur l'unité*, p. 209.
105. For an account of the role that this example of the local–global conceptual couple plays in Deleuze see Simon Duffy, 'The Mathematics of Deleuze's Differential Logic and Metaphysics', in Duffy (ed.), *Virtual Mathematics: The Logic of Difference.*
106. Lautman, *Essai sur l'unité*, p. 288.
107. Lautman, *Essai sur l'unité*, p. 209.
108. Lautman, *Essai sur l'unité*, p. 288.
109. Loi in Lautman, *Essai sur l'unité,* p. 12.
110. Petitot, 'La dialectique de la vérité', p. 99.
111. Lautman, *Essai sur l'unité*, p. 22.
112. Petitot, 'La dialectique de la vérité', p. 113.
113. Petitot, 'La dialectique de la vérité', p. 80.
114. Petitot, 'La dialectique de la vérité', p. 113. See also Barot, 'L'objectivité mathématique selon Albert Lautman', pp. 6, 16 n. 1. For a Deleuzian account of an alternative speculative logic to the Hegelian dialectical logic, one that implicates the work of Lautman, see Simon Duffy, *The Logic of Expression: Quality, Quantity and Intensity in Spinoza, Hegel and Deleuze* (Aldershot: Ashgate, 2006), pp. 74–91, 254–60.
115. Jean-Michel Salanskis, 'Idea and Destination', in *Deleuze: A Critical Reader*, edited by P. Patton (Cambridge: Blackwell, 1996); Salanskis, 'Pour une épistémologie de la lecture', *Alliage* 35–6 (1998) <http://www.tribunes.com/tribune/alliage/accueil.htm>; Daniel W. Smith, 'Mathematics and the Theory of Multiplicities: Deleuze and Badiou Revisited', *Southern Journal of Philosophy* 41:3 (2003), pp. 411–49;

Duffy, 'The Mathematics of Deleuze's Differential Logic and Metaphysics'.

116. Petitot, 'La dialectique de la vérité', p. 87 n. 14.
117. See Salanskis, 'Pour une épistémologie de la lecture' (in particular the section entitled 'Contre-temoinage').
118. Salanskis, 'Idea and Destination', p. 64.
119. When Deleuze and Guattari comment on 'the "intuitionist" school (Brouwer, Heyting, Griss, Bouligand, etc.),' they insist that it 'is of great importance in mathematics, not because it asserted the irreducible rights of intuition, or even because it elaborated a very novel constructivism, but because it developed a conception of problems, and of a calculus of problems that intrinsically rivals axiomatics and proceeds by other rules (notably with regard to the excluded middle)' (TP 570 n. 61). Deleuze extracts this concept of the calculus of problems itself as a mathematical problematic from the episode in the history of mathematics when intuitionism opposed axiomatics. It is the logic of this calculus of problems that he then redeploys in relation to a range of episodes in the history of mathematics that in no way binds him to the principles of intuitionism. See Duffy, 'Deleuze and Mathematics', in Duffy (ed.), *Virtual Mathematics: The Logic of Difference*, pp. 2–6.
120. For a brief account of Deleuze's enagement with Galois see Gilles Châtelet, 'Interlacing the Singularity, the Diagram and the Metaphor', trans. S. Duffy, in Duffy (ed.), *Virtual Mathematics: The Logic of Difference*, p. 41; Salanskis, 'Mathematics, Metaphysics, Philosophy', in *Virtual Mathematics* pp. 52–3; Salanskis, 'Pour une épistémologie de la lecture'; Daniel W. Smith, 'Axiomatics and Problematics as Two Modes of Formalisation: Deleuze's Epistemology of Mathematics', in Duffy (ed.), *Virtual Mathematics: The Logic of Difference*, pp. 159–63.
121. See Salanskis, 'Pour une épistémologie de la lecture'.
122. See Salanskis, 'Pour une épistémologie de la lecture'.
123. See Duffy, *The Logic of Expression*, where the complex concept of the logic of differen*t*/*c*iation is demonstrated to be characteristic of Deleuze's 'philosophy of difference'.
124. Lautman, *Essai sur l'unité*, p. 209.
125. For a critical account of Lautman's engagement with Kant see Petitot, 'La dialectique de la vérité'. See also Salanskis, 'Idea and Destination', for an account of the significance of Kant for Deleuze's engagement with Lautman. See Nathan Widder, 'The rights of Simulacra: Deleuze and the Univocity of Being', *Continental Philosophy Review* 34 (2001), pp. 437–53 for an account of Deleuze's reversal of Platonism and its implied idealism.

126. See Smith, 'Axiomatics and Problematics', for an account of the operation of the relation between Royal and nomad science and between axiomatics and problematics in Deleuze's work.
127. Lautman, *Essai sur l'unité*, p. 87.
128. See Duffy, 'Schizo-Math', *Angelaki* 9: 3, 2004, pp. 199–215 and 'The Mathematics of Deleuze's Differential Logic and Metaphysics'.
129. Lautman, *Essai sur l'unité des mathématiques,* p. 195.