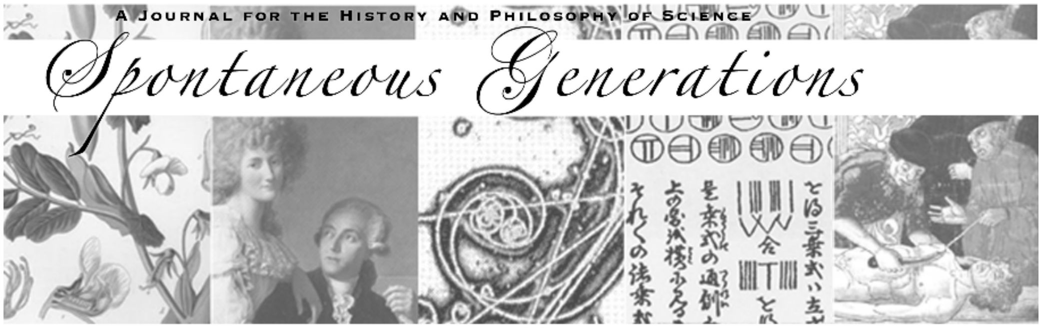

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On the Intrinsically Ambiguous Nature of Space-Time Diagrams*

Elie During†

When the German mathematician Hermann Minkowski first introduced the space-time diagrams that came to be associated with his name, the idea of picturing motion by geometric means, holding time as a fourth dimension of space, was hardly new. But the pictorial device invented by Minkowski was tailor-made for a peculiar variety of space-time: the one imposed by the kinematics of Einstein's special theory of relativity, with its unified, non-Euclidean underlying geometric structure. By plotting two or more reference frames in relative motion on the same picture, Minkowski managed to exhibit the geometric basis of such relativistic phenomena as time dilation, length contraction or the dislocation of simultaneity. These disconcerting effects were shown to result from arbitrary projections within four-dimensional space-time. In that respect, Minkowski *diagrams* are fundamentally different from ordinary space-time *graphs*. The best way to understand their specificity is to realize how productively *ambiguous* they are.

I. Relativity and geometry

The primary motivation behind Albert Einstein's first theory of relativity—known as the "special theory of relativity"—was to work out a kinematical framework common to mechanics and electromagnetism. The trick was to make sense of the troublesome fact that the speed of light was constant in empty space without giving up the principle of relativity inherited from Galilean physics, i.e. the equivalence of all kinematical perspectives attached to reference frames in uniform motion relative to each other ("inertial frames"). It is

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now well-established that Henri Poincaré had hit upon the geometric structure underlying such a framework some time before Einstein's revolutionary 1905 paper, although, for reasons that need not concern us here, he did not bother to figure out all its physical implications. The coordinate transformations ("Lorentz transformations") induced in space and time measurements by the mere fact of moving from one inertial frame (or group of inertially co-moving observers) to another could be shown to result directly from postulating an invariant, finite and therefore maximum velocity in every frame. One only had to examine the form of the equations to realize that the algebraic structure exhibited by the coordinate transformations admitted a straightforward geometric interpretation in four-dimensional space. Hermann Minkowski, a German mathematician of the Göttingen school, popularized this idea in his famous 1908 Köln lecture by showing that space-time transformations could be assimilated to rotations in a four-dimensional pseudo-Euclidean space with a special metric signature (Galison 1979; Walter 1999a). The essence of relativity theory, he claimed, revealed itself to geometric intuition in four-dimensional space, with time playing the role of a fourth, imaginary dimension. "Space by itself and time by itself are doomed to fade away into mere shadows," Minkowski wrote rather pompously, "and only a kind of union of the two will preserve an independent reality" (1952, 75).

II. "Space-time"

"Space-time" (*Raumzeit*) was the name of the resulting geometrical structure. Minkowski presented this four-dimensional manifold as a new absolute, able to yield direct insight about the underlying substance of the physical universe ("the absolute world"). However, for all the buzz surrounding its birth, this glorious but inscrutable entity could only be properly introduced by the means of more modest pictorial representations involving points, lines, and basic geometrical figures. These representations soon came to be known as "Minkowski diagrams," although Minkowski himself—who died shortly after, in 1909—never used the word "diagram." For reasons which will soon become apparent, these diagrams may well be his most lasting contribution to the philosophical understanding of relativity, more important in this regard than his controversial appeal to substantival space-time.

The idea of plotting time against space in order to represent motion was hardly a revelation when Minkowski first introduced the diagrams that came to be associated with his name. As early as 1698, Pierre Varignon recommended that spatial and temporal intervals be combined into a single picture, provided that space and time be treated as "homogeneous magnitudes" (quoted in Stachel 2006, 19). This proviso, as one suspects, encapsulates the main philosophical difficulty behind the very idea of space-time. But the habit of treating time as the fourth dimension of space is by no means peculiar to relativistic space-time.

In an article of the *Encyclopédie* ("Dimension," published in 1754), d'Alembert alludes to this extra dimension. Lagrange's *Mécanique analytique* (1788) also relies heavily on the possibility of viewing motion in terms of static geometric figures in *n*-dimensional space. Kant, in the *First Principles of Natural Science* (1786), devotes a lengthy footnote to what appears as a *schematismus* of spatio-temporal becoming, involving a layering of snapshots of space—maximal extensions of simultaneous events—along the time dimension. As for the vocable of "space-time," it pre-dates the introduction of Minkowski diagrams within the context of relativity physics. The German expression "*Raumzeit*" was coined by Novalis in his *Allgemeine Brouillon* (1798-1799); Maurice Boucher's 1903 popular essay on "hyperspace" and n-dimensional geometry refers to "*Espace-Temps.*" This being said, the radical novelty of relativistic space-time may be reason enough to credit Minkowski with the invention of space-time as we know it—that is, the truly interesting concept of space-time. For what sets Minowski's space-time apart from earlier spatio-temporal constructions is that space and time coordinates appear to be genuinely fused together rather than being merely juxtaposed or combined in the form of the Cartesian product of three dimensions of space and a fourth, temporal dimension. The very possibility of identifying Lorentz transformations—the coordinate transformations required when switching reference frames—with rotations in four-dimensional space attests to the unifying power of the deep geometrical structure of space-time. Coordinate transformations in space-time have a group structure—the Poincaré-Lorentz group. The associated notion of orthogonality and pseudo-Euclidean metric open the way for a genuine *chronogeometry*: not a geometry of space-plus-time but an integrated geometry of space-time. Therein lies the mathematical rationale behind the puzzling idea of a "union" or "fusion" of space and time.

As a result of this new entanglement between spatial and temporal concepts, there is no unique way to slice space-time into instantaneous spaces of simultaneous events: each inertial frame of reference has its own way of describing the unfolding of things across time. In other words, any space-plus-time rendition of becoming turns out to be a particular projection of four-dimensional configurations into their spatial and temporal components. As Minkowski (1909, 7) wrote: "projection in space and in time may still be undertaken with a certain degree of freedom." But it is no more than that: an arbitrary projection of invariant space-time configurations according to a particular perspective. This new situation seems to have eluded Henri Bergson, who viewed relativistic space-time as just another instance of the age-old tendency to "spatialize" time by treating becoming as something extended at once through time as well as space. In *Duration and Simultaneity*, his essay on Einstein's theory, he misleadingly claims that Minkowski's geometric representation of relativistic transformations only generalizes the "cinematographical" analysis of motion as a juxtaposition of instantaneous slices along a temporal dimension (Bergson 2009, chap. 6), when in fact such a representation suggests the opposite, namely that any rendering of this kind could, at best, have relative value. Upon reflection, the image of a hyper-cinematograph embedding an indefinite number of possible projections (as many as there are reference frames) amounts to an extenuation of the cinematographical method, rather than its generalization. As for the suggestion that becoming would be "frozen" by the mere fact of being represented by graphical means, it is rather misguided. On closer inspection, it seems to draw much of its appeal from an interpretation of diagrammatic representation that tends to overemphasize its referential or iconic features. But, as Quine and others have argued, the space-time view, while captured in a static picture, does not subtract anything from change: "Change is still there, with all its fresh surprises. It is merely incorporated" (Quine 1987, 197). A more pragmatic account of space-time should remove further doubts. If we believe that space-time is strictly unobservable when considered apart from spatio-temporal happenings (Harré 1991, 55) and that its apparent ontological weight is in fact inseparable from the variety of representational models and diagrams which abide by its rule, then the possibility of picturing dynamic evolutions as geometric patterns laid out in four-dimensional space appears as no more than a built-in feature or formal condition of our mode of representation. This acknowledgement need not raise any metaphysical worry regarding the disappearance of time or the negation of becoming.

III. How diagrams work

So much for space-time. What about diagrams? Einstein humorously said that since mathematicians had taken up relativity, he could not understand it anymore. But there is no doubt that visual representations of the kind provided by Minkowski are of invaluable help in reaching a proper mathematical understanding of the fundamental physical situation at the core of relativity theory. My contention is that, besides the obvious iconic efficiency of the graphical depiction of motion, the main function of these toy-models is to organize symbolic cues leading to a visual grasp of the inner consistency, not of the theory as a whole, but of its kinematical framework, the implications of which can only appear as counter-intuitive and even baffling to the untrained mind. The fact that introductory textbooks still commonly resort to such pictorial devices is a proof of their ongoing pedagogical relevance. As early as 1914, Ludwik Silberstein recommended Minkowski's "graphic representations" as "very advantageous, especially for the trained geometer of our days" (130-31). Granted, some authors still do not wish to consider them as anything more than visual aids. As Synge (1965, 63) puts it: "We are not embarking on a programme of 'graphical relativity.' Our space-time diagrams are to be used as a mathematician or physicist uses rough sketches, rather than as an architect or engineer uses blueprints. The diagrams are to serve as guides for the mind." But even then, "anyone who studies relativity without understanding how to use simple space-time diagrams is as much inhibited as a student of functions of complex variable who does not understand the Argand diagram" (Synge 1965, 63).

The point, however, is that space-time diagrams are not mere space-time *graphs*. While representing physical events as points and particle tracks as lines or curves traced across space-time, these diagrams offer more than a graphical shorthand for the analysis of motion. What is basically expected from them is not so much a visual rendition of motion through space, but rather a consistent depiction of relativistic phenomena from the perspective of at least two reference frames in relative motion (each with their own coordinate system). In other words, a space-time diagram must provide a geometric equivalent of the Lorentz transformations involved in the substitution of one reference frame to another. In this respect, space-time representations function as "conceptual maps," as David Bohm (1965, 140) suggests: "with the aid of a good map having a proper structure, one can relate what is seen from one perspective to what is seen from another, in this way abstracting out what is invariant under change of perspective."

Space-time coordinates naturally play a fundamental part in the story, and this is what we should examine now. A simplified model of the Lorentz transformations is obtained by retaining only one dimension of space in addition to the time dimension. But what is peculiar about Minkowski diagrams is that one can, in principle, superimpose as many coordinate systems as one wishes upon the same representation space, as simplified as it is. Applying the familiar trigonometric identities to the ensuing geometrical configurations, one can extract the relevant information needed to make sense of the spatio-temporal distortions associated with shifting kinematical perspectives. Hence, Minkowski diagrams provide a rather straightforward illustration of various relativistic effects such as the shortening of lengths in the direction of motion (known as "Lorentz contraction"), the "dilation" of time in moving clocks, and the dislocation of simultaneity relations induced by the adoption of a new reference frame. The images printed below (see Figure 1 and Figure 2) illustrate the basic relativistic situation by focusing on the contraction affecting a moving electron. The coordinate axes (Ox, Ot) and (Ox', Ot') appear tilted according to their relative speed. As a result, the picture may seem to be centered on (Ox, Ot), but this visual privilege is really irrelevant for each frame provides an equally legitimate description of the situation from its own perspective. Accordingly, the diagram could be constructed around any of them. This sketch was initially drawn in three colors on a transparent slide by Minkowski himself for his 1908 lecture; it was reproduced in the publications that followed (Minkowski 1909).

Fig. 1.

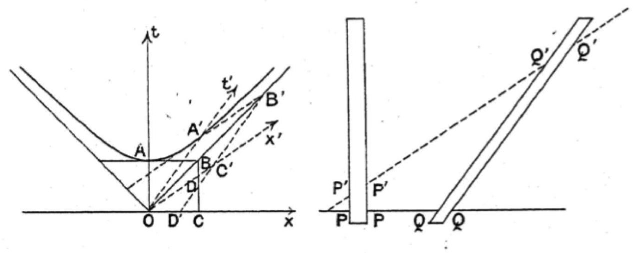

Figure 1. Reprinted from Hermann Minkowski. "Raum und Zeit," *Jahresberichte der Deutschen Mathematiker-Vereinigung*, (Leipzig: B.G. Teubner, 1909: 3). In this diagram, OA and OA′ are world line segments of equal space-time length, although their spatial and temporal components take different values in the (Ox, Ot) frame and in the (Ox', Ot') frame. The hyperbola intercepting A and A′ is a curve of equal proper time for all events propagating from O at uniform speed. Its role is analogous to that of a circle in the Euclidean plane: as the locus of points that are at a fixed spatio-temporal distance from O, it can be used to calibrate both sets of axes.

The moral of relativity, however, is that what is physically objective is not what varies, but rather what remains invariant under such relativistic transformations. And it is of course essential that the diagram should enable us to grasp this aspect of things dynamically, through the changes of perspective suggested by the diagram when it is actually put to work or performed. In Minkowski's parlance, the "world line" representing the motion of a particular object can be referred to various coordinate systems, just as a geometrical figure in space can be described according to various systems of spatial coordinates (the analogy is spelled out in detail in Taylor and Wheeler 1992, 1-11). Intervals of time and space are measured differently in each frame according to different perpendicular projections on the axes. Yet, as in the case of the Euclidean plane, there exists a metric invariant whose expression is given by a combination of squares of the projected magnitudes (analogous to Pythagoras's formula). A measure of that invariant space-time interval (or space-time length) is given by the "proper time" calculated along the space-time path of an object or process unfolding through time. As opposed to the "coordinate" (or projected) times attached to different systems of reference, proper time is not subject to distortions: it remains invariant and can be retrieved under any particular kinematical perspective. This notion of local time—distinct from global, coordinate time—lies behind the space-time distortions entailed by the introduction of non inertial (accelerated) frames, as in the famous "twin paradox."

IV. A duplicitous geometric intuition

The analogy with instances of geometric invariance in Euclidean space should not obscure the fact that the relations reference frames bear to one

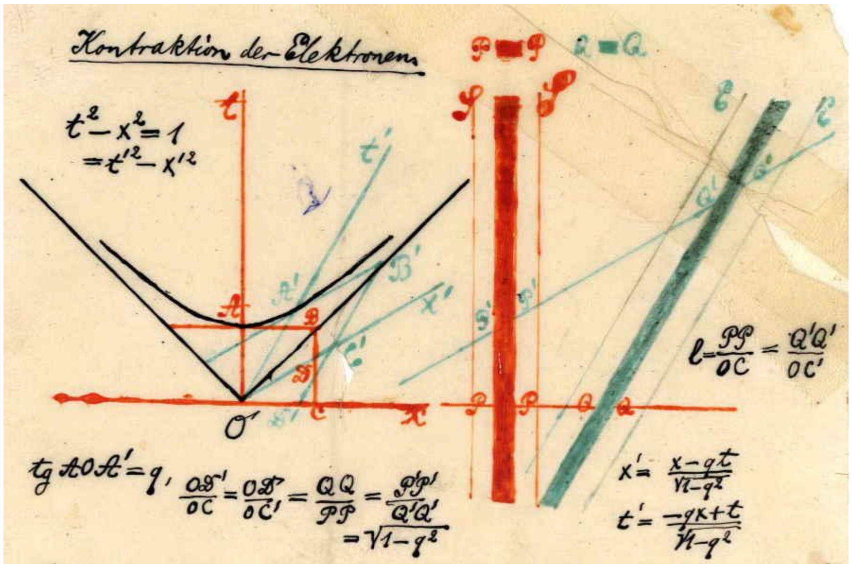

Figure 2. Hermann Minkowski. Hand-colored transparent slide (10x15 cm) used for the address delivered in Köln on the occasion of the 80th Assembly of German Natural Scientists and Physicians (September 21, 1908).

another are not static: contrary to coordinate systems in space, inertial frames differentiate themselves by their relative *speed*; shifting from one frame to another involves a space-time "boost." In this respect, a Minkowski diagram can best be described as the representation of the kinematical space of relativistic physics. The time axes and the segments of proper time, inclined at various angles, make up a "velocity space" (Penrose 2004, 426) characterized by its horizon structure (the unattainable speed of light) and its fundamental metric invariants (space-time intervals). These intrinsic features of the diagram are directly inherited from the geometry of space-time: invariant under any coordinate transformation, they are unaffected by speed, in the same way that Euclidean three-dimensional invariants (angles and lengths) are preserved under rotation or translation. The only reason why we are tempted to think that "each space-time map represents data from a particular reference frame" (Taylor and Wheeler 1992, 138) is that the graphical status of diagrams implies that they are, in fact, embedded in plain Euclidean two-dimensional space (the space of the page), thus admitting a natural—if partial—interpretation within the geometry of the Euclidean plane. As J. L. Synge (1965, 65) puts it: "The geometry of space-time is Minkowskian geometry, or briefly M-geometry. On the other hand, when we have constructed a space-time diagram on a sheet of paper, we begin very naturally to think in terms of the ordinary geometry of the sheet of paper, that is, Euclidean geometry, or E-geometry." This ambiguity may seem to us a sheer artefact of the diagrammatical method which we may want to overlook in favour of geometrical invariants, yet it turns out to be one of its most interesting features, because it forces us to convert the immediate geometric reading of the diagram into a dynamic intuition of the superposition of perspectives: it forces us to move back and forth from one projection to another, while mobilizing two different varieties of geometrical intuition.

By definition, the time axis and the spatial axis of a particular reference frame intersect orthogonally, although this may not seem obvious in the Euclidean plane when the two axes (such as Ox' and Ot') seem to be symmetrically tilted towards the bisecting diagonal which depicts space-time paths of a light ray conventionally set at 45°. Since the speed of light is invariant, it is only natural that all systems of axes should share the same bisecting diagonal. Hence, (OB) and (OB′) refer to the same line. But it is still somewhat disturbing to have to reorganize our intuition of orthogonality around that invariant light-path in space-time. Let us remember therefore that the diagram is supposed to express a geometric situation proper to *pseudo*-Euclidean space. The geometrical underpinnings of this mode of representation are more subtle than one may imagine: they involve elements of hyperbolic geometry.[1] When we look at the page on which the diagram is printed, it is difficult to refrain from interpreting certain shapes as distorted, when in fact they should strike us as invariant. An interval that looks obviously shorter than another may turn out to be longer. The method chosen for calibrating the axes partly compensates for these very natural visual "illusions": instead of a circle of unit radius, we must use a hyperbola (hence the designation "hyperbolic geometry") and spheres are similarly replaced by "pseudospheres." From a Euclidean point of view, this means that Minkowski diagrams require scaling factors when going between scales. In order to dispense with this burdensome procedure, mathematicians and physicists have come up with alternative diagrammatic methods whose interest lies in the fact that they only exhibit quantities that are real and in proper proportion, involving no distortion, and requiring no scale conversion. Thus, besides Minkowski diagrams, one should mention Loedel, Brehme and Gudermann diagrams.[2] Epstein diagrams are also rather appealing because, in addition to unit lengths being the same on all axes, all systems of axes are orthogonal in the Euclidean plane; on the other hand, an event does not correspond to a single point in space-time and the whole depiction revolves around the unfamiliar intuition that we all move through time at the speed of light, albeit in different directions in space-time (Epstein 1983). More recently, David Mermin (2005, 103-42) gave his own version of space-time diagrams, relying on properties of triangles in elementary plane geometry.

1 See I.M. Yaglom, *A Simple Non-Euclidean Geometry and its Physical Basis* (New York: Springer Verlag, 1979); and E.A. Robinson, *Einstein's Relativity in Metaphor and Mathematics* (Englewood Cliffs, N.J.: Prentice Hall, 1990).

2 For Loedel and Brehme diagrams, see A. Shadowitz, *Special Relativity*, (Philadelphia: Saunders Co.: chap. 1); and for Gudermann diagrams, see Robinson (1990).

Ingenious as they are, these attempts have not really shaken the popularity of Minkowski diagrams. But the latter must be handled with care in order to distinguish between the merely apparent distortions due to the method of depiction, the real but perspectival distortions due to the use of particular reference, and the real and objective distortions due to the intrinsic properties of space-time (such as those illustrated by the famous twin paradox). Comparisons between the measurements made by different observers are somewhat indirect. One must train one's eyes to see what is distorted as undistorted. But once we realize that Minkowski diagrams are really governed by a geometry which is not Euclidean, that they draw on one type of geometrical object to represent another, the impression that they are necessarily centered on a particular reference frame (the one whose axes seem to be crossing orthogonally in the ordinary plane geometry of the page) dissipates. The illusion of a privileged point of view appears, in retrospect, as an expression of the indexical ambiguity inherent in this form of visual representation. A good deal of philosophical misunderstanding could have been avoided if one had focused on the capacity of diagrams to exhibit the transformations themselves, rather than any particular kinematical perspective. Granted, it is impossible to hold two perspectives *at the same time*, as Bergson and Merleau-Ponty repeatedly argued in a way that echoes Wittgenstein's dictum that one may not speak two languages or play two games at the same time. But diagrams call for a duplicitous intuition that may achieve such a feat on a more abstract level, by inventing a new game in which one does not need to be "in" a reference frame in order to know how things would look from there. In fact, each diagram virtually encapsulates an indefinite number of equivalent space-time projections from every possible perspective. In that sense space-time itself may be described as the abstract space of all the possible *mappings* from one space-time projection (or one particular instantiation of a space-time diagram) to another (Sartori 1996, 144).

V. Productive ambiguity

To sum things up, the real value of space-time diagrams, what sets them apart from the ordinary space-time graphs used to represent motion in space, is the kind of ambiguity which arises from the superposition, within one space, of different perspectives on the same state of affairs. This is reminiscent of similar attempts made in the domain of visual arts: one thinks of the use of embedded perspectives in cubist paintings, in certain works by Escher, or in van Doesburg's and El Lissintzky's axonometric architectural drawings. A more familiar analogy would be the Necker cube and its systematically ambiguous spatial orientation. However, in our case, the ambiguity runs deeper than the mere fact of allowing for different perspectival readings, for as we have seen, what ultimately counts is the invariant features revealed in the play of perspectives. In fact, we may say that this interplay of perspectivity and objectivity—or if one prefers, of covariance and invariance—exhibited by the actual working of space-time diagrams is the clearest expression of their constitutive ambiguity. Perspective itself, and the distortions observed as a result of projecting a given situation from a particular point of view in a particular frame, is to a large extent governed by the grammar of iconic representation and so is the graphical method in general when it sets out to depict an evolution in time as an extension in space. The *coordination* of perspectives, however, is another matter. Here what counts is the procedure that manages to establish an isomorphic relation between the seemingly diverging representations yielded by two different framings of the same situation. If a diagram is a space-time map, it is so in a very special way, involving as it does a coordination or coordinatization procedure. In this capacity, it is symbolic. Or if one prefers, it calls for a relational or structural concept of objectivity (Daston and Galison 2007). As Emily Grosholz (2007, 202) puts it, "geometrical diagrams may be used in a way that is mostly symbolic and only barely iconic. Icons, especially icons that must be read in two or three different ways, do not wear their meaning on their faces; the interpretation of icons is not direct or 'intuitive.' And a geometrical figure can be used not as an icon but as a symbol." Clearly, a pragmatic approach to space-time diagrams as pedagogical and maybe heuristic tools should not focus too narrowly on one single mode of representation or on a particular dimension of that mode of representation. If ambiguity is indeed constitutive of diagrammatic representation as such, it must be present at every level. Thus, it is important to remember that a space-time diagram is *also* a graph and more often than not works simultaneously as a book-keeping or tracking device—after all, that is what coordinate systems were originally made for—and as a more abstract space displaying the coexistence (compatibility, equivalence) of several kinematical perspectives.

Thus, in addition to deploying a two-fold regime of geometric intuition adjusted to a 2-flat Euclidean model of pseudo-Euclidean geometry, Minkowski's constructions combine the iconic logic of graphs (still apparent in the study of space-time curves by the means differential geometry) with the more symbolic logic of so-called "representation spaces." This abstract orientation is already present in the very idea of visualizing the correspondences between equivalent coordinate systems in order to grasp their underlying symmetries. It is even more obvious when one focuses on the topological structure upon which the whole edifice of relativity physics is built: once the universe has been reduced to point-like events and their local connections through space and time, the whole space-time structure—including its metrical properties—can be derived from the order defined by relations of causal connectibility alone. It so happens that the invariant topology of the "light-cone" can be directly read off from space-time diagrams. For example, in the 2-flat model, events lying beyond the threshold of the bisecting diagonal cannot interact in any way with the point-event (O) where the axes meet, for that would involve supra-luminal velocities. But this situation holds for any point in space-time. The fact has universal scope; it expresses a structural feature of all diagrams, of all space-time maps, regardless of their particular referential content.

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