Knowing who occupies an office; purely contingent, necessary and impossible offices

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Abstract. This paper examines different kinds of definite descriptions denoting purely contingent, necessary or impossible objects. The discourse about contingent/impossible/necessary objects can be organised in terms of rational questions to ask and answer relative to the modal profile of the entity in question. There are also limits on what it is rational to know about entities with this or that modal profile. We will also examine epistemic modalities; they are the kind of necessity and possibility that is determined by *epistemic* constraints related to knowledge or rationality.

Definite descriptions denote so-called *offices*, roles, or *things to be.* We explicate these α-offices as partial functions from possible worlds to chronologies of objects of type α, where α is mostly the type of individuals. Our starting point is Prior's distinction between a 'weak' and 'strong' definite article 'the'. In both cases, the definite description refers to at most one object; yet, in the case of the weak 'the', the referred object can change over time, while in the case of the strong 'the', the object referred to by the definite description is the same forever, once the office has been occupied.

The main result we present is the way how to obtain a Wh-knowledge about who or what plays a given role presented by a hyper-office, i.e. *procedure* producing an office. Another no less important result concerns the epistemic necessity of the impossibility of knowing who or what occupies the impossible office presented by a hyper-office.

Keywords. Wh-knowledge; individual offices and hyper-offices; Transparent Intensional Logic; wh-questions and answers

1 Introduction

There is a lot of dispute about the semantics of definite descriptions. There are extensionalist and intensionalist approaches to definite description; some philosophers and linguists treat definite descriptions as referential expressions, others treat them as quantificational expressions, and some treat them as predicational expressions. Though there has been a long-term dispute about the character of definite descriptions, whether they are Russellian or Strawsonian,[1] we presume that definite descriptions denote offices, roles, or 'things to be'. This is in accordance with our background theory, which is Tichý's Transparent Intensional Logic (TIL). Tichý distinguishes between the semantics of proper names and definite descriptions. Independently of any particular theory of proper names, it should be granted that a *proper* proper name (as opposed to a definite description grammatically masquerading as a proper name) is a rigid designator of a numerically particular individual. On the other hand, definite descriptions like 'the richest man' do offer an *empirical criterion* that both enables and forces us to establish which individual, if any, plays the role of the richest man in a particular world/time pair.

Some descriptions can be inconsistent in the sense of necessarily not referring to any object; these are impossible objects. Our analyses rest on Tichý's Transparent Intensional Logic (TIL) with its *procedural* semantics, where hyperintensions are positively defined as abstract, algorithmically structured procedures that are the meanings of terms, including definite descriptions. The TIL hyperintensional approach enables us to differentiate between different hyper-offices that produce the same impossible object explicated as the necessarily vacant office or one and the same necessary object, i.e., a constant office. By a *hyperoffice*, we mean Frege's mode of presentation of the office, which we define as the TIL procedure producing the office. In general, hyper-offices make it possible to differentiate between different modes of presentation of the same intensional office. We also examine epistemic modalities; they are the kind of necessity and possibility that is determined by *epistemic* constraints related to knowledge or rationality.[2] We illustrate by examples how to reason with and answer Wh-questions concerning the objects referred to by definite descriptions. The agents can thus obtain Wh-knowledge to deal with and reason about. Dealing with hyper-offices is plausible, as one can know the answer to the question of who occupies this or that office, yet one does not know the answer if the Wh-question is presented via another mode of presentation of the same office. In addition, we propose a classification of the objects that trigger the necessity/impossibility of an office.

Duží (2009) and (2014) proposed a solution to the dilemma 'Russellian vs Strawsonian' descriptions by pointing out that sentences of the form "The *F* is a *G*" are systematically ambiguous between topic-focus articulation. If 'the *F*' is articulated as the topic, then the property *G* is ascribed to the only object (if any) that is the *F*. If there is none, then the sentence has a truth-value gap. This is a Strawsonian view. On the other hand, articulating a '*G*' as the topic, the sentence can be read as "Among those who are a *G* is the only *F*". Then the sentence gets Russellian truth-conditions right. If there is no *F* then the sentence is simply false. Duží does not adhere to Russell's dealing with definite descriptions as those that do not have meaning in isolation and with the analysis of sentences in which definite descriptions occur as including a quantifier phrase plus a propositional function rather than a singular term. We agree with Duží's stance and analyse definite descriptions as uniquely denoting an α-office that can be occupied by at most one α-object.

The preliminary goal of this paper is to introduce different kinds of definite descriptions. As a result, we show that some of them are *purely contingent* in the sense that the objects they refer to (if any) can differ in time and possible worlds. For example, 'the President of Slovakia' is such a contingent

[1] See, for instance, (Russell, 1905, 1957); (Strawson, 1950, 1964); (Donnellan, 1966); (Fintel, 2004); (Neale, 1990).

[2] See Brandov (2021).

description. Currently, it refers to Mrs Čaputová.[3] But she had not always been and will not always be the President of Slovakia (time index *t*). And though she is now the President, it might have been otherwise; somebody else might have been made the President of Slovakia (which is the modal index of possible worlds *w*).

Another category is formed by definite descriptions that are inconsistent and thus do not refer to any object in any time and world. As an example, Duží, Jespersen and Glavaničová adduce in (2021) the description (the ancient paradox of Achilles and Tortoise) 'The quickest runner who can never overtake the slowest runner if the former allows the latter the head start of $n > 0$ metres and both run at a constant speed' or examples like 'the divorced bachelor', 'the fake banknote that is a banknote', and 'the wooden horse that is a horse'.[4] These are *impossible objects* (accounted for without invoking impossibilia).[5] Hence, they are offices that necessarily go vacant.

Finally, there are definite descriptions that denote offices whose value is the same object at any time since the moment the office becomes occupied but possibly by different objects in different possible worlds. These are *epistemically necessary* entities that we will coin '*semi-necessary*'. For instance, 'Wimbledon 2023 women's singles winner' is a 'semi-necessary' office. Till the summer of 2023, this role was vacant; as Ons Jabeur missed the chance to finally become the winner, Vondroušová will play this role since then forever. Yet, it might have been otherwise. Jabeur, Świątek, or any other player might have won. Therefore, it is not an analytic necessity; instead, it is an epistemic necessity, not much unlike nomic necessity. While analytic necessity holds in all possible worlds and times, nomic necessity is empirical yet eternal, e.g., physical laws. While the validity of the laws of nature has to be empirically investigated, to know that in no possible world at no time can any individual be a divorced bachelor, it suffices to understand the term 'the divorced bachelor'.

Our starting point is Prior's distinction between the strong and weak definite article 'the'. We are inspired by Číhalová and Rybaříková (forthcoming), who compare Tichý's and Prior's approach to definite descriptions. On the linguistic level, a definite description is usually highlighted by the definite article 'the'. Prior argued that the definite article applied in definite descriptions has two distinct interpretations. It can stand for the weak 'the' or the strong 'the'. He says:

> In the weak sense 'The *a* is a *b'* is true so long as an *a* is a *b* when it is the only *a* at the time of utterance. This is the 'the' which we use in common speech in phrases like 'The President of the United States'. […] But in the strong sense, 'The *a* is a *b'* only if the *a* which is a *b* is the only *a* there is or has been or will be. Prior (1957, p. 76)

Prior further mentions this distinction in (1967, p. 164 and 172), where he describes the use of the strong 'the' as 'the only thing *ever* to be an *a*'.[6] Hence, in the case of the weak 'the', the denoted office is purely contingent, while in the case of the strong 'the', the denoted office is 'semi-necessary' or 'sempiternal'. This kind of necessity might be compared to Kripke' *temporal rigidity*, designation of the same individual with respect to other times.[7] There are also some ontological commitments; recognition of temporally rigid designation carries with it a commitment to some sort of "transtime" or transtemporal identity.

[3] Written in March 2024.

[4] These descriptions are inconsistent because the property modifier 'fake' is privative with respect to the property of being a banknote and the modifier wooden is privative with respect to the property of being a horse. Hence, no fake banknote is a banknote and no wooden horse is a horse. For details, see Jespersen, Carrara and Duží (2017).

[5] As Jespersen characterized it in the talk in the University of Connecticut, Logic Group Colloquia, April 30, 2021. See https://logic.uconn.edu/2021/04/26/impossibility-without-impossibilia/. See also Jespersen, Duží, Carrara (2024).

[6] See also Rybaříková (2023).

[7] See LaPorte (2022, §1.4)

The weak ‘the’ appears when the definite description refers to an individual that can change in time, such as ‘the President of the Czech Republic’, ‘the King of France’, etc. In every moment, there is at most one individual to which the description refers, but the reference can change in time. On the other hand, there are definite descriptions that refer to one and the same individual (if any) in the entire history of the world, such as ‘the first President of the Czech Republic’, ‘the first follower of Pavel Tichý’s ideas’, etc. In this case, the definite article ‘the’ is furnished by the strong interpretation. Tichý did not deal with this distinction in his analysis of definite descriptions denoting individual offices. In this paper, we are going to fill this gap.

As the first two categories of offices, namely purely contingent and impossible ones, have been analysed by previous papers,[8] we just briefly summarise these results, expand them and explain how they are related to this research. Our main goal and novelty is the analysis of the third category of necessary offices. To this end, we apply Tichý’s Transparent Intensional Logic (TIL) with its procedural semantics. In TIL, hyperintensions are positively defined as abstract, algorithmically structured procedures that are being assigned to language terms as their meanings.[9] These procedures are encoded by lambda terms in the logical apparatus of TIL and are typed within the ramified hierarchy of types. TIL hyperintensional approach enables us to differentiate between different hyper-offices that share one and the same impossible object explicated as the necessarily vacant office or one and the same necessary object, i.e. a constant office. We illustrate this by examples of reasoning with and answering questions about sentences that contain as a constituent a particular definite description. In addition, we propose a classification of the terms that trigger the necessity/impossibility of a description.

The remaining parts of the paper are structured as follows. In Section 2, TIL is briefly introduced. The summary of the analyses and reasoning with purely contingent and impossible offices is presented in Section 3. The main results on semi-necessary offices are presented in Section 4. Finally, concluding remarks and summary can be found in Section 5.

2 Fundamentals of Transparent Intensional Logic

Pavel Tichý, the founder of Transparent Intensional Logic (TIL), was inspired by Frege’s semantic triangle. Frege characterised the sense of an expression as the ‘mode of presentation’. Tichý defines this mode of presentation as an abstract, algorithmically structured *procedure* that produces the object denoted by the expression or, in rigorously defined cases, fails to produce a denotation if there is none.[10] These failing procedures are encoded by non-referring terms like ‘the value of the cotangent function at the number π’. This description is meaningful, as mathematicians obviously had to understand the sense of this term first and only then could they prove that there is no such number. Hence, In TIL, the meaning of an expression is understood as a context-invariant *procedure* encoded by a given expression. By context invariant, we mean this. The procedure encoded by an unambiguous expression is the same (up to procedural isomorphism), independent of the context in which the

[8] See, for instance, Duží, Jespersen, Glavaničová (2021), Duží (2009) and Duží (2014).

[9] For detail, see Tichý (1988), Duží (2019a) and Duží et al. (2010).

[10] See Tichý (1988). A similar philosophy of meaning as a ‘generalized algorithm’ can be found in (Moschovakis 2006); this conception has been further developed by Loukanova (2009). TIL procedural viewpoint is also not far from the idea of algorithmic logic, see Li, B. (2022).

expression is used.[11] If the expression is ambiguous, it is furnished with more than one procedure corresponding to its different meanings.[12]

Tichý defined six kinds of meaning procedures and called them *constructions.*[13] Two kinds of atomic constructions supply objects to be operated on by molecular constructions. They are *Trivialisation* and *Variable.* Trivialisation presents an object X without the mediation of any other procedures. Using the terminology of programming languages, the Trivialisation of X, denoted by '^0X', is just a *pointer* or *reference* to X. Trivialisation can present an object of any type, even another construction C. Hence if C is a construction, ^0C is said to *present* the construction C, whereby C occurs *hyperintensionally*, i.e. in the *non-executed* mode. Variables produce objects dependently on valuations; they *v-construct* the objects. The execution of a Trivialisation or a variable never fails to produce an object. However, since TIL is a logic of *partial functions*, the execution of some of the molecular constructions can fail to present an object of the type they are typed to produce. When this happens, we say that a given construction is *v-improper*.

There are two kinds of *molecular* constructions, which correspond to *λ-abstraction* and *application* in the λ-calculi, namely *Closure* and *Composition*, respectively; λ-*Closure*, $[\lambda x_1 \ldots x_n\ X]$, is the very procedure of producing a function with the values *v*-produced by the procedure X, by abstracting over the values of the variables $x_1, \ldots, x_n$ to provide functional arguments. No Closure is *v*-improper for any valuation *v*, as a Closure always *v*-constructs a function (which may be, in an extreme case, a degenerate function undefined at all its arguments, in case X is *v*-improper for any valuation *v*). *Composition*, $[X\ X_1 \ldots X_n]$, is the very procedure of applying a function f produced by X (if any) to the tuple argument $\langle a_1, \ldots, a_n \rangle$ (if any) produced by the procedures $X_1, \ldots, X_n$. A Composition is *v*-improper as soon as f is a *partial function* not defined at its tuple argument or if one or more of its constituents $X, X_1, \ldots, X_n$ are *v*-improper.

TIL being a *hyperintensional* system, each construction C can occur not only in execution mode to produce an object (if any) when being executed but also as an object in its own right on which other (higher-order) constructions operate. The Trivialisation of C causes C to occur just presented as an argument, as mentioned above. Yet sometimes, we need to cancel the effect of Trivialisation and trade the mode of C for execution mode. *Double Execution*, ^2C, does just that; it executes C twice over. If C *v*-constructs a construction D that in turn *v*-constructs an entity E, then ^2C *v*-constructs E. Otherwise, ^2C is *v*-improper. Hence, for any construction C, this law is valid: $^{20}C = C$.

Definition 1 (*construction*)

(i) *Variables* x, y, … are *constructions* that construct objects (i.e., elements of their respective ranges) dependently on a valuation function *v*; they *v*-construct.

(ii) Where X is an object whatsoever (even a *construction*), ^0X is the *construction Trivialisation* that constructs X without any change.

(iii) Let $X, Y_1, \ldots, Y_n$ be arbitrary *constructions*. Then the *Composition* $[X\ Y_1 \ldots Y_n]$ is the following *construction*. For any *v*, the Composition $[X\ Y_1 \ldots Y_n]$ is *v-improper* if one or more of $X, Y_1, \ldots, Y_n$ are *v*-improper, or if X does not *v*-construct a function that is defined at the *n*-tuple of objects *v*-constructed by $Y_1, \ldots, Y_n$. If X does *v*-construct a *v*-proper function, then $[X\ Y_1 \ldots Y_n]$ *v*-constructs the value of this function at the *n*-tuple.

(iv) (λ-) *Closure* $[\lambda x_1 \ldots x_m\ Y]$ is the following *construction*. Let $x_1, x_2, \ldots, x_m$ be pair-wise distinct variables and Y a *construction*. Then $[\lambda x_1 \ldots x_m\ Y]$ *v*-constructs the function f that takes any members $B_1, \ldots, B_m$ of the respective ranges of the variables $x_1, \ldots, x_m$ into the object (if any) that is

[11] For the definition of procedural isomorphism, see (Duží 2019a).

[12] Though TIL has become a well-known logical system, we include this section for completeness to make the paper easier to read for those who are not acquainted with TIL. The presentation of TIL technicalities appeared in many papers; here we refer in particular to Duží et al. (2010).

[13] In this paper, we use the terms 'procedure' and 'construction' as being synonymous.

$v(B_1/x_1,\ldots,B_m/x_m)$-constructed by Y, where $v(B_1/x_1,\ldots,B_m/x_m)$ is like v except for assigning B_1 to x_1, …, B_m to x_m.

(v) Where X is an object whatsoever, ${}^1X$ is the *construction Single Execution* that v-constructs what X v-constructs. Thus, if X is a v-improper construction or not a construction at all, ${}^1X$ is v-improper.

(vi) Where X is an object whatsoever, ${}^2X$ is the *construction Double Execution*. If X is not itself a *construction*, or if X does not v-construct a *construction*, or if X v-constructs a v-improper *construction*, then ${}^2X$ is v-improper. Otherwise, ${}^2X$ v-constructs what is v-constructed by the *construction* v-constructed by X.

(vii) Nothing is a *construction* unless it follows from (i) through (vi).

With constructions of constructions, constructions of functions, functions, and functional values in TIL stratified ontology, we need to keep track of the traffic between multiple logical strata. The *ramified type hierarchy* discharges this task. The type of first-order objects includes all non-procedural objects that are not constructions. Therefore, it includes not only the standard objects of individuals and truth values but also sets, functional mappings and functions defined on possible worlds (i.e., the *intensions* germane to possible-world semantics). The type of second-order objects includes constructions of first-order objects and functions that have such constructions in their domain or range. The type of third-order objects includes constructions of first- or second-order objects and functions that have such constructions in their domain or range; and so on ad infinitum.

DEFINITION 2 (*ramified hierarchy of types*)**.** Let B be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

$\mathbf{T_1}$ (*types of order 1*).

i) Every member of B is an elementary *type of order 1 over B.*

ii) Let $\alpha, \beta_1, \ldots, \beta_m$ ($m > 0$) be types of order 1 over B. Then the collection $(\alpha\ \beta_1 \ldots \beta_m)$ of all m-ary partial mappings from $\beta_1 \times \ldots \times \beta_m$ into α is a functional *type of* order 1 *over B.*

iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

$\mathbf{C}_n$ (*constructions of order n*)

i) Let x be a variable ranging over a type of order n. Then x is a *construction of order n over B.*

ii) Let X be a member of a type of order n. Then ${}^0X$, ${}^1X$, ${}^2X$ are *constructions of order n over B.*

iii) Let $X, X_1, \ldots, X_m$ ($m > 0$) be constructions of order n over B. Then $[X\, X_1 \ldots X_m]$ is a *construction of order n over B.*

iv) Let $x_1, \ldots, x_m, X$ ($m > 0$) be constructions of order n over B. Then $[\lambda x_1 \ldots x_m\, X]$ is a *construction of order n over B*.

v) Nothing is a *construction of order n over B* unless it so follows from $\mathbf{C}_n$ (i)-(iv).

$\mathbf{T}_{n+1}$ (*types of order n* + 1)

Let $*_n$ be the collection of all constructions of order n over B. Then

i) $*_n$ and every type of order n are *types of order n* + 1.

ii) If $m > 0$ and $\alpha, \beta_1, \ldots, \beta_m$ are types of order $n + 1$ over B, then $(\alpha, \beta_1, \ldots, \beta_m)$ (see T_1 ii)) is a *type of order* $n + 1$ *over B*.

iii) Nothing is a *type of order n + 1 over B* unless it so follows from (i) and (ii).

For the purposes of natural-language analysis, we are usually assuming the following base of ground types:

ο: the set of truth-values {**T, F**};
ι: the set of individuals (the universe of discourse);[14]
τ: the set of real numbers (doubling as times);
ω: the set of logically possible worlds (the logical space).

Empirical expressions denote *empirical conditions*, which may or may not be satisfied at the world/time pairs selected as points of evaluation. These empirical conditions are modelled as intensions. *Intensions* are entities of type $(\beta\omega)$: mappings from possible worlds to an arbitrary type β. The type β is frequently the type of the *chronology* of α-objects, i.e., a mapping of type $(\alpha\tau)$. Thus α-intensions are frequently functions of type $((\alpha\tau)\omega)$, abbreviated as '$\alpha_{\tau\omega}$'. *Extensional entities* are entities of a type α where $\alpha \neq (\beta\omega)$ for any type β. Where the variable w ranges over ω and t over τ, the following outline of a Closure essentially characterises the logical syntax of empirical language: $\lambda w \lambda t$ [...w....t...].

Examples of frequently used α-intensions are: *propositions* of type $o_{\tau\omega}$, *properties of individuals* of type $(o\iota)_{\tau\omega}$, *binary relations-in-intension between individuals* of type $(o\iota\iota)_{\tau\omega}$, *offices* of type $\iota_{\tau\omega}$ and hyperintensional *attitudes* of type $(o\iota *_n)_{\tau\omega}$. Logical objects like *truth-functions* and *quantifiers* are extensional: $\wedge, \vee, \supset$ are of type (ooo), and $\neg$ of type (oo). The *quantifiers* $\forall^\alpha$, $\exists^\alpha$ are type-theoretically polymorphic total functions of type $(o(o\alpha))$, for an arbitrary type α, defined as follows. The *universal quantifier* $\forall^\alpha$ is a total polymorphic function that associates a class A of α-elements with **T** if A contains all elements of the type α, otherwise with **F**. The *existential quantifier* $\exists^\alpha$ is a total polymorphic function that associates a class A of α-elements with **T** if A is a non-empty class, otherwise with **F**.

Notational conventions. Below, all type indications will be provided outside the formulae in order to make the notation easier to read. Moreover, the outermost brackets of Closures will be omitted whenever no confusion can arise. Furthermore, 'X/α' means that an object X is (a member) of type α. '$X \rightarrow \alpha$' means that X is typed to *v*-construct an object (if any) of type α. Throughout, it holds that the variables $w \rightarrow \omega$ and $t \rightarrow \tau$. If $C \rightarrow \alpha_{\tau\omega}$ then the frequently used Composition [[C w] t], which is the *extensionalization* of the α-intension *v*-constructed by C, is encoded as 'C_{wt}'. When no confusion arises, we use the standard infix notation without Trivialisation to apply logical objects like truth functions and quantifiers. Hence, instead of '$[^0\forall\lambda x\ B]$', '$[^0\exists\lambda x\ B]$', we often write '$\forall x\ B$', '$\exists x\ B$' for any $B \rightarrow o$ to make quantified formulas easier to read.

The *general semantic schema* involving the *meaning* (i.e., a construction) of an expression E, *denotation* (i.e., the object, if any, denoted by E) and *reference* (i.e., the value of an intension, if the denotation is an intension, in the actual world at the present time) is depicted by Fig. 1.

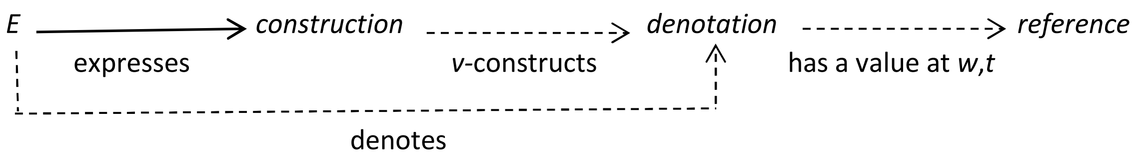

Fig. 1. TIL *General semantic schema*

Once the meaning construction of a term or expression has been given, what the construction produces (if anything) can be logically derived, i.e., what the denotation of E is. Provided the denotation is not a trivial (i.e., constant) intension or a mathematical function, the reference cannot be logically derived; instead, it must be established by extra-logical and extra-semantic means (i.e., empirical inquiry or mathematical calculation) what the reference, if any, is.

[14] We assume that the universe of discourse ι is multivalued and consists of at least two elements, though we leave aside the cardinality of this basic type.

Note that in TIL, we distinguish between *denotation* and *reference*. The *denotation* of an empirical term is always an intension. The *reference* relation then holds between the denoted intension and its value (if any) in a possible world and time of evaluation. For instance, the term 'President of the Czech Republic' denotes an individual office and refers to its value. Currently, it happens to be Petr Pavel. *Co-referential* terms refer to the same object in the world and time of evaluation. For instance, 'the fourth President of CR' and 'the chair of NATO military committee from 2015 to 2018' are co-referring terms; they both refer to Petr Pavel. However, they are not co-denoting, as the above roles are distinct.

As mentioned above, TIL is a logic of partial *functions*. Therefore, sets and relations are modelled by their characteristic functions. For instance, $(o\tau)$ is the type of a set of numbers, while $(o\tau\tau)$ is the type of a binary relation-in-extension between numbers. That an element *v*-constructed by $a \to \iota$ belongs to a set $M \to (o\iota)$, which in set-theoretical notation is written as '$a \in M$', in TIL is recorded as an application of the function M to a: $[M\ a]$. For instance, having the set of prime numbers $Prime/(o\tau)$, the sentence "2 is a prime number" is furnished with this simple construction as its meaning:

$$[{}^0Prime\ {}^02].$$

Note that any non-procedural entities must be supplied to molecular constructions by Trivialisation (or a variable, as the case may be). The reason is this. Parts or constituents of procedures can be only their (sub)procedures. No non-procedural abstract or concrete object can be a constituent part of a procedure. The objects on which procedures operate are beyond them. Thus, while *John* is an individual who cannot be executed and thus cannot be a part of a procedure, ${}^0John$ is a procedure, albeit trivial.[15]

Properties of individuals are intensions, objects of type $(o\iota)_{\tau\omega}$. A functional application is used to apply a property to an individual. However, properties are not type-theoretically proper entities to be directly applied to an individual. They have to be extensionalized first. For instance, the sentence

"John is a painter."

ascribes the property of being a painter to John. As any other non-procedural objects to be operated on, the individual John, as well as the property of being a painter, are supplied by their Trivialisation, ${}^0John$, ${}^0Painter$. Since the property is an intension of type $(((o\iota)\tau)\omega)$, or $(o\iota)_{\tau\omega}$ for short, the property must be applied to a possible world (type ω) first and then to time (type τ). To this end, we have variables $w \to \omega$ and $t \to \tau$; thus, we get $[[{}^0Painter\ w]\ t]$, or ${}^0Painter_{wt}$, for short. In this way, we obtain the population of painters in the world and the time in which we will evaluate the truth value of the sentence. That John belongs to this population is expressed simply by the application of this population to John: $[{}^0Painter_{wt}\ {}^0John] \to o$. Finally, we abstract over the values of the variables w and t to obtain the proposition that John is a painter.

$$\lambda w \lambda t\ [{}^0Painter_{wt}\ {}^0John] \to o_{\tau\omega}$$

So much for the basic technicalities of TIL.

3 Purely contingent objects

Definite descriptions with Prior's weak 'the' denote purely contingent offices; values of such offices are in no way necessary and change in time. As an example, we analyse the description 'the President of the USA'. The meaning of this description is the procedure that produces an individual office. Currently, this office is held by Joe Biden, but it might have been otherwise. Donald Trump, or any

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[15] In this sense, TIL is neo-Fregean and deviates from Russell's conception of structured propositions. Mont Blanc, that mound of rocks and boulders, cannot be a constituent of a procedure; ${}^0Mont\text{-}Blanc$ can.

other candidate, might have won the last elections. Besides, Joe Biden was not and will not be the US president forever. The meaning procedure comes down to this TIL construction.

$$\lambda w \lambda t\ [{}^0\mathit{President\text{-}of}_{wt}\ {}^0\mathit{USA}] \rightarrow \iota_{\tau\omega}$$

Types. $\mathit{President\text{-}of}/(\iota\iota)_{\tau\omega}$: empirical attribute, i.e. a function-in-intension that dependently on worlds w and times t associates an individual (USA in this case) with at most one individual (its president); USA/ι; ${}^0\mathit{President\text{-}of}_{wt} \rightarrow (\iota\iota)$; $[{}^0\mathit{President\text{-}of}_{wt}\ {}^0\mathit{USA}] \rightarrow \iota$; $\lambda t\ [{}^0\mathit{President\text{-}of}_{wt}\ {}^0\mathit{USA}] \rightarrow (\iota\tau)$; $\lambda w \lambda t\ [{}^0\mathit{President\text{-}of}_{wt}\ {}^0\mathit{USA}] \rightarrow ((\iota\tau)\omega)$, or $\iota_{\tau\omega}$ for short.

Much has been written about definite descriptions. It mainly concerns the long-term debate about their character, whether they are Russellian or Strawsonian. Says Ludlow (2022):

> The analysis of descriptions has played an important role in debates about metaphysics, epistemology, semantics, psychology, logic and linguistics ever since the publication of Bertrand Russell's paper "On Denoting," in 1905. Despite the apparent simplicity of definite and indefinite descriptions, the past 100+ years have seen heated debates about their proper analysis. For example, some philosophers and linguists treat definite descriptions as referential expressions, others treat them as quantificational expressions, and some treat them as predicational expressions. Other analyses of descriptions have held that the determiners 'the' and 'a' do not make a semantical contribution but rather a pragmatic contribution to what is communicated.

In this section, we introduce the results obtained by applying TIL analysis to the problem of the semantics of definite descriptions. We do not deal with pragmatic issues of their use and concentrate on the semantic difference between treating them as referential vs quantificational terms. We also make a few remarks on the ambiguity of sentences containing definite descriptions; this ambiguity stems not only from their topic-focus articulation (Russellian vs Strawsonian reading) but also concerns sentences with time reference and descriptions occurring with supposition *de dicto* or *de re*.[16] We will show that sentences of the schematic form "The *F* met with the *G* in time *T*" are systematically ambiguous and have at least four different non-equivalent meanings. Hence, if an agent wants to know who the *F* is, where 'the *F*' denotes a purely contingent office, they must ask and disambiguate; we deal with a Wh-question. In TIL, a method of answering such questions has been developed, and we briefly introduce its details.

3.1 Russell vs Strawson on definite descriptions

Concerning Russellian vs Strawsonian analysis of definite descriptions, the long-term dispute concerns these issues.[17] Russell, who followed his conception of structured propositions where concrete or abstract objects referred to by the terms of a sentence are constituents of a proposition, could not analyse sentences like "The *F* is a *G*" employing the singularizer $\iota x\ F$ ('the only *x* such that *F*') in case the term 'the *F*' is non-referring. Hence, he deprives the description of its meaning in isolation and says that we must *discover* the logical form of the sentence. Using Russell's favourite example, the 'hidden form' of the sentence "The King of France is bald" is "There is an *x* with the property of being a king of France, and this *x* is the unique one with this property, and this *x* is bald". In its simple, non-technical form, Russell's analysis of "The *F* is a *G*" is shorthand for the conjunction of three claims:

[16] The de re occurrence is an extensional occurrence; it means that the *value* of a denoted function (office in our case) is the object of predication, i.e. an argument of another function. The de dicto occurrence is an intensional occurrence, i.e. the whole function (office) is an object of predication or an argument of another function. For details, see Duží et al. (2010, §1.5.2).

[17] For details, see, for instance, Donnellan (1966), Ludlow (2022), or Neale (1990). The contention seems to be still alive, see Fintel (2004). Here we summarise the results obtained in TIL, see Duží (2009) and (2014).

(a) There is an *F*
(b) At most one entity is an *F*
(c) Something that is an *F* is a *G*

Hence, if there is currently no *F* (as when there is no King of France), the sentence is simply false. In TIL, the following procedure gets Russellian truth conditions right.

$$\lambda w\lambda t\ [^0\exists\lambda x\ [[^0KF_{wt}\ x] \wedge {}^0\forall\lambda y\ [[^0KF_{wt}\ y] \supset x{=}y] \wedge [^0Bald_{wt}\,x]]]$$

Types. $x, y \to \iota$; $KF, Bald/(o\iota)_{\tau\omega}$; $\exists, \forall/(o(o\iota))$: quantifiers.

The above procedure can be simplified in an equivalent way by making use of the singular attribute $King\text{-}of/(\iota\iota)_{\tau\omega}$. It is a function-in-intension that dependently on worlds and times associates an individual with at most one individual. The office of the King of France is then produced by this construction: $\lambda w\lambda t\ [^0King\text{-}of_{wt}\ {}^0France] \to \iota_{\tau\omega}$. The equivalent simplified construction is this:

$$\lambda w\lambda t\ [^0\exists\lambda x\ [[x = \lambda w'\lambda t'\ [^0King\text{-}of_{w't'}\ {}^0France]_{wt}] \wedge [^0Bald_{wt}\,x]]],$$

or by applying a restricted β-reduction, we get[18]

$$\lambda w\lambda t\ [^0\exists\lambda x\ [[x = [^0King\text{-}of_{wt}\ {}^0France]] \wedge [^0Bald_{wt}\,x]]]$$

On the other hand, Strawsonian analysis comes down to this procedure:

$$\lambda w\lambda t\ [^0Bald_{wt}\,\lambda w'\lambda t'\ [^0King\text{-}of_{w't'}\ {}^0France]_{wt}] \text{ or}$$

$$\lambda w\lambda t\ [^0Bald_{wt}\,[^0King\text{-}of_{wt}\ {}^0France]]$$

The dispute on which of these analyses is the right one was triggered by Strawson's (1950) paper. Strawson criticised Russell's analysis, saying that it gets the truth conditions wrong if there is no King of France. According to Strawson, in such a case, the sentence is not false, but it has a truth-value gap, as there is no individual to whom the property of being bald could be ascribed. If it were false, then the sentence "The King of France is *not* bald" would have to be true, which is not possible either because there is no King of France. Hence, according to Strawson, sentences like "The *F* is a G" not only entail but also *presuppose* the existence of the only *F.*

Russell was upset by this criticism; in response to Strawson, he argued that, despite Strawson's protests, the sentence was in fact false:

> Suppose, for example, that in some country there was a law that no person could hold public office if he considered it false that the Ruler of the Universe is wise. I think an avowed atheist who took advantage of Mr. Strawson's doctrine to say that he did not hold this proposition false would be regarded as a somewhat shifty character. (Russell 1957, p. 389)

Duží (2009) and (2014) proposed a solution to this dilemma. The point of departure is that sentences of the form "The *F* is a *G*" are systematically ambiguous. Their ambiguity is not rooted in a shift of meaning of the definite description 'the *F*'. Rather, the ambiguity stems from different *topic-focus articulations* of such sentences. Whereas articulating the topic of a sentence activates a presupposition, articulating the focus usually yields merely an entailment.[19] The point is this. If 'the *F*' is the topic phrase, this description occurs with *de re* supposition, and Strawson's analysis appears to be what is wanted. In this reading that corresponds to Donnellan's *referential use* of 'the *F*', the sentence *presupposes* the existence of the reference of 'the *F*'. The other option is '*G*' occurring as a topic and 'the *F*' as a focus. The sentence could then be reformulated as "Among the *G*s, there is the *F*". This reading corresponds to Donnellan's *attributive* use of 'the *F*', and the description occurs with

[18] Restricted β-reduction simply substitutes variables for variables of the same type (see, e.g., Duží (2019a)). Here we substitute the variables *w, t* for *w'* and *t'*, respectively.

[19] This assumption is based on Hajičová (2008), and supported by other linguists as well. See, for instance Gundel (1999), Gundel and Fretheim (2006) and Strawson (1952, esp. p. 173ff.).

de dicto supposition. In this case, the Russellian analysis gets the truth-conditions of the sentence right. The existence of a unique *F* is merely entailed. If there is no *F*, the sentence is simply false. Hence, Russellian and Strawsonian truth conditions of "The *F* is a *G*" are 'almost the same'; they differ only in those worlds and times where 'the *F*' is a non-referring term. In such states of affairs, the Strawsonian proposition has a truth-value gap, while the Russellian one has the value **F**.

Yet, there is another difference between these two positions. It concerns the way of *negating*; while Strawson applies a *narrow-scope negation,* "The *F* is *not* a *G*", Russell applies the *wide-scope negation,* "It is *not true* that the *F* is a *G*". In the logic of partial functions, these two negations are not equivalent. To show that, we apply TIL analysis.

Russellian *wide-scope negation* is this:

$$\lambda w \lambda t \neg[{}^0True_{wt}\, \lambda w \lambda t\, [{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]],$$

which is equivalent with

$$\lambda w \lambda t \neg[{}^0\exists \lambda x\, [[x = [{}^0King\text{-}of_{wt}\, {}^0France]] \wedge [{}^0Bald_{wt}\, x]]].$$

Additional type: $True/(o o_{\tau\omega})_{\tau\omega}$: the property of a proposition of being true in a given world w and time t. This property is defined as follows. Let $P \rightarrow o_{\tau\omega}$. Then

$$[{}^0True_{wt}\, P] = {}^0\mathbf{T} \text{ iff } P_{wt}\ v\text{-produces } \mathbf{T}, \text{ otherwise } [{}^0True_{wt}\, P] = {}^0\mathbf{F}$$

For completeness, here are the other two important properties of propositions, *False* and *Undef*, both of type $(o o_{\tau\omega})_{\tau\omega}$:

$$[{}^0False_{wt}\, P] = {}^0\mathbf{T} \text{ iff } P_{wt}\ v\text{-produces } \mathbf{F}, \text{ otherwise } [{}^0False_{wt}\, P] = {}^0\mathbf{F}$$

$$[{}^0Undef_{wt}\, P] = \neg[{}^0True_{wt}\, P] \wedge \neg[{}^0False_{wt}\, P]$$

Hence, the proposition that the King of France is bald is not true in two cases: either the King of France does not exist, and the proposition is undefined, or the King of France is not bald, and the proposition is false.

On the other hand, Strawsonian narrow-scope negation is simply

$$\lambda w \lambda t \neg[{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]].$$

Hence, if the King of France does not exist, i.e. the Composition $[{}^0King\text{-}of_{wt}\, {}^0France]$ is *v*-improper in the given world *w* and time *t* of evaluation, then the Compositions $[{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]$ and $\neg[{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]$ are also *v*-improper. The produced (negated) proposition has a truth-value gap.

Note that on the Strawsonian analysis, the meaning of 'the King of France', i.e. the Closure $\lambda w \lambda t\, [{}^0King\text{-}of_{wt}\, {}^0France]$, occurs with *de re* supposition (unlike Russellian analysis). Hence, *two principles de re* are valid for Strawson. They are the principle of *existential presupposition* and the *substitution of co-referring terms*; thus, the following arguments are valid:

<u>The King of France is (is not) bald</u>
The King of France exists

The King of France is bald
<u>Louis XVI is the King of France</u>
Louis XVI is bald

To put the above results on a more solid ground, we must define the difference between logical *entailment* and *presupposition.* Following Frege and Strawson in treating survival under (narrow-scope) negation as the most important test for presupposition, we define:

Definition 3 (***presupposition vs mere entailment***) Let P, S be propositional constructions ($P, S/*_n \rightarrow o_{\tau\omega}$) and let $\models / (oo_{\tau\omega}o_{\tau\omega})$. Then

P is analytically entailed by S, denoted '$S \models P$', iff $\forall w \forall t\ [[{}^0True_{wt}\, S] \supset [{}^0True_{wt}\, P]]$
P is a *presupposition* of *S* iff $\forall w \forall t\ [[[{}^0True_{wt}\, S] \vee [{}^0False_{wt}\, S]] \supset [{}^0True_{wt}\, P]]$

Gloss. If *P* is a presupposition of *S* and *P* is not true at a given $\langle w, t\rangle$-pair, then *S* is neither true nor false. Hence, *S* has no truth value at such a $\langle w, t\rangle$-pair at which its presupposition is not true. On the other hand, if *P* is merely entailed by *S*, then if *S* is not true, we cannot deduce anything about the truth-value, or lack thereof, of *P.*

To prove the existential presupposition argument, we need to say a few words about TIL's view of *existence.* As mentioned above, TIL does not deal with impossibilia, and the set of individuals, i.e. the discourse domain, is fixed once the base is voted for. Hence, bare individuals trivially exist. *Non-trivial existence* is a property of functions, namely the property of having a value at a given argument. For instance, when claiming that Cotangent of the number π does not exist, we do not speak about a non-existing number. Rather, we say that the function cotangent has the property of being undefined at the number π. Similarly, when saying that the King of France does not exist, we claim that the individual office does not have a value (is not occupied) at a given world and time of evaluation. Hence, we deal with $Exist/(o\iota_{\tau\omega})_{\tau\omega}$, the property of an office. This property is defined as follows. Let $O \rightarrow \iota_{\tau\omega}$, $x \rightarrow \iota$, then:

$$[{}^0Exist_{wt}\, O] = [{}^0\exists \lambda x\ [x = O_{wt}]]$$

The proof of existential presupposition makes use of the two rules for *True* and *False* introduction, namely (*True-I*) and (*False-I*), respectively: $P_{wt} \mid\!\!— [{}^0True_{wt}\, P]$, $\neg P_{wt} \mid\!\!— [{}^0False_{wt}\, P]$.

In any world *w* and time *t*, the following steps are truth-preserving:

1) $[{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]] \vee \neg[{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]$ — assumption
    a. $[{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]$ — hypotheses
    b. $[{}^0True_{wt}\, \lambda w \lambda t\, [{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]]$ — *True-I*
    c. $[{}^0\exists \lambda x\ [x = [{}^0King\text{-}of_{wt}\, {}^0France]]]$ — $\exists$-I
    d. $[{}^0\exists \lambda x\ [x = \lambda w \lambda t\ [{}^0King\text{-}of_{wt}\, {}^0France]_{wt}]]$ — β-expansion
    e. $[{}^0Exist_{wt}\, \lambda w \lambda t\ [{}^0King\text{-}of_{wt}\, {}^0France]]$ — def. of *Exist*
2) $[{}^0True_{wt}\, \lambda w \lambda t\, [{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]] \supset [{}^0Exist_{wt}\, \lambda w \lambda t\ [{}^0King\text{-}of_{wt}\, {}^0France]]$
    a. $\neg[{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]$ — hypotheses
    b. $[{}^0False_{wt}\, \lambda w \lambda t\, [{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]]$ — *False-I*
    c. $[{}^0\exists \lambda x\ [x = [{}^0King\text{-}of_{wt}\, {}^0France]]]$ — $\exists$-I
    d. $[{}^0\exists \lambda x\ [x = \lambda w \lambda t\ [{}^0King\text{-}of_{wt}\, {}^0France]_{wt}]]$ — β-expansion
    e. $[{}^0Exist_{wt}\, \lambda w \lambda t\ [{}^0King\text{-}of_{wt}\, {}^0France]]$ — def. of *Exist*
3) $[{}^0False_{wt}\, \lambda w \lambda t\, [{}^0Bald_{wt}\, [{}^0King\text{-}of_{wt}\, {}^0France]]] \supset [{}^0Exist_{wt}\, \lambda w \lambda t\ [{}^0King\text{-}of_{wt}\, {}^0France]]$

In both cases, step (c), namely existential quantifier introduction, is justified even in the logic of partial functions due to the definition of Composition. If a given proposition takes the value **T** or **F**, then none of the constituents of its construction can be *v*-improper. Hence, the Composition $[{}^0King\text{-}of_{wt}\, {}^0France]$ is not *v*-improper, which in turn means that there is an *x* such that $[x = [{}^0King\text{-}of_{wt}\, {}^0France]]$.

The proof of substitution is obvious. It is a one-step application of Leibniz's law of substitution of identicals.

The received view still tends to be that there is room for at most one of the two positions since they are incompatible. Above, we showed that there is no incompatibility between Strawson's and

Russell's positions, as they simply do not talk about the same meaning of the sentence "The King of France is bald". Russell argued for the attributive use of 'the King of France', whereas Strawson argued for its referential use. Which of these two non-equivalent meanings is the intended one is a pragmatic issue.

Hence, when asking whether the sentence the King of France is bald, Russell would answer that it is false, while Strawson would answer that it is not true, as the denoted proposition is gappy. Formally, here is the analysis of both readings.

(R) $\lambda w \lambda t\ [{}^0 Know_{wt}\, {}^0 Russell\ {}^0[{}^0 False_{wt}\ \lambda w \lambda t\ [{}^0\exists \lambda x\ [[x = [{}^0 King\text{-}of_{wt}\ {}^0 France]] \wedge [{}^0 Bald_{wt}\, x]]]]]$

(S) $\lambda w \lambda t\ [{}^0 Know_{wt}\, {}^0 Strawson\ {}^0[{}^0 Undef_{wt}\ \lambda w \lambda t\ [{}^0 Bald_{wt}\, [{}^0 King\text{-}of_{wt}\ {}^0 France]]]]$

Note that the complements occur hyperintensionally, as they occur within the scope of Trivialisation. We analyse *Knowing-that* as being of the type $(o\iota{*}_n)_{\tau\omega}$ to avoid problems with logical/mathematical omniscience.

3.2 Ambiguities concerning time reference

As a sample example of the ambiguities of sentences with purely contingent definite descriptions with respect to time reference, Duží (2019b) introduces the sentence

"The US President met with the Czech President in the Reduta Jazz Club, Prague, in 1994".

There are at least five different, non-equivalent readings of this sentence. In prose, these readings can be disambiguated like this:

(R1) "The *then* US President met with *then* Czech President in the Reduta Jazz Club, Prague, in 1994".

Both the terms (or rather their meaning constructions) 'the US President' and 'the Czech President' occur with *de re* supposition with respect to the *year 1994*. Hence, both the presidents had to exist in 1994. If not, the sentence has a truth-value gap.

(R2) "The *current* US President met with the *then* Czech President in the Reduta Jazz Club, Prague, in 1994".

The meaning of 'the US President' occurs with *de re* supposition with respect to time *t* of evaluation, while 'the Czech President' occurs with *de re* supposition with respect to 1994. Hence, the current US president must exist now, and the Czech President had to exist in 1994 in order the sentence have a truth value.

(R3) "The *then* US President met with *the current* Czech President in the Reduta Jazz Club, Prague, in 1994".

The meaning of 'the Czech President' occurs with *de re* supposition with respect to time *t* of evaluation, while 'the US President' occurs with *de re* supposition with respect to 1994. The truth conditions are similar to those of (R2).

(R4) "The *current* US President met with *the current* Czech President in the Reduta Jazz Club, Prague, in 1994".

Both the meanings of 'the Czech President' and 'the US President' occur with de re supposition with respect to the time of evaluation. Hence, if one of the presidents does not exist now, the sentence has a truth-value gap.

(R5) "*In 1994*, the then US President met with the then Czech President in the Reduta Jazz Club, Prague".

Both the meanings of 'the Czech President' and of 'the US President' occur with *de dicto* supposition, as the topic phrase is 'in 1994'.

Duží (2019b) introduces and proves different arguments connected with these distinct meanings. An epistemic agent may start with limited evidence that leaves open many epistemic possibilities. As the agent obtains more information, various possibilities are ruled out until some propositions are epistemically necessary and so must be true.

For instance, if we present an additional assumption that in 1994, the President of CR was Václav Havel, and the US president was Bill Clinton, we can easily deduce from (R1) that Havel met with Clinton at the Reduta Jazz Club Prague in 1994. Indeed, it was so, as those who are well acquainted with the history, or the older people who remember this significant event, know.

3.3 Wh-questions on purely contingent offices

The agents in a multi-agent system frequently need to know the answer to the question on the object referred to by a given definite description, particularly if the description denotes a purely contingent office.[20] Hence, we have to deal with Wh-questions that are even more frequent than Yes-No questions traditionally dealt with by erotetic logics. Wh-questions denote α-intensions the value of which the inquirer would like to know. Unlike Yes-No questions, the variety of possible answers to Wh-questions is much greater depending on the type α of an α-intension the value of which is asked for. Here, we are interested in questions on the value of an *individual office* (or *role*) of type $\iota_{\tau\omega}$, like "Which is the highest mountain in New Zealand?", "Who is the mayor of the city of Dunedin?", "Who is the No.1 player in ATP tennis singles"? A possible direct answer to such a question is a unique individual (an object of type ι) who happens to play a given role.[21]

For instance, the analysis of the question "Who is the No.1 player in ATP tennis singles"? comes down to this construction.

$$\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0\mathit{ATP\text{-}ranking}_{wt}\ x] = {}^01]] \rightarrow \iota_{\tau\omega}$$

Types: $\mathrm{I}/(\iota(o\iota))$: the singularizer, i.e. the function that associates a set S of individuals with the only member of S provided S is a singleton, and otherwise (if S is an empty or a multi-valued set) the function I is undefined; $x \rightarrow \iota$: the variable ranging over individuals such that the direct answer would be provided by the valuation of this variable; $\mathit{ATP\text{-}ranking}/(\tau\iota)_{\tau\omega}$: an attribute, i.e. a function-in-intension that associates a given individual with a number that is its value in ATP ranking singles.

Hence, Wh-questions transform into constructions with λ-bound variables, the value of which is asked for. The answers are then obtained by the technique of suitable substitutions, i.e. unification known from the general resolution method.[22] To answer the above question, assume that in an agent's knowledge base, there are these formalised sentences.

ATP ranking of Novak Djokovic is 1:	$\lambda w \lambda t\ [[{}^0\mathit{ATP\text{-}ranking}_{wt}\ {}^0\mathit{Djokovic}] = {}^01]$
ATP ranking of Carlos Alcaras is 2:	$\lambda w \lambda t\ [[{}^0\mathit{ATP\text{-}ranking}_{wt}\ {}^0\mathit{Alcaras}] = {}^02]$
ATP ranking of Daniil Medvedev is 3:	$\lambda w \lambda t\ [[{}^0\mathit{ATP\text{-}ranking}_{wt}\ {}^0\mathit{Medvedev}] = {}^03]$
And so on.	

The answer to the question "Who is the No.1 player in ATP tennis singles"? that obtains the analysis

$$\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0\mathit{ATP\text{-}ranking}_{wt}\ x] = {}^01]] \rightarrow \iota_{\tau\omega}$$

is derived like this.

(1) $[{}^0\mathrm{I}\ \lambda x\ [[{}^0\mathit{ATP\text{-}Ranking}_{wt}\ x] = {}^01]]$ Question

[20] By an agent in a multi-agent system, we mean a software agent of a software system or a human agent in a multi-cultural world.

[21] For TIL analysis of Yes-No questions and answers, see Duží, Číhalová (2015), or Tichý (1978).

[22] In TIL, the method for answering Wh-questions has been developed. For details, see Duží, Fait (2021).

(2) $\lambda x\ [[{}^0ATP\text{-}Ranking_{wt}\ x] = {}^01]$ 1, E-I
(3) $[[{}^0ATP\text{-}Ranking_{wt}\ {}^0Djokovic] = {}^01]$ assumption
(4) $x = {}^0Djokovic$ 2, 3, unification (${}^0Djokovic\ /\ x$)

The direct answer to the above question is Djokovic.

Comments. In the proof, we omitted the first proof steps that consist in the elimination of the left-most $\lambda w\lambda t$. It is the standard way of proving in TIL that is justified due to this: As defined above (Def. 3), the relation of *entailment* obtains between constructions of propositions such that in all possible words and times, whenever the propositions of assumptions are true, the proposition produced by the conclusion is true as well. Hence, in any world w' and time t' of evaluation, the derivation sequence must be truth-preserving from premises to the conclusion. Thus, the typical series of derivation steps is this. We have assumptions of the form $\lambda w\lambda t\ [\dots w \dots t \dots] \to o_{\tau\omega}$, and we assume that the propositions produced by these constructions are true in the world w' at time t' of evaluation. Using the detailed notation, we have the Composition

$$[[[\lambda w\ [\lambda t\ [\dots w \dots t \dots]]]\ w']\ t'] \to o.$$

By applying restricted β-reduction twice,[23] we eliminate the left-most $\lambda w\lambda t$, thus obtaining $[\dots w' \dots t' \dots] \to o$. Now we proceed with derivation steps until the conclusion $[\dots w' \dots t' \dots] \to o$ is derived. Since we are to derive a proposition, we finally abstract over the values of the variables w', t', thus introducing the left-most $\lambda w\lambda t$ back to construct a proposition:

$$\lambda w\lambda t\ [\dots.\ w \dots t \dots] \to o_{\tau\omega}.$$

3.4 Knowing-Wh the value of a contingent office

Assume that the answer to a Wh-question has been derived and the questioner knows the answer; we have the case of *knowing-Wh.*[24] In the early days of epistemic logic, Hintikka (1962) elaborated theories of knowing-wh and its relation to questions in terms of the first-order modal logic. For example, "knowing who the Mayor is" is formalised as $\exists xK(Mayor{=}x)$, where K stands for 'knowing that'. However, we do not analyse knowing-wh and the corresponding wh-question in Hintikka's way within quantified epistemic logic as an existentially quantified formula. The existence of the known object is the consequence, or rather the *presupposition*, of knowing-wh. Hence, our analysis of Wh-questions deviates from Hintikka's one. On the other hand, we agree with Wang (2018) that there is a constant domain of individuals in all the possible worlds and that *knowing-wh* relates an agent to the *value* of an intension or, in mathematical cases, the value produced by a given procedure. Yet, we are not going to apply a formal axiomatic approach without specifying the meaning of 'knowing-wh' first.

To obtain the meaning of *Knowing-wh*, we have to examine what it means for an agent to know the answer to the corresponding Wh-question. For instance, if John obtains the answer to the question "Who is the No.1 player in ATP tennis singles?" then John knows who the No. 1 in ATP ranking is. How do we analyse this knowledge? There are two possibilities, namely implicit (intensional) knowledge and explicit (hyperintensional) knowledge.

[23] Restricted β-reduction consists just in substitution of variables for variables; hence, it is a 'safe' reduction that transforms the redex into an equivalent contractum. For more details on β-conversions in the logic of partial functions such as TIL, see Duží, Kosterec (2017).

[24] There is a necessary condition for agent's knowing-wh, i.e. knowing the value of the intension asked for, that the agent has got a *conclusive answer* to the corresponding wh-question in the form of a *description* that rigorously refers to the value. Most frequently, such a conclusive answer is provided by a *proper* name. In this section we partly draw on the results from Duží (2023); however, we substantially adjust and correct those results.

Intensional *Know-wh*/$(o\iota_{\tau\omega})_{\tau\omega}$ is a relation-in-intension of an individual to an office. Hence, John is related to the office itself, and we have

$$\lambda w\lambda t\ [{}^0\mathit{Know\text{-}wh}_{wt}\ {}^0\mathit{John}\ \lambda w\lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0\mathit{ATP\text{-}ranking}_{wt}\ x] = {}^01]]].$$

However, assume that 'the No.1 ATP tennis player' and 'the best current male tennis player' denote one and the same individual office. It is thinkable that John knows who the No.1 ATP tennis player is without knowing who the best male tennis player is. But on the above assumption, we would obtain a contradiction. Therefore, hyperintensional *Know-wh**/$(o\iota *_n)_{\tau\omega}$ relating an agent to the mode of presentation, i.e. *construction* of an office, is certainly more plausible, and we are going to vote for this variant.

$$\lambda w\lambda t\ [{}^0\mathit{Know\text{-}wh}^*_{wt}\ {}^0\mathit{John}\ {}^0[\lambda w\lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0\mathit{ATP\text{-}ranking}_{wt}\ x] = {}^01]]]]$$

To *specify* the relation of *Know-wh** in more detail, we have to refine this concept. First, there is a *presupposition* that the No.1 in ATP ranking exists. If it were not so, then the answer to the question "Who is the No.1 player in ATP tennis singles?" would be 'nobody', which actually is the negated presupposition.[25] Second, if John knows who the No.1 player in ATP tennis singles is, then he must have identified a particular individual as the value of this office. Hence, we can explicate the relation *Know-wh** as *knowing the value* of the intension asked for and define this relation as follows.

Let $K^{*v} \to (o\iota *_n)_{\tau\omega}$ be the relation of knowing the value of an office $R/\iota_{\tau\omega}$ produced by a construction $C_R/*_n \to \iota_{\tau\omega}$, $a \to \iota$ an agent who knows the value, $x \to \iota$, and let *Ident(ified)*/$(o\iota\iota *_n)_{\tau\omega}$ be the relation between an agent, an individual, and the construction of an office such that the agent has identified that individual as the value of the produced office. Then, in any world w and time t of evaluation, the equivalence of the following definition holds:

Definition 4 (*knowing hyperintensionally the value of an office***)**

$$[K^{*v}_{wt}\ a\ {}^0C_R] =_{\mathrm{df}} \exists x\ [[x = C_{Rwt}] \wedge [{}^0\mathit{Ident}_{wt}\ a\ x\ {}^0C_R]]$$

Using Def. 4, we can specify the rules for such knowing.

(R1)

$$\frac{[K^{*v}_{wt}\ a\ {}^0C_R]}{\exists x\ [x = C_{Rwt}]}$$

Obviously, the second rule is this.

(R2)

$$\frac{[K^{*v}_{wt}\ a\ {}^0C_R]}{\exists x\ [{}^0\mathit{Ident}_{wt}\ a\ x\ {}^0C_R]}$$

Above, we said that we do not adhere to Hinttika's proposal to explicate knowing-wh by means of an existentially quantified formula and transforming knowing-wh into knowing-that because these are consequences of a proper definition of knowing-wh. Hence, the question arises: what does it mean 'to *identify* an individual as something'?[26] To answer this question, we are going to explicate the relation *Ident*/$(o\iota\iota *_n)_{\tau\omega}$ by means of *knowing that*.

Assume that John knows who the No.1 player in ATP tennis singles is, and the No.1 is Mr Djokovic. Do these assumptions entail John's knowing that Djokovic is the No.1 player in ATP tennis singles?

[25] Duží, M., Číhalová, M. (2015) deal with presuppositions of questions. They distinguish between a direct and complete answer to a question. The direct answer directly refers to the object asked for. The complete answer is a proposition that this or that object is the value of an intension asked for. The main idea is this. If the presupposition of a question is not true, then there is no direct answer. Instead, a plausible answer is a complete one, to wit, negated presupposition.

[26] As we deal with individual offices, we are going to define 'identifying the value of an office'. Generalization for properties or other intensions is obvious.

Though it seems undoubtedly, it depends on John's *deduction abilities*. According to Def. 4, John identified an individual x as the value of this office in the world w and time t of evaluation:

$$[K^{*v}_{wt}\ {}^0John\ {}^0[\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]]] \text{ iff}$$
$$\exists x\ [[x = [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]] \wedge [{}^0Ident_{wt}\ {}^0John\ x\ {}^0[\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]]]]$$

Thus, in any $\langle w,t \rangle$-pair of evaluation, we have:

1) $\exists x\ [[x = [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]] \wedge [{}^0Ident_{wt}\ {}^0John\ x\ {}^0[\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]]]]$ $\varnothing$
2) $[{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]] = {}^0Djokovic$ $\varnothing$
3) $[{}^0Ident_{wt}\ {}^0John\ x\ {}^0[\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]]]$ $\wedge$E, 1

Note that now we cannot simply substitute ${}^0Djokovic$ for x into the second conjunct of (1). To this end, we must *define* the relation-in-intension *Ident.* Above, we characterised it as the relation between agent a, an individual x and the construction C_R of an office such that a has identified that individual x as the value of the office. Hence, if $b \rightarrow \iota$ is the holder of the office, we can define:

Definition 5 (*identifying the holder of an office***)**. Let $C_R/*_n \rightarrow \iota_{\tau\omega}$ be the construction of an office R, $a \rightarrow \iota$ the agent who identified the holder of R, $x \rightarrow \iota$, $b \rightarrow \iota$. Then

$$[{}^0Ident_{wt}\ a\ x\ {}^0C_R] =_{\mathrm{df}} [[C_{Rwt} = b] \supset [{}^0Ident_{wt}\ a\ b\ {}^0C_R]]$$

Having defined *Ident*, we can finish the derivation of the proposition that John identified Djokovic as the individual playing the role of the No.1 in ATP ranking singles:

4) $[[{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]] = {}^0Djokovic] \supset$
$[{}^0Ident_{wt}\ {}^0John\ {}^0Djokovic\ {}^0[\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]]]$ Def.5
5) $[{}^0Ident_{wt}\ {}^0John\ {}^0Djokovic\ {}^0[\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]]]$ MP 2,4

To derive that John (hyperintensionally) knows that Djokovic is the No.1 in ATP ranking singles, i.e.

$$\lambda w \lambda t\ [{}^0Know^*_{wt}\ {}^0John\ {}^0[\lambda w \lambda t\ [{}^0Djokovic = [\lambda w \lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0ATP\text{-}ranking_{wt}\ x] = {}^01]]]_{wt}]],$$

we have to *postulate* that identifying b as the value of an office amounts to knowing that the value of the office is b. Hence, we specify the *Meaning Postulate*:

$$[{}^0Ident^*_{wt}\ a\ b\ {}^0C_R] = [{}^0Know^*_{wt}\ a\ {}^0[\lambda w \lambda t\ [b = C_{Rwt}]]]$$

Without this postulate, it is *not logically* derivable that knowing the value of an office is equivalent to knowing that this or that individual is the holder of the office. In order to ensure these desirable results even in case the agent does not have the capacity to derive the conclusion, we again specify the rules for transforming *Know-Wh* ($K^{*v} \rightarrow (o\iota *_n)_{\tau\omega}$) into *Know-that* ($K^* \rightarrow (o\iota *_n)_{\tau\omega}$) and vice versa.

Knowing-wh* $\Rightarrow$ *Knowing-that

$$\frac{[K^{*v}_{wt}\ a\ {}^0C_R] \wedge [b = C_{Rwt}]}{[K^*_{wt}\ a\ {}^0[\lambda w \lambda t\ [b = C_{Rwt}]]}$$

Similarly, if in a $\langle w, t \rangle$-pair of evaluation the agent a knows that b is the value of the office R produced by C_R, then by applying the following rule, a should know who is the holder of R in $\langle w, t \rangle$.

Knowing-that* $\Rightarrow$ *Knowing-wh

$$\frac{[K^*_{wt}\ a\ {}^0[\lambda w \lambda t\ [b = C_{Rwt}]]}{[K^{*v}_{wt}\ a\ {}^0C_R]}$$

As these rules capture basic patterns of reasoning with knowing-wh and knowing-that, they might contribute to a smooth communication of agents in a multi-agent system and to avoiding misunderstandings and inconsistencies among the agents.

3.5 Impossible objects

We have seen that the attitudes *Knowing-wh,* like 'knowing who', 'knowing what', etc., presuppose the existence of the object to be identified. In other words, the office in question must be occupied. But there are *impossible objects* denoted by descriptions '*the F*' such that nobody can know who or what the *F* is. For instance, nobody can know who the King of France, who is not a monarch, is or who is the divorced bachelor, as there are no such objects.[27]

As mentioned above, in TIL we do not deal with impossible individuals. The universe of discourse is a fixed element of the system's base, and thus, individuals trivially exist; we could pick a different base *B,* but then we would work in another system. In the case of individual offices, existence is the property of an office, to wit, the property of being occupied in a given world and time of evaluation. Yet, in the case of impossible objects, there is just one office that is necessarily vacant, i.e. not occupied in any world and time. However, the impossible objects denoted by descriptions like 'the divorced bachelor', 'the fake banknote that is a banknote' or 'the King of France who is not a monarch' are distinct. To differentiate between these distinct impossible objects, we deal with *hyper-offices*, i.e. distinct constructions of one and the same necessarily vacant individual office. These constructions must be in some way *inconsistent* to produce the necessarily vacant office. Hence, Duží, Jespersen and Glavaničová say in (2021):

Impossible individuals are explicated as inconsistent hyper-offices presenting the impossible office.

To develop a logical theory of offices, we examine the necessary relation *Requisite*/$(o(o\iota)_{\tau\omega}\iota_{\tau\omega})$ between an office and the properties such that any individual that happens to occupy the office must have these properties.[28] The collection of all the requisites of a given office is the *essence* of the office. The requisite relation is defined as follows.

Definition 5 (*Requisite relation between a property and an office***).** Let $P \rightarrow (o\iota)_{\tau\omega}$, $R \rightarrow \iota_{\tau\omega}$ be constructions of a property and an office, respectively, $x \rightarrow \iota$. Then, the property produced by P is a *requisite* of an office produced by R iff

$$\forall w \forall t\ [\forall x\ [[^0True_{wt}\ \lambda w \lambda t\ [x = R_{wt}]] \supset [^0True_{wt}\ \lambda w \lambda t\ [P_{wt}\, x]]]]$$

Assume now that R produces the impossible office, i.e. the office that is not occupied in any possible world w and time t. As a straightforward corollary of Def. 5, we obtain the result that *any* property is a requisite of the impossible office. It is so because the proposition $\lambda w \lambda t\ [x = R_{wt}]$ is not true in any world and time; it is gappy everywhere. Such an explosion of requisites simply highlights the fact that the impossibility inherent to the impossible office fails to impose any sort of restriction on what must be true of the occupant because there is none. Yet, we want to restrict the proliferation of requisites to those that are *conceptually relevant* to a given construction of the impossible office. We want to be able to infer that the fake banknote that is a banknote must be both a banknote and a fake banknote, but it is not a lion, a unicorn, a smoker, or what else not. In other words, the concept of the fake banknote that is a banknote subsumes the properties of being a banknote and being a fake banknote, but it does not subsume the properties of being a lion, etc. Since the properties of being a

[27] In this section we partly draw on material from Duží M., Jespersen B., Glavaničová D. (2021). It is included here for completeness to show their relevance to hyperintensional epistemology.

[28] Hence, we vote for individual anti-essentialism yet intensional essentialism. A requisite relation obtains between intensions of any type; for details, see Duží et al. (2010, Ch. 4).

banknote and being a fake banknote are *contrary* and logically exclusive, this hyper-office produces the impossible office.[29]

To provide a solution to the problem of the explosion of requisites, Duží, Jespersen and Glavaničová developed in (2021) a method of inferring conceptually relevant hyper-requisites of a given hyper-office. In order not to reinstate the problem of explosion to hyper-requisites, they do not apply 'ex falso quodlibet'. Not applying ex falso quodlibet is justified by the fact that the goal of the derivation is to prove the inconsistency in a given description. Hence, as soon as we find some witness of inconsistency, the derivation is stopped. The method is applied step by step. First, *primary* hyper-requisites of a hyper-office are derived; they are those constructions of properties that can be directly derived from the hyper-office as it immediately presents itself. If there is a pair of inconsistent properties, the derivation is terminated. Otherwise, *secondary* hyper-requisites are derived from primary ones in the same way, and so on, until the witness of inconsistency is found.

Definition 6. (*Primary hyper-requisite of a hyper-office*). Let $*R/*_n \rightarrow \iota_{\tau\omega}$; $*Req/*_n \rightarrow (o(o\iota)_{\tau\omega}\iota_{\tau\omega})$. Then the *primary hyper-requisites* $*Req$ of the hyper-office $*R$ are those property-producing constructions that are provably derivable from $*R$ without applying ex falso quodlibet.

Remark. In TIL, several kinds of proof calculus have been developed. They include, inter alia, a general resolution method adjusted to TIL, the sequent calculus and natural deduction.[30] Hence, by 'provably derivable', we mean the application of any of these methods.

As the goal is to track down an inconsistency in a given definition of the impossible office, we go on to derive secondary hyper-requisites of a given hyper-office $*R$. Secondary hyper-requisites of $*R$ are primary hyper-requisites of another hyper-office $*R'$, where $*R'$ is obtained by refining $*R$. Since the refined construction is provably equivalent to the original one in the sense of producing the same office, in case the office in question is impossible, we arrive after a finite number of steps at a pair of contradictory hyper-requisites, at which point we terminate the process.

To illustrate the application of the method, we now prove the inconsistency of the description 'the only bachelor who is divorced'. The hyper-office is this construction:

$$\lambda w\lambda t\ [{}^0\mathrm{I}\ \lambda x\ [[{}^0Bachelor_{wt}\ x] \wedge [{}^0Divorced_{wt}\ x]]]$$

Types. $\mathrm{I}/(\iota(o\iota))$: the singularizer; $x \rightarrow \iota$; $Bachelor, Divorced/(o\iota)_{\tau\omega}$.

Step 1. Derivation of primary hyper-requisites $*Req_0$: $\{{}^0Bachelor; {}^0Divorced\}$

1)	$[{}^0\mathrm{I}\ \lambda x\ [[{}^0Bachelor_{wt}\ x] \wedge [{}^0Divorced_{wt}\ x]]]$	∅
2)	$[[{}^0Bachelor_{wt}\ x] \wedge [{}^0Divorced_{wt}\ x]]$	IE, λE, 1
3)	$[{}^0Bachelor_{wt}\ x]$	$\wedge$E, 2
4)	$[{}^0Divorced_{wt}\ x]$	$\wedge$E, 2

Step 2. We refine the atomic concepts ${}^0Bachelor$ and ${}^0Divorced$ by replacing them with the ontological definitions of these properties. For the sake of simplicity, these definitions should do: "A bachelor is a man who has never been married." "Someone divorced is someone whose marriage has been dissolved". Since the property of being previously married is a pre-requisite of the property of being a man whose marriage has been dissolved, we can utilise the former to derive a subset of secondary hyper-requisites. To this end, here is the construction of both these properties:

'never been married': $\lambda w\lambda t\ [\lambda x\ \neg\exists t'\ [[t' \leq t] \wedge [{}^0Married_{wt'}\ x]]]$

'been previously married': $\lambda w\lambda t\ [\lambda x\ \exists t'\ [[t' < t] \wedge [{}^0Married_{wt'}\ x]]]$

[29] The properties of being a banknote and being a fake banknote are contrary because the modifier fake is *privative* with respect to the property of being a banknote. No fake banknote is a banknote but there are many individuals that are neither a banknote nor a fake banknote. For details, see Duží (2017) or Jespersen, Carrara, Duží (2017).
[30] See Duží, Fait (2021).

This gives us secondary hyper-requisites
$*Req_1 = \{\lambda w\lambda t\ [\lambda x\ \neg\exists t'\ [[t' \leq t] \wedge [{}^0Married_{wt'}x]]];\ \lambda w\lambda t\ [\lambda x\ \exists t'\ [[t' < t] \wedge [{}^0Married_{wt'}x]]]\}$.

Since these properties are contradictory, we have discovered an inconsistency that makes the hyperoffice impossible, and we finish at this point.

Again, an epistemic agent might have limited evidence that leaves many epistemic possibilities open. As the agent acquires more evidence, various possibilities are ruled out; in this case, the agent obtains a piece of knowledge that asking about who or what occupies an impossible office is futile.

4. Necessary objects

In this section, we examine definite descriptions that denote necessary offices. We have seen that purely contingent objects are offices denoted by descriptions involving Prior's weak 'the'. If an agent wants to know who, if any, occupies such an office, they need to regularly ask or otherwise empirically investigate whatever is the current state-of-affairs obtaining in the world, at which they are posing their question or carrying out their inquiry. Such an office is a revolving door of occupants. On the other hand, descriptions involving the strong 'the' denote necessary offices. Once the office becomes occupied, it is held by one and the same individual forever after. If an agent gets to know who the inventor of the zip is, they do not have to ask again. If the inventor of the zip is Gideon Sundback, then by applying the above rules for the transformation of knowing-wh into knowing-that, the agent knows that Gideon Sundback is the inventor of the zip and can insert this snippet into their knowledge base.

However, there is more to these offices than merely being necessary. One can ask which kind of necessity is involved. It is not an analytical necessity because if it were, then purely understanding the meaning of the description would be sufficient to know which individual (if any) occupies the office. Yet, to know, e.g., who is the Wimbledon 2023 female winner, one has to empirically explore objective reality to obtain the piece of information that it is Marketa Vondroušová. Any other female tennis player might, logically or epistemically/doxastically speaking, have won the tournament, and just a few lucky balls made Marketa win against Ons Jabeur.

Hence, this is 'semi-necessity' not unlike the nomic necessity of empirical physical laws; we assume that if a law obtains, then it obtains eternally. But there is eternity from the beginning to the end of time ('forever'), and there is eternity from a given moment onward and until the end of time ('ever since'). Bolzano makes the difference between eternal and sempiternal truth. Unlike eternal, sempiternal truth is everlasting, having a beginning but not an end. Hence, our semi-necessary offices could be characterised as sempiternal.[31]

The schematic form of such necessities is $\lambda w\forall t\ C$, where $C \to o$. To specify this semi-necessity more rigorously, here is the law.

Let $R \to \iota_{\tau\omega}$ produce an office, $x \to \iota$. Then the office is *semi-necessary* iff

$$\lambda w\forall t\ [\exists x\ [x = R_{wt}] \supset \forall t'\ [[t' \geq t] \supset [R_{wt} = R_{wt'}]]]$$

Note that the above construction formalises the characterisation we have specified above; if office R is occupied, then its holder will remain constant for all times in the future. It does not specify the time when office R was first occupied. This is as it should be because this information is not conveyed by the description.

In a multi-agent world of queries and answers, it is helpful to distinguish between purely contingent, impossible, and semi-necessary offices. If an office R is impossible, then we can apply the method specified above to prove the inconsistency in the description in order to know that it would be a futile activity to ask who or what the R is. On the other hand, if the office is semi-necessary, then once an agent obtains the piece of information that an individual a is an R, the agent can extend their

[31] See Bolzano (1972, §147). For more details, see Betti (2006).

knowledge base with this information and does not have to ask again.[32] But how do we differentiate between a purely contingent and a necessary office? At the linguistic level, syntactically, there is no hint as to whether 'the' is a strong or weak article. Číhalová, Rybaříková (forthcoming) has made a first attempt to analyse definite descriptions tagged by the strong 'the' and specify the list of typical categories of such descriptions. Here, we substantially expand and adjust those results by pointing out that these descriptions do not denote 'necessary' offices that are occupied by the same individual eternally; rather, using Bolzano's term, they are sempiternally necessary; in addition, we adjusted and extended the list of typical categories of such descriptions. Our proposal is not exhaustive, yet it can serve as a useful device in the communication of agents in a multi-agent world, and it can be extended as needed.

1. Specification of the *unique order* in the sequence of occupants of an office;
   examples: 'the first (second, third) President of Slovakia'
2. Specification of the *unique order* in the sequence of bearers of a property;
   Examples: 'the first child born in 2023'; 'the first man to run 100m under 9 s'
3. Descriptions with a fixed time reference;
   Examples: 'the US President in 2010', 'the female Wimbledon 2023 winner'.
4. The timeless uniqueness of the value of an empirical attribute like 'the inventor of something', 'the murderer of somebody', 'the author of something', 'the (biological) father/mother of somebody', etc.;
   Examples: 'the inventor of the zip', 'the founder of Transparent Intensional Logic', and 'the author of *Waverley*'.

In order for agents to use this classification when making decisions, we need to specify the general patterns of the analysis for each category.

First category. Here, the terms 'first', 'second', 'third', etc., denote modifiers of offices that produce modified offices. Hence, they are entities of type $(\iota_{\tau\omega}\ \iota_{\tau\omega})$, i.e. functions that associate a given office with another office.

Property modifiers are defined and analysed in Duží et al. (2010, § 4.4). A summary of different kinds of modifiers can be found in Duží (2017) or Jespersen et al. (2017). These deal with property modifiers, i.e. functions of type $((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$, and distinguish two basic categories of modifiers, namely *subsective* and *privative.* If M^s is a subsective modifier concerning a property P, then it holds for each individual x that if x is an $[M^sP]$ then x is a P. For instance, a skilful surgeon is a surgeon. On the other hand, if M^p is privative with respect to P, then for each x, it holds that if x is an $[M^pP]$ then x is a *non-P*, where *non-P* is a property contrary to P. It means that no individual can be both P and *non-P,* but many individuals are neither P nor *non-P*. The above papers define these modifiers in terms of the *essence* of a property P, where the essence is the set of all the requisites of P. While a subsective modifier enriches the essence of P with another property P' compatible with all the elements of the essence (so that, necessarily, i.e. in each world w and time t, the population of a modified property is a subset of the population of the root property), a privative modifier extends the essence with a property Q that contradicts some of the elements of the essence of the root property P. Hence, in each world and time, the population of P and the population of the modified property are disjunctive.

[32] There might be more individuals who occupy such an office; for instance, more inventors of something who worked together. Anyway, the same holds in this case though the value of an office would be a set of individuals. The office is semi-necessary. For the sake of simplicity, we deal here with offices whose value is a unique individual.

A similar approach can be applied to modifiers of intensions of any type. Here, we deal with modifiers of offices, which are entities of type $(\iota_{\tau\omega}\iota_{\tau\omega})$. The modifiers denoted by 'first', 'second', 'third', etc. are subjective. Each of the first, second, third, … President of Slovakia is a President of Slovakia. The analysis of these descriptions are these constructions:

$$[{}^0First\ \lambda w\lambda t\ [{}^0President\text{-}of_{wt}\ {}^0Slovakia]]$$
$$[{}^0Second\ \lambda w\lambda t\ [{}^0President\text{-}of_{wt}\ {}^0Slovakia]]$$
$$\ldots$$
$$[{}^0n^{th}\ \lambda w\lambda t\ [{}^0President\text{-}of_{wt}\ {}^0Slovakia]]$$

Note that though the office of the President of Slovakia is occupied now (writing in early 2024) by Zuzana Čaputová, the office of the 7th President of Slovakia is vacant. However, upon the expiry of her term, the office of the 7th President of Slovakia can become occupied. Hence, once an agent obtains the information that the 7th President of Slovakia is individual *a*, it is superfluous to investigate the situation again, as the agent would know that this office is semi-necessary.

Second category. The terms 'first', 'second', 'third', etc. denote *modifiers of properties* which produce offices. Hence, they are entities of type $(\iota_{\tau\omega}(o\iota)_{\tau\omega})$, functions that associate a property with an office. These modifiers are again subsective, as the first child born in 2023 is a child born in 2023. The schema of analysis is a construction of the form $[{}^0First\ P]$, $[{}^0Second\ P]$, …, $[{}^0n^{th}\ P]$, where *First*, *Second, …,* $n^{th}/(\iota_{\tau\omega}(o\iota)_{\tau\omega})$, $P \rightarrow (o\iota)_{\tau\omega}$.

In both of these categories, there is a question of how to obtain the value of the semi-necessary office. This issue is beyond the topic of this paper, as it is a pragmatic matter. For instance, agents can aim to insert the chronology of the holders of a given office into their knowledge base. It will be a sequence of time intervals $T_1, T_2, \ldots, T_n$, all of type $(o\tau)$ such that T_i precedes T_{i+1} for all i ($1 \leq i < n$). Then, it is easy to choose the ith-interval and determine who (if anybody) the ith-holder is.

Third category. Descriptions with a fixed time reference are invariably semi-necessary. Hence, once an agent knows the answer to the question of who the holder of such an office is, they also know that this holder is going to hold the office forever. However, an answer can be obtained only when querying about the *past* unless the agent has a reliable fortune-teller at their disposal.[33] For instance, one can hardly rely on the answer to the question of who will win the Wimbledon tennis tournament in 2053. Hence, the schematic analysis of reasonable Wh-questions such that the reference time interval is set in the past is this. Let $T/(o\tau)$ be the reference time interval; $R \rightarrow \iota_{\tau\omega}$; $t, t' \rightarrow \tau$; $x \rightarrow \iota$. Then, the following construction produces the office of the holder of *R* in *T.*

$$\lambda w\lambda t\ [{}^0\mathrm{I}\lambda x\ \exists t'\ [[t' \leq t] \wedge [{}^0T\ t'] \wedge [x = R_{wt'}]]]$$

For instance, the Wh-question about the US President in 2010 is transformed into this construction:

$$\lambda w\lambda t\ [{}^0\mathrm{I}\lambda x\ \exists t'\ [[t' \leq t] \wedge [x = [{}^0President\text{-}of_{wt'}\ {}^0USA]] \wedge [{}^02010\ t']]]$$

To answer the question, one can apply the method described in Section 3.3.

[33] There are two views on sentences in the future tense, *deterministic* and *indeterministic*. According to the determinists (for instance, Tichý), a sentence in the future tense is true, false or gappy now, only we do not yet *know* which one it is. For instance, imagine that in 1980 the future-teller says: "Václav Havel will be the last president of Czechoslovakia and the first president of the Czech Republic". In 1980, when Havel was imprisoned and severely persecuted by the communist regime, hardly anybody would have believed the fortune-teller. Yet, in 1992 or later, we would say that the future-teller was right. According to the indeterminists (for instance, Prior), such sentences cannot be true (or false) now. For our purposes, though, it is not important which of the two opinions is accepted, as the agent *cannot know* whether the proposition denoted by a sentence in the future tense is true now or will be true in the future. Therefore, here we consider as reasonable only questions about the present and the past.

Fourth category. This is a complicated case because, from the linguistic point of view, there is no hint as to whether a given empirical attribute is timelessly unique. For instance, the attributes ‘the (biological) mother of somebody’, ‘the author of something’, and ‘the inventor of something’ come with a property so as to create a semi-necessary office when applied to an argument, like *the inventor of the zip* or *the author of Waverley*, while the other attributes do not come with such a property. For instance, ‘the mayor of something’ and ‘the president of something’ do not make the office necessary. A linguistically competent human agent can decide which category the attribute belongs to. Yet, if we want to automatize reasoning about such attributes, we must note that a software agent lacks this ability. Such an agent does not know whether the resulting office is purely contingent or semi-necessary. Hence, the only way forward would be to tag such descriptions with ‘the only forever’ and build up a list of them. This is a task for computational linguistics, exemplifying how logic and linguistics work hand in hand.

5 Conclusion

In this paper, we have investigated the notion of knowing the respective values of the offices, which are denoted by various kinds of definite descriptions. We distinguished three basic categories: those denoting purely contingent offices, those denoting the impossible office, and those denoting semi-necessary offices. We analysed each category and proposed in each case a method for answering Wh-questions about the holder of an office so that the agents can enrich their knowledge base with such knowledge. As a result, we have shown that in the case of purely contingent objects, the agents have to ask again whenever needed, as the holder can change over time. By contrast, in the case of impossible objects, asking for a holder is futile. Moreover, by applying the hyperintensional approach of Transparent Intensional Logic, we demonstrated how to distinguish between different hyper-offices that nevertheless produce one and the same necessarily vacant intensional office, and also how to obtain a witness of inconsistency in a description of an impossible object. Finally, we dealt with semi-necessary objects, i.e. those offices that, if once occupied, are then occupied by the same individual forever after. We put forward four categories of such offices. These results contribute toward agents’ reasoning in a multi-agent world.

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