Quantum Bayesianism Assessed

John Earman
Dept. HPS
University of Pittsburgh

Abstract: The idea that the quantum probabilities are best construed as the personal/subjective degrees of belief of Bayesian agents is an old one. In recent years the idea has been vigorously pursued by a group of physicists who fly the banner of quantum Bayesianism (QBism). The present paper aims to identify the prospects and problems of implementing QBism, and it critically assesses the claim that QBism provides a resolution (or dissolution) of some of the long standing foundations issues in quantum mechanics, including the measurement problem and puzzles of non-locality.

1 Introduction

Bruno de Finetti, the patron saint of the personalist (a.k.a. subjectivist) interpretation of probability, famously proclaimed that "THERE ARE NO PROBABILITIES" (de Finetti 1990, p. x). Decoding the bombast, what de Finetti meant to convey is the claim that there are no probabilities in nature, no objective physical probabilities (a.k.a. chances); there are only the degrees of belief of individual agents who, if rational, regiment their credences in conformity with the axioms of probability theory. Although an appendix in Vol. 2 of de Finetti's *Theory of Probability* (1990, pp. 313-321) sketches some of the formalism of quantum mechanics (QM), de Finetti himself did not take up the challenge of showing how physicists' deployment of quantum probabilities can be understood in personalist terms. Philosophers of science have an abiding concern with the nature and meaning of probability, and it is natural that some of them have taken up this challenge (see, for example, Pitowsky 2006). But in recent years the major push for a personalist reading of quantum probabilities has come not from philosophers but from physicists who march under the banner of quantum Bayesianism (or QBism for short). In the vanguard are Carleton Caves, Christopher Fuchs, and Rüdiger Schack, and David Mermin.[1] Hans Christian von Baeyer, a recent convert to QBism,

[1] A presentation of QBism intended for a philosophy of science audience is to be found in Caves, Fuchs, and Schack (2007). Presentations aimed at a general physics au-

has offered a manifesto intended for the educated lay audience (von Baeyer 2016).

What the QBians propose is a program for understanding quantum physics that promises to resolve (or dissolve) some of the long standing puzzles that emerge from quantum theory. Although still in the development stage, the program has made considerable progress, and it is mature enough that it can benefit from critical assessment. And, indeed, critical reviews have already appeared, an especially insightful and evenhanded one supplied by Timpson (2008).[2] I will offer an assessment of QBism that is at once more sympathetic and more critical: QBians create faux difficulties for themselves; they refuse to accept victories that are readily at hand and, thereby, fail to convey some of the strengths of their stance; more importantly, they fail to recognize lacunae in and awkward features of their program; and they oversell their attempted resolution of foundations puzzles.

At the outset let me stipulate that, for present purposes, I accept one form of quantum Bayesianism; namely, the inductive inferences that physicists make about quantum events is best discussed in a Bayesian personalist framework. But quantum events have a non-commutative structure, and this necessitates appropriate adjustments in the familiar Bayesian framework for classical probabilities—in particular, adjustments are required in the classical conditionalization rule for updating personal probabilities. This is something that apparently de Finetti did not recognize and is not clearly explained in QBism literature. This mild form of quantum Bayesianism is, of course, perfectly compatible with a rejection of the monism of de Finetti and the QBians in favor of a pluralism with respect to the interpretation of probability in quantum physics: more specifically, a pluralism that sees the need both for a personalist interpretation of the probabilities used to reconstruct the inferences of quantum physicists *and* for an objectivist interpretation of the probabilities delivered by the apparatus of QM, probabilities that ostensively are about the tendencies exhibited by quantum systems rather than the credences of quantum physicists.

Such pluralists owe us an account of how the two types of probability

dience are to be found in Mermin (2012, 2014) and Fuchs, Mermin, and Schack (2014). Versions of Fuchs' QBian manifesto are to be found in Fuchs (2002, 2010). More technical presentations of various aspects of QBism include Caves, Fuchs, and Schack (2002a, 2002b, 2002c), Fuchs and Schack (2004, 2013), Fuchs, Schack, and Scudo (2004), and Schack, Brun, and Caves (2001).

[2] See also Bacciagaluppi (2013), Palge and Konrad (2008), and Stairs (2011).

are related. Philosophers' instinct is to postulate, the most famous example being David Lewis' Principal Principle (PP) which postulates a principle of rationality linking objective chance and rational credence.[3] Mathematicians' instinct is to prove rather than postulate. In the case of quantum probabilities it turns out that proof suffices in that a form of the Principal Principle is a theorem of quantum probability, a result the QBians can use to argue that so-called objective quantum probabilities are just the objectification of personalist probabilities. But, curiously, QBians turn their backs on such result—an example of what I mean by refusing to accept victories readily at hand.

The plan of the paper is as follows. Section 2 sketches an approach to quantum probabilities that facilitates a comparison between the objectivist vs. subjectivist interpretation of quantum probabilities, and it sets out the grounds for the former. The subjectivist interpretation promoted by the QBians is discussed in Section 3. It is able to get a foothold through the basic representation theorem of quantum probabilities, a theorem that is much more powerful than de Finetti's representation theorem for classical probabilities in that it does not rely on exchangeability or other substantive requirement on credence functions, save for a condition on additivity. And it is shown how the QBians can gain firmer footing by means of some corollaries of the basic representation theorem. Thus far I have tried to help the QBians put the best face on their program. But in Section 4 I air some qualms about the QBian approach to quantum probabilities, none of which is even near fatal but together may dampen enthusiasm for pursuing the QBian program. This enthusiasm is further dampened by a critical examination in Section 5 of claims that QBism delivers a resolution of the quantum measurement problem and puzzles about quantum non-locality. Conclusions are contained in Section 6. An Appendix discusses the so-called operational approach to QM to which the QBians sometimes appeal.

[3] The original formulation of PP is in Lewis (1980). Exactly how to formulate and justify Lewis' idea has generated considerable controversy in the philosophical literature. For present purposes it is not necessary to wade into this controversy.

2 The formalism of QM and the objectivist reading of quantum probabilities

2.1 The algebraic formulation of QM

In the approach to quantum theory adopted here a quantum system is characterized by an algebra of observables and a set of states on the algebra. For present purposes the algebra is assumed to be a von Neumann algebra $\mathfrak{N}$ acting on a Hilbert space $\mathcal{H}$.[4] To make the discussion manageable the case of ordinary non-relativistic QM (sans superselection rules) will be the focus of discussion, which is to say that $\mathfrak{N}$ is the Type I factor algebra $\mathfrak{B}(\mathcal{H})$, the von Neumann algebra of all bounded operators acting on $\mathcal{H}$.[5] In many applications it suffices to use a Hilbert space that is separable (i.e. has a countable orthonormal basis), but applications requiring higher dimensional spaces can be contemplated.

A projection $E \in \mathfrak{B}(\mathcal{H})$ is a self-adjoint element such that $E^2 = E$. The range $Ran(E)$ of E is a closed subspace of $\mathcal{H}$, and for $\mathfrak{B}(\mathcal{H})$ the projections are in one-one correspondence with the closed subspaces of $\mathcal{H}$. The projections $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ have a lattice structure that derives from a partial order whereby $E_1 \leq E_2$ iff $Ran(E_1) \subseteq Ran(E_2)$.[6] That $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ forms a lattice means that it is closed under meet $E_1 \wedge E_2$ and join $E_1 \vee E_2$ of $E_1, E_2 \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$, which are defined respectively as the greatest lower bound and the least upper bound and are respectively the projections corresponding to $Ran(E_1) \cap Ran(E_2)$ and the closure of $Ran(E_1) \cup Ran(E_2)$. Projections E_1 and E_2 are said to be mutually orthogonal iff $Ran(E_1) \cap Ran(E_2) = \varnothing$. When E_1 and E_2 are mutually orthogonal $E_1 \wedge E_2 = E_1 E_2 = E_2 E_1 = E_2 \wedge E_1 = 0$ and $E_1 \vee E_2 = E_1 + E_2$.

The elements of the projection lattice $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ are referred to as quantum propositions (also yes-no questions, or quantum events). Quantum probability theory may be thought of as the study of quantum probability functions Pr on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ (see Hamhalter 2003). Pr is required to satisfy

(a) $\text{Pr}: \mathcal{P}(\mathfrak{B}(\mathcal{H})) \rightarrow [0, 1]$

[4] The relevant operator algebra theory can be found in Bratelli and Robinson (1987) and Kadison and Ringrose (1997).

[5] A factor algebra $\mathfrak{N}$ has a trivial center, i.e. $\mathfrak{N} \cap \mathfrak{N}' = \mathbb{C}I$, where $\mathfrak{N}'$ denotes the commutant of $\mathfrak{N}$. When superselection rules are present the center is non-trivial.

[6] This is equivalent to requiring that $E_1 \leq E_2$ iff $E_2 - E_1$ is a positive operator.

(b) $\Pr(I) = 1$, where I is the identity operator

(c) $\Pr(E \vee F) = \Pr(E + F) = \Pr(E) + \Pr(F)$ for all mutually orthogonal pairs $E, F \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$.

The condition (c) of finite additivity may be strengthened to require complete additivity

(c*) $\Pr(\sum_{a\in\mathcal{I}} E_a) = \sum_{a\in\mathcal{I}} \Pr(E_a)$ for any family $\{E_a\}$ of mutually orthogonal projections.[7]

When $\mathcal{H}$ is separable any family of mutually orthogonal projections is countable, in which case (c*) reduces to the condition of countable additivity.

Quantum states are normed positive linear functionals mapping elements of $\mathfrak{B}(\mathcal{H})$ to $\mathbb{C}$. Most standard texts on QM assume that the physically realizable states are the normal states $\mathcal{N}$, states with a density operator representation, i.e. there is a density operator ϱ (a trace class operator on $\mathcal{H}$ with $Tr(\varrho) = 1$) such that $\omega(A) = Tr(\varrho A)$ for all $A \in \mathfrak{B}(\mathcal{H})$.[8] This assumption requires justification; for present purposes suffice it to say that several lines of argument converge to provide a strong motivation,[9] but occasionally there are rumblings in the literature supporting the use of non-normal states in interpreting features of QM. A vector state is a state ω such that there is a vector $\psi \in \mathcal{H}$ with $\omega(A) = \langle\psi|A|\psi\rangle$ for all $A \in \mathfrak{B}(\mathcal{H})$. Vector states are normal, and for $\mathfrak{B}(\mathcal{H})$ vector states coincide with the pure states, i.e. states ω such that there are no distinct states ω_1 and ω_2 and real numbers $0 < \lambda_1, \lambda_2 < 1$, $\lambda_1 + \lambda_2 = 1$, such that $\omega = \lambda_1\omega_1 + \lambda_2\omega_2$. Impure states are also referred to as mixed states.

Quantum states on $\mathfrak{B}(\mathcal{H})$ induce quantum probability functions on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$. It is easily checked that for any quantum state ω on $\mathfrak{B}(\mathcal{H})$, $\Pr^{\omega}(E) := \omega(E)$ for $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is a quantum probability, i.e. satisfies the conditions (a)-(c). Furthermore, if ω is a normal state then $\Pr^{\omega}$ is completely additive, whereas if ω non-normal then $\Pr^{\omega}$ will be merely countably or merely finitely additive depending on the dimension of $\mathcal{H}$. It is natural to ask whether the process of moving from quantum states on $\mathfrak{B}(\mathcal{H})$ to probabilities on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ can be

[7] If $\mathcal{I}$ is infinite, the convergence of $\sum_{a\in\mathcal{I}} E_a$ is taken in the weak operator topology.

[8] This is what physicists call the Born rule for calculating probabilities see below.

[9] For a discussion of arguments in favor of identifying the normal states with the physically realizable states, see Ruetsche (2011) and Arageorgis et al. (2017).

reversed so that quantum states can be recovered from quantum probabilities. This question will be answered below.

2.2 The objectivist reading of quantum probabilities

On the objectivist interpretation of quantum probabilities quantum states codify observer independent physical features of quantum systems. Thus, on this interpretation, the probabilities induced by quantum states are, pace de Finetti, objective physical probabilities. The objectivist reading of quantum probabilities has more going for it than merely postulating theoretical entities and sticking the label 'objective' on them; for it is supported by infrastructure of the theory which gives an account of state preparation, at least for the normal pure states. A basic interpretational tenet of QM holds that the elements of the projection lattice $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ are in principle verifiable/refutable by appropriate 'yes-no' experiments. The theory itself does not provide a manual for how to construct a laboratory device for carrying out the experiment—that belongs to the experimental practicum of QM.

Among the elements of $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is the support projector S_φ for a normal state, defined as the smallest projection in $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ to which φ assigns probability 1. Since normal pure states on $\mathfrak{B}(\mathcal{H})$ are coextensive with the vector states and $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ contains all projectors, the support projector S_φ for a normal pure state φ is the projector onto the ray spanned its vector representative. As a result, S_φ for a normal pure state serves as a filter for φ in the set $\mathcal{N}$ of all normal states, viz. for any $\omega \in \mathcal{N}$ (pure or impure) such that $\omega(S_\varphi) \neq 0$, $\omega(S_\varphi A S_\varphi)/\omega(S_\varphi) = \varphi(A)$ for all $A \in \mathfrak{B}(\mathcal{H})$ (see Ruetsche and Earman 2011). By the von Neumann projection postulate, when a measurement of $F \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is made on a system initially in state ω and the measurements returns a Yes answer, the new state of the system is $\omega_F(\bullet) := \omega(F \bullet F)/\omega(F)$. Hence, whatever the initial state ω, as long as $\omega(S_\varphi) \neq 0$, a Yes answer to a measurement of S_φ for a normal pure state φ ensures that the new state ω_{S_φ} is φ. A normal impure (or mixed) state does not have a filter, and so there is no preparation procedure in this sense for impure states (see Ruetsche and Earman 2011).

So much for the formalism. It is now time for Nature to weigh in. Prepare a quantum system in a normal pure state φ. Conduct a yes-no experiment for some $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$, and record the result. Reset the system (or a similar system) in the same state φ, and repeat the yes-no experiment for E. The probability for a Yes outcome on any trial is $\varphi(E)$ regardless of the outcomes on any other trials, which is to say the experimental protocol gives i.i.d. trials. The strong law of large numbers thus implies that as the number of trials tends to infinity the frequency of Yes responses almost surely tends to $\varphi(E)$. In actual realizations of such experiments expectations are fulfilled in that there is rapid apparent convergence to the value supplied by the quantum formalism. Of course, the inductive skeptic will caution that the apparent convergence may disappear if trials are continued into the indefinite future, but such skepticism if pushed too far would undermine all scientific inquiry. In short, it is hard to resist the notion that the probabilities delivered by the quantum formalism latch on to objective features of the physical world.

3 The personalist interpretation of quantum probabilities

The most natural way to try to implement de Finetti's personalism in QM is to reject the idea that quantum probabilities on the projection lattice $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ arise from quantum states on $\mathfrak{B}(\mathcal{H})$ construed as codifying objective features of physical systems (call this the top-down perspective); rather the starting point should be probability functions on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ construed as the credence functions of actual or potential Bayesian agents (call this the bottom-up perspective). To make such reading viable three things are required: a representation theorem relating probability measures on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ to quantum states on $\mathfrak{B}(\mathcal{H})$ (this provides the route 'up' for the bottom-up perspective); a rule for updating a credence function on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$; and a personalist alternative to the objectivist account of state preparation. All three are available (almost entirely) off-the-shelf, although the QBians convey the impression that there is a need to reinvent the wheel.

3.1 The Gleason representation theorem and the status of quantum states

Perhaps mimicking de Finetti's "THERE ARE NO PROBABILITIES, Christopher Fuchs writes that "THERE ARE NO QUANTUM STATES" (Fuchs 2002, Sec. 9 and 2010, Sec. II). Decoding the bombast of the latter, Fuchs' slogan is not meant to assert the absurdity that there are no quantum states qua expectation functionals on the algebra of observables, but rather that these mathematical objects do not carry the ontological clout that the objectivists assign them. That is, for QBians quantum states do not codify objective features of the quantum system, and they do not induce objective chances on the projection lattice of quantum propositions; rather, quantum states are to be viewed as bookkeeping devices that can be used to keep track of credences that QBian agents assign to elements of $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$.

This proposed shift of perspective gains its initial purchase through Gleason's theorem, the basic representation theorem for quantum probabilities:

> *Prop. 1.* Let $\mathcal{H}$ be a separable Hilbert space and suppose that $\dim(\mathcal{H}) \geq 3$. For any probability measure Pr on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ there is a unique extension of Pr to a quantum state ω^{Pr} on $\mathfrak{B}(\mathcal{H})$. Further, if Pr is countably additive (respectively, merely finitely additive) then ω^{Pr} is a normal (respectively, non-normal) quantum state.

Gleason's theorem has been extended to quite general von Neumann algebras, including the Type III algebras encountered in the algebraic formulation relativistic QFT (see Maeda 1990 and Hamhalter 2003).

Thus far there is nothing in the mathematics that favors the objectivist top-down point of view—wherein quantum probabilities are objective chances that arise from normal quantum states which codify objective features of quantum systems–vs. the QBian bottom-up perspective—wherein probabilities on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ are to be interpreted as arising from the credence functions of QBian agents, and normal quantum states are merely devices that represent these completely additive credence functions. This is about to change to the detriment of QBism.

3.2 Updating quantum probabilities

Suppose that a QBian agent learns that $F \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is true. How should she update her personal probability function Pr to accommodate this new knowledge? The most commonly accepted rule is motivated by the following proposition:

> *Prop. 2.* Let Pr be a countably additive quantum probability measure on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ for separable $\mathcal{H}$ with $\dim(\mathcal{H}) \geq 3$, and

let $F \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ be such that $\Pr(F) \neq 0$. Then there is a unique functional $\Pr_F(\bullet)$ on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ such that (a) $\Pr_F(\bullet)$ is a quantum probability, and (b) for all $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ such that $E \leq F$, $\Pr_F(E) = \Pr(E)/\Pr(F)$.[10]

Condition (b) is the direct quantum analog of the condition that uniquely picks out classical conditionalization as the rule for updating in classical probability (aka Bayes updating). The unique $\Pr_F(\bullet)$ is called Lüders conditionalization or, as I will refer to it, Lüders updating.

What is $\Pr_F(\bullet)$? The answer is something of an embarrassment to the QBians because it requires a detour though quantum states. It is easy to verify that if ω is a normal state and $F \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is such that $\omega(F) \neq 0$ then $\omega_F(E) := \omega(FEF)/\omega(F)$, $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$, defines a normal state ω_F. Hence, $\Pr^{\omega_F}(E) = \omega_F(E)$, $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$, defines a countably additive quantum probability if $\mathcal{H}$ is separable and ω is normal. The functional $\Pr_F(\bullet)$ picked out by Prop. 2—the Lüders F-updating of Pr—is the probability $\Pr^{\omega_F}(\bullet)$ induced by ω_F where ω is the unique normal state that by Gleason's theorem corresponds to the countably additive Pr.

Why is this detour through states necessary? When $E, F \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ commute $FEF = EF = FE = E \wedge F \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ and, thus, Lüders updating can be expressed as $\Pr_F(E) = \Pr(E \wedge F)/\Pr(F)$, which agrees with classical Bayes updating. However, when E and F don't commute $FEF \notin \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ and Lüders updating cannot be expressed as $\Pr_F(E) = \Pr(FEF)/\Pr(F)$ since $\Pr(FEF)$ is undefined. If quantum states were merely bookkeeping devices used to track the degrees of belief QBians agents they should be dispensable. But apparently they are needed for something as basic as updating the QBian belief functions.

3.3 Objectification

The interpretation of quantum states I attributed to the QBians has the consequence that there are as many quantum states are there are actual or potential Bayesian agents. This seems to fly in the face of quantum state preparation which ostensibly establishes a unique objective, observer independent state. A reconciliation can be achieved by means of some (almost) off-the-shelf results.

[10] See Bub (1977) and Cassinelli and Zanghi (1983). The latter authors generalize this result to arbitrary von Neumann algebras.

Prop. 3. Let Pr be a countably additive probability function on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ where $\dim(\mathcal{H}) \geq 3$ and $\mathcal{H}$ separable. If φ is a normal pure state such that $\Pr(S_\varphi) \neq 0$ (S_φ the support projector for φ) then $\Pr_{S_\varphi}(\bullet) = \varphi(\bullet)$.

The proof is straightforward. By Gleason's theorem Pr extends uniquely to a normal state $\omega^{\Pr}$ on $\mathfrak{B}(\mathcal{H})$. If $\Pr(S_\varphi) \neq 0$ then $\omega(S_\varphi) \neq 0$ and, thus,

$$\Pr_{S_\varphi}(E) = \frac{\omega^{\Pr}(S_\varphi E S_\varphi)}{\omega^{\Pr}(S_\varphi)} = \varphi(E), \quad E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$$

where the last equality follows by the filter property of S_φ.

From the objectivist perspective Prop. 3 can be viewed as a special case of Lewis' PP: when an agent learns that S_φ is true she learns that the objective chances are given by the state φ; updating by Lüders conditionalizing on this knowledge brings her credences into line with the objective chances assigned by φ. Contra what the philosophical literature on PP assumes, there is no need here for an additional principle of rationality; for the alignment of credence and chance is guaranteed as a theorem of quantum probability, at least for agents whose credence functions are countably countably.[11]

QBians will put a different spin on Prop. 3, emphasizing an immediate corollary:

Cor. Let φ be a normal pure state. All quantum Bayesian agents who have countably additive credence functions on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$, with $\dim(\mathcal{H}) \geq 3$, experience merger of opinion when updating on S_φ, provided that these credence functions give non-zero prior probability to S_φ; specifically, they all concur that the updated credence for any $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is $\varphi(E)$.

QBians can now sermonize as follows: We QBians can speak with the vulgar without being vulgar; that is, we can speak of 'objective' quantum probability without admitting that it is the probability induced by quantum states construed as characterizing objective features of a quantum system (the top-down perspective). Rather, 'objective' quantum probability emerges from the bottom-up perspective as the objectified or merged opinion that results when Bayesian agents update their prior opinions by conditionalizing on the proposition S_φ expressed by the support projection of a normal pure state

[11] For more on Lewis' Principal Principle in QM see Earman (2018).

φ. To be sure this merger of opinion only applies to the subset of Bayesian agents who give S_φ a non-zero prior; but that is as it should be since those who give S_φ a flatly zero prior will not be swayed by evidence that other agents take as decisive in favor of S_φ. The lack of universality of merger only goes to underscore, the QBians will say, the subjectivist foundation of so-called objective probability.

To my knowledge, QBians nowhere propose even this modest form of reconciliation, perhaps because they think it compromises the staunch form of personalism they favor. For example, Caves, Fuchs, and Schack (2007) seem to think that they need to perform the following modus tollens move: If facts determined the quantum state then QBism would be wrong; but it is not wrong and, therefore, facts do not determine the quantum state.

> The objective-preparations view [of quantum states] supports the seeming need for a PP-style account by positing that classical facts about a preparation device determine the prepared quantum state ... The subjective Bayesian interpretation of quantum probabilities contends, in contrast, that facts alone never determine a quantum state. What the objective-preparations view leaves out is the *essential* quantum nature of the preparation device, which means that the prepared quantum state always depends on prior beliefs in the guise of a quantum operation that describes the preparation device. (Caves, Fuchs, and Schack 2007, pp. 262-263; italics in original).

A "PP-style account" of state preparation is not something imposed by disciples of David Lewis: the quantum PP encapsulated in Prop. 3 is a theorem of quantum probability and, thus, is not something the QBians can choose to ignore. Nor does the account of state preparation using Prop. 3 posit classical facts or deny the quantum nature of the preparation device.

Caves et al. proceed to repeat (as if repetition can induce truth) the conclusion of their modus tollens:

> In the Bayesian view, a prepared quantum state is not determined by facts alone, but always depends on prior beliefs ... Facts, in the form of measurement outcomes, are used to update the prior state ... but they never determine a quantum state ... (ibid, p. 266)

It is quite true that when state preparation is seen through the lens of Bayesianism, facts in the form of measurement outcomes never determine a quantum state *in the sense wanted by the objectivist, i.e. an objective physical feature of the system that dictates probability assignments in a top-down manner.* For a quantum state is seen through the QBian lens as simply a device for representing credences, and what happens in so-called state preparation is that the prior credences of a Bayesian agent are Lüders updated on the measurement outcome, producing new credences and, ipso facto, a new state which is (as the QBians rightly note) dependent on the prior credences. But facts, in the form of measurement outcomes, can determine a quantum state *in the sense of Bayesian personalism* since, as seen above, conditionalizing on said facts can lead to the same credence function (= quantum state on the QBian understanding) for *all* those agents who do not exclude a priori the measurement outcomes (by assigning them flatly zero credence). In sum, I find it baffling that QBians should refuse to accept the obvious way in which their view can accommodate state preparation.

4 Some speed bumps on the road to QBism

In the preceding section I attempted to help the QBians frame their view of quantum probabilities in a manner that would make it seem a viable alternative to the objectivist view. I now turn to worries about the QBian program: two concern gaps in my framing of QBism; a third concerns the Dutch book justification for axioms of probability in the case of high dimensional Hilbert spaces; and the fourth concerns a disquieting implication of the QBian instrumentalist treatment of quantum states.

4.1 Dimension 2

For $\dim(\mathcal{H}) = 2$ and indeed for any $\mathcal{H}$ such that $\dim(\mathcal{H}) < \infty$ all quantum states on $\mathfrak{B}(\mathcal{H})$ are normal. But for $\dim(\mathcal{H}) = 2$ Gleason's theorem does not hold, and there are probability functions on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ that are not extendible to any quantum state on $\mathfrak{B}(\mathcal{H})$.[12] On the objectivist top-down approach

[12]Example: For $\dim(\mathcal{H}) = 2$ define a probability function on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ as follows. Choose two orthogonal rays R_1, R_2 in $\mathcal{H}$ and denote their corresponding projections by E_1, E_2. Set $\Pr(E_1) = \Pr(E_2) = 1/2$. For any other pair of orthogonal rays R_3, R_4 and their corresponding projections E_3, E_4 set $\Pr(E_3) = 0$ and $\Pr(E_4) = 1$ or vice versa.

this failure of Gleason's theorem is no cause for alarm; the rogue probability functions can simply be dismissed as mathematical oddities because they are not induced by any quantum state and, therefore, cannot codify objective chances. But the failure of Gleason's theorem should be a cause for alarm on the bottom-up personalist reading of probabilities. For there do not seem to be any non-question begging grounds of rationality on which a Bayesian agent with such a quantum non-compatible credence function can be excluded from the QBian camp as having irrational credences. And to exclude such an agent simply because he has a non-quantum compatible credence function is to compromise Bayesian personalism and, thus, QBism insofar as it embodies personalism.

This is only a minor embarrassment for the QBians since the real world is not described by a q-bit space; indeed, to describe a system as simple as a single spinless particle moving in space requires an infinite dimensional Hilbert space, where Gleason's theorem applies. But it is an embarrassment nevertheless since q-bit spaces are often used by QBians themselves to illustrate quantum probabilities. When one moves from finite dimensional to infinite dimensional Hilbert spaces a different set of worries assault QBism.

4.2 The Born rule, normal states, and countable additivity

Some QBians struggle mightily over the status of the 'Born rule.' For example, in "QBism, the Perimeter of Quantum Bayesianism" Fuchs likens the Born rule to one of the Biblical Ten Commandments.

> The Born Rule is not like the other classic laws of physics. Its normative nature means, if anything it is more like the Biblical Ten Commandments ... The Born Rule guides, 'Gamble in such a way that all your probabilities mesh together through me.' The agent is free to ignore the advice, but if he does so, he does so at his own peril. (Fuchs 2010, p. 8)

And later the Born rule is said to be an addition to Bayesian probability

And finally set $\Pr(I) = 1$. This Pr is not induced by any state on $\mathfrak{B}(\mathcal{H})$. For all states on $\mathfrak{B}(\mathcal{H})$ with $\dim(\mathcal{H}) = 2$ are normal; and any normal state and, thus, the probability function it induces on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$, is continuous in the weak, strong, and ultra-weak operator topology (see Brattelli and Robinson 1987, Theorem 7.1.12). But the Pr in question is obviously not continuous.

> not in the sense of a supplier of more-objective probabilities, but in the sense of giving extra normative rules to guide the agent's behavior when he interacts with the physical world. (ibid, p. 12).

But there is no mystery here, biblical or otherwise; nor is extra normative guidance required. By Gleason's theorem a countably additive (respectively, completely additive) measure Pr on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ with $\mathcal{H}$ separable (respectively, non-separable), whether or not it is given a personalist interpretation, corresponds to a normal state ω^{Pr} on $\mathfrak{B}(\mathcal{H})$ and, therefore, the probabilities are given by the Born rule (a.k.a. trace rule), viz. there is a density operator $\rho_{\omega^{\mathrm{Pr}}}$ on the Hilbert space on which $\mathfrak{B}(\mathcal{H})$ acts such that $\mathrm{Pr}(E) = \omega^{\mathrm{Pr}}(E) = Tr(\rho_{\omega^{\mathrm{Pr}}}E)$ for all $E \in \mathcal{P}(\mathfrak{N})$.

Still the QBians fret about the status of $\rho_{\omega^{\mathrm{Pr}}}$ in the Born rule—does its use allow the reviled objectivism to come through the back door? But QBians should stop fretting since they have already answered this concern: for them the quantum state ω^{Pr} is just a bookkeeping device for a Bayesian agent's credences codified in Pr and, further, the density operator $\rho_{\omega^{\mathrm{Pr}}}$ is just a computational device the book keeper can employ in calculating probabilities. I am baffled again: Why find problems for your view where there are none?

Actually there is a problem lurking underneath the above discussion. It arises from the assumption that Pr is countably or completely additive. Perhaps this is the true source of QBians' swivet about the Born rule. On the top-down objectivist approach to quantum probabilities the Born rule is equivalent to the assertion that only normal states are physically realizable. As mentioned above there are strong but not definitive arguments for this assertion. On the bottom-up personalist approach to quantum probabilities when $\dim(\mathcal{H}) = \infty$ (the real world case) and $\mathcal{H}$ is separable (respectively, non-separable) the Born rule holds for a Bayesian agent just in case that agent's credence function on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is countably additive (respectively, completely additive). So if the QBians want to hew to the Born rule they must exclude from their camp agents with non- countably or non-completely additive credence function, and seemingly the only non-question begging grounds on which they can justify such an exclusion is to maintain that such credence functions violate the norms of rationality of belief. Pursuing this line would, by the way, cast out their patron saint de Finetti who hewed to finite additivity[13], and it would enmesh the QBians in the on going

[13] For de Finetti's views on finite vs. countable additivity, see his (1972, Ch. 5), (1990, Vol. 1, Sec. 3.11), (1990, Vol. 2, Sec. 18.3). I read de Finetti as saying not only that ra-

debate in the Bayesian statistics literature on merits of finite vs. countable additivity. This is not the place to review the debate, and I simply refer the interested reader to the relevant literature (see Kadane et al. 1986 and Seidenfeld 2001). But in the next subsection I will note that there is a problem lurking here for QBians who want to run the Dutch book argument for quantum probabilities.

4.3 Dutch book and quantum probabilities

QBians try to launch their program by transferring to the quantum context the Dutch book argument originally used by de Finetti and others in the classical setting in order to show that rational degrees of belief ought to satisfy the axioms of probability (see Caves et al. 2002). The argument consists of two parts: the Dutch book theorem, whereby it is shown that if an agent's degrees of belief fail to conform to the axioms of probability then there is a family of bets each of which the agent finds fair or favorable but which taken together guarantee that the agent will lose money in every possible outcome (Dutch book); and the converse Dutch book theorem, whereby it is shown that conforming to the axioms of probability confers immunity to Dutch book. De Finetti used the Dutch book argument to motivate finite additivity, but the argument can be extended to cover countable additivity as well.

So far so good. But problems arise when the Hilbert space $\mathcal{H}$ is non-separable. An example is given by an infinite spin chain consisting of a countable infinity of spin sites each of which may be a state of either spin up or spin down, requiring a Hilbert space of dimension $2^{\aleph_0}$ which, if the contunuum hypothesis is correct, is the power of the continuum. The developments discussed above are easily rewired to cover such cases if the probability Pr is completely additive. For example, Gleason's theorem holds in that a completely additive Pr on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ has a unique extension to a normal state on $\mathfrak{B}(\mathcal{H})$. Similar extensions to non-separable Hilbert spaces hold for Props. 2 and 3 if Pr is completely additive. The trouble comes in trying to use Dutch book considerations to justify complete additivity. If (as seems plausible) a rational agent should only be required to stand ready to accept

tional credence does not require countable additivity but also that countable additivity ought to be rejected in favor of finite additivity. But even if de Finetti is read as taking the less harsh position that countable additivity is acceptable but finite additivity is good enough for rational credence, he and his followers create a problem for the QBians.

bets that she regards as favorable (as opposed to merely fair) then although the Dutch book theorem shows that a failure of countable additivity means that the agent is subject to a sure loss, a failure of complete additivity does not entail a sure loss (see Skyrms 1992).

A possible out for the QBians lies in the fact that for ordinary QM, with $\mathfrak{B}(\mathcal{H})$ the algebra of observables, complete additivity on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ reduces to countable additivity unless $\mathcal{H}$ has a dimension as great as the least measurable cardinal. Since no known applications of QM require Hilbert spaces of such huge dimension, countable additivity suffices FAPP. However, this reduction of complete additivity to countable additivity does not hold for the observable algebras encountered in relativistic QFT (see Arageorgis et al. 2017).

4.4 Dynamics: Schrödinger and Heisenberg evolution

In the conventional quantum mechanical treatment, the state dynamics is given by Schrödinger evolution. Assuming that the Hamiltonian H of the system is essentially self-adjoint, its exponentiation to $U(t) := \exp(-i\hbar Ht)$, $-\infty < t < +\infty$, is a strongly continuous unitary group in the parameter t, which is interpreted as time. If the state on the algebra $\mathfrak{B}(\mathcal{H})$ of observables is ω_0 at $t = 0$ then the state at $t > 0$ is $\omega_t(\bullet) := \omega_0(U^{-1}(t) \bullet U(t))$. If ω_0 is a vector state corresponding to the vector $|\psi_o\rangle$ then ω_t is a vector state corresponding to the vector $|\psi_t\rangle = U(t)|\psi_o\rangle$, per the familiar "Schrödinger picture." On the top-down objectivist interpretation of quantum probabilities, according to which quantum probabilities arise from quantum states, the temporal evolution of probabilities is the evolution induced by the Schrödinger evolution of states, viz. $\Pr_t(E) := \Pr^{\omega_t}(E) := \omega_t(E)$ for $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$.

In QBism where the quantum state is merely a device for representing the credence function of a Bayesian agent, the state changes only when the agents's credence function changes. Changes in the credence function can happen because the agent updates her credence function on new information, or because of some sort of drift in beliefs uninformed by new information. QBians do not discuss the latter possibility, and for good reason since uninformed drift does not have a rational explanation. The upshot for QBians is that, as far as rational agents are concerned, there is no Schrödinger state evolution between updating events.

How then does QBism account for the conventional expectation—whose success is borne out in the statistics of measurement outcomes—that if the probabilities at $t = 0$ are those induced by the state ω_0 then at $t > 0$ the probabilities are those induced by the state ω_t Schrödinger evolved from ω_0? The answer, of course, is that the QBians can adopt a form of Heisenberg evolution: if at $t = 0$ an observable is represented by a self-adjoint operator $A_0 \in \mathfrak{B}(\mathcal{H})$ then at $t > 0$ the observable is represented by $A_t := U^{-1}(t)A_0U(t)$. In the conventional treatment of quantum evolution, Heisenberg and Schrödinger evolution are flip sides of the same coin since $\omega_0(A_t) = \omega_t(A_0)$. But in QBism Heisenberg evolution has primacy since the right hand side of this equality makes no sense for the QBian unless it is understood as a notational variant of the left hand side. Similarly, restated in the language of probability functions on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ the conventional treatment makes the Heisenberg and Schrödinger evolution of probabilities flips side of the same coin since for $E_0 \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$, $\Pr_0(E_t) = \Pr_t(E_0)$ where $\Pr_0(\bullet) := \Pr^{\omega_0}(\bullet)$, $\Pr_t(\bullet) := \Pr^{\omega_t}(\bullet)$, and $E_t := U^{-1}(t)E_0U(t)$. But the QBian will insist that the Heisenberg expression $\Pr_0(E_t)$ is the preferred expression for probability dynamics and the Schrödinger $\Pr_t(E_0)$ expression is be understood as a notational variant.

What is disquieting about the QBian stance here is the dualism it implies: there is something like a realist/objectivist commitment to the structure of quantum observables and their temporal evolution but an instrumentalist/subjectivist attitude towards quantum states. But reasons for a realist/objectivist commitment on observables are of a piece with reasons for a realist/objectivist commitment on states. The QBians may reply that the dualism serves them well in resolving puzzles that have bedeviled discussions of the foundations of QM. This claim is addressed in the following section.

5 Does QBism tame foundations puzzles?

QBism deals with puzzles in the foundations of QM mainly by sidestepping them. Sidestepping a problem can be an honorable tactic, but in the present instance the QBian form of this tactic comes with costs. One of the main means of avoidance is the QBian rejection of the value assignment rule that is typically adjoined to the objectivist take on quantum states, and the price for the rejection is a loss in explanatory power. The discussion below is confined to the case of ordinary QM.

The objectivist value assignment rule is often stated for a special case of an observable with a discrete spectrum: If the state of the system is a vector state $|\psi\rangle$ and $|\psi\rangle$ is an eigenstate of a self-adjoint operator O with eigenvalue o then the system possesses a value o of the observable corresponding to O. The natural generalization of this rule in the algebraic formulation is:

> (R) If ω is the state of the system is a pure state and $\omega(E) = 1$, where $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$, then E is true.

A converse rule is often bruited but remains controversial because it helps to generate the measurement problem in QM (see below): If the state of the system is a vector state $|\psi\rangle$ then the system possesses a value o of the observable corresponding to O only if $|\psi\rangle$ is an eigenstate of O with eigenvalue o. The natural generalization to the algebraic formulation is:

> (C) If the state ω of the system is a pure state then $E \in \mathcal{P}(\mathfrak{B}(\mathcal{H}))$ is true only if $\omega(E) = 1$.

It should be clear why the QBians must reject (R): for them quantum states are merely ways to represent the degrees of beliefs of agents, and the move from '$\Pr(E) = 1$ for me (or for any agent)' to 'E is true' would lead to contradictions. QBians must reject (C) as well; for 'E is true' does not imply that any particular Bayesian agent assigns degree of belief 1 to E.

Such rejectionism, coupled with the refusal—and the apparent inability—to provide alternative valuation rules, allows the QBians to avoid issues that may leave the objectivist perplexed. For example, before Schrödinger's infamous box is opened, is the cat alive, or is it dead, or is it neither? And what does 'neither' mean? When the QBian asked such questions he simply smiles and replies: 'I do not bother with such matters since I have no means of answering one way or another because I reject the objectivist's value assignment rules and decline to provide others. All I am concerned with is assigning degrees of belief to propositions about what will be found when Schrödinger's box is opened.' Although it initially seems liberating, the appeal of such sidestepping quickly wears thin.

5.1 The measurement problem

In conventional quantum mechanics the measurement problem arises from a refusal to take "measurement" as a primitive, unexplained term of the theory and the insistence that a measurement is to be treated as an interaction between an object system and a measurement instrument—and, perhaps, also an interaction between the measuring instrument and an observer who reads the indicator dial on the measuring instrument. The problem arises because, as any number of no-go theorems show, the description of the interaction in the composite system (observer + measuring instrument + object system) in terms of Schrödinger/Heisenberg evolution seems incapable of accounting for observed measurement outcomes and, indeed incapable of accounting for the fact that measurements have determinate outcomes if the objectivist value assignment rules (R) and (C) are applied.

The proposed solutions to the measurement problem are so various as to defy a neat classification, but for present purposes it suffices to take the main divide to be between collapse vs. non-collapse solutions. Some non-collapse solutions, such as the family of modal interpretations (see Lombardi and Dieks (2014)) maintain an objectivist stance on quantum states but reject (C) and assign a value to an observable even when the state of the system is not an eigenstate of the observable being measured.[14] Here I ignore such interpretations and concentrate on collapse solutions.

On the objectivist reading of quantum states, "state vector collapse" involves a physical change in the state of the quantum system wherein Schrödinger evolution is interrupted, and the superposition of the pre-measurement state collapses into an eigenstate of the observable being measured.[15] The QBian dissolution of the measurement problem can be classified as a species of collapse interpretation, but with the seemingly attractive feature that no physical explanation of state vector collapse is required. Since for the QBian the quantum state is nothing but a representation of a Bayesian agent's credences, state vector collapse is nothing but a change in the mathematical representation of the agent's credence function that takes place when the agent updates on the information about the measurement outcome.

So far so good. But QBism does nothing to help resolve the core issue of why measurements have definite outcomes or, in more QBian friendly

[14]Other non-collapse interpretations include many worlds interpretations (see Vaidman 2015) and Bohmian mechanics (see Goldstein 2013).

[15]Among the collapse interpretations is the now almost forgotten proposal of Wigner (1961, 1963) whereby the action of the consciousness of the observer produces a change in the state of the instrument and object systems, as well as the more highly regarded family of GRW interpretations (see Ghirardi 2011) which propose physical mechanisms for producing state vector collapse.

terms, why QBian agents experience definite outcomes. As long as a QBian agent is treated as an abstract, disembodied probability calculator that is fed information by an oracle the issue can be avoided. But it resurfaces for physically embodied observers, such as ourselves, whose information acquisition has to be treated quantum mechanically in terms of an interaction with the (measurement apparatus + object system). QBians may respond that they are concerned only with their own personal experiences but not with explaining why they have these experiences or, indeed, why they have any definite experiences at all. The experiences are what they are, and QBism is content to organize and relate them through the probably calculus. One way of pursuing this line would lead to a solipsistic phenomenalism. That at least would be an interesting position. Most QBians deny that what they are aiming for is phenomenalism. But their subjectivist interpretation of quantum states deprives them of the resources to tackle questions about the relation of agents to a non-phenomenalistic world. Trying to make a virtue out of this seems a stretch.

5.2 Non-locality puzzles

Worries about spooky action-at-a-distance are generated by stirring together three elements: entangled quantum states, an objectivist interpretation of states, and a collapse account of measurement. To illustrate, consider the singlet state for a two-particle system. This is a vector state whose generating vector takes the form

$$|\Psi(1,2)\rangle = \frac{1}{\sqrt{2}}[|\psi_1^{\uparrow}\rangle \otimes |\psi_2^{\downarrow}\rangle - |\psi_1^{\downarrow}\rangle \otimes |\psi_2^{\uparrow}\rangle]$$

where '$\uparrow$' and '$\downarrow$' indicate respectively spin-up and spin-down along the z-axis.[16] Two agents, Ted and Alice interact with the system, with Ted making a measurement on particle 1 and Alice making a measurement on particle 2, their respective measurements being made at relatively spacelike positions.[17] If Ted measures spin along the z-axis then according to the collapse interpretation the singlet state collapses to $|\psi_1^{\uparrow}\rangle \otimes |\psi_2^{\downarrow}\rangle$ or to $|\psi_1^{\downarrow}\rangle \otimes |\psi_2^{\uparrow}\rangle$ according

[16] This is an example of a quantum entangled state that violate the Bell inequalities. For an overview of the meaning and implications quantum entanglement see Earman (2015).

[17] Such talk can be made precise in the setting of the algebraic formulation of relativistic QFT where there is an explicit association of observables with spacetime regions.

as Ted obtains spin-up or spin-down. And on the objectivist interpretation of quantum states this change produces an objective change at Alice's location. To be sure, no-go theorems ensure that this procedure cannot be used by Ted to send a faster-than-light message to Alice (essentially because Ted cannot control the outcome of his measurement on particle 1). But nevertheless, there does seem to be some spooky action-at-a-distance involved, although commentators have a hard time characterizing the spookiness and articulating why it should be disturbing (but never mind).

Some QBians think that their reading of quantum probabilities offers simple and pain-free remedy for the spookiness:

> QBist quantum mechanics is local because its entire purpose is to enable any single agent to organize her own degrees of belief about the contents of her own personal experience ... Quantum correlations, by their very nature, refer only to time-like separated events: the acquisition of experiences of any single observer. Quantum mechanics, in the QBian interpretation, cannot assign correlations, spooky or otherwise, to space-like separated events since they cannot be experienced by any single agent. Quantum mechanics is thus *explicitly* local in the QBian interpretation. (Fuchs, Mermin, and Schack 2014, pp. 750-751; italics in original)

This is a possible form of QBism, but a very crabbed and disappointing one. On the more expansive version of QBism sketched above, QBian agents are not concerned merely with the contents of their own personal experiences; for they assign degrees of belief to all the elements of the projection lattice, and for a multiparticle system this means assigning degrees of belief to observables associated with relatively spacelike regions of spacetime. To put the point the other way around, in relativistic QFT states entangled over the algebras associated with relatively spacelike regions have been shown to be generic among the normal states. It is thus tantamount to abandoning the interpretational game to adopt a form of QBism that excludes agents whose credence functions are represented by such states on the grounds that such states do not represent degrees of belief about the contents of the personal experience of any one Bayesian agent.

Let us see how a less extreme form of QBism might confront locality worries. For sake of discussion let us suppose, contra Fuchs, Mermin, and

Schack (2014), that the singlet state represents the initial credence function of both Ted and Alice regarding the two-particle system. Ted measures spin along the z-axis for particle 1 and gets, say, spin-up. He Lüders updates on this information, and his updated credence function is represented by the state $|\psi_1^\uparrow\rangle \otimes |\psi_2^\downarrow\rangle$—state vector collapse to be be sure, but nothing mysterious about it on the QBist interpretation of quantum states. Alice may receive word of Ted's result by means of a conventional non-superluminal telegram, but until then or until she makes her own measurement on particle 2 there is no change in her credence function for the two-particle system and, a fortiori, no change in her state for the two particle system. When she does receive word from Ted or makes her own measurement on particle 2 then her credence function changes and, thus, her state for the two particle system undergoes collapse; but again the collapse is completely non-mysterious on the QBian reading of quantum states. At no stage is there any spooky action-at-a-distance.

This initially looks like a success story for QBism. But the success comes at a price. Suppose that Ted and Alice don't exchange telegrams but simply carry out their respective spin measurements on the two particles and record their results. After repeating the procedure many times they meet to compare results, and they find that the measured spins of the two particles are always anti-correlated. The objectivist interpretation of quantum states explains why: the initial state of the object system is the objective (observer independent) singlet state; upon measurement this state collapses either to $|\psi_1^\uparrow\rangle \otimes |\psi_2^\downarrow\rangle$ or to $|\psi_1^\downarrow\rangle \otimes |\psi_2^\uparrow\rangle$ for both Ted and Alice (and for the whole damn army too); and by the value assignment rule (R) the collapsed state $|\psi_1^\uparrow\rangle \otimes |\psi_2^\downarrow\rangle$ (respectively, $|\psi_1^\downarrow\rangle \otimes |\psi_2^\uparrow\rangle$) implies that particle 1 has spin-up and particle 2 has spin-down (respectively, particle 1 has spin-down and particle 2 has spin-up). But, of course, the price paid for this explanation is some spookiness.

On the QBian story, by contrast, the collapsed states are simply representations of the post-measurement epistemic states of Ted and Alice. The QBian story explains why both Ted and Alice *expect*, with degree of belief one, to find anti-correlated spins, but since a QBians agent cannot validly move from '$\Pr(E) = 1$ for me' to 'E is true' the QBian story does not explain why the measured spins *are in fact* anti-correlated. Retreating to the position that QBism is concerned only the experiences of individual agents does not provide a safe haven. For each of our agents has the experience of finding a spin outcome for one of the particles coupled with the experience of the other agent reporting the opposite spin outcome for the other particle. Again, each agent expects this with personal probability 1; but QBism does not explain why the agents have these experiences.

In sum, on the considered version of QBism there is no spooky action-at-a-distance, but also no explanation of striking patterns of correlated events. On the objectivist interpretation there is an explanation of the correlation, but it brings with it the threat of spooky action-at-a-distance. Take your pick.

6 Conclusion

As presented above, the formalism QM lends itself to both pluralist and monist stances on quantum probability: the pluralists, who recognize both subjective and objective probability, can say that the formalism allows credence and chance to smoothly mesh; the monists of the de Finetti stripe, who only countenance probability as subjective degree of belief, can (with some caveats noted in Section 4) equally say that the formalism allows them to treat so-called objective chance as objectified credence. Something outside the formalism is needed to break the standoff. Those of a realist bent will tend to side with the pluralists, the idea being that the impressive empirical success of the probabilistic predictions of QM calls for an explanation that the monist-subjectivists are seemingly incapable of providing. But as philosophers of science are all too aware, this line of argumentation leads directly into the swamp of the realism vs. instrumentalism debate from which none who enter ever return.[18]

A more productive line of inquiry is to contrast how the opposing views on the nature of quantum probabilities impact on foundations issues in QM. The objectivist stance generates puzzles aplenty. The QBian subjectivist stance "resolves" these puzzles by side stepping them; specifically, by rejecting the objectivist value assignment rules linking quantum states to possessed properties and refusing to provide alternative rules, pleading that QBians are concerned only with organizing their expectations about their experiences with quantum systems. Adopting the objectivist stance requires taking the foundations puzzles seriously and then searching either for a better understanding of how the quantum formalism maps onto the world or for a modification of

[18] See Chakravartty (2014) for an overview of the status of scientific realism.

the formalism that preserves the empirical success of QM without generating the puzzles. Adopting the QBian stance avoids all this angst—from the QBian perspective QM requires no modification or new interpretation rules. It is tempting to choose complacency over hard work. But be aware that such complacency will never to lead to progress in understanding a non-phenomenalistic quantum world.

Appendix: Operational QM

A limitation of the present inquiry is its allegiance to the once standard view that observables in QM correspond to self-adjoint operators and that outcomes of measurements correspond to projection operators. This orthodoxy has been challenged by the advocates of so-called operational approach to QM (see Busch et al. 1995) which describes measurements in terms of "effects." One immediate benefit of this approach is that it yields a Gleason type theorem even for the case of $\dim(\mathcal{H}) = 2$. Define the effect algebra $\mathcal{A}(\mathfrak{N})$ associated with a von Neumann algebra $\mathfrak{N}$ acting on $\mathcal{H}$ by $\mathcal{A}(\mathfrak{N}) := \{A \in \mathfrak{N}_{sa} : 0 \leq A \leq I\}$ where $\mathfrak{N}_{sa}$ denotes the self-adjoint elements of $\mathfrak{N}$ and $\leq$ is the usual partial order relation whereby $A \leq B$ iff $B - A$ is a positive operator, i.e. $\langle (B-A)\psi, \psi\rangle \geq 0$ for all $\psi \in \mathcal{H}$. A "generalized probability measure" on $\mathcal{A}(\mathfrak{N})$ is a measure that satisfies analogs of axioms (a), (b), (c*) for *all* effects. For the case of ordinary QM with $\mathfrak{N} = \mathfrak{B}(\mathcal{H})$ and $\mathcal{H}$ separable Busch (2003) shows that a generalized probability measure on the effect algebra $\mathcal{A}(\mathfrak{B}(\mathcal{H}))$ has, regardless of $\dim(\mathcal{H})$, a unique extension to a quantum state, which is normal when the generalized probability measure is countably additive.

QBians occasionally make use of this operational approach, but it is unclear how it lends itself to something that deserves to be called quantum Bayesianism. For Bayesians, classical or quantum, probability is degree of belief, the objects of which are propositional entities. In contrast to the projection lattice $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$, the effect algebra does not have a natural propositional structure. For one thing, $\mathcal{A}(\mathfrak{B}(\mathcal{H}))$ does not form a lattice under $\leq$, i.e. the meet and join of two elements of $\mathcal{A}(\mathfrak{B}(\mathcal{H}))$ may not lie in $\mathcal{A}(\mathfrak{B}(\mathcal{H}))$. The effect algebra $\mathcal{A}(\mathfrak{B}(\mathcal{H}))$ is a lattice under an alternative partial order $\leq_s$(called the spectral order) that is coarser than $\leq$ (see de Groote 2005).[19] However, if negation for effects is defined by $\neg A := I - A$ then the relations $A \vee_s \neg A = I$ and $A \wedge_s \neg A = 0$ hold iff A is a projection.

[19] I am grateful to Tom Pashby for this reference.

Next note that the principle of finite additivity for effects requires that $\Pr(A+B) = \Pr(A) + \Pr(B)$ for all $A, B \in \mathcal{A}(\mathfrak{B}(\mathcal{H}))$. How can this principle to justified by a Dutch book construction—to which the QBians appeal (see Caves et al. 2003a)—when A and B are non-commuting and there is no possibility of settling simultaneous bets on A, B, and $A + B$?

Finally, consider the rule for updating on an effect $A \in \mathcal{A}(\mathfrak{B}(\mathcal{H}))$ that has been proposed by the the advocates of the operational approach, viz., updating on $A \in \mathcal{A}(\mathfrak{B}(\mathcal{H}))$ is given by $\Pr(\bullet) \longrightarrow \Pr_A(\bullet) =: \Pr(A^{1/2} \bullet A^{1/2}) / \Pr(A)$. Note that the difficulty discussed above in Section 3.2 for Lüders updating probabilities on $\mathcal{P}(\mathfrak{B}(\mathcal{H}))$ does not arise for affects; for if $A, B \in \mathcal{A}(\mathfrak{B}(\mathcal{H}))$ then $ABA \in \mathcal{A}(\mathfrak{B}(\mathcal{H}))$ even if A and B don't commute, so a detour through states is not needed. However, the proposed updating rule does not produce what is comfortably interpreted as a conditional probability; in particular, $\Pr_A(A) \neq 1$ unless A is a projection. Perhaps the QBians will reply that QM teaches us that the notion of what counts as the objects of belief has to be liberalized to include non-propositional entities. Pursuing this matter is a project for another occasion.[20]

Acknowledgment: I am grateful to Aristidis Arageorgis, Gordon Belot, Tom Pashby, and Laura Ruetsche for helpful comments on this project. Needless to say, I alone am responsible for the opinions express herein.

[20] Although POVMs may not further the case of QBism, they are useful in studying the foundations of QM. See, for example, Pashby (2014) for a discussion of how POVMs can be used to illuminate various aspects of time in QM.

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