What is Complexity? - The philosophy of complexity *per se* with application to some examples in evolution.

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Abstract²

It is argued that complexity has only a limited use as a paradigm against reductionist approaches and that it has a much richer potential as a comparable property.

What can complexity be usefully said to be a property of is discussed. It is argued that it is unlikely to have any useful value as applied to "real" objects or systems. Further that even relativising it to an observer has problems. It is proposed that complexity can usefully be applied only to constructions within a given language.

It is argued that complexity is usefully differentiated from the concepts of size, ignorance, variety, minimum description length and order.

A definition of complexity is proposed which can be summarised as "that property of a language expression which makes it difficult to formulate its overall behaviour even when given almost complete information about its atomic components and their inter-relations.". Some of the consequences of this definition are discussed.

It is shown that this definition encompasses several existing varieties of complexity measures and is then applied to some examples pertaining to the evolution of "complex" systems including: "What is the complexity that has evolved in organisms and has it increased?".

1 Introduction

The label of "complexity" often performs much the same role as that of the name of a desirable residential area in Estate Agent's advertisements. It is applied to many items beyond the original area but which are still somewhere in the vicinity. It thus helps in the item's promotion by ensuring that a sufficient number of people will enquire into the details, but this does not mean that this wider use is ideal if you want to perform a more precise analysis. In this paper I concentrate on the use of complexity

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^{2.} This paper is a little expanded and altered since the original abstract.

as a tool for analysis. I hope to clarify the concept and thus make it a more useful tool in the analysis of evolution and its effect on the evolved.

In Section 2, I will discuss the broad use of complexity as a paradigm for new approaches to intractable systems; in Section 3, I will discuss the useful scope of the idea of complexity; in Section 4, I will try to clear some of the surrounding clutter of associated concepts paving the way to a definition in Section 5. Then the relation of this definition to some other definitions used in the field are discussed in Section 6, followed by the application of the definition to broader questions concerning the connection of complexity and evolution in Section 7.

Some of the ideas contained here have been at least partially expressed elsewhere, but not brought together in this form. To credit all of them would make the paper very unwieldy, for some surveys of the field see [4, 6, 7, 9, 10, 23, 13, 28, 29].

2 Complexity as a paradigm

Some authors³ use complexity as a paradigm of some sort of holism. In this case it indicates that the authors consider many systems to be unameanable to a reductionist/Newtonian scientific method or analysis. ⁴ They suggest that any attempt to cope with complexity using such traditional tools is doomed to failure. Their remedies vary from a complete abandonment of scientific method to introducing new techniques and approaches.

The problem with this approach to complexity is that is does not allow for many meaningful comparisons w.r.t. complexity; a whole spectrum of statements of the form "x is *more* complex than y" are rendered trivial. From this point of view of paradigmatic complexity, the starting point of evolution is simple, as it is amenable to a reductionist approach, while the end point (us) is complex. At some point (presumably) there is a transition from simple to complex. We are left with a single catastrophe from a system amenable to reductionist science to one that is not, with no prior or subsequent development of this characteristic being counted as meaningful. Since we are interested in discovering the mechanism of complexity emergence, this is very unhelpful. Here I will take the pragmatic approach that many systems are *practically* intractable by such traditional methods and likely to remain so for some time, so that the broader question becomes a purely philosophical issue, which I will henceforth ignore.

Of course, any constructive suggestions for dealing with "complexity" are welcome, from whatever source. Thus all the techniques of the new sciences of "Complexity" are welcome techniques for studying what have been considered intractable systems. Here we have a second, softer, paradigm of complexity as a banner under which a clutch of new techniques and methodology is making some headway with hitherto intractable systems. Many of these techniques, however, have nothing to do with complexity *per se*. Thus many papers with the word "complexity" in the title refer merely to some techniques for dealing with rather difficult systems.

3 What can complexity usefully refer to?

The first question is about the scope of complexity. Since I am writing from a pragmatic perspective, I am only concerned with the *useful* application of this concept. If complexity could *theoretically* be applied to a particular system but would be completely impractical, this is of no help.

^{3.} e.g. [16, 21, 27]

^{4.} This often seems coupled with a denial of the Church-Turing thesis.

The most obvious use is with reference to real systems: "The brain is complex." or "Humans are more complex than amoeba.". This approach has several severe problems:

- The "true complexity" of real objects (if it existed) would probably be totally beyond us;
- The complexity of things seems to vary depending on which aspect you are concerned with;
- Any "real" complexity is irrelevant, as we never interact with the total systems, only certain aspects of them at any one time;
- The complexity of an object is only revealed though interaction with the complexity of another system (typically us);
- There would be little connection between this "real" complexity and the complexity of that object as represented in communications about it.

All of these problems are connected. We are ourselves systems that can deal with limited complexity (as are all the systems we have created). In fact, I would argue that, the reduction in complexity of our inter-action with any part of reality is an *essential* component of our existence (see Section 7.3).

In response to these problems many authors⁵ have stressed that complexity only makes sense when considered as relative to a given observer. Thus there is an established split between the "system complexity" and the "observer complexity". The system complexity is the complexity of the system w.r.t. the observer and the observer complexity is the complexity of the observer w.r.t. the system. There are still some problems with this approach, namely:

- It is still difficult to ascribe useful meaning to the complexity of the observer w.r.t. the system unless the system is an observer too, otherwise the observer is itself unobserved and so undefined in the sense of the "real" systems above;
- The complexity of the system w.r.t. the observer will still vary according to which aspect of the observed system is being considered by that observer;
- If the system as observed is not representable in terms of a communicable language (i.e. it is restricted to an internal model of some kind) then this model suffers from many of the above difficulties w.r.t. ascribing complexity to real objects.

For these reasons I, like others⁶, am going to restrict my application of the concept of complexity to representations within finitely presented languages. Thus talking about complexity will necessitate indicating the language of representation that this is relative to. This will help to clarify some of the confusions around this concept and make the study of the evolution of complexity more productive.

4 Some concepts related to complexity

Before I consider what complexity is, I would like to talk about some ideas that surround the concept. All of these have some *connection* with complexity but would make for very weak *definitions* of it. Examining these cases will help motivate and illustrate my definition of complexity (in Section 5).

4.1 Size

The use of complexity as size is usually casual and not meant as anything more than an *ad hoc* definition. Examples include "the size of a genome" and "the number of species in an ecology". Here size is an *indication* of the general difficulty in dealing with the system and the *potential* for a complex

^{5.} e.g. [7]

^{6.} e.g. [15]

system. To see that this is not a sufficient definition of complexity compare a microprocessor with a chest of nails, nobody would exclaim "My, how complex!" on opening the chest of nails. Here the parts (represented in most normal ways) are not inter-related and hence the size of the system is insufficient for it to be counted as complex. Often there is an assumption that all such parts are inter-related in situations where this definition is used. Making this assumption clear by specifying the language of representation and the relationship of size with complexity would help make this assumption transparent.

To take a biological example: in terms of a language of chemical inter-actions, the genome of a human being may well be more complex than that of wheat, despite the fact that the latter is about 10 times bigger.

4.2 Ignorance

The use of complexity to describe systems of which we are necessarily ignorant has close connections to the use of complexity as a paradigm. Examples might be "The brain is too complex for us to understand" or "as we understand more about chaotic systems, the boundary of complexity recedes". Complexity can be a *cause* of ignorance, but to *completely* associate the two assumes that there are not any other significant causes. It is not very helpful to describe the internal state of an electron as complex just because we are ignorant about it.

4.3 Minimum description size

One of the most interesting of the serious definitions of complexity is that of the minimum possible length of a description in some language, also called Kolmogorov⁷ complexity⁸. The language is frequently taken to be that of a Turing machine. Thus if a description can be greatly compressed without loss, then this is counted as simpler than one that cannot. By this definition highly ordered expressions come out as simple and random expressions as maximally complex.

This measure has more to do with the amount of *information* in an expression than its complexity. For example it has been shown [5] that you can not prove that expressions are (much) more complex than the original axioms in a formal system, despite the fact that there is no intuitive limit to the complexity of possible theorems in the system. This reading is confirmed by the close connection of this with other (entropic) measures of information like Shannon's [31]. Expressions with little information will be limited in their complexity, but that is not to say that all expressions with a high information content are complex; one can surely imagine a situation in which you are given a lot of information, but where most of it is unrelated, so that the whole is incompressible and large but ultimately simple. In fact the more interrelations there are, the more compressible the expression is likely to be, but this would also tend to make it *more* rather than *less* complex.

4.4 Variety

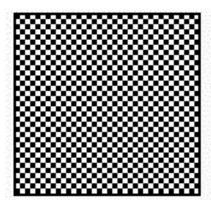
Variety is a key component of evolution. Examples might be "the bone structure is more complex as there is a greater variety of basic shapes" or "this species markings are complex due to their great variety". As with the above ideas the implication is not both ways; some variety is necessary for complexity but it is not sufficient for it. One can imagine a piece of electronic music that went through all possible variations of a set of notes, without this being a complex piece of music in its perceived form.

^{7.} Alternatively called various combinations of Algorithmic/Solomonof/Kolmogov/Chaitin complexity.

^{8.} For a biological use of this see [12] or [15].

4.5 Order and disorder

Complexity is sometimes posited as a mid-point between order and disorder. Grassberger [10] considered three diagrams similar to those in figure 1





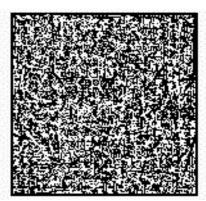


Figure 1: Complete Order, Chaos and Complete Disorder

The immediate reaction is to judge that the first and last diagrams are simple and the middle one relatively complex (leading to diagrams such as figure 2), but this is due to the facts that our perceptions 'filter out' the complexity of the right-hand diagram and that we interpret it as *representing* a situation with *no rules*. Thus, we are not judging these uniformly; it may well be that the right hand diagram represents *such* a complex situation that we don't recognise it. To illustrate this, consider the possibility that there may be a small version of the left-hand diagram included in the middle and a small version of the middle diagram included in the right-hand diagram (as in figure 3).

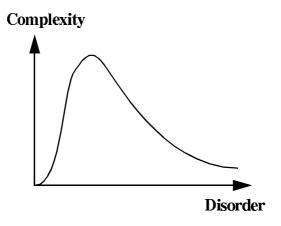


Figure 2: Presumed graph of disorder against complexity

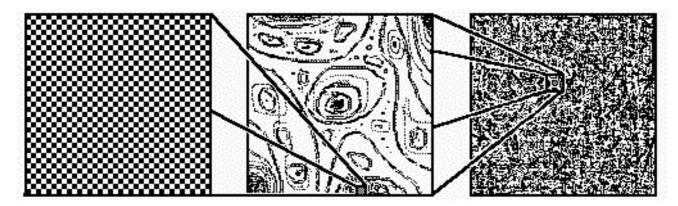


Figure 3: Possible diagrammatic inclusions

If this is the case, we are forced to judge the diagrams in order of increasing complexity from left to right. Again, we see the importance of the language of representation. If we were considering the complexity of some (assumed) rules to generate these diagrams, then the original intuitions would be preserved. The confusion comes because such diagrams do not have an inherent language 9 - we have to impose one on them.

5 A proposed definition of complexity

My proposed working definition of complexity is:

"That property of a language expression which makes it difficult to formulate its overall behaviour, even when given almost complete information about its atomic components and their inter-relations.".

This is a very general definition, which is intended to have different interpretations in different contexts, especially with respect to the phrase "difficult to formulate its overall behaviour". Here "language" is meant in a general sense, to include diagrams ¹⁰. "Atomic components" are defined by what are irreducible signs in the chosen language of representation; this would correspond to undefined signs, functions, predicates and constants in a formal logic ¹¹. "The formulation of the overall behaviour" is assumed to be of much the same type as that which specifies the atomic components and their interrelations.

This definition relates the difficulty in formalisation of the whole compared to that of its fundamental parts (from the point of view of the language). It is only applicable in cases where there is at least a possibility of gaining almost complete information about the components, thus clearly separating ignorance from complexity. You will get different concepts of complexity depending on the base language chosen, the type of difficulty focused on and the type of formulation desired within that language.

For example, consider a hypothetical herd of animals, where the flight behaviour of each is fairly well understood in terms of its reaction to danger, its wish to follow others of its kind, etc., where we are trying to understand the flight behaviour of the whole herd in terms of the direction and path. If it turned out that all the animals always followed one specified leader, or all went in a direction represented by

^{9.} Except may be an assumed one, which may be different for different observers.

^{10.} But not, of course, pictures without a language of representation.

^{11.} I am glossing over the possibility that there are no such atomic symbols here.

the average of the separate directions they would have gone individually then we would be justified in calling the flight behaviour of the herd simple. If the behaviour turned on the precise configuration of the herd at the time of attack, so that the animals followed different individuals at different instances as the configuration developed, we would be justified in calling the behaviour more complex.

To illustrate how the language can make a difference consider a case where the language did not take into account the direction of travel (of individual or herd) but only the average speed. In this case, the behaviour might be simple, despite the fact that the behaviour is complex from the point of view of direction.

The type of "difficulty" can also be crucial. Given a language which includes direction and speed, a predator might be concerned with the difficulty of predicting in which direction the herd will initially set off, an ecologist might only be concerned with the eventual direction and distance a herd travelled until it settled down. The first might be difficult to predict, the other easy; in the first case the behaviour of the herd would be complex while in the second it would be simple. The types of difficulty considered are obviously limited by the base language chosen.

6 Relation to other definitions of complexity

There is not space here for a full study of the relationship of this definition with others in the literature, so I will concentrate only on some of those used in the literature on evolution and ignore some other important approaches (for some of the others see [9]).

6.1 Computational Complexity

Computational complexity is the amount of computational resource (usually time or memory) that it takes to solve a class of problem ¹². Thus the *difficulty* here is the limited supply of these resources once the appropriate program is supplied. This is now a very well studied measure ¹³. For our purposes this is a weak definition of complexity as applied to evolving entities, as the time to perform a program or the space that the program (e.g. DNA) takes is often not a very pressing difficulty compared with the problem of providing the program itself (through evolution).

6.2 Kolmogorov Complexity

This is the minimum length of a Turing machine program needed to generate a pattern and has thus been discussed above (Section 4.3). It is used by several authors (e.g. [12, 15, 22]) to talk about the complexity of organisms. This is a weak definition of complexity because it corresponds to the difficulty of compression of a representation which is limited by storage difficulties. This difficulty has little direct connection with the practical aspects of a functioning organism, and is presumably used because it is a well-understood formalism.

6.3 Bennett's Logical Depth

Logical depth [2] is the computational resources (chiefly time) taken to calculate the results of a program of minimal length. It is thus a combination of computational complexity (Section 6.1) and Kolmogorov complexity (Section 6.2). Bennett uses it to formalise the level of organisation in systems. All present-day organisms can be viewed as the result of a very long computation from an incompressible program ¹⁴ and are thus *by this definition* complex. Using this definition where the

^{12.} More strictly it is the order of the asyptotic use of those resources compared with problem size.

^{13.} For an introduction see [14].

"difficulty" is essentially the difficulty of computation from a random starting point, evolution necessarily produces complexity however intuitively simple the results would appear to be. This is because this measure is aimed at the complexity of the process and not the results.

6.4 Löfgren's Interpretation and Descriptive Complexity

Löfgren [20] ascribes complexity to the two processes of interpretation and description (see figure 4). The interpretation process is the translation from the description to the system and the descriptive process is the other way around. For example the "description" could be the genotype and the "system" the phenotype. The interpretation process would correspond to the decoding of the DNA into the effective proteins that control the cell and the descriptive process the result of reproduction and selection on the information there encoded.

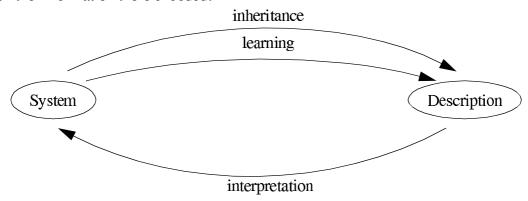


Figure 4: Description and interpretation processes

Löfgren then goes on to associate descriptive complexity with Kolmogorov Complexity (as in Section 6.2) and interpretational complexity with logical strength and computational complexity (Section 6.1). In order to talk about these processes he uses a base language to represent his "description", "system" and processes between them. Thus these can be seen as different "difficulties" with different interpretations of parts and overall behaviour of the same meta-system expressed in a wider language.

6.5 Kauffman's number of conflicting constraints

Kauffman [17], is less concerned with complexity than with order, but introduces a working definition of complexity for the formal models of self-organisation he investigates. He uses *complexity* for the "number of conflicting constraints". This fits neatly into my above definition because it represents the difficulty of specifying a successful evolutionary walk given the constraints imposed.

7 Complexity and biological evolution

Here I consider some aspects of the questions about the evolution of complexity with the help of the above thoughts on what complexity can usefully mean.

7.1 What is the complexity that has evolved in organisms?

To answer this question we need to pin down the definition given in Section 5 as to what is relevant to our intuitions about the complexity that has arisen in organisms (as far as possible).

^{14.} Disordered states are almost always incompressible, and hence minimal.

Firstly the relevant sorts of language needs to be addressed. Such a language would have to be able to include a full description of the genotype and the processes of interpretation and evolution that occurred to them, but might greatly vary in the detail given to the phenotype. The atomic parts in this representation would have to include the genes or at least the unit codes used in encoding them. Also the reproductive, selective, decoding and self-organising (if there is any) would need to be covered.

This still leaves two broad overall frameworks: one where we take the genes as the atomic parts and one where we take the essential features of evolution as basic. In the former we are concerned with the formulation of the characteristics of the biology (or more generally the phenotype) in terms of decoding the genetic code and in the later we are concerned with the process of arriving at the gene population in terms of what is known about the base processes of evolution that we postulate.

Within these frameworks we still need to specify the sort of overall *difficulty* in the overall formulation of behaviour we intend. Two distinct approaches suggest themselves for types of difficulty: the difficulty in analysing how an organism functions and the difficulty that a proto-typical predator would have in being able to usefully predict the overall behaviour of an organism, i.e. *our* view as scientists versus the *environment's* view (via a suitable representative) ¹⁵.

This gives us four possibilities to which I have given some obvious working names (table 1). I am *not* claiming these are the only ones, merely singling them out as important examples.

	Genes as atomic parts	Evolution processes as atomic
Difficulty in analysis	biological analytic complexity	evolutionary analytic complexity
Difficulty in prediction	biological fitness complexity	evolutionary fitness complexity

Table 1: Four possible frameworks for complexity in evolution

We are immediately concerned with evolutionary analytic complexity, it is the immediate problem that faces us as inquiring humans; the difficulty of formulating an overall understanding of organisms given the processes of evolution. This does not have a direct connection with the most concrete case, which is biological fitness complexity; the difficulty an environmental agent has is exploiting an organism given its genetic make-up. This *has* a direct relationship with the evolutionary selection process as we understand it. It is part of our task to discover the relationship. If we conflate the two kinds of complexity we may miss something important.

7.2 Does complexity increase over evolution?

It seems almost certain that at some time in the past there were only organisms of (presumably) small biological analytic complexity while now there are complex beings which are difficult to understand (us). This does not mean, however, that complexity never decreases in any part of a species' evolutionary trajectory. In fact there is evidence that it does. ¹⁶

^{15.} There is a relationship between what I have (ineptly) called the *environment's view* and efforts towards an evolutionary epistemology [11, 25].

^{16.} For a critique of the naive expectation of strict local complexity increase see [24], and for some theoretical reasons see [17].

A very weak definition of complexity (such as size of genome) would be sufficient to explain an increase in the maximum complexity of all species under evolution. This is due to the fact that an incremental search for new ecological niches would necessitate this (albeit with a sharply decreasing rate of this increase over time as the space of new reachable niches increases without requiring a much larger genome). This would also imply a general increase in average complexity which would also tend to increase (by this explanation) increasingly slowly.

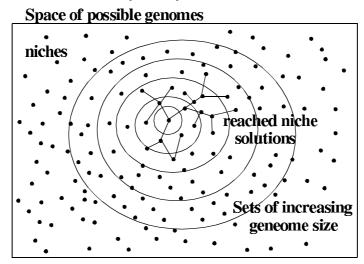


Figure 5: Incremental search over a space of possible genomes of increasing size

This is, however, a very weak notion of complexity increase. A stronger question is "Is there a *general tendency* for complexity to increase in each species over evolution?". I will not presume to answer this question, but only comment on some the suggested avenues of progress w.r.t. the types of complexity they refer to.

7.2.1 Co-evolution

This is the idea ¹⁷ that the result of species interacting will imply that the evolutionary development of one will make the environment of the others more complex as it forms part of the environment. This in turn will make it advantageous (according to something akin to Ashby's *law of requisite variety* [1]) for it to evolve more complex processes to cope. This in turn puts the 'pressure' on the first species and so on.

Such a process would necessarily increase the analytic complexity of the *ecology* w.r.t. the species, as well as the evolutionary fitness complexity of 'predicting' the behaviour w.r.t. the evolutionary processes that led to it. It is unclear, however, if this would increase any of the other versions of complexity.

7.2.2 Co-adaptation

Wimsatt [30], suggests that the co-adaption of one of an organism's mechanisms for several purposes (or as sub-mechanisms of other mechanisms) is a source of the evolution of its 'descriptive' complexity. He relates the dividing up a description of such an organism into different levels as an attempt to simplify such a description (albeit at an inevitable cost to the complete picture). This closely corresponds to what I have called biological analytic complexity, with a simple (in the evolutionary analytic sense) explanation. Further he is pointing out how we attempt to adopt a base language that will actually simplify our description for a particular purpose and why this may be inherently difficult.

^{17.} See for example [2].

7.2.3 Inherent Self-organisation

Kauffman [17], suggests that the order manifest in organisms is a result of selection acting upon a system that is basically self-organising and that this self-organisational ability depends critically on the complexity of conflicting constraints or connections at the base level. This complexity (see Section 6.5) is a kind of biological fitness complexity (conflicting constraints between the genes affecting fitness) that causes both the evolutionary analytic complexity and the evolutionary fitness complexity. The evolutionary analytic complexity follows from the constraints on evolution which create structures (attractors etc.) that are difficult to specify. Evolutionary fitness complexity is implied by the difficulty of predicting the behaviour of the structure.

7.2.4 Meta-system transitions

Heylighen ¹⁸ suggests that the complexity of organisms can increase with the emergence of meta-levels. The truth of this might critically depend on what variety of complexity was intended. The biological fitness complexity may well increase with the introduction of meta-levels, because such stacked meta-levels can produce very unpredictable behaviour (based on knowledge of the levels), but it will also accompany a *decrease* in the biological analytic complexity because a structured system of hierarchies is more decomposable and hence more easily analysed than one that has no separable hierarchies. Perhaps this forms the basis of the evolutionary advantage of such an emergence ¹⁹.

7.3 Do systems necessarily filter out complexity?

Heylighen [11] and Luhman²⁰ stress the importance of the reduction of complexity necessary for the effective reduction of environmental complexity for organisms and institutions respectively. Here we are talking about the evolutionary fitness complexity, but *of* the environment *from* the organism's (or instituition's) point of view. In the short term the only response is somehow to ignore or filter out most of it, since otherwise the entity's computational capacity would be overwhelmed. This ability will be closely connected to the evolutionary fitness of the organism w.r.t. the environment but the exact nature of this co-evolution (see Section 7.2.1) is unclear.

Perhaps it is in the nature of the *identity* of such entities that it must do so. If such an entity did not filter out (much) complexity from its environment, then it becomes more difficult to separate it from that environment. This raises the possibility that we only recognise entities that do perform such a filtration, and so the observation is a tautology.

One often sees arguments of the type "entities evolve to the edge of chaos, since in the chaotic region it would be *overwhelmed* with change and in the frozen region it would not be able to sufficiently adapt". This can be roughly translated as *the complexity of an entity's pattern of adaption can not be either too simple nor too complex*. This sort of argument should be used with great care; it may well be that in one language of representation, an entity seems to be overwhelmed by change so that it is no longer recognisable, while still having sufficient continuity to be recognised as a specific individual in another. Again, the relativisation of complexity to a particular language is very important.

^{18.} In the abstract of his talk at this workshop he says "the higher the number of meta-levels, the more complex the system...".

^{19.} Hey lighen hints at this when he says that meta-system transitions are the only way to increase both constraint and variety

^{20.} I have unfortunately lost the exact reference.

8 Conclusion

It should be clear from the above that there is no one appropriate measure of complexity appropriate to evolution in general, but that it is desirable to be clear about the *language of representation*, the type of *difficulty* and the *overall formulation* that each variety refers to. At the very least we should distinguish concepts like size, order and variety from complexity. In this way we can be clearer about the strength and context of any such use for clearer analysis and communication.

Acknowledgements

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