“MATH HAS ONLY ONE LANGUAGE”

A series of meetings with Sber Science Award winners

February 15, 2024. Main Hall of Lomonosov Moscow State University

Sber Science Award 2023 winner[1] in the “Digital Universe” category, full member of the Russian Academy of Sciences, Doctor of Physics and Mathematics, Head of the Chair of Computational Technology and Modeling of the Department of Computational Mathematics and Cybernetics of Moscow State University, Director of the Marchuk Institute for Computational Mathematics of the Russian Academy of Sciences **Evgeny Evgenyevich Tyrtyshnikov** dedicated his lecture entitled ***“Dimension: Is it a curse or a blessing?”*** to methods of presentation of multi-dimensional data based on the idea of separation of variables and to algorithms that even now successfully help resolve problems that are beyond the capabilities of supercomputers.[2]

The lecture was followed by a panel discussion[3] attended by:

Evgeny Evgenyevich Tyrtyshnikov, full member of the Russian Academy of Sciences, Doctor of Physics and Mathematics, Director of the Institute for Computational Mathematics of the Russian Academy of Sciences;

Alexander Vladimirovich Gasnikov, Doctor of Physics and Mathematics, President of Innopolis University;

Gleb Gennadyevich Gusev, PhD of Physics and Mathematics, Head of Sberbank’s Artificial Intelligence Laboratory.

The discussion was moderated by Albert Ruvimovich Efimov, PhD of Philosophy, Vice President, Director of the Research and Innovation Division of Sberbank.

Efimov: Evgeny Evgenyevich, while listening to you, I thought of a quote from the diary of Russian mathematician Dmitry Evgenyevich Menshov: ***“And then came the year 1917. It was a very memorable year in our lives, as the most important event that affected our entire subsequent lives happened then: We started working on trigonometric series.”*** We, on the other hand, remember 2023 as a milestone because it was then that Evgeny Evgenyevich Tyrtyshnikov won the Sber Science Award for tensor trains. There was a sense of importance of this discovery and of importance of increasing the dimension, which we should always keep in mind: ***“When face to face we cannot see the face. We should step back for better observation.”***

We are talking about computations today because we have computers. But if you think about ancient history, writing was invented just some 4,500 years ago, while the oldest of

[1] https://www.sberbank.com/promo/sberscienceaward
[2] https://youtu.be/y3slOjQATB4
[3] https://youtu.be/wprQu1Zef-o

mathematical tools known to us, the Lebombo bone, dates back some 37,000 years. So calculations are much older than [written] language. This is an astonishing fact about our history and our understanding of what we are doing, and it must not be forgotten.

Algorithms and calculations are thought to be able to help model the world at large. In fact, everything that Gleb Gennadyevich and Alexander Vladimirovich are doing at labs and universities is required to model the surrounding world in one way or another. But can we model everything? Galileo has said that nature's book is written in the language of mathematics. In our own times, Professor Alexander Borovik of Manchester University wrote an article entitled ***"A mathematician's view of the unreasonable ineffectiveness of mathematics in biology."***[4] Indeed, while math explains physics perfectly, it helps with biology not so much. A question arises whether we are in fact modeling the world? And is there a limit to mathematical modeling of the world? Evgeny Evgenyevich, let's talk about this. You've mentioned astronomy, and that the total number of atoms in the universe is approximately 10^{83}. I think you've left out the dark matter, which might still increase that number somewhat. But anyway, is there a limit to such modeling?

Tyrtyshnikov: I meant something slightly different. Why can't math explain biology? I'd say math doesn't even have to explain biology. Biology must be explained by biologists that use mathematical tools. There are certain fields of science, above all physics, where fundamental laws have been discovered. These are used to build mathematical models for many phenomena or processes. That much is clear. But there are many fields of science where the situation is different: there are no paramount fundamental principles, or perhaps they have not yet been discovered, but there is a lot of phenomenological material, there is systematization, and there are specific concepts, which are natural things for any science to have. Yet there are no principles like those found in physics. So what do we do?

The opportunities presented by artificial intelligence (AI) come to mind. In my opinion, that's an area where data models are proposed and their parameters are researched through training. Plus, the models that are offered are quite universal, although there is some diversity. The question of why exactly such models are used still awaits answers from mathematicians, and it's not such an easy thing to do. So what I'm saying is that in a situation of great uncertainty, which is typical for various fields of science, we are using universal models and trying to fine-tune them through training. That's not what science usually aspires to achieve. Science normally seeks to minimize the number of parameters and to discover their meaning and the laws that govern them. But the meaning that is created by neural networks does not make a lot of sense.

Efimov: The physics Nobel Prize winner Konstantin Novoselov explained that this way: Physicists like simple problems very much, they do not like complex problems[5].

Tyrtyshnikov: And yet, even when dealing with complex problems, we need to strive to

[4] **Borovik A., A mathematician's view of the unreasonable ineffectiveness of mathematics in biology, https://arxiv.org/abs/2103.04190**

[5] *K. Anokhin, K. Novoselov, S. Smirnov, A. Efimov, F. Matveev.* Artificial intelligence for science and science for artificial intelligence // Voprosy Filosofii. 2022. V. No. 3. P. 93–105.

understand. I believe that artificial intelligence tools offer tremendous opportunities, but whether they give us an understanding is a big question.

Efimov: That's what we are going to find out today. Alexander Vladimirovich, can we model the world? As a mathematician, do you see a limit to the modeling of the world? Or are we indeed building models that don't really understand the world around us?

Gasnikov: I don't know. I'd look at this matter from my own point of view. Suppose we need to solve a problem of designing a ship that would properly withstand a storm of certain strength. We could do calculations and start designing. Or, we could try a ***"bathroom experiment"*** and use elementary considerations of dimensional similarity, something that is called the π theorem in physics. So we'll run an experiment in a bath and compute the strength of a storm that ship would withstand. What do I mean by that? As a matter of fact, human brain is much more efficient than modern supercomputers (roughly speaking, our brain is 10^5 times more efficient than a typical GPU video card, even though the video card consumes 10 times more power than the brain). So the answer to the question of whether we can model anything in particular is likely to be yes, in principle, if it is truly indispensable, and we could spend a lot of resources doing that. And perhaps many questions regarding Gödel's incompleteness theorem module could also be answered. Math helps here, of course. But the question that needs to be asked is: Is this worth it? Does this need to be modeled versus the effort it requires? Sometimes life or nature itself helps solve problems. I will give you an example of researchers from Japan[6]. They needed to evaluate the quality of a railway network layout design. This is a problem that's classified as complex, that is, in fact, a brute-force search problem. However, it was found that *Physarum polycephalum* plasmodia (which is a single large eukaryotic cell with multiple nuclei) tend to choose the shortest distance between sources of food as they feed themselves. The plasmodium was given a nutritional substrate and placed where the planners wanted to locate railway stations, and it started growing. As a result, paths that were most heavily used in this feeding process served as a prototype for railway routes. Surprisingly, this closely matched the network that was actually built (i. e. it was designed in an optimal manner, except that the *Physarum* plasmodia solved this problem much faster, and with significantly less energy consumption). So what's my point here? To what extent should we look at modeling through the lens of existing computer or algorithm architectures to be able to solve problems? Sometimes, it can be easier just to run an experiment like that.

Efimov: To select another substrate for modeling.

Gasnikov: Let's circle back to the Gödel's theorem. There exist, after all, results for Diophantine equations: the kind of $x^3+5xy+3z^5+7xyz+...$ is equal to …a free variable. So the answer to the question "Can this equation be solved in integers for each integer value on its right side?" – can be positive, but there is no way to prove it.

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[6] Tero A. et al. Rules for biologically inspired adaptive network design // Science. 2010. V. 327, No. 5964. P. 439–442.

There’s also an interesting case of Fermat’s last theorem. Back then, people were struggling to solve a particular Diophantine equation. It so happened that the English mathematician Andrew Wiles solved this problem in 1996. Yet it may well have been not solved at all. Not every theorem can be proven.

So on the one hand, history demonstrated that people will learn anything, will do anything if they really need it. But on the other hand, there are limits, in principle, to what can be accomplished. And mathematics helps understand that. It gives “lowest estimates” that describe what Evgeny Evgenyevich has called a “mountain”. So that we understand what mountains it’s better not to climb, but to bypass and look for workarounds or alternatives. I believe that one of the objectives of mathematics is exactly to give us correct guidance so that we don’t beat our heads against those mountains.

But let’s get back to what Evgeny Evgenyevich was saying. On the one hand, that’s very cool, but at the same time we can’t say that absolutely any problem can be solved this way. It is very important to understand that this all works fine and well provided the rank is low. In that case, mathematics helps us understand how to compress the representation of such problems and reduce their dimension significantly, but this is by no means a “universal screwdriver”. There can be problems that are complex indeed. The question is: can they be modeled if the corresponding rank is high? The answer will likely be negative. That said, a high rank may not be typical, and many important problems may well have a low rank.

Tyrtyshnikov: Not all matrices are low-ranked, that’s true. Take the identity matrix; would you say that it’s close to a low-rank matrix? You’d think it’s not. This depends on the norm. And norms vary. In particular, the elementwise norm is interesting. Here’s a mathematical statement: the identity matrix is approached by a matrix whose rank increases logarithmically with its order and is inversely proportional to margin of error squared. If you make the margin of error fixed, even the identity matrix turns out to be close to a low-rank matrix. That’s how unexpected things can happen.

Efimov: Gleg Gennadyevich, I had a wonderful and very lively discussion with your employees at the Forum of Future Technologies a couple of days ago, exactly on the subject of limits to what can be modeled, including in the fields of biology and medicine. What’s your opinion on this? Are there such limits? Are we in fact developing models that describe the surrounding world in a random way, including medical devices?

Gusev: I think this question is multi-pronged. On the one hand, have we embedded mathematical and algorithmic tools in our models that can be embedded there in principle? I think the answer to that question is “not yet.” The existing models and methods for training AI algorithms are pretty limited at the moment in terms of overall algorithmic diversity. On the other hand, we have a classic philosophical problem: Can an algorithm be used to model the world? Much depends on whether we believe in determinism, among other things. To what extent are we ready to believe in it in principle? On the other hand, just how long would such a description be and why is it needed at all? If its length exceeds the number of atoms in the universe by an unthinkable number of times, what are we going to do with it? How would we train such an algorithm? This calls for a lot of different questions.

Large neural networks today look mostly like a giant sponge that has soaked up everything that's available on the Internet, processed it somehow and is able to regurgitate it word by word, like a parrot. A parrot that thinks at every step, flips a coin and decides what to say at any given moment.

On the other hand, there's talk about emergence these days. Notably, there's a lot of discussions going on this topic. It is said that as the number of parameters, the amount of computing resources and the volumes of data grow, at some point a generative neural network would obtain certain new qualitative properties that it lacked before. It's not a quantitative transformation into some new state – rather, it's a leap, a gap between what it was and what it has become. This transition is called the property of emergence. For example, neural networks can now generate very long texts, which they weren't able to do before. Or, inversely, they can be fed a long text and output a brief summary, or have a dialog, albeit not very long, but nevertheless a meaningful one, with a person.

The matter of the description length is yet another question. How much science are we really talking about in case of a very large mathematical model, or a big algorithm? As far as I understand, knowledge and science are characterized by their ability to produce certain brief descriptions of complex phenomena that can have a sufficiently good power of generalization. In that sense, the model clearly does not contain this kind of knowledge from the point of view of human perception. But on the other hand, it is able to generate that knowledge.

Efimov: Should the model be transparent and easy to understand in this context? You've mentioned ***"brief descriptions,"*** but it seems to me that the descriptions should also be clear.

Gusev: We'll never be able to understand what's going on inside it, what with the enormous number of parameters built into it. But we can aspire to understanding its properties. What a model can say, what questions and how it answers, what it cannot answer. It needs to be studied as a new object. Studied as something we have discovered but don't quite understand what it is – as something new.

Efimov: In this case, I always recall the words of Garry Kasparov[7]. When he lost to DeepBlue, he said: ***"Thank God the machine does not understand that it has won."*** In principle, it can discover something, but it still can't understand what that is.

We've discussed models. Now I would like to ask you about mathematicians themselves. In his lecture, Evgeny Evgenyevich mentioned Gilbert, the last mathematical universalist, many times; he understood literally everything (Gilbert's 23 problems are a product of his creativity, because he had understood each of them). It looks like over the last 80 years that Gilbert is no longer with us, we have not seen any such widely knowledgeable mathematicians. Current winners of the Fields Prize admit that they are experts in one or two areas, and that's it[8]. As I was preparing for this panel, I took a brief look at what is happening to mathematical proof. The biggest proof took 200 terabytes of data in 2016[9]. Nobody can read it at all. It would take a

[7] Declared a foreign agent in Russia.
[8] See e. g. https://gowers.wordpress.com/2010/08/31/icm2010-spielman-csornyei-lurie/
[9] A proof of the triples theorem took up 200 terabytes. N+1, 27.05.2016.

supercomputer a few days just to process it. Does this loss of integrity constitute a problem for further development of the mathematical science? Could something, such as AI, help overcome this contradiction and assist mathematicians in moving the science forward?

Tyrtyshnikov: There is a remarkable quote on the topic you mentioned from Andrey Nikolayevich Kolmogorov. I don't remember it verbatim, but he certainly regretted the loss of integrity. This loss is inevitable. But Andrey Nikolayevich was of the opinion that after all, ***we need mathematicians who try to embrace the entire science.*** Of course, it's nice to understand what's going on in different fields, but you are absolutely right: Very narrow specialization is the norm.

Efimov: Can artificial intelligence help overcome mathematicians' loneliness?

Tyrtyshnikov: I believe that AI technologies are not yet mature enough to be able to play this role. In fact, we are often unable to solve certain problems because we haven't learned something in other fields of mathematics. And had we known that, we could have elaborated on the right idea and accomplished what we've been thinking about.

This is, by the way, yet another reminder of how dangerous it is to waste time. Suppose you missed a lecture and you think, "Well, I'm a strong person, I'll study this later on when I need it." Yet later, when you really have to use what you've missed, it turns out that you no longer have the time to study it. So someone would miss a remarkable idea of which they had not learned at the right time. Of course, AI as a tool can help us look into fields in which we are not competent enough.

It should also be noted that the world has changed. A huge number of academic papers are published, maybe not an astronomically large number, but nevertheless a very large one. Yet there are not so many good ideas around as there are papers.

Efimov: How many papers can a good mathematician read a day?

Tyrtyshnikov: You're lucky if you can finish just one paper in a few days, but you also need to understand it and get to the bottom of it.

Efimov: This is the same as with biology. I've recently taken a look at the stats, and it turned out that 100,000 papers on biology were published during COVID-19 alone. A good biologist can read one paper a day. You can do the math yourselves.

Tyrtyshnikov: I think that for a mathematician, one paper a day sounds too good to be true. Sometimes you need a few days.

Gusev: Speaking of the number of papers. It was Vladimir Igorevich Arnold, I think, who said that when you read a paper, you go for the findings before anything else. Then you try to understand what it means, what it is for, examples, implications, open questions. And only then, if you still want to dig deeper, you try to understand the ideas, the proof, the tools that the author used to achieve the findings. It seems to me that as you try to understand math right now, every next step requires increasingly more energy. Oftentimes you can understand the findings fast enough, and then you may spend a lot more time understanding the implications, and even more time to penetrate, to understand the technique – but that's something only true professionals are able to do.

Efimov: Gleb, this is a great quote, and we have to keep it in mind when we read yet another press release about the development of artificial intelligence or something similar. Alexander Vladimirovich, what do you think of practical application? We are talking a lot of this topic right now. Steve Jobs once said that ***"the computer is the bicycle of the mind."*** Can we say that AI is the bicycle of a mathematician's mind?

Gasnikov: Me and Evgeny Evgenyevich, we are often on the same wavelength, and I immediately thought of Kolmogorov too when Gilbert's name came up. He may have been the last universalist indeed after all. Kolmogorov was engaged in many fields and was the world leader in all of them. But speaking of AI, let me give you an example. Back in the Soviet times, there were comics in newspapers: If someone was a runner, they had to have big (pumped up) legs, or arms if they were a shot putter. What's my point here? As a matter of fact, when a person specializes in some area, they develop a specific group of muscles, just like an athlete. If you want to be successful, to become a world champion, then probably you should focus on such training. A decathlete would lose to a specialized athlete in a particular discipline.

Of course, brain is not a muscle, but a set neural chains. Depending on what you are genetically predisposed to and what you study, the relevant portion of your brain is boosted. But if you want to boost another portion, this would lead to a partial loss of skills that are no longer actively supported by training. Which is to say that knowledge and the ability to use it do not come free. It is very difficult to absorb a lot of different skills, like a knowledge base, and maintain them all in working order. This was possible two centuries ago, or maybe, to some extent, a century ago, when the amount of information and competition were altogether different from what they are today. The situation is changing dramatically at the moment. Take, for example, data analysis (AI) and the development of many other fields of science spurred by the advent of AI over the past twenty years. And take, for example, a historical period of two centuries ago and of twenty years. That's not the same. We live at a unique time, when we have witnessed (and are still witnessing) several cultural revolutions, including the advent of the Internet and of large generative neural networks, as Albert Ruvimovich has said. As scientists, we are actually asking ourselves right now: To what extent can our work of writing papers or preparing reports be automated?

For instance, a few days ago we used large language models of generative neural networks to write an internal report. This is a game changer. We can prove secondary claims in our papers (lemmas) using those models. Of course it doesn't always work, and of course the first iteration might still fail, but this is really a game changer. So the questions are: What's next? How is this working? Can we predict what will be happening in ten years' time? How different would our work be from ten years ago?

I'll digress a little. Yesterday, I visited the office of full member of the Russian Academy of Sciences Albert Nikolayevich Shiryaev (a prominent specialist in mathematics who works in the fields of the probability theory, random processes and their applications, a student of A.N.

Kolmogorov). His students told me that Albert Nikolayevich will be ninety years old soon. Nevertheless, he still works from dawn to dusk and writes new books from memory as a clean copy, with hundreds of pages and hundreds of references. These are clearly unique capabilities of a unique person (to keep it all in one's head and reproduce it almost in real time). But where are we going right now? Today, we will probably be relying on large language models to a notable extent to accomplish this kind of work. For instance, we get a paper and ask a language model to give us a summary of what's most important there – this can be used even now. The quality that's currently available is another matter.

By the way, I'm just offering some talking points right now rather than say something definite. I'm playing a kind of ping-pong with Albert Reubenovich – maybe someone else will join us too.

Tyrtyshnikov: I will join you. We've made progress in terms of substance by using artificial intelligence. Suppose a government ministry requests a note. A note is written, and the ministry rejects it. We then resort to artificial intelligence, and it writes the text of a note for us that the ministry it happy to accept. And everyone's happy.

Gusev: I'd like to add my five cents on the essence of the question raised by Albert Ruvimovich. He said that math is becoming increasingly complex. Can artificial intelligence help? I can share my intuition and experience, and my knowledge of mathematics. Looking back at history, we will notice that when new fields emerge in mathematics – and quite a lot of existing fields are new, as a very large portion of mathematics has been written over the past fifty years – these fields were first described using quite a complex language and very complex techniques were used. Once a century has passed, the language will become much simpler. AI can play the role of an accelerator. Suppose there's a complex something: We know for certain that a theorem is correct. We need to think about how to reduce the amount of computation or logical reasoning, to come up with shorter and higher-capacity lemmas, or how to generalize a certain technique.

AI has an important role to play here. Yet there is a limit to such simplifications.

Take, for instance, topology. Henri Poincaré and others originally developed it in the late 19th and early 20th centuries. The language was far more complex back then. I think that today, the language of topology, if we are talking about homology computations, has been simplified to the limit. It just cannot be simplified any further. But when I try to explain what a homology group is even to mathematicians who are not specialists in the field, it turns out to be very challenging. AI is not going to help here, because there are limitations to the cognitive ability of humans themselves.

Efimov: Why wouldn't it help if there are limitations to human cognitive abilities? Perhaps it's just the opposite. Please elaborate.

Gusev: That is, I cannot even imagine how artificial intelligence would explain homology in a simpler language. I believe that the language that is currently used is already as simple as possible. It cannot be simplified any further. AI cannot make our minds perceive this complex structure any better.

Tyrtyshnikov: There is a very important thing. What is understanding? A person is thinking, trying to understand. What does that mean? A moment comes when that person decides for themselves, "I get it." When exactly that moment comes is not quite clear yet. There are complex theories, actually. For understanding to take place, perhaps a couple words are still missing that at some point should be spoken. Who will find those words?

Efimov: I have discussed the matter of using AI in mathematics with Stanislav Smirnov, a winner of the Fields Prize. I told him my favorite joke about the difference between mathematicians and philosophers at a university. Mathematicians needs paper, a pencil and a trash can. And philosophers don't even need a trash can – I always use up everything indeed. Stas responded that the joke was no longer relevant, because mathematicians now need a supercomputer. According to him, it is indispensable. Gleb Gennadyevich is a mathematician with a supercomputer. So, Gleb, does having a supercomputer help?

Gusev: Having a supercomputer is necessary for further development of mathematics. We are perhaps not yet using it in our respective fields to get new results. But everything can change.

Gasnikov: Evgeny Evgenyevich has shown us today that super mathematicians just need a regular computer, if I understand it correctly. Is that so, Evgeny Evgenyevich? It looks like "super" refers to mathematicians.

Tyrtyshnikov: I remember a statement that is attributed to Andrey Nikolayevich Kolmogorov, but I am not quite sure that this is actually his statement: ***"A theorem is either wrong or trivial."***[10]

Gusev: That was Kolmogorov's particular feature. Sometimes his colleagues would come to him and tell him that they were writing a doctoral thesis. Andrey Nikolayevich would ask them about their key ideas and about the essence, they would reply, and in five minutes Andrey Nikolayevich would say, "Well, I roughly understand how this should be proven."

Tyrtyshnikov: We are used to thinking that math is a rigorous science, the queen of all sciences. It's nice to think that way about what you do yourself. But, as it's already been mentioned today, some proofs are so long and complicated that even reading them is difficult, let alone interpreting them. It's all but impossible. So what does that mean? It means that mathematicians, too, tend to accept results in blind faith. So how is our wonderful science then different from any other science? By the way, Arnold used to call mathematics an experimental science. Because science has become more complex, we too are starting to trust previous research without reviewing it ourselves.

Gasnikov: There is a remarkable story on this subject. Exactly ten years ago, a paper was published whose authors called it "Adam." As in Adam and Eve. It is dedicated to a technique of stochastic optimization with momentum that is used to train GigaChat, among other things. So it's essentially an optimization algorithm. The paper has received some 200,000 citations. It has had more citations than all of the main optimizers in this field combined. A good optimizer

[10] Mentioned in a blog at https://kolmogorov.livejournal.com/55190.html

has some 10,000 citations. Some may have 40,000–60,000. And here you have 200,000 citations for just one paper. This paper has proven what is essentially an incorrect statement. But this has not stopped it from continuing to gain citations, and it is offered as part of every package. The paper is cited, but it just contains an error. That is to say that the method described in the paper works pretty well in practice, but there is a counter-example that it may not work even for convex problems.

I have asked some of my colleagues who are professional optimizers: Why don't you work with neural networks? One of the answers was, "Well, I opened the first paper (one of the main papers on the subject), and there are automatic differentiation (back propagation) and ReLu there. But that's not a differentiable function, so what mathematical differentiation, what back propagation are we talking about?" A mathematician reads a paper and understands that something is not right. It can't be like that. And yet, it works. If we take a look at a large number of papers published as a result of A* conferences: There are nine pages in a paper itself and some 30 pages of appendices. A peer reviewer is given two weeks to review five or six papers. Of course, they don't have the time to read all those appendices properly. And that's the bleeding edge right now, everyone is watching those conferences. I'm not saying that this is bad – that's the reality we live in. Many of such papers result in experiments or successful observations that are grounded in mathematics. Of course, a system of automatic verification of proofs will be of great help for authenticity of results. If you come up with a proof, verifying it with the help of a computer is an infinitely easier task than obtaining it in the first place.

Efimov: This problem exactly was discussed at the latest meeting of the Nikitsky Club. Mathematicians tend not to understand proofs, except the few people who invented those proofs in the first place.

But I'd like to mention the following. On December 23 of last year *Nature* published a remarkable article that argued that the top three countries with most retractions from that journal were Saudi Arabia, Pakistan and Russia[11]. And last week, there was another news item. China (ranked fourth on that list) said that the first author who is unable to prove that *Nature* has retracted their paper wrongly, must be cast out of science. This is a serious problem for biologists and medical researches. Math doesn't even make this ranking, I think, but still. A crisis of trust in experts is a major preoccupation in science.

Tyrtyshnikov: This discussion has brought back a memory. Math is an amazing science. One good mathematician told me once that some kind of algebraic system was studied for a while by a group of researchers who were even publishing reviews. This went on for about ten years, after which it was discovered that the system had been based on contradictory axiomatics. In other words, the object of their study simply did not exist. Stuff happens.

Efimov: I'd like to move on to the next important topic of our discussion. We've talked about models, about how mathematicians work, and now I'd like to talk about the math education, and specifically about the training of mathematicians. I'm an engineer and a mathematician by training. All of us sitting here on this stage are graduates of physics and math schools. What's

[11] *Nature* **624**, 479-481 (2023)

wrong with math education today? What can be improved? Should equations be taught in high schools at all? Alexander Vladimirovich and I have recently taken part in a brainstorm that offered a hypothesis that equations have no place in schools.

Tyrtyshnikov: I can still remember the time when I went in school. I remember being in sixth grade when algebra first came up. It made a huge impression on me at first sight. There were only numbers before, and now I could suddenly add and multiply letters too. Arithmetic problems for fifth grade at the time when I went to school were quite difficult. They had to be solved by asking questions and answering them sequentially, and there could be five or six steps. That was hard. But once letters appeared, these problems could now be solved instantly. Because they were equations. What can we do without equations? They always existed, they exist now and they will always exist. Naturally, once letters appear – and they do appear, – equations begin to emerge. This direct question has only one answer: Of course they should [be taught in high schools].

Gasnikov: I have so much to say, I don't even know where to begin. Let me start by saying that we've recently had a closed-doors strategic session initiated by the Ministry of Science and Higher Education of the Russian Federation. It lasted one day, and tables were being prepared for weeks before it. As one of the session organizers, I will tell you about some suggestions proposed there.

To begin with, there are quite a lot of challenges. Frankly, most schools are simply understaffed with teachers of mathematics and physics. Schoolchildren's interest in STEM, particularly physics, is dropping fast. Fewer and fewer eleventh-grade students pass the Unified State Exam in physics. To become a teacher of, for example, computer science, one needs to graduate from a teachers' college. If a student is a good programmer, it's extremely unlikely that they will choose to work as a teacher rather than at a large IT company.

Meta-gaming mechanisms are needed to resolve the problem described above. For example, universities should promote the engagement of their senior students in teaching at secondary schools or colleges as an indicator of their academic success. So that we have a system where the best students/post-graduates/young scientists could not only make good money in the industry, but also pass their experience on to next generations, i. e. teach someone.

Another suggestion was to support leading academic schools. For example, we are starting work on a mega-grant program sponsored by the Ministry of Education and Science of Russia. Its rules include inviting leading Western academics and paying them 100 million rubles a year each to teach us something. But there are some concerns about this program specifically. Under the current circumstances, would there be many truly leading scientists willing to come here and teach us something? Given that we have world-class scientists right here, in our country. Evgeny Evgenyevich sits right here on my right. Perhaps we should allow our own scientists to compete for the honor of being a leading scientist, along with their Western counterparts? On the one hand, at a meeting with Vladimir Vladimirovich as part of AI Journey 2023[12], I said that the experience of the first waves of mega grants played a very important role, for example, in

[12] http://kremlin.ru/events/president/news/72811

shaping the youth’s optimization and stochastic landscape in our country. But the situation has changed noticeably. That said, I’d only be happy to learn that my concerns are baseless and good Western specialists will be able to come to us.

Another suggestion involved the role of companies such as Sber or Yandex in the formation of IT and AI specialists. This role has already become quite visible. Our IT giants are shaping the interests of an entire generation of students. They propose problems to be solved. That is, on the one hand, they possess sufficient resources, and on the other, they understand which problems need to be solved. If properly organized, for example, in collaboration with the Academy of Sciences or the Ministry of Science and Higher Education, this could become a fruitful endeavor. I like it that Sber organizes academic meetings. Yandex is doing this too. These are good trends. I believe students feel this way too. I know that some students who work at Sber or Yandex teach, do science and write papers. I think this path is quite interesting. It’s important that they be not enticed out of science entirely or become poached. Ideally, interaction would be in a “friendly” format: For example, a student of Evgeny Evgenyevich writes papers, but for a time solves problems, say, for Yandex, and gets the necessary practical experience. If this could be done, it would be very cool.

Efimov: That’s what’s happening at Sber and Yandex. We are open to any type of collaboration with science. Even now, some 600 scientists from our Artificial Intelligence Centers[13] work with us.

Approximately 50% of them have academic degrees and are PhDs in physics and mathematics. This is a huge army, but they’re not prohibited from collaborating on research with Tinkoff or Yandex. All of them are engaged in science. It is also very important to encourage Sber employees to be scientifically curious and eager to study the world around them[14].

Gusev: Now it’s my turn to speak in my capacity as an AI lab lead and as an ex-mathematician (because lately I’ve been writing papers on artificial intelligence). I see and hear all the time about the shortage of math teachers everywhere, even in Moscow, even at very strong schools. Often times it’s just impossible to recruit an educated person who can teach mathematics. They have to hire people who are not specialists at all, neither teachers nor mathematicians. Just to fill a position. This problem is acute. I’ve encountered it myself. My son has enrolled into a specialized school, and there’s no math teacher there. I had to rise up to the challenge and help resolve the situation myself.

Another matter that I’m very concerned about is that the world has been obviously changing a lot recently, and so has been the environment in which schoolchildren grow up, and as a result motivation has been changing too. When I went to school in the 1990s, I didn’t have to ask for a reason why I have to study math. I just had a desire to study it: It was cool, it was fascinating, there were authoritative academics who knew it, members of the Academy of Sciences. I wanted to know it too, and I knew it would be useful.

[13] As of December 2023, these included HSE, Skoltech and MIPT.
[14] More details at https://sberlabs.com

Today the situation is different. When explained a new area, high school seniors and college freshmen tend to ask, “Why is this even necessary? Why study it at all? Please prove that it’s required first.”

Tyrtyshnikov: This is a very dangerous question.

Gusev: It stems from objective changes. First of all, there’s a lot of information, a lot of different competing areas in science. People have to choose. They understand that they cannot master them all, they need to focus on something specific, and they thus need to understand why. The young are no longer ready to just trust the authority. Also, what was authoritative 20 years ago is no longer today. Trends and fads have changed. People have to find their own way and choose their path on their own. We have to understand this and respect this view. Teachers’ motivation has to change too because of this.

Motivation is paramount. Why would somebody chose to become a scientist? Above all, because of curiosity. If someone is not curious, they will burn out at some point. Okay, I did something for science, and they said thank you, but often there’s no one around to share what you did with. If someone is curious, they can become a scientist. But that person must be aware of that, and the initiative must come from within. And we have to help cultivate and support this curiosity.

And now for more applied fields of mathematics and science. Applied science labs are interested in hiring educated people. We discourage students who come to us and ask for a full time job. We can’t hire them full-time, and we don’t need post-graduates who have not yet completed their training. We need people who have already defended a thesis or are planning to do this in parallel with their day job. We need institutions and mechanisms that would enable us to support people who carry on with scientific activity, but at the same time want to deal with science applications and be motivated by them. That’s because today, unfortunately, in the post-Soviet space science is poorly integrated into industry. There’s no culture of industry-specific R&D institutes or entire industry-focused government ministries. The situation in this area is really bad. Either you work in science, but then there will be no applications for you. Or you go to a corporation, where they’ll ask you to write code for a specific product. We should be compensating for this through joint efforts.

Efimov: From my vantage point as moderator, I’ll ask the last question of our remarkable discussion today. We’ve mentioned the Gödel’s theorem. Few people know that Gödel almost stopped writing scientific papers in the later years of his life. He did a lot of math philosophy though. His biographer mentions one of Gödel’s “twisted pronouncements”: ***“Either our mind is mechanical, or math, and even arithmetic, is not our own construct.”*** Your brain starts boiling as you think about it. Hence my usual question, which I ask of all mathematicians: Have we discovered mathematics or have we invented it?

Tyrtyshnikov: Math is so abstract that it creates an impression that it emerges inside our heads. I think that those who do math professionally are convinced that mathematicians discover what actually exists. Laymen think that mathematicians have invented axioms and derive effects from them. But it doesn’t really work like that. Those axioms were not invented in the blink of an eye and everything didn't suddenly begin with them – they were worked on for years, if not decades or even longer. I'm sure that math reveals something that exists independently of humans. But there's one more thing than I can add to this. Even though I think that this statement of mine is not provable.

Gasnikov: Math is worth doing because it puts the mind in order, if not for any other reason. I believe that this paraphrase of a quote by M. V. Lomonosov is a good response to your question. But I'm looking at it from a slightly different perspective. As humans, we started evolving dramatically faster when language first appeared – that's a proven fact. Language is a way to transmit information between generations very fast.

In a certain sense, math is the language of nature. That's next level. Human language makes communication between us possible. While mathematics helps us communicate with nature and understand it. It's a constant attribute. Please note that mathematics is the same everywhere: be it in England or in Russia. It's something special – a way to understand the world, a language with which we try to describe the world in a concise and easy-to-understand way. Consider 16th century mathematics, where formulas were written down in words. It has been evolving ever since. The evolution has been taking place gradually and has now reached unprecedented heights. Of course, if this language describes nature adequately, it's like harmony, we are developing in step with the world around us, and mathematics is an intermediary between us and the world at large. I'm not sure if I have answered your question, but that's how I understand things.

Gusev: This matter is complex and needs to be pondered in all seriousness. I also think that mathematicians discover and learn about new things rather than invent something. But what does it mean – to invent? We can see that there are no different beliefs in mathematics and there are no irreconcilable feuds. It's one language. There are no different dogmas – there are preferences regarding formal constructs and methods or areas of logic, but these are mere variations within the single language of mathematics. It's like making a choice between explicit or clean speech, but it's still our common Russian language. Same in mathematics. It seems like we are studying something objective, because mathematics does not depend on language, culture or religion. I think that mathematical ideas are part of the spiritual world, and that they are the most obvious proof of existence of spiritual entities, the Platonic ideas. They exist independently of humans. Whoever the human being, Pythagoras's theorem stays the same. What role do these ideas play? This may be a universal law of the way our thinking works, or a reflection of reality. I tend to believe that our thinking is isomorphic of the universe: What we have inside is the same as on the outside.

Efimov: We are discovering the laws of nature, we are discovering mathematics that exists as an objective reality, perhaps somewhere outside the limits of Plato's cave. We must go beyond its limits where those images exist. Carl Popper talked about a third world where those ideas supposedly exist.

Tyrtyshnikov: There are no contradictions between mathematicians, there are really no different beliefs, but there are some dramatic stories though. Take Hilbert's famous basis theorem. When it first appeared, certain prominent mathematicians went so far as to say that this was theology rather than mathematics. But then an understanding came. Language is developing all the time, it is a living organism, and not everyone immediately understands or accepts new things that emerge within it.

Gusev: Or another example where Cantor was reflecting on contradictions that arose in his set theory and was unable to resolve them. My understanding is that this was a root cause of his mental troubles.

Tyrtyshnikov: There is also non-constructive mathematics, but it is also of importance for this science. For example, any set can be ordered in some way. That's a totally non-obvious statement. And if you take the axiom of choice, it seems to be harmless. But this supposedly harmless axiom leads to absolutely non-obvious statements. Any set can be ordered in some way, but no algorithm is offered for how to achieve that! That's a totally non-constructive statement, but it is very useful.

Gasnikov: I'd like to end on a more unifying note. As my colleague Andrey Mikhailovich Raygorodsky likes to say: ***"Mathematics is beautiful not because it has applications, but applications exist because mathematics is beautiful."***[15] It seems appropriate to me to quote him one more time.

Efimov: Talk about beautiful. Mathematics is beautiful, that's why we are all engaged in it. We could talk about beauty to no end, but we now have to put a full stop. This could well be an ellipsis for our next discussion. I'd like to thank Gleb Gennadyevich, Alexander Vladimirovich and, of course, Evgeny Evgenyevich for this wonderful evening. Thank you!

[15] Andrey Raygorodsky and Alexey Savvateyev discussed what kind of math should be studied in the 21st century