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Smooth Neutrosophic Topological Spaces

M. K. EL Gayyar

Physics and Mathematical Engineering Dept., Faculty of Engineering, Port-Said University, Egypt.- mohamedelgayyar@hotmail.com

Abstract. As a new branch of philosophy, the neutrosophy was presented by Smarandache in 1980. It was presented as the study of origin, nature, and scope of neutralities; as well as their interactions with different ideational spectra. The aim in this paper is to introduce the concepts of smooth neutrosophic topological space, smooth neutrosophic cotopological space, smooth neutrosophic closure, and smooth neutrosophic interior. Furthermore, some properties of these concepts will be investigated.

Keywords: Fuzzy Sets, Neutrosophic Sets, Smooth Neutrosophic Topology, Smooth Neutrosophic Cotopology, Smooth Neutrosophic Closure, Smooth Neutrosophic Interior.

1 Introduction

In 1986, Badard [1] introduced the concept of a smooth topological space as a generalization of the classical topological spaces as well as the Chang fuzzy topology [2]. The smooth topological space was rediscovered by Ramadan [3], and El-Gayyar et al. [4]. In [5], the authors introduced the notions of smooth interior and smooth closure. In 1983 the intuitionistic fuzzy set was introduced by Atanassov [[6], [7], [8]], as a generalization of fuzzy sets in Zadeh's sense [9], where besides the degree of membership of each element there was considered a degree of non-membership. Smarandache [[10], [11], [12]], defined the notion of neutrosophic set, which is a generalization of Zadeh's fuzzy sets and Atanassov's intuitionistic fuzzy set. The words "neutrosophy" and "neutrosophic" were invented by F. Smarandache in his 1998 book. Etymologically, "neutro-sophy" (noun) [French *neutre* < Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom] means knowledge of neutral thought.

While "neutrosophic" (adjective), means having the nature of, or having the characteristic of Neutrosophy.

Neutrosophic sets have been investigated by Salama et al. [[13], [14], [15]]. The purpose of this paper is to introduce the concepts of smooth neutrosophic topological space, smooth neutrosophic cotopological space, smooth neutrosophic closure, and smooth neutrosophic interior. We also investigate some of their properties.

2 PRELIMINARIES

In this section we use X to denote a nonempty set, I to denote the closed unit interval $[0\,,1]$, I_o to denote the interval $(0\,,1]$, I_1 to denote the interval $[0\,,1)$, and I^X to be the set of all fuzzy subsets defined on X. By $\underline{0}$ and $\underline{1}$ we denote the characteristic functions of ϕ and X, respectively. The family of all neutrosophic sets in X will be denoted by $\aleph(X)$.

2.1 Definition [11], [12], [15]

A neutrosophic set A (NS for short) on a nonempty set X is defined as:

$A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$, $x \in X$

where $T, I, F: X \to [0\,,1]$, and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ representing the degree of membership (namely $T_A(x)$), the degree of indeterminacy (namely $I_A(x)$), and the degree of non-membership (namely $F_A(x)$); for each element $x \in X$ to the set A.

2.2Definition [13], [14], [15]

The Null (empty) neutrosophic set 0_N and the absolute (universe) neutrosophic set 1_N are defined as follows:

$$\text{TypeI} \;:\; 0_N = \langle x,0,0,1\rangle ,\; x \in X$$
$$\text{TypeII} \;:\; 0_N = \langle x,0,1,1\rangle ,\; x \in X$$
$$\text{TypeI} \;:\; 1_N = \langle x,1,1,0\rangle ,\; x \in X$$
$$\text{TypeII} \;:\; 1_N = \langle x,1,0,0\rangle ,\; x \in X$$

2.3Definition [13], [14], [15]

A neutrosophic set A is a subset of a neutrosophic set B, ($A \subseteq B$), may be defined as:

$$\text{TypeI} : A \subseteq B \Leftrightarrow T_A(x) \le T_B(x), \; I_A(x) \le I_B(x), F_A(x) \ge F_B(x), \quad \forall x \in X$$

$$\text{TypeII} : A \subseteq B \Leftrightarrow T_A(x) \le T_B(x), \; I_A(x) \ge I_B(x), F_A(x) \ge F_B(x), \quad \forall x \in X$$

2.4Definition [13], [14], [15]

The Complement of a neutrosophic set A, denoted by coA, is defined as:

$$\text{TypeI} : coA = \langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle$$

$$\text{TypeII} : coA = \langle x, 1 - T_A(x), 1 - I_A(x), 1 - F_A(x) \rangle$$

2.5Definition [13], [14], [15]

Let $A, B \in \aleph(X)$ then:

$$\text{TypeI} : A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$$

$$\text{TypeII} : A \cup B = \langle x, \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$$

$$\text{TypeI} : A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$$

$$\text{TypeII} : A \cap B = \langle x, \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$$

$$[\;]A = \langle x, T_A(x), I_A(x), 1 - T_A(x) \rangle$$

$$\langle\;\rangle A = \langle x, 1 - F_A(x), I_A(x), F_A(x) \rangle$$

2.6Definition [13], [14], [15]

Let $\{A_i\}, i \in J$ be an arbitrary family of neutrosophic sets, then:

$$\text{TypeI} : \underset{i \in J}{\cup} A_i = \left\langle x, \sup_{i \in j} T_{A_i}(x), \sup_{i \in j} I_{A_i}(x), \inf_{i \in j} F_{A_i}(x) \right\rangle$$

$$\text{TypeII} : \underset{i \in J}{\cup} A_i = \left\langle x, \sup_{i \in j} T_{A_i}(x), \inf_{i \in j} I_{A_i}(x), \inf_{i \in j} F_{A_i}(x) \right\rangle$$

$$\text{TypeI} : \underset{i \in J}{\cap} A_i = \left\langle x, \inf_{i \in j} T_{A_i}(x), \inf_{i \in j} I_{A_i}(x), \sup_{i \in j} F_{A_i}(x) \right\rangle$$

$$\text{TypeII} : \underset{i \in J}{\cap} A_i = \left\langle x, \inf_{i \in j} T_{A_i}(x), \sup_{i \in j} I_{A_i}(x), \sup_{i \in j} F_{A_i}(x) \right\rangle$$

2.7Definition [13], [14], [15]

The difference between two neutrosophic sets A and B defined as $A \setminus B = A \cap coB$.

2.8Definition [13], [14]

Every intuitionistic set A on X is NS having the form $A = \langle x, T_A(x), 1 - (T_A(x) + F_A(x)), F_A(x) \rangle$, and every fuzzy set A on X is NS having the form $A = \langle x, T_A(x), 0, 1 - T_A(x) \rangle$, $x \in X$.

2.9Definition [5]

Let Y be a subset of X and $A \in I^X$; the restriction of A on Y is denoted by $A_{/Y}$. For each $B \in I^Y$, the extension of B on X, denoted by B_X, is defined by:

$$B_X = \begin{cases} B(x) & \text{if } x \in A \\ 0.5 & \text{if } X - Y \end{cases}$$

2.10Definition [1],[3]

A smooth topological space (STS, for short) is an ordered pair (X, τ) where X is a nonempty set and $\tau : I^X \to I$ is a mapping satisfying the following properties:

(O1) $\tau(\underline{0}) = \tau(\underline{1}) = 1$

(O2) $\forall A_1, A_2 \in I^X, \; \tau(A_1 \cap A_2) \ge \tau(A_1) \wedge \tau(A_2)$

(O3) $\forall A_i, i \in J, \; \tau(\underset{i \in J}{\cup} A_i) \ge \underset{i \in J}{\wedge} \tau(A_i)$

2.11Definition [1],[3]

A smooth cotopology is defined as a mapping $\Im : I^X \to I$ which satisfies:

(C1) $\Im(\underline{0})=\Im(\underline{1})=1$

(C2) $\forall B_1, B_2 \in I^X,\ \Im(B_1 \cup B_2) \geq \Im(B_1) \wedge \Im(B_2)$

(C3) $\forall A_i, i \in J,\ \Im(\bigcap_{i\in J} B_i) \geq \bigwedge_{i\in J} \Im(B_i)$

3.Smooth Neutrosophic Topological spaces

we will define two types of smooth neutrosophic topological spaces, a smooth neutrosophic topological space (SNTS, for short) take the form $(X, \tau^T, \tau^I, \tau^F)$ and the mappings $\tau^T, \tau^I, \tau^F : I^X \to I$ represent the degree of openness, the degree of indeterminacy, and the degree of non-openness respectively.

3.1 Smooth Neutrosophic Topological spaces of type I

In this part we will consider the definitions of typeI.

3.1.1Definition

A smooth neutrosophic topology (τ^T, τ^I, τ^F) of typeI satisfying the following axioms:

($SNOI_1$) $\tau^T(\underline{0}) = \tau^I(\underline{0}) = \tau^T(\underline{1}) = \tau^I(\underline{1}) = 1$,

and $\tau^F(\underline{0}) = \tau^F(\underline{1}) = 0$

($SNOI_2$) $\forall A_1, A_2 \in I^X$,

$\tau^T(A_1 \cap A_2) \geq \tau^T(A_1) \wedge \tau^T(A_2)$,

$\tau^I(A_1 \cap A_2) \geq \tau^I(A_1) \wedge \tau^I(A_2)$, and

$\tau^F(A_1 \cap A_2) \leq \tau^F(A_1) \vee \tau^F(A_2)$

($SNOI_3$) $\forall A_i \in I^X, i \in J,\ \tau^T(\bigcup_{i\in J} A_i) \geq \bigwedge_{i\in J} \tau^T(A_i)$,

$\tau^I(\bigcup_{i\in J} A_i) \geq \bigwedge_{i\in J} \tau^I(A_i)$, and

$\tau^F(\bigcup_{i\in J} A_i) \leq \bigvee_{i\in J} \tau^F(A_i)$

3.1.2Definition

Let $\Im^T, \Im^I, \Im^F : I^X \to I$ be mappings satisfying the following axioms:

($SNCI_1$) $\Im^T(\underline{0}) = \Im^I(\underline{0}) = \Im^T(\underline{1}) = \Im^I(\underline{1}) = 1$,

and $\Im^F(\underline{0}) = \Im^F(\underline{1}) = 0$

($SNCI_2$) $\forall B_1, B_2 \in I^X$,

$\Im^T(B_1 \cup B_2) \geq \Im^T(B_1) \wedge \Im^T(B_2)$,

$\Im^I(B_1 \cup B_2) \geq \Im^I(B_1) \wedge \Im^I(B_2)$, and

$\Im^F(B_1 \cup B_2) \leq \Im^F(B_1) \vee \Im^F(B_2)$

($SNCI_3$) $\forall B_i \in I^X, i \in J,\ \Im^T(\bigcap_{i\in J} B_i) \geq \bigwedge_{i\in J} \Im^T(B_i)$,

$\Im^I(\bigcap_{i\in J} B_i) \geq \bigwedge_{i\in J} \Im^I(B_i)$, and

$\Im^F(\bigcap_{i\in J} B_i) \leq \bigvee_{i\in J} \Im^F(B_i)$

The triple $(\Im^T, \Im^I, \Im^F)$ is a smooth neutrosophic cotopology of typeI, $\Im^T, \Im^I, \Im^F$ represent the degree of closedness, the degree of indeterminacy, and the degree of non-closedness respectively.

3.1.3Example

Let $X = \{a, b\}$ .Define the mappings $\tau^T, \tau^I, \tau^F : I^X \to I$ as:

$$\tau^T(A) = \begin{cases} 1 & \text{if } A = \underline{0} \\ 1 & \text{if } A = \underline{1} \\ \min(A(a), A(b)) & \text{if } A \text{ is neither } \underline{0} \text{ nor } \underline{1} \end{cases}$$

$$\tau^I(A) = \begin{cases} 1 & \text{if } A = \underline{0} \\ 1 & \text{if } A = \underline{1} \\ 0.5 & \text{if } A \text{ is neither } \underline{0} \text{ nor } \underline{1} \end{cases}$$

$$\tau^F(A) = \begin{cases} 0 & \text{if } A = \underline{0} \\ 0 & \text{if } A = \underline{1} \\ \max(A(a), A(b)) & \text{if } A \text{ is neither } \underline{0} \text{ nor } \underline{1} \end{cases}$$

Then $(X, \tau^T, \tau^I, \tau^F)$ is a smooth neutrosophic topological space on X.

3.1.4Proposition

Let (τ^T, τ^I, τ^F) and $(\Im^T, \Im^I, \Im^F)$ be a smooth neutrosophic topology and a smooth neutrosophic cotopology, respectively, and let $A \in I^X$,

$\tau^{T}_{\Im^{T}}(A)=\Im^{T}(coA)$, $\tau^{I}_{\Im^{I}}(A)=\Im^{I}(coA)$,

$\tau^{F}_{\Im^{F}}(A)=\Im^{F}(coA), \Im^{T}_{\tau^{T}}(A)=\tau^{T}(coA)$,

$\Im^{I}_{\tau^{I}}(A)=\tau^{I}(coA)$, and $\Im^{F}_{\tau^{F}}(A)=\tau^{F}(coA)$, then

(1) $(\tau^{T}_{\Im^{T}}, \tau^{I}_{\Im^{I}}, \tau^{F}_{\Im^{F}})$ and $(\Im^{T}_{\tau^{T}}, \Im^{I}_{\tau^{I}}, \Im^{F}_{\tau^{F}})$ are a smooth neutrosophic topology and a smooth neutrosophic cotopology, respectively.

(2) $\tau^{T}_{\Im^{T}_{\tau^{T}}}=\tau^{T}, \tau^{I}_{\Im^{I}_{\tau^{I}}}=\tau^{I}, \tau^{F}_{\Im^{F}_{\tau^{F}}}=\tau^{F}$,

$\Im^{T}_{\tau^{T}_{\Im^{T}}}=\Im^{T}, \Im^{I}_{\tau^{I}_{\Im^{I}}}=\Im^{I}, \Im^{F}_{\tau^{F}_{\Im^{F}}}=\Im^{F}$,

Proof

(1) (a) $\tau^{T}_{\Im^{T}}(\underline{0})=\tau^{T}_{\Im^{T}}(\underline{1})=\tau^{I}_{\Im^{I}}(\underline{0})=\tau^{I}_{\Im^{T}}(\underline{1})=1$, and

$\tau^{F}_{\Im^{F}}(\underline{0})=\tau^{F}_{\Im^{F}}(\underline{1})=0$

(b) $\forall A_1, A_2 \in I^{X}$, $\tau^{T}_{\Im^{T}}(A_1 \cap A_2)=$

$\Im^{T}(co(A_1 \cap A_2)=\Im^{T}(coA_1 \cup coA_2)\geq$

$\Im^{T}(coA_1)\wedge \Im^{T}(coA_2)=\tau^{T}_{\Im^{T}}(A_1)\wedge \tau^{T}_{\Im^{T}}(A_2)$

,similarly, $\forall A_1, A_2 \in I^{X}$,

$\tau^{I}_{\Im^{I}}(A_1 \cap A_2)\geq \tau^{I}_{\Im^{I}}(A_1)\wedge \tau^{I}_{\Im^{I}}(A_2)$,and

$\tau^{F}_{\Im^{F}}(A_1 \cap A_2)\leq \tau^{F}_{\Im^{F}}(A_1)\vee \tau^{F}_{\Im^{F}}(A_2)$

(c) $\forall A_i \in I^{X}, i \in J, \tau^{T}_{\Im^{T}}(\underset{i\in J}{\cup} A_i)=\Im^{T}(co \underset{i\in J}{\cup} A_i)$

$=\Im^{T}(\underset{i\in J}{\cap} coA_i)\geq \underset{i\in J}{\wedge}\Im^{T}(coA_i)=\underset{i\in J}{\wedge}\tau^{T}_{\Im^{T}}(A_i)$

,similarly, $\forall A_i \in I^{X}, i \in J$,

$\tau^{I}_{\Im^{I}}(\underset{i\in J}{\cup} A_i)\geq \underset{i\in J}{\wedge}\tau^{I}_{\Im^{I}}(A_i)$,and

$\tau^{F}_{\Im^{F}}(\underset{i\in J}{\cup} A_i)\leq \underset{i\in J}{\vee}\tau^{F}_{\Im^{F}}(A_i)$ .Hence, $(\tau^{T}_{\Im^{T}}, \tau^{I}_{\Im^{I}}, \tau^{F}_{\Im^{F}})$ is a smooth neutrosophic topology.Similarly, we can prove that $(\Im^{T}_{\tau^{T}}, \Im^{I}_{\tau^{I}}, \Im^{F}_{\tau^{F}})$ is a smooth neutrosophic cotopology.

(2) the proof is straightforward.

3.1.5Proposition

Let $\{(\tau^{T}_{i}, \tau^{I}_{i}, \tau^{F}_{i})\}_{i\in J}$ be a family of smooth neutrosophic topologies on X .Then their intersection $\underset{i\in J}{\cap}(\tau^{T}_{i}, \tau^{I}_{i}, \tau^{F}_{i})$ is a a smooth neutrosophic topology.

Proof

The proof is a straightforward result of both definition(2.6) and difintion (3.1.1).

3.1.6Definition

Let $(\tau^{T}, \tau^{I}, \tau^{F})$ be a smooth neutrosophic topology of type I, and $A \in I^{X}$. Then the smooth neutrosophic closure of A , denoted by $\overline{A}$ is defined by:

$$\overline{A}=\begin{cases} A & ,(\Im^{T}_{\tau^{T}}(A), \Im^{I}_{\tau^{I}}(A), \Im^{F}_{\tau^{F}}(A))=(1,1,0) \\ \cap\{H : H \in I^{X}, A \subseteq H, \Im^{T}_{\tau^{T}}(H)>\Im^{T}_{\tau^{T}}(A), & \\ \Im^{I}_{\tau^{I}}(H)>\Im^{I}_{\tau^{I}}(A), \Im^{F}_{\tau^{F}}(H)<\Im^{F}_{\tau^{F}}(A)\} , & \\ & (\Im^{T}_{\tau^{T}}(A), \Im^{I}_{\tau^{I}}(A), \Im^{F}_{\tau^{F}}(A))\neq(1,1,0) \end{cases}$$

3.1.7Proposition

Let $(\tau^{T}, \tau^{I}, \tau^{F})$ be a smooth neutrosophic topology on X, and $A, B \in I^{X}$. Then

(1) $\underline{0}=\overline{\underline{0}}$, $\underline{1}=\overline{\underline{1}}$

(2) $A \subseteq \overline{A}$

(3) $\Im^{T}_{\tau^{T}}(\overline{A})\geq \Im^{T}_{\tau^{T}}(A), \Im^{I}_{\tau^{I}}(\overline{A})\geq \Im^{I}_{\tau^{I}}(A)$, and

$\Im^{F}_{\tau^{F}}(\overline{A})\leq \Im^{F}_{\tau^{F}}(A)\}$, $\forall A \in I^{X}$

(4) $B \subseteq A, \Im^{T}_{\tau^{T}}(A)\geq \Im^{T}_{\tau^{T}}(B), \Im^{I}_{\tau^{I}}(A)\geq \Im^{I}_{\tau^{I}}(B)$

and $\Im^{F}_{\tau^{F}}(A)\leq \Im^{F}_{\tau^{F}}(B)\Rightarrow \overline{B}\subseteq \overline{A}$, $\forall A, B \in I^{X}$

(5) $\overline{A} \subseteq \overline{\overline{A}}$

(6) $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

Proof

(1) Obvious

(2) Directly from definition (3.1.6)

(3) (a) if $A = \overline{A}$, the proof is straightforward .

(b) if $A \neq \overline{A}$, we have from the definition (3.1.2) and the

definition (3.1.6):

$$\Im^{T}_{\tau^{T}}(\overline{A}) = \Im^{T}_{\tau^{T}}(\cap\{H : H \in I^{X}, A \subseteq H,$$
$$\Im^{T}_{\tau^{T}}(H) > \Im^{T}_{\tau^{T}}(A), \Im^{I}_{\tau^{I}}(H) > \Im^{I}_{\tau^{I}}(A),$$
$$\Im^{F}_{\tau^{F}}(H) < \Im^{F}_{\tau^{F}}(A)\}) \geq \wedge\{\Im^{T}_{\tau^{T}}(H) : H \in I^{X}, A \subseteq H, \text{ we}$$
$$\Im^{T}_{\tau^{T}}(H) > \Im^{T}_{\tau^{T}}(A), \Im^{I}_{\tau^{I}}(H) > \Im^{I}_{\tau^{I}}(A),$$
$$\Im^{F}_{\tau^{F}}(H) < \Im^{F}_{\tau^{F}}(A)\} \geq \Im^{T}_{\tau^{T}}(A)$$

can prove that $\Im^{I}_{\tau^{I}}(\overline{A}) \geq \Im^{I}_{\tau^{I}}(A)$ in a similar way.

$$\Im^{F}_{\tau^{F}}(\overline{A}) = \Im^{F}_{\tau^{F}}(\cap\{H : H \in I^{X}, A \subseteq H,$$
$$\Im^{T}_{\tau^{T}}(H) > \Im^{T}_{\tau^{T}}(A), \Im^{I}_{\tau^{I}}(H) > \Im^{I}_{\tau^{I}}(A),$$
$$\Im^{F}_{\tau^{F}}(H) < \Im^{F}_{\tau^{F}}(A)\}) \leq \vee\{\Im^{F}_{\tau^{F}}(H) : H \in I^{X}, A \subseteq H,$$
$$\Im^{T}_{\tau^{T}}(H) > \Im^{T}_{\tau^{T}}(A), \Im^{I}_{\tau^{I}}(H) > \Im^{I}_{\tau^{I}}(A),$$
$$\Im^{F}_{\tau^{F}}(H) < \Im^{F}_{\tau^{F}}(A)\} \leq \Im^{F}_{\tau^{F}}(A)$$

(4) (a) if $B = \overline{B}$, then $A = \overline{A}$ and $\overline{B} \subseteq \overline{A}$.

(b) if $B \neq \overline{B}$, and $A = \overline{A}$

$$\overline{B} = \cap\{H : H \in I^{X}, B \subseteq H, \Im^{T}_{\tau^{T}}(H) > \Im^{T}_{\tau^{T}}(B),$$
$$\Im^{I}_{\tau^{I}}(H) > \Im^{I}_{\tau^{I}}(B), \Im^{F}_{\tau^{F}}(H) < \Im^{F}_{\tau^{F}}(B)\},$$

this family contains A, hence, $\overline{B} \subseteq A = \overline{A}$

(c) if $B \neq \overline{B}$, and $A \neq \overline{A}$

From definition (3.1.6) every element in the family $\overline{A}$ will be an element in the family $\overline{B}$, hence $\overline{B} \subseteq \overline{A}$.

(5) From (2) , (3) and the definition (3.1.6) we have

$\overline{A} \subseteq \overline{\overline{A}}$.

(6) (a) if $A = \overline{A}$, and $B = \overline{B}$, then

$$\overline{A \cup B} = A \cup B \supseteq A \cap B = \overline{A} \cap \overline{B}$$

(b) if $A = \overline{A}$, $B \neq \overline{B}$, and $\overline{A \cup B} \neq A \cup B$,

from (4) $\overline{B} \subseteq \overline{A \cup B}$, hence $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

(c) if $A = \overline{A}$, $B \neq \overline{B}$, and $\overline{A \cup B} = A \cup B$,

then $\overline{A} \subseteq \overline{A \cup B}$, hence $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

(d) if $A \neq \overline{A}$, $B = \overline{B}$, and $\overline{A \cup B} \neq A \cup B$,

similar to(6b)

(e) if $A \neq \overline{A}$, $B = \overline{B}$, and $\overline{A \cup B} = A \cup B$,

similar to(6c)

(f) if $A \neq \overline{A}$, $B \neq \overline{B}$,and $\overline{A \cup B} = A \cup B$, it follows

from(4)that $\overline{A} \subseteq \overline{A \cup B}$, hence $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$.

(g) if $A \neq \overline{A}$, $B \neq \overline{B}$,and $\overline{A \cup B} \neq A \cup B$

$$\overline{A \cup B} = \cap \{H : H \in I^X, A \cup B \subseteq H, \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(A \cup B), \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(A \cup B), \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(A \cup B)\}$$

$$\supseteq \cap \{H : H \in I^X, A \cup B \subseteq H, \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(A) \wedge \Im^T_{\tau^T}(B), \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(A) \wedge \Im^I_{\tau^I}(B), \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(A) \vee \Im^F_{\tau^F}(B)\}$$

$$= \cap \{H : H \in I^X, A \subseteq H, B \subseteq H, \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(A) \text{ or } \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(B), \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(A) \text{ or } \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(B), \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(A) \text{ or } \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(B)\}$$

$$\supseteq \cap [\ \{H : H \in I^X, A \subseteq H, \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(A), \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(A), \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(A)\} \cup \{H : H \in I^X, B \subseteq H, \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(B), \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(B), \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(B)\}\ ]$$

$$= [\ \cap \{H : H \in I^X, A \subseteq H, \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(A), \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(A), \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(A)\}\ ] \cap [\ \cap \{H : H \in I^X, B \subseteq H, \Im^T_{\tau^T}(H) > \Im^T_{\tau^T}(B), \Im^I_{\tau^I}(H) > \Im^I_{\tau^I}(B), \Im^F_{\tau^F}(H) < \Im^F_{\tau^F}(B)\}\ ]$$

$$= \overline{A} \cap \overline{B}$$

3.1.8 Definition

Let (τ^T, τ^I, τ^F) be a smooth neutrosophic topology of type I, and $A \in I^X$. Then the smooth neutrosophic interior of A, denoted by A^o is defined by:

$$A^o = \begin{cases} A & ,(\tau^T(A), \tau^I(A), \tau^F(A)) = (1,1,0) \\ \cup \{H : H \in I^X, H \subseteq A, \tau^T(H) > \tau^T(A), \tau^I(H) > \tau^I(A), \tau^F(H) < \tau^F(A)\}, & (\tau^T(A), \tau^I(A), \tau^F(A)) \neq (1,1,0) \end{cases}$$

3.1.9 Proposition

Let (τ^T, τ^I, τ^F) be a smooth neutrosophic topology on X, and $A, B \in I^X$. Then

(1) $\underline{0} = \underline{0}^o$, $\underline{1} = \underline{1}^o$

(2) $A^o \subseteq A$

(3) $\tau^T(A^o) \geq \tau^T(A), \tau^I(A^o) \geq \tau^I(A)$, and $\tau^F(A^o) \leq \tau^F(A)\}$, $\forall A \in I^X$

(4) $B \subseteq A, \tau^T(B) \geq \tau^T(A), \tau^I(B) \geq \tau^I(A)$ and $\tau^F(B) \leq \tau^F(A) \Rightarrow B^o \subseteq A^o$, $\forall A, B \in I^X$

(5) $(A^o)^o \subseteq A^o$

(6) $(A \cap B)^o \subseteq A^o \cup B^o$

Proof

Similar to the procedure used to prove Proposition (3.1.7)

3.2. Smooth Neutrosophic Topological spaces of type II

In this part we will consider the definitions of typeII. In a similar way as in typeI, we can state the following definitions and propositions. The proofs of the propositions of typeII, will be similar to the proofs of the propositions in typeI.

3.2.1Definition

A smooth neutrosophic topology (τ^T, τ^I, τ^F) of typeII satisfying the following axioms:

$(SNOII_1)$ $\tau^T(\underline{0}) = \tau^T(\underline{1}) = 1$, and

$\tau^I(\underline{0}) = \tau^I(\underline{1}) = \tau^F(\underline{0}) = \tau^F(\underline{1}) = 0$

$(SNOII_2)$ $\forall A_1, A_2 \in I^X$,

$\tau^T(A_1 \cap A_2) \geq \tau^T(A_1) \wedge \tau^T(A_2)$,

$\tau^I(A_1 \cap A_2) \leq \tau^I(A_1) \vee \tau^I(A_2)$, and

$\tau^F(A_1 \cap A_2) \leq \tau^F(A_1) \vee \tau^F(A_2)$

$(SNOII_3)$ $\forall A_i \in I^X, i \in J, \tau^T(\bigcup_{i \in J} A_i) \geq \bigwedge_{i \in J} \tau^T(A_i)$,

$\tau^I(\bigcup_{i \in J} A_i) \leq \bigvee_{i \in J} \tau^I(A_i)$, and

$\tau^F(\bigcup_{i \in J} A_i) \leq \bigvee_{i \in J} \tau^F(A_i)$

3.2.2Definition

Let $\mathfrak{I}^T, \mathfrak{I}^I, \mathfrak{I}^F : I^X \to I$ be mappings satisfying the following axioms:

$(SNCII_1)$ $\mathfrak{I}^T(\underline{0}) = \mathfrak{I}^T(\underline{1}) = 1$, and

$\mathfrak{I}^I(\underline{0}) = \mathfrak{I}^I(\underline{1}) = \mathfrak{I}^F(\underline{0}) = \mathfrak{I}^F(\underline{1}) = 0$

$(SNCII_2)$ $\forall B_1, B_2 \in I^X$,

$\mathfrak{I}^T(B_1 \cup B_2) \geq \mathfrak{I}^T(B_1) \wedge \mathfrak{I}^T(B_2)$,

$\mathfrak{I}^I(B_1 \cup B_2) \leq \mathfrak{I}^I(B_1) \vee \mathfrak{I}^I(B_2)$, and

$\mathfrak{I}^F(B_1 \cup B_2) \leq \mathfrak{I}^F(B_1) \vee \mathfrak{I}^F(B_2)$

$(SNCII_3)$ $\forall B_i \in I^X, i \in J, \mathfrak{I}^T(\bigcap_{i \in J} B_i) \geq \bigwedge_{i \in J} \mathfrak{I}^T(B_i)$,

$\mathfrak{I}^I(\bigcap_{i \in J} B_i) \leq \bigvee_{i \in J} \mathfrak{I}^I(B_i)$, and

$\mathfrak{I}^F(\bigcap_{i \in J} B_i) \leq \bigvee_{i \in J} \mathfrak{I}^F(B_i)$

The triple $(\mathfrak{I}^T, \mathfrak{I}^I, \mathfrak{I}^F)$ is a smooth neutrosophic cotopology of typeII, $\mathfrak{I}^T, \mathfrak{I}^I, \mathfrak{I}^F$ represent the degree of closedness, the degree of indeterminacy, and the degree of non-closedness respectively.

3.2.3Example

Let $X = \{a, b\}$. Define the mappings $\tau^T, \tau^I, \tau^F : I^X \to I$ as:

$$\tau^T(A) = \begin{cases} 1 & \text{if } A = \underline{0} \\ 1 & \text{if } A = \underline{1} \\ \min(A(a), A(b)) & \text{if } A \text{ is neither } \underline{0} \text{ nor } \underline{1} \end{cases}$$

$$\tau^I(A) = \begin{cases} 0 & \text{if } A = \underline{0} \\ 0 & \text{if } A = \underline{1} \\ 0.5 & \text{if } A \text{ is neither } \underline{0} \text{ nor } \underline{1} \end{cases}$$

$$\tau^F(A) = \begin{cases} 0 & \text{if } A = \underline{0} \\ 0 & \text{if } A = \underline{1} \\ \max(A(a), A(b)) & \text{if } A \text{ is neither } \underline{0} \text{ nor } \underline{1} \end{cases}$$

Then $(X, \tau^T, \tau^I, \tau^F)$ is a smooth neutrosophic topological space on X.

Note that: the Propositions (3.1.4) and (3.1.5) are satisfied for typeII.

3.2.4Definition

Let (τ^T, τ^I, τ^F) be a smooth neutrosophic topology of type II, and $A \in I^X$. Then the smooth neutrosophic closure of A, denoted by $\overline{A}$ is defined by:

$$\overline{A} = \begin{cases} A & ,(\mathfrak{I}^T_{\tau^T}(A), \mathfrak{I}^I_{\tau^I}(A), \mathfrak{I}^F_{\tau^F}(A)) = (1,1,0) \\ \cap \{H : H \in I^X, A \subseteq H, \mathfrak{I}^T_{\tau^T}(H) > \mathfrak{I}^T_{\tau^T}(A), \\ \mathfrak{I}^I_{\tau^I}(H) < \mathfrak{I}^I_{\tau^I}(A), \mathfrak{I}^F_{\tau^F}(H) < \mathfrak{I}^F_{\tau^F}(A)\}, \\ \quad (\mathfrak{I}^T_{\tau^T}(A), \mathfrak{I}^I_{\tau^I}(A), \mathfrak{I}^F_{\tau^F}(A)) \neq (1,1,0) \end{cases}$$

Also, the smooth neutrosophic interior of A, denoted by A^o is defined by:

$$A^o = \begin{cases} A & ,(\tau^T(A), \tau^I(A), \tau^F(A)) = (1,1,0) \\ \cup \{H : H \in I^X, H \subseteq A, \tau^T(H) > \tau^T(A), \\ \tau^I(H) < \tau^I(A), \tau^F(H) < \tau^F(A)\}, \\ \quad (\tau^T(A), \tau^I(A), \tau^F(A)) \neq (1,1,0) \end{cases}$$

Note That: the Propositions (3.1.7) and (3.1.9) are satisfied for typeII.

4. Conclusion and Future Work

In this paper, the concepts of smooth neutrosophic topological structures were introduced. In two different types we've presented the concepts of smooth neutrosophic topological space, smooth neutrosophic cotopological space, smooth neutrosophic closure, and smooth neutrosophic interior. Due to unawareness of the behaviour of the degree of indeterminacy, we've chosen for τ^I to act like τ^T in the first type, while in the second type we preferred that τ^T behaves like τ^F. Therefore, the definitions given above can also be modified in several ways depending on the behaviour of τ^I. Moreover, as a consequence of our choices of the performance of τ^I, one can see that: In typeI, booth τ^T and τ^I defined in (3.1.1) with their conditions are smooth topologies; while in typeII, only τ^T defined in (3.2.1) with its conditions is a smooth topology.

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