Interference and memory capacity limitations

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Abstract

Working memory (WM) is thought to have a fixed and limited capacity. However, the origins of these capacity limitations are debated, and generally attributed to active, attentional processes. Here, we show that the existence of interference among items in memory mathematically guarantees fixed and limited capacity limits under very general conditions, irrespective of any processing assumptions. Assuming that interference (i) increases with the number of interfering items and (ii) brings memory performance to chance levels for large numbers of interfering items, capacity limits are a simple function of the relative influence of memorization and interference. In contrast, we show that time-based memory limitations do not lead to fixed memory capacity limitations that are independent of the timing properties of an experiment. We show that interference can mimic both slot-like and continuous resource-like memory limitations, suggesting that these types of memory performance might not be as different as commonly believed. We speculate that slot-like WM limitations might arise from crowding-like phenomena in memory when participants have to retrieve items. Further, based on earlier research on parallel attention and enumeration, we suggest that crowding-like phenomena might be a common reason for the three major cognitive capacity limitations. As suggested by Miller (1956) and Cowan (2001), these capacity limitations might thus arise due to a common reason, even though they likely rely on distinct processes.

Keywords: Working memory; Interference; Temporary Memory; Memory Capacity

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Introduction

A central tenet of the cognitive neurosciences holds that working memory (WM) capacity is severely limited, typically to about four items (Miller, 1956; Cowan, 2001; for reviews, see e.g., Conway et al., 2005; Cowan, 1995, 2005). These memory capacity limitations have profound effects on many aspects cognitive processing, and WM capacity correlates with various measures of language comprehension, reasoning, educational achievement and even general intelligence (e.g., Barrouillet, 1996; Conway, Kane, & Engle, 2003; Daneman & Carpenter, 1980; Daneman & Green, 1986; Engle, Tuholski, Laughlin, & Conway, 1999; Engle, Carullo, & Collins, 1991; Engle, Cantor, & Carullo, 1992; Fukuda, Vogel, Mayr, & Awh, 2010; King & Just, 1991; Kyllonen & Christal, 1990).

While there is a general agreement *that* WM is limited, there is substantial debate on *why* it is limited. For example, is WM better characterized as a set of discrete memory "slots" in which items can be placed (see, among many others, Cowan, 2001; Hartshorne, 2008; Luck & Vogel, 1997; Piazza, Fumarola, Chinello, & Melcher, 2011; Rouder et al., 2008; W. Zhang & Luck, 2008) or rather as a continuous resource (e.g., Alvarez & Cavanagh, 2004; Bays & Husain, 2008; van den Berg, Shin, Chou, George, & Ma, 2012; Just & Carpenter, 1992; Ma, Husain, & Bays, 2014)?

Specifically, many authors hold that memory capacity limitations stem from active processes that allow a limited number of items to be encoded in an all-or-none fashion, and that these processes rely on some form of attention (e.g., Cowan, 1995, 2005; Conway & Engle, 1994; Kane & Engle, 2002; Oberauer, 2002; Rouder et al., 2008). For example, according to Cowan's (1995) model of WM, we can remember exactly 3 or 4 items (depending on the person) because we remember them by attending to them, and we have an attentional capacity of only 3 or 4 items. When we have to remember more items, we pick 3 or 4 of them, attend to those, while the rest does not enter WM. This is because, following Miller's (1956) suggestion that different cognitive capacity limitations might have a common source, WM, subitizing and parallel individuation might all rely on a system of parallel attention such as the one proposed by Trick and Pylyshyn (1994) (Cowan, 2001; see Cowan, 2015 for a historic overview, and Piazza et al., 2011, for a recent discussion of this model). According to such views, WM is better characterized as a set of discrete memory slots, and these slots are made available by a parallel attention system.

However, the evidence for the role of such active, attentional processes in WM is mixed, and seems to depend on the type of experiment used to measure WM capacities. While there is little doubt that WM tasks such as complex span tasks are effortful and require active manipulation of items (see e.g. Engle, 2002, for a review, but see Oberauer, Lewandowsky, Farrell, Jarrold, & Greaves, 2012, for an interference-based model of such tasks), other WM tasks such as the change detection paradigm (see below) show little to no interference with tasks testing parallel attention (e.g., Fougnie & Marois, 2006; Hollingworth & Maxcey-Richard, 2013; H. Zhang, Xuan, Fu, & Pylyshyn, 2010).

Accordingly, other scholars propose that WM is a continuous resource that can be utilized either for processing or for temporarily storing a limited number of items with varying precision, and that this resource can hold fewer items when more detailed memory representations are required (e.g., Alvarez & Cavanagh, 2004; Bays & Husain, 2008; van den Berg et al., 2012; Just & Carpenter, 1992; Ma et al., 2014). Still other scholars propose that there is no real WM system, and that observations attributed to WM have other interpretations (Öztekin & McElree, 2007; Öztekin, Davachi, & McElree, 2010). Specifically, these authors propose that we can keep a single item in the focus of attention (and hence in WM), but that all other items need to be retrieved from LTM.

Based on these discrepant data, we believe that is important to identify the psychological mechanisms that are responsible for memory capacity limitations. Here, we contribute to this goal by showing what is *not* diagnostic of the underlying psychological mechanisms: the existence of capacity limitations and their mathematical form. Specifically, we provide mathematical proofs that, in the presence of interference among memory items, fixed and finite memory capacity limitations are largely a mathematical consequence of the presence of interference even in the absence of active maintenance mechanisms (that are generally assumed to explain capacity limitations), and that interference-based capacity limitations can mimic the signatures of both slot-based and resource-based capacity limitations. We will then speculate under which psychological conditions either type of memory limitation might be observed, and suggest that the three major cognitive capacity limitations — WM, parallel attention and subitizing — might have a common source even if the underlying mechanisms are different: All three capacity limitations might be due to interference among items, but the interference might take place among distinct representations that might not be shared across capacities.

Slot- vs. resource-based models of WM

In the last years, the strongest evidence for fixed WM limitations in terms of the number of items arguably came from research on visual WM, especially since Luck and Vogel's (1997) seminal report of their change detection paradigm. In this task, participants view an array of items (e.g., color patches) for 100 ms, and then view another array. They have to decide whether or not the two arrays are identical. Results show that performance is at ceiling for array-sizes up to 3, and then decreases for greater set-sizes. Participants' memory thus seems to be capacity-limited as predicted by a slot-model. In this and the following literature, fixed and limited capacities have thus been used as diagnostic of slot-like memory representations vs. continuous resource-like representations, often based on sophisticated mathematical analyses (e.g., van den Berg et al., 2012; Rouder et al., 2008; W. Zhang & Luck, 2008).

However, the psychological interpretation of either type of model is not always clear. First, despite the sophisticated mathematical analyses, such analyses can sometimes be ambiguous. For example, W. Zhang and Luck's (2008) influential slot-based model turns out to be mathematically equivalent to a continuous resource-based model with slightly different assumptions (see Appendix A). Second, and as mentioned above, the slot-based model has a clear, parsimonious and elegant interpretation, where WM, subitizing and parallel individuation might all rely on the system of parallel attention proposed by Trick and Pylyshyn (1994). However, given that taxing visual parallel attention does not seem to impair visual WM (e.g., Fougnie & Marois, 2006; Hollingworth & Maxcey-Richard, 2013; H. Zhang et al., 2010), a parallel attention system cannot be the basis for memory limitations either.

With respective to resource-based models, it is not always clear what these resources represent psychologically. In fact, memory capacity limitations can have a variety of sources, and sometimes appear in surprising contexts. For example, Nairne and Neath (2001) asked participants to rate words in lists of various lengths. After they had rated all lists and had completed a filled 5-min retention interval, participants were given a surprise memory test for the lists. They had a span of about 5 items for each list — although the lists were encoded into long-term memory (LTM).

We thus believe that it is important to identify the cognitive mechanisms responsible for WM limitations. To start this endeavor, we show below what is *not* diagnostic of the underlying cognitive mechanisms: the mathematical shape of the capacity limitations. Based on assumptions that amount to little more than the definition of interference, we provide mathematical proofs that fixed and finite capacity limitations are a natural consequence of the existence of interference among items, even in the absence of active maintenance mechanisms. We then show that interference-based capacity limitations can mimic both continuous-resource models and slot-based models, suggesting that these models might not be as distinct as commonly believed, and that the same psychological mechanism can result in either response profile. We then speculate about the psychological conditions under which interference might lead to slot-like limitations, and suggest that capacity limitations in WM and parallel attention might be due to a common principle — representations that are nearby in (representational) space might interfere with each other. We also prove that memory limitations based on decay do not lead to fixed capacity limitations across retention intervals, though they do lead to fixed an finite capacity limitations as long as the timing properties of an experiment are kept constant.

The claim that interference is sufficient to produce memory capacity limitations even in the absence of active maintenance should not be interpreted as more general claims that attention plays no role in WM. In fact, there is substantial evidence that attention is beneficial for and used in memorization (see, among many others, e.g., Chen & Cowan, 2009;

Craik & Lockhart, 1972; Lepsien & Nobre, 2007; Majerus et al., 2014; Morey & Bieler, 2013; Vergauwe, Camos, & Barrouillet, 2014). Rather, we show that attention is not *necessary* to account for fixed and finite memory limitations, and that the involvement of attention, as well as the cognitive mechanisms that can lead to slot-like representations, need to be established from psychological manipulations. Further, while our work is primarily inspired by research on visual WM, the proofs apply to memory in other modalities as well.

Memory and interference

Interference has long been known to play a crucial role for memory limitations. For example, in an experiment testing memory for numbers, participants might hear the list 4 7 2 3 9... and then the list 5 3 8 6 1..., and have to decide whether the number '2' was in the second list. Participants thus face the problem to decide that '2' was not in the current list, although it was in previous lists. In other words, there is interference with other items in memory. This type of "proactive interference" (PI) is such a pervasive feature of WM experiments that some authors proposed that the function of WM processes is to counteract it (Engle, 2002), and the role of interference is central to several computational models of WM (e.g., Oberauer et al., 2012; Oberauer & Lin, 2017; Sengupta, Surampudi, & Melcher, 2014). Further, even a single previous item can lead to massive interference (Keppel & Underwood, 1962), while, when interference is minimized, virtually no capacity limitations are observed (Endress & Potter, 2014a). Accordingly, memory capacity estimates are much larger when observers can recall objects embedded in a naturalistic scene (e.g., the objects found in a kitchen presented in a picture of a kitchen), presumably because the objects' context helps memorization as opposed to leading to interference (e.g., Hollingworth, 2004;

Melcher, 2001). Further, it has been shown that the ability to deal with interference in WM task predicts measures of intelligence (e.g., Braver, Gray, & Burgess, 2008; Burgess, Gray, Conway, & Braver, 2011). Hence, it seems clear that interference has a major influence on how many items can be retained in memory.[1]

Goals and limitations of the current proofs

Before presenting our model in more detail, it is important to be clear about its goals. We will show that fixed and finite memory capacities are a largely automatic consequence of inter-item interference, even without positing capacity-limited active maintenance mechanisms (e.g., Cowan, 1995, 2005; Conway & Engle, 1994; Kane & Engle, 2002; Oberauer, 2002; Rouder et al., 2008), or a finite resource (e.g., Alvarez & Cavanagh, 2004; Bays & Husain, 2008; van den Berg et al., 2012), and that interference functions can mimic both slot-like and continuous memory limitations. We do not claim, however, that attention is not involved in memorizations, and there is a wealth of evidence showing that attention (or at least active processing) is used in memorization, starting with the levels of processing theory (see, among many others, e.g., Chen & Cowan, 2009; Craik & Lockhart, 1972; Lepsien & Nobre, 2007; Majerus et al., 2014; Morey & Bieler, 2013; Vergauwe et al., 2014). What our proofs show is that such mechanisms are not *necessary* to explain fixed and finite capacity limitations, and that fixed capacities are not necessarily diagnostic of active mechanisms such as a parallel attention system.

Further, given the generality of our proofs, the model makes few processing predic-

[1] Some authors propose that PI has only a limited effect on WM capacity estimates (Hartshorne, 2008; see also Lin & Luck, 2012; Makovski & Jiang, 2008). However, except in some of Hartshorne's (2008) analyses, these conclusions were based on comparisons of high interference with slightly lower interference, and, in all of these studies, capacity estimates were mathematically limited to 4 or 5, limiting the potential size of the interference effect (see Endress & Potter, 2014a, for discussion).

tions on its own. Rather, we see it as a theoretical baseline with minimal assumptions against which more elaborate models should be compared: our analyses reveal the logical consequences of assumptions that are little more than the very definition of interference.

Such abstract descriptions are quite common in other fields. For example, in many physical systems, we just need to know the relative energies and the temperature of two states of a system to calculate their relative probabilities (i.e., we can use the Boltzmann distribution); the system's internal details are often (but not always) irrelevant. Similarly, our model reveals the conditions under which interference leads to capacity limitations, irrespective of how interference "works" in terms of the underlying psychological mechanisms.

In fact, there is a wealth of models of memory that describe different aspects of memory, many of which account for interference effects. We will discuss some of these models after having presented our analysis. We will argue that models that account for interference effects are special cases of our analyses with respect to the specific relation between memory capacity and interference. However, these models account for many other memory phenomena about which our analysis is completely silent.

The model

We consider a simple model, where participants are presented with P items such that they can usefully retrieve M_P of them. (We consider here some form of short-term memory. If this type of memory reflects the activated part of LTM (e.g., Endress & Potter, 2014b; Ranganath & Blumenfeld, 2005), M_P reflects the expected number of items activated from LTM, and not all items in LTM.) When an additional item is presented, it has a probability R to be retrieved if there is no interference. Hence, if we already remember M_P out of P

items, and are presented with an additional item, we will remember $M_{P+1} = M_P + R$ items. For example, if $R = 1$ (i.e., perfect encoding), and if we already remember 3 out 3 presented items, presenting an additional items will lead to remembering 4 items.

However, this formula ignores interference. The number of retrievable items M_{P+1} will thus be reduced by some interference function $\mathcal{I}(M_P)$, and is given by

$$M_{P+1} = M_P + R - \mathcal{I}(M_P) \tag{1}$$

with $M_1 = R$. For example, if encoding is perfect ($R = 1$), if $\mathcal{I}(M) = .25 \times M$, and if we remember already (on average) 2.3 out of 3 presented items, we will remember (on average) $2.3 + 1 - .25 \times 2.3 \approx 2.7$ items after viewing an additional item. (We use a recurrence relation to calculate the number of items in memory out of mathematical convenience, but we do not propose that actual humans represent such a recurrence relation.) [2]

We note that this model is agnostic as to whether interference reduces the number of items that can be maintained in memory, or acts only at retrieval (see Nairne, 2002a, for arguments that remembering involves both the status of the memory trace and retrieval), as to whether there is a separate short-term memory store or not, and about many other important distinctions. For the proofs below, we just need to assume that interference is a growing function of the number of items in memory.[3] Further, while we assume that

[2] It should be noted that the function $\mathcal{I}$ depends on M_P and not on P, and it describes the cumulative effect of all the previously presented items, taking into account their various memory strengths.

[3] While we assume that items are presented sequentially rather than simultaneously, most WM paradigms use sequential procedures, and where visual items are presented simultaneously, it has been argued that observers encode them one after the other (Liu & Becker, 2013; Vogel, Woodman, & Luck, 2006; but see Mance, Becker, & Liu, 2012).

interference is additive, the model covers multiplicative interference as well.[4]

It should be noted that our model might not be applicable to phenomena of verbal working memory such as time-based capacity limitations (e.g., Schweickert & Boruff, 1986). This might be because verbal memory has at least partially different properties from memory in other domains (e.g., Baddeley, 1996; though see Nairne, 2002b, for an alternative interpretation), and it is important to ask whether memory limitations in verbal memory can also be derived from minimal assumptions such as those presented here. However, we will prove below that pure time-based memory constraints do not lead to fixed capacity limitations in terms of the number of retrievable items, and that, with time-based decay, memory capacity is a function of the timing properties of an experiment, at least in the absence of active refreshing mechanisms.

Three cases of memory capacity limitations — theory

In the proofs below, we make only very basic assumptions about the interference function $\mathcal{I}$. First, and most importantly, we assume that $\mathcal{I}$ is strictly increasing (i.e., $\forall M \geq 0 : \mathcal{I}'(M) > 0$). In other words, we assume that interference is more pronounced when more items are active in memory, which, we believe, is an uncontroversial assumption and might be considered to be part of the definition of interference. Second, we assume that there is no interference when there are no active memory items, i.e. $\mathcal{I}(0) = 0$. However, this assumption is not necessary[5], and just a matter of convenience. Third, we assume that $\mathcal{I}$

[4]For multiplicative interference, the equation corresponding to equation (1) would be $M_{P+1} = M_P + R\mathcal{I}(M_P)$, where our assumptions translate to $\mathcal{I}(0) = 1$ and a negative derivative of $\mathcal{I}$. By considering the function $\hat{\mathcal{I}}(M) = 1 - \mathcal{I}(M)$, this equation becomes $M_{P+1} = M_P + R - R\hat{\mathcal{I}}(M_P)$. Given that $R\hat{\mathcal{I}}(0) = 0$ and that $R\hat{\mathcal{I}}'(M_P) > 0$, $R\hat{\mathcal{I}}(M_P)$ is an interference function as defined below.

[5]The proofs below remain valid also for $\mathcal{I}(0) = \beta > 0$; if so, we simply rewrite equation 1 by replacing $\mathcal{I}$ with $\hat{\mathcal{I}}(M) = \mathcal{I}(M) - \beta$ and R with $\hat{R} = R + \beta$.

is continuously differentiable. This is just a technical assumption that does not restrict the psychological generality of our model.

With these assumptions, we consider three situations that result in different memory capacities. First, $\mathcal{I}$ might cross R once a certain number of items is active in memory. Psychologically, this means that, if the number of interfering items is large enough, interference becomes so strong that memory retrieval performance drops to the chance level in a recognition memory task (although our model is agnostic as to whether a recognition task or a recall task is chosen).[6] This assumption is implicitly present in many models of WM. In all-or-none models (Cowan, 2001; Rouder et al., 2008), this is because only a limited number of items (e.g., 4) is placed in WM. When more items are presented, those additional items cannot be memorized, which, in turn, will bring retrieval performance eventually to chance. In continuous resource models (e.g., Alvarez & Cavanagh, 2004; Bays & Husain, 2008; van den Berg et al., 2012; Just & Carpenter, 1992), this is because the memory resource will be depleted at some point.

Under these conditions, memory capacity is fixed and finite. It is given by $K = \mathcal{I}^{-1}(R)$, where $\mathcal{I}^{-1}$ is the inverse function of $\mathcal{I}$. In other words, no matter how many items are presented, observers will remember only $\mathcal{I}^{-1}(R)$ of them (though, for some implausible interference functions, there might be oscillations around K, with sometimes slightly more and sometimes slightly fewer items in memory, but always less than $K + R$).

Second, $\mathcal{I}$ might converge to R without crossing it. Practically, this means that

[6] Specifically, we can convert the number of retrievable items into the percentage of correct responses in a two-alternative forced choice task by inverting the two-high-threshold formula that is generally used to calculate memory capacities (e.g., Cowan, 2001; Rouder et al., 2008), as calculated for a recognition experiment, where participants are shown a single test item on each trial, and have to decide whether it was part of the memory set, and where there is an equal number of "old" and "new" test items. The formulae will be presented later.

performance almost goes to chance when the number of presented items is increased, but there is always a (very) small residual ability to store additional information in memory. Under this condition, the memory capacity is what we call "*practically limited.*" In principle, the number of retained items converges to infinity, but so slowly that it has no relevance in practice. For instance, if for remembering a single additional item, we need to present 10 million additional items, the number of retained items is constant for all practical purposes.

Third, and unsurprisingly, if interference remains low, and never comes in the vicinity of the memory strength of a new item, it will never prevent new items from being added to memory. Hence, the number of remembered items will clearly grow indefinitely as more items are presented. We will now prove these assertions in turn. Hence, the remainder of this section does not add any new information. All theorems cited below are well known in analysis, and can found in handbooks such as Bronstein and Semendjajew (1996). In Appendix B, we provide some preliminary proofs that we will use below.

Before proving these assertions, it is critical to ask whether our *psychological* interpretation of these properties in terms of interference is the only one, or whether there are plausible alternative psychological interpretations of a function that impairs memory performance and grows with the number of items in memory. For example, in principle, one might interpret Equation (1) in terms of a finite resource or a finite set of slots that get filled up. If so, the "interference function" would reflect that it becomes increasingly hard to add items to memory as more items already reside in memory. However, while such interpretations are consistent with our formalism from a mathematical perspective, they are more problematic from a psychological perspective. After all, why should items that already reside in memory make it harder to memorize other items — even when the resources or slots are still largely empty? It is thus not clear why a limited resource or a limited number of slots would predict a per-item cost for storing information, nor how a limited resource or a limited number of slots should be interpreted when the per-item cost is small (i.e., when there is little interference), because the storage capacity is fundamentally unlimited. In contrast, these properties fit well with a psychological interpretation of interference functions in terms of interference, based on known and probably uncontroversial properties of interference.

Definition. *Interference function. A function $\mathcal{I}$ is called an interference function if it is continuously differentiable, strictly monotonic and increasing, with $\mathcal{I}(0) = 0$. This definition just gives a name to the assumptions discussed above (i.e., more interference with more items, and no interference with no items).*

Memory capacity for interference functions that cross R

Theorem 1. *Let $\mathcal{I}$ be an interference function as defined above. Further, let $\mathcal{I}$ cross R (i.e., the memory efficacy) for some M, i.e., $\lim_{M\to\infty} \mathcal{I}(M) > R$. As mentioned above, this means that, for a large enough number of items, memory performance is at chance.*[7] *Under these conditions, M_P will tend to a fixed and finite capacity. This memory capacity is given by $K = \mathcal{I}^{-1}(R)$, where $\mathcal{I}^{-1}$ is the inverse function of $\mathcal{I}$. (We will illustrate this result with example functions below.) In the case of some psychologically implausible functions (where adding one item to memory removes several items from memory; see proof below), the number of remembered items oscillates around K, but is still bounded by $K + R$.*

[7] This condition is also necessary. If a K exists such that $\lim_{n\to\infty} M_n = K$, equation (1) implies that $K = K + R - \mathcal{I}(K)$, and thus that $\mathcal{I}(K) = R$. Hence, if the limit exists, $\mathcal{I}$ necessarily crosses R.

Proof. For the proof, we define a function g of M:

$$g(M) = M + R - \mathcal{I}(M) \tag{2}$$

With this notation, equation (1) can be written as $M_{P+1} = g(M_P)$. For the proof, we will use a well-know theorem about fixed-points of functions. (Fixed-points are numbers for which the value of a function is equal to its argument.) If the function g has such a fixed-point K where $g(K) = K$, it is at $K = \mathcal{I}^{-1}(R)$. This can be seen by plugging K into equation (2) above. The fixed-point K exists, as $\mathcal{I}(M)$ crosses R. Further, it is unique, as $\mathcal{I}$ is strictly increasing. From Banach's fixed-point theorem, the condition $|g'(K)| < 1$ is sufficient for the sequence M_1, M_2, M_3, ... to converge to K. In other words, if we start with some value for M_1 and repeatedly apply g, the sequences M_1, $M_2 = g(M_1)$, $M_3 = g(g(M_1))$, $M_4 = g(g(g(M_1)))$, ... will tend to K if this condition is fulfilled. Given that applying the function g just means calculating the number of remembered items after presenting an additional item, this sequence corresponds to the number of remembered items after each number of presented items. We show below that the condition $|g'(K)| < 1$ is fulfilled for psychologically plausible interference functions:

$$\begin{aligned} |g'(K)| &= |1 - \mathcal{I}'(K)| \\ &= \begin{cases} 1 - \mathcal{I}'(K) & \text{if } 0 < \mathcal{I}'(K) \leq 1; \\ \mathcal{I}'(K) - 1 & \text{if } 1 < \mathcal{I}'(K). \end{cases} \end{aligned} \tag{3}$$

In the first case, $|g'(K)|$ is clearly smaller than 1, since $\mathcal{I}'(M)$ is strictly positive by assumption. In the second case, $\mathcal{I}'(K) - 1$ can only be greater or equal to 1 if $\mathcal{I}'(K)$ is at least

2, which means that, for any additional item that is presented for memorization, two items stop being retrievable, and we would consider such interference functions psychologically implausible.

However, even if such interference functions seem psychologically implausible, we will now show that the number of remembered items is still bounded for $\mathcal{I}'(K) \geq 2$ as well. The lower bound of the sequence $\{M_P\}$ is clearly zero, as we cannot remember fewer than zero items (but see below). Lemma 1 (see Appendix) shows that for any $M_{\hat{P}} > K$, subsequent M_P's will be smaller (if and until they cross K again). As a result, if such a $\hat{P}$ exists, the local maxima of the sequence are elements M_P such that $M_{P-1} < K$. From Weierstrass' theorem, we further know that the continuous function g has a finite maximum U in the closed interval $[0, K]$. Further, for any $M_{P-1} < K$, we know that $g(M_{P-1}) \leq U \leq K + R$. As a result, if $\hat{P}$ exists, $M_{\hat{P}} \leq K + R$. Hence, M_P will remain below $K + R$ in this case as well.

For $\mathcal{I}'(K) > 2$, we now show that the sequence $\{M_P\}$ oscillates around K. We will first show that a $\hat{P}$ exists such that $M_{\hat{P}} > K$, and then that the sequence $\{M_P\}$ will drop below K at some point after crossing it. The proof proceeds by contradiction. Assume that $\{M_P\}$ never crosses K. Then $\{M_P\}$ is bounded and increases monotonically, and should converge according to the monotone convergence theorem. However, in Lemma 3, we proved that $\{M_P\}$ does *not* converge for $\mathcal{I}'(K) > 2$. Hence, the assumption that $\{M_P\}$ never crosses K leads to a contradiction. As a result, for some $\hat{P}$, $M_{\hat{P}} > K$.

We will now show that $\{M_P\}$ will drop below K again after crossing it. Assume that $\hat{P}$ exists, and that $\{M_P\}$ does not cross K for some $P > \hat{P}$. Then the subsequence of $\{M_P\}$ starting at $\hat{P}$ is bounded and monotonic. (We showed in Lemma 1 that it is monotonically decreasing.) By the monotone convergence theorem, this subsequence converges. This, however, contradicts the result of Lemma 3 that $\{M_P\}$ does *not* converge. Hence, $\{M_P\}$ crosses K again for some $P > \hat{P}$. A similar argument shows that $\{M_P\}$ will cross K again after it dropped below K. Hence, the sequence oscillates around this fixed point for $\mathcal{I}'(K) > 2$.

For $\mathcal{I}'(K) \geq 2$, it is possible that, for some pathological interference functions (that are *massively* accelerated after crossing K), there exists an M_P below zero. This is only a mathematical problem of course, as actual humans cannot remember fewer than zero items. In such case, we thus replace the function g with

$$\hat{g}(M) = \begin{cases} g(M) & \text{for} \quad M \leq K \\ \max(g(M), 0) & \text{otherwise} \end{cases} .$$

For such interference functions, we thus end up with an oscillation that starts with $M_0 = 0$, crosses K at some point (but remains below $K + R$), and then returns to 0 at some point, to start over again.

For the case $\mathcal{I}'(K) = 2$, the divergence proof from Lemma 3 is not applicable. In this case, $\{M_P\}$ might stay below K, converge to K, or oscillate around K, potentially with convergence to K. Importantly, however, $\{M_P\}$ remains bounded by $K + R$. □

Technical remark 1. *From Banach's fixed-point theorem, we can also estimate the speed* *of convergence towards K, with $q = \mathcal{I}'(K)$:*

$$\begin{aligned} |K - M_P| &\leq \frac{q^P}{1-q}\,|M_1 - M_0| \\ &= \frac{q^P}{1-q} R \end{aligned} \tag{4}$$

Technical remark 2. *As we assume that $\mathcal{I}$ crosses R, it might seem as if we retained fewer items upon presentation of more items once $\mathcal{I}$ crossed R. This is because $\mathcal{I}$ crossing R just means that the interference becomes stronger than the memory strength of a new item. This situation seems implausible from a psychological perspective. However, it does not arise in practice. Indeed, as long as M is not larger than K (i.e., in the interval $[0, K]$), $\mathcal{I}$ does not exceed R either. If the convergence is monotonic, M will not exceed K, and thus $\mathcal{I}$ will not cross R either. This is guaranteed as long as $I'(K) \leq 1$, and thus for most "reasonable" interference functions. In fact, if $\mathcal{I}'$ exceeds 1 for some value below K, g can momentarily have a negative slope, which is just to say that there is an overshoot and a correction towards K. Convergence can thus become oscillatory. However, interference functions with this property are implausible to begin with, because they predict that sometimes the number of items we have in memory is reduced upon encountering an additional item.*

Be that as it might, it easy to handle such cases as well. For $I'(K) > 1$, one can consider the function

$$\hat{\mathcal{I}}(M) = \begin{cases} \mathcal{I}(M) & \textit{for} \;\; M < K \\ R & \textit{otherwise} \end{cases}.$$

This function simply caps $\mathcal{I}$*. Hence, if the* Q^{th} *presented item is the first item for which* M_Q *overshoots (i.e., for which* $M_Q > K$*), then all* M_P *will be equal to* M_Q *for* $P \geq Q$ *(because* $\mathcal{I}$ *has been capped). In other words, replacing* $\mathcal{I}$ *with* $\hat{\mathcal{I}}$ *leads to convergence to the first* $M_Q > K$*. Further,* M_Q *will differ from* K *by at most* R*, as* $M_Q = M_{Q-1} + R - \mathcal{I}(M_{Q-1}) \leq K + R - \mathcal{I}(0) = K + R$*.*

Memory capacity for interference function that converge to, but do not cross R

In the previous section, we considered the case where the interference function crosses R for some finite number of presented items, which is just to say that recognition performance will reach chance for some finite number of presented items. Our analysis shows that, in this case, participants can retain a fixed and finite number of items.

We will now consider a slightly more complex situation, where $\mathcal{I}$ converges to R but does not cross it. As mentioned above, this means that, as many items are presented, average memory performance will almost be at chance, but performance will remain (very) slightly and decreasingly above chance (though this above-chance component is unlikely to be detected except in experiments with extremely high statistical power). Under these assumptions, the memory capacity is *practically limited*: While the number of retained items converges to infinity, the convergence is so slow that it is practically irrelevant. To take the example above, if for remembering a single additional item, we need to present 10 million additional items, the number of retained items is constant for all practical purposes. Technically speaking, we show that one can always find a number of presented items P such that the number of retained items changes by less than some threshold (1 in the example above) when some number of additional items beyond P is presented (10 million in the example), no matter how small the threshold and how large the number of additional items.

Theorem 2. *Let $\mathcal{I}$ be an interference function as defined above. Further, let $\mathcal{I}$ converge to, but never cross R (i.e., the memory efficacy), i.e., $\lim_{M\to\infty}\mathcal{I}(M) = R$, with $\mathcal{I}(M) < R$ for all R. Under these assumptions, the memory capacity is practically limited: for any number of additionally presented items A and any threshold ϵ, there is a number of presented items P such that the number of retained items M_P changes by less than ϵ if A more items are presented:*

$$\forall A > 0, \forall \epsilon > 0 \exists P : M_{P+A} - M_P < \epsilon \tag{5}$$

Proof. To make this proposition plausible, we first note that the difference between subsequent M_P goes to zero. Indeed, the difference between M_{P+1} and M_P is just $R - \mathcal{I}(M_P)$. As $\mathcal{I}$ converges to R, this difference converges to zero. Next, we rewrite equation (1) for $P > 1$:

$$M_P = R + \sum_{k=1}^{P-1} R - \mathcal{I}(M_k) \tag{6}$$

Using equation (6), we can rewrite the difference $M_{P+A} - M_P$:

$$\begin{aligned}
M_{P+A} - M_P &= \left[R + \sum_{k=1}^{P+A-1} R - \mathcal{I}(M_k)\right] - \left[R + \sum_{k=1}^{P-1} R - \mathcal{I}(M_k)\right] && (7)\\
&= \sum_{P}^{P+A-1} R - \mathcal{I}(M_k) && (8)\\
&\leq A\,(R - \mathcal{I}(M_P)) && (9)
\end{aligned}$$

Since $\mathcal{I}$ converges to R, there is an x such that $R - \mathcal{I}(x) < \epsilon/A$. This follows from the definition of convergence. Further, since $\mathcal{I}$ does not cross R, we know from the previous section that M_P is unbounded. Hence, to satisfy equation (5), we just have to pick a P such that M_P is greater than x. $\square$

Technical remark 3. *From equation (9), it is clear that M_P will be practically constant for smaller numbers of presented items when $\mathcal{I}$ reaches the vicinity of R earlier.*

A simple interpretation of the memory capacities

Our analyses so far suggest that people can retain a fixed and finite number of items if their recognition performance converges to chance for a finite number of presented items, and that their memory capacity is practically limited if their recognition performance converges to chance while retaining a (very) small residual above chance component. We will now show that these memory capacities have a very simple interpretation. They are a function of the relative efficiency of committing items to memory and the interference between items in memory. For example, with linear interference $\mathcal{I}(M) = \alpha M$, K is R/α. We will now prove this assertion.

Proof. In the situation where $\mathcal{I}$ crosses R, the derivative of $\mathcal{I}^{-1}$ is given by $\left(\mathcal{I}^{-1}\right)'(x) = 1/\mathcal{I}'(\mathcal{I}^{-1}(x))$. As $\mathcal{I}'(M) > 0\ \forall M \in [0, K]$, $\left(\mathcal{I}^{-1}\right)'$ is strictly positive as well. Hence, K is a growing function of R.

To see the influence of the strength of interference on the memory capacity, we will use the Taylor expansion of $\mathcal{I}^{-1}$ around zero. The first order approximation is given by:

$$\begin{aligned}\mathcal{I}^{-1}(x) &\approx \mathcal{I}^{-1}(0) + \left(\mathcal{I}^{-1}\right)'(0) \times x \\ &= 0 + \frac{1}{\mathcal{I}'(\mathcal{I}^{-1}(0))}x \\ &= \frac{1}{\mathcal{I}'(0)}x \end{aligned} \tag{10}$$

Hence, the larger the slope of the interference function, the smaller the resulting memory capacity. Since the interference with zero items is zero, this is equivalent to say that, the larger the interference at R, the smaller the memory capacity.

In the situation where $\mathcal{I}$ does not cross R, it is clear from equation (6) that, for all P, M_P is a growing function of R. Further, it is clear that M_P will be larger if $\mathcal{I}$ grows more slowly. □

In sum, the memory capacity is a function of the relative efficiency of committing items to memory and the interference between items in memory.

Can decay result in fixed memory capacities?

So far we analyzed the role of interference in memory capacity limitations. However, it is also interesting to consider the consequences if the main limitation of memory is time-based decay. We consider a model similar to equation (1), except that we replace the interference function by some decay function $\mathcal{D}$ that denotes the loss in retrievability of an *individual* item from memory. That is, if an item has some probability of being retrieved at some time point, this probability is reduced after a delay Δt by $\mathcal{D}(\Delta t)$. We make no other assumption about the decay function, except for the definition of decay, i.e., that the magnitude of decay depends on the time during which items are kept in memory. For simplicity, we assume that memory items are presented at some regular interval Δt. The model then becomes:

$$M_{P+1} = M_P + R - M_P \times \mathcal{D}(\Delta t) \tag{11}$$

$$\tag{12}$$

In the equation above, $\mathcal{D}(\Delta t)$ must be multiplied with M_P, since $\mathcal{D}$ is the decay function for an *individual* item, and decay applies to all items in memory. A fixed-point such that $M_{P+1} = M_P$ exists only if $R - M_P \times \mathcal{D}(\Delta t)$ is zero. If it exists, it is at $M^* = R/\mathcal{D}(\Delta t)$. Hence, memory capacity is constant for a given presentation rate, but not for different presentation rates. Under the (implausible) assumption that R does not depend on the presentation rate, more items than M^* can be remembered for faster presentation rates, and fewer items for slower presentation rates. However, given that both R and $\mathcal{D}(\Delta t)$ likely depend on the presentation rate, there is no capacity limits in terms of the number of items that applies across presentation rates.[8] Maybe more surprisingly, for a given presentation rate, there is convergence towards the memory capacity. Formally, we can

[8]Empirically, memory performance is often better with slower presentation rates, in both vision and audition (e.g., Endress & Siddique, 2016; Intraub, 1980; Mackworth, 1962a, 1962b; McReynolds & Acker, 1959; Matthews & Henderson, 1970; Melcher, 2001; Roberts, 1972). However, in some experiments, serial recall performance is better for fast presentation rates, at least when recall is unpaced (Conrad & Hille, 1958; Mackworth, 1964). This is likely due to recall strategies. For example, Posner (1964) showed that performance is better for fast presentation rates in a serial recall task for auditory digits only when participants are forced to recall the digits in the order in which they have been presented; when they are instructed to recall the last digits first, and only then the first digits, this advantage disappears. In support of this view, Jahnke (1968) showed that, when participants fill out the recall sheets, they generally don't start with the items that have presented first (except for very slow presentation rates), and have better memory for the last few items in the list with faster presentation rates.

treat decay *as if* it were an interference term in equation (1); if so, $\mathcal{I}(M_P) = \mathcal{D}(\Delta t)M_P$, and thus $\mathcal{I}'(M_P) = \mathcal{D}(\Delta t)$, which meets the condition for convergence to M^* as long as $0 < \mathcal{D}(\Delta t) < R \leq 1$.

Hence, in the absence of refreshing mechanisms, pure time-based decay leads to constant capacity at a given presentation rate, but cannot account for constant memory capacities across presentation rates, a conclusion that fits well with the observation that interference is much more important for memory processes than decay in the first place (e.g., Berman, Jonides, & Lewis, 2009; da Costa Pinto & Baddeley, 1991; Wixted, 2004, though decay sometimes seems to take place as well, see e.g., McKeown & Mercer, 2012). However, in a situation where only decay but not interference is operational, and the presentation rate is kept constant, constant memory capacities in terms of the number of retained items would be expected.

Three cases of memory capacity limitations — prior empirical evidence

We derived three situations in which memory capacity behaves differently as a function of the number of items we try to store in memory. First, if interference becomes so strong that it reduces recognition performance to chance after a certain number of items is presented, we can remember only a fixed number of items that is a simple function of the relative strength of interference and memorization.[9]

Second, if interference impairs recognition performance as more items are presented, bringing it almost to chance while remaining very slightly above chance (though it is doubtful that this could be measured experimentally), memory capacity is *practically limited*. In

[9]As mentioned above, we can convert the number of remembered items to the percentage of correct responses in a recognition memory task by using the two-high-threshold formula (e.g., Cowan, 2001; Rouder et al., 2008).

principle, the number of remembered items grows indefinitely as more items are presented, but the growth is so slow that memory capacity will appear constant for all practical purposes.

Third, if interference never grows enough to come in the vicinity of the memory strength of a newly encoded item, memory capacity is unbounded.

Hence, the interplay between memorization and interference can produce limited memory capacities in the absence of the specialized mechanisms or resources that are usually postulated for its explanation. That said, attentional mechanisms might well be used to counteract interference (e.g., Kane & Engle, 2000; Engle, 2002), and there is substantial evidence that active, attentional processes are used in and helpful for memorization (see, among many others, e.g., Chen & Cowan, 2009; Craik & Lockhart, 1972; Lepsien & Nobre, 2007; Majerus et al., 2014; Morey & Bieler, 2013; Vergauwe et al., 2014). However, what our model shows is that one does not *need* to appeal to limited-capacity attentional mechanisms to explain limited memory capacities. Rather, such limited memory capacities might be an inevitable consequence of the presence of interference. Interestingly, we also showed that pure time-based memory constraints lead to convergence to a memory capacity for a given presentation rate, but they cannot account for fixed memory capacity limitations across presentation rates.

Importantly, all three situations are experimentally attested. First, when interference among items is massive, capacity estimates are constant irrespective of how many items are presented, at least in the change detection experiments described above (though see e.g. Bays & Husain, 2008; Bays, Catalao, & Husain, 2009; van den Berg et al., 2012, for evidence that memory capacity might not be constant in this situation). Specifically, and as mentioned above, in Luck and Vogel's (1997) classic change-detection Experiment 1, stimuli were drawn from just seven different colors, and were then reused throughout the experiment, and even within trials. As a result, participants had to remember that a given test item was presented in the current trial rather than a prior trial, which likely led to substantial PI across trials. Accordingly, memory capacity estimates were severely limited (see Endress & Potter, 2014a, for discussion).

The other extreme case — that of apparently open-ended memory capacities — is illustrated by Endress and Potter's (2014a) experiments. In these experiments, participants viewed rapid sequences of everyday objects at 250 ms per image, and completed a yes/no recognition test after each sequence. Importantly, interference was minimized by presenting new images on every trial, such that no image was repeated across trials. This manipulation contrasts markedly with traditional WM tasks such as the change detection paradigm, where items are repeated in numerous trials and thus lead to substantial proactive interference. When interference was minimized in this way, Endress and Potter (2014a) observed no clear capacity limitations. For example, when 100 items were presented, participants remembered about 30 of them. These results should not be taken as evidence for a memory capacity of 30. If participants had a memory capacity of 30, they should show ceiling performance when presented with less than 30 items. However, this was not the case. Rather, Endress and Potter (2014a) showed that, when interference is minimized, the probability of remembering any single item shows only a weak dependence on the total number of items to be memorized (see also Banta Lavenex, Boujon, Ndarugendamwo, & Lavenex, 2015 for similar results in spatial WM, and Sands & Wright, 1980, for similar observations in rhesus monkeys).

Endress and Potter (2014a) also provided evidence for our second situation above, where interference brings recognition performance almost to chance, but does not completely prevent the addition of new information to memory (see also Sands & Wright, 1980). Endress and Potter (2014a) included a condition where, in each trial, they picked the memory items from a set of only 22 items. As a result, the items were repeated across trials, but less so than in Luck and Vogel's (1997) experiments. Under these conditions, the number of retained items remained in the range of traditional capacity limitations, but increased slowly as more items were added. It thus seems that the situations we derived using rather general assumptions describe different experimental situations that are observed empirically.

Illustrations

So far, we presented our results in terms of abstract classes of functions. We will now choose some example interference functions to illustrate the emergence of capacity limitations in the first of our situations. Specifically, and as shown in Table 1, we will use logarithmic, linear and exponential interference, respectively. Following these general illustrations, we turn to the debate among proponents of slot-based vs. resource-based memory limitations, and show that there are interference functions that can mimic either type of memory model, and that suggest different psychological interpretations.

Table 1
Details of the functions used in Figures.

Fig. 1		**Fig. 2**	
logarithmic	$\mathcal{I}(M) = \frac{\log(1+M)}{\log(5)}$	(A, B)	$\mathcal{I}_k(M) = 2^{\left(\frac{M}{4}\right)^k} - 1$
linear	$\mathcal{I}(M) = \frac{M}{4}$	(C)	$\mathcal{I}_\alpha(M) = 2^{\alpha\frac{M}{4}} - 1$
exponential	$\mathcal{I}(M) = 2^{\frac{M}{4}} - 1$		

Importantly, we use these functions just to illustrate the present results, but do not claim that they reflect actual interference functions. For illustrative purposes, we will set R to 1 (i.e., perfect remembering), and K to 4. Hence, the memory capacities converge to 4 not due to some intrinsic property of the model, but because we chose the functions accordingly. As mentioned above, we converted the number of retrievable item to the percentage of correct responses in a recognition experiment, by inverting the two-high-threshold formula that is generally used to calculate memory capacities (e.g., Cowan, 2001; Rouder et al., 2008), as calculated for a recognition experiment, where participants are shown a single test item on each trial, and have to decide whether it was part of the memory set, and where there is an equal number of "old" and "new" test items.[10]

As shown in Figures 1(A) through (C), for all three functions, the number of retrievable items asymptotically approaches a capacity of 4; conversely, the percentage of correct responses fell as more items were presented, and fell to 50% for large numbers of presented items.

However, even when just two items are presented, this model predicts that not all presented items are fully retrievable, which might underestimate human performance for small numbers of presented items. This issue can be addressed in two ways. First, and as discussed below, this behavior depends on the interference function, and can be changed with suitably chosen interference functions; as discussed in more detail below, the interference function just needs to be sufficiently upwardly curved, which, we suggest, has a straightforward psychological interpretation in terms of crowding of items in memory. Second, we

[10]This model assumes that participants have a certain probability to successfully recognize an "old" item and to successfully recognize a "new" item; in-between, participants guess. With these assumptions, the estimated memory capacity K is given by $P \times (H - FA)$, where P is the total number of presented items, and H and FA are the hit rate and the false alarm rate, respectively (see e.g. (Rouder et al., 2008)). With an equal number of "old" and "new" test items, the formula can be written as $K = P(2p_{\text{cor}} - 1)$, where p_{cor} is the proportion of correct responses. Inverting this formula yields $p_{\text{cor}} = \frac{1}{2}\left(1 + \frac{M_P}{P}\right)$.

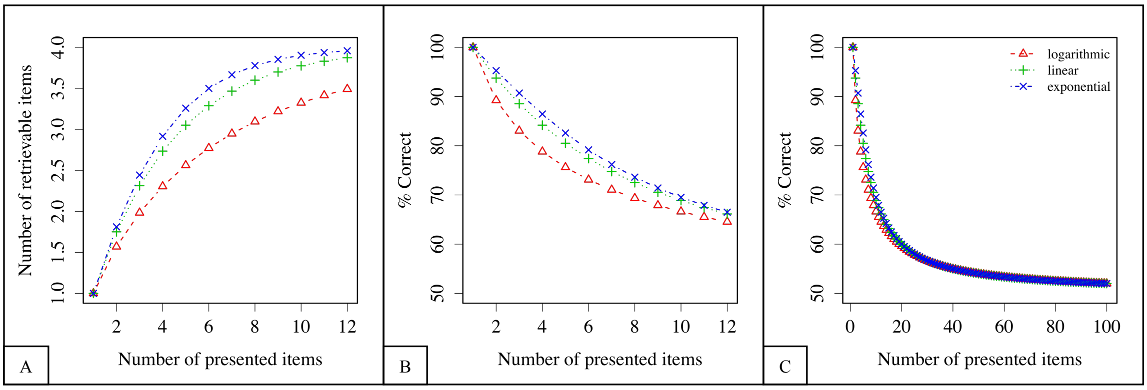

Figure 1. Number of retained items and percentage of correct responses in a recognition experiment when no focus of attention is included in the model. The figures show three different interference functions (logarithmic, linear and exponential).

note that it has long been recognized that people can keep at least one item in the focus of attention while retaining memory items (e.g., Lewis-Peacock, Drysdale, Oberauer, & Postle, 2012; Oberauer, 2002; Öztekin & McElree, 2007; Öztekin et al., 2010), which might or might not be related to rehearsal in the case of memory for verbal items (Baddeley, 1996). Our model can thus be amended by using the most conservative estimate of the focus of attention, namely a single item. In other words, the model is identical to the one above, except that the first memory item is assumed to be placed in the focus of attention and, therefore, does not interact with the other items that are retained.

The results with the same interference functions as before, but including the focus of attention, are shown in Figures 2(A) through (C). As in the previous model, the number of retrievable items converges to 4 items. In this model, however, performance is still at about 95% correct for 3 items (except for logarithmic interference, where it reached only 92%). Hence, this model accounts for the hypothesis that people can retain between 3 and 4 items (Cowan, 2005).

Our analyses are consistent with different memory phenomena and past models. Most

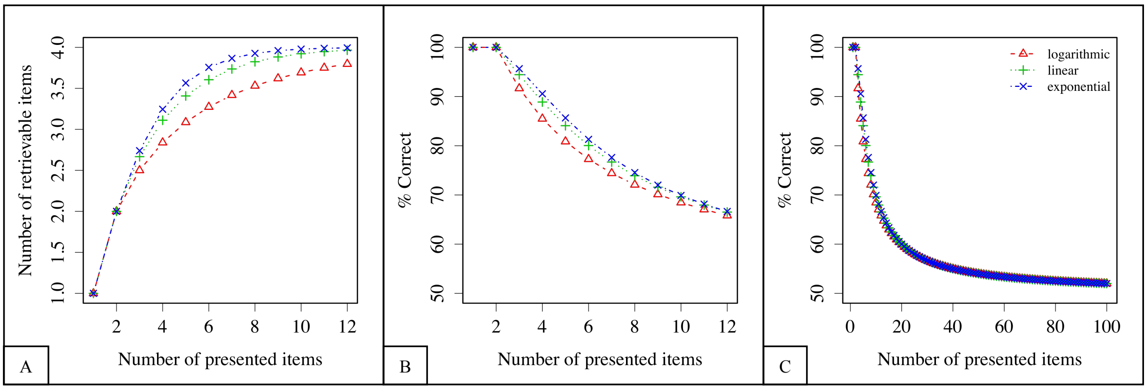

Figure 2. Number of retained items and percentage of correct responses in a recognition experiment when one item is placed in the focus of attention. The figures show three different interference functions (logarithmic, linear and exponential).

importantly, they show that interference can mimic a slot-like all-or-none model and a continuous resource model. We also show that it can mimic chunking.

Modeling an all-or-none encoding in WM

Our model can approximate a slot-like all-or-none WM model (Cowan, 1995, 2005; Luck & Vogel, 1997; Rouder et al., 2008; W. Zhang & Luck, 2008). According to this model, we can remember 3 or 4 items. If we have to remember more items, we pick 3 or 4 of them, and do not place the rest in WM. This is illustrated in Figures 3(A) and (B). The prediction of Cowan's (2001) all-or-none model is that the number of retrievable items should be equal to the number of presented items up to the capacity, and constant afterwards (thick solid line). In other words, for $P < K$, $M_P \approx P$, and for $P \geq K$, $M_P \approx K$.

Setting R to 1 (i.e., perfect encoding of the memory items), and writing out equation (1) for the different iterations reveals that this condition is equivalent to assuming negligible interference below the memory capacity, and full interference when more items are presented than fit in memory, i.e. $\mathcal{I}(M) \approx 0$ for $P < K$, and $\mathcal{I}(M) \geq 1$ otherwise. In other words, the interference function should have an upwardly curved slope.

This is illustrated in Figure 3(A) and (B). We plotted interference functions with increasingly upward-curved slopes (parameter k, see Table 1 for details). By choosing an interference function with a suitably upward-curved slope, it is possible to approximate the all-or-none view of memory (black thick line) arbitrarily closely. In other words, to approximate the all-or-none encoding in WM, interference needs to be benign when few items are presented, and should have a noticeable effect only once the number of presented items approaches the memory capacity.

Such behavior is well known from other areas of perception where neighboring items interfere with each other. For example, visual crowding refers to impaired recognition of visual objects when these objects are surrounded by other, similar, objects. However, crowding effects are observed only when the objects are separated by less than a critical distance (that tends to be half the objects' eccentricity; see e.g., Pelli, Palomares, & Majaj, 2004; van den Berg, Roerdink, & Cornelissen, 2007), while crowding does not occur beyond that distance. We thus suggest that, in experimental conditions that yield the slot-like all-or-none pattern of WM encoding, interference might similarly be noticeable only once a critical number of items has been presented so that they can interfere in memory. In other experimental situation, the interference function might be different.

Importantly, this conjecture suggests a simple psychological interpretation of the all-or-none pattern of WM encoding: in such situations, there might be crowding of memory representations, especially if these representations overlap in features that are relevant for retrieval, and thus possibly comprise a combination of object features and of a representation of *when* an object has been encountered (as in distinctiveness models of memory; see e.g.

Brown, Neath, & Chater, 2007; Glenberg & Swanson, 1986; Neath & Crowder, 1990; Neath, 1993; Unsworth, Heitz, & Parks, 2008). In analogy to the well-documented difficulties for identifying (but not detecting) items in crowded visual displays (e.g., Whitney & Levi, 2011), the all-or-none pattern of performance might be due to retrieval processes that find it hard to correctly identify a target item among crowded memory representations. In contrast to current slot-based WM models, however, this pattern might arise at retrieval, and not due to slot-like memory representations. In line with this view, other authors have suggested that WM performance might be best conceived of as a decision problem that evaluates the evidence provided by memory representations (e.g., Pearson, Raskevicius, Bays, Pertzov, & Husain, 2014).

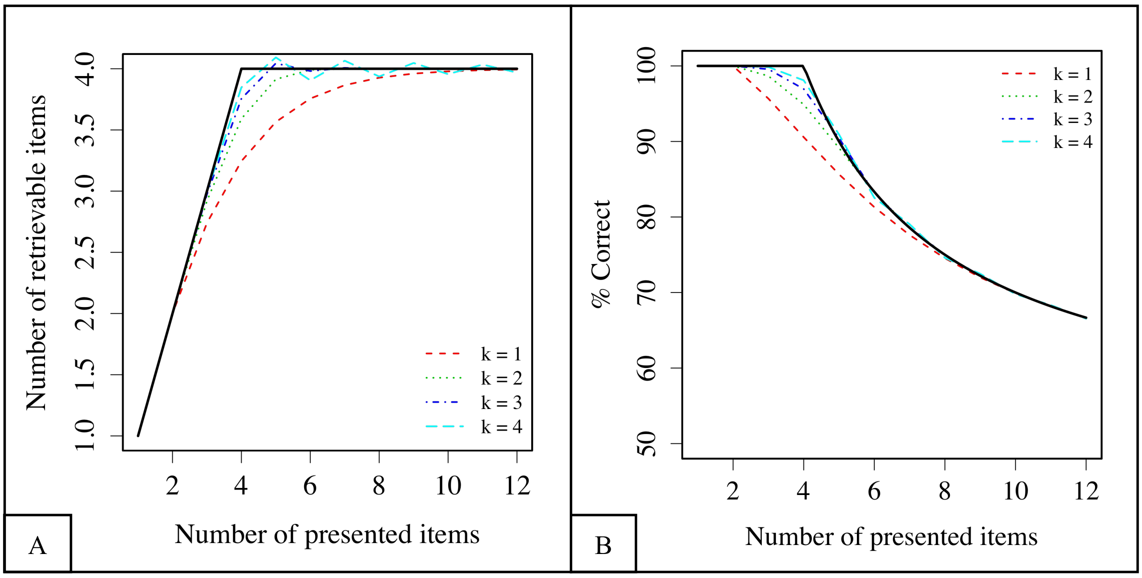

Figure 3. Approximations of an all-or-none memory encoding. With increasingly upward-curved interference functions (parameter k), it is possible to approximate an all-or-none memory encoding (thick black line). The curves are based on models including the focus of attention.

However, these considerations raise an important question. According to slot-based models of WM, a critical feature of the slots is that items in these slots do *not* interfere with other items, while we suggest that slot-like performance profiles might result from crowding-like interference among items. As a result, while the performance profile as a function of the number of memory items is similar here and in traditional slot-based models, the psychological interpretations are radically different, and need to be teased apart empirically.

Modeling a continuous resource model

As mentioned above, some prominent models of WM hold that the size of WM capacity depends on the amount of detail that needs to be stored (Alvarez & Cavanagh, 2004; Bays & Husain, 2008). As shown in Figure 4, this can be modeled by manipulating the strength of interference (parameter α). After all, we need to encode more detail about items when the items are more similar to each other, which, in turn, might increase interference among those items because they are more confusable (e.g., Baddeley, 1966, 1968; Conrad, 1963; Conrad & Hull, 1964; Walker, Hitch, & Duroe, 1993). With stronger interference (or weaker memory encoding), capacities are smaller than with weaker interference. This follows from the fact that the memory capacity in our model is a function of the relative efficacy of memorization and interference. If interference is relatively stronger, the resulting capacities will be reduced.

This idea also readily accounts for the observation that the precision of memory encoding is variable (see Ma et al., 2014, for a review). The more confusable memory items are, the higher the precision required to keep them apart. Hence, when a memory task requires a high precision, we would expect relatively few items to be retained.

Importantly, while slot-based models and resource-based models are often treated as theoretical alternatives depending on how well they fit a given pattern of data, these illustrations show that an interference account can mimic either pattern of data. Crowding-like phenomena during retrieval might lead to WM performance that fits slot-like models while varying levels of interference might lead to the more flexible response profile of resource-based models. In the General Discussion, we will suggest that this view also leads to a new interpretation of Miller's (1956) suggestion that different cognitive capacity limits might share a common underlying reason, even when the different capacities do not form a single system.

Chunking

Chunking strategies have long been recognized as central to memory (e.g., Chase & Simon, 1973; Cowan, Chen, & Rouder, 2004; Feigenson & Halberda, 2008; Rosenberg & Feigenson, 2013; Simon, 1974). For example, chunking the letters C, I and A into the chunk CIA substantially facilities memorization. According to an interference-based account of memory limitations, chunking might increase apparent memory capacity by reducing interference, both by reducing the number of items that need to be memorized, and by making the items that are effectively memorized (i.e., the chunks) more distinct.

The chunking view also presents an alternative interpretation to previous results that lead to the suggestion that humans might have distinct memory stores, such as for language vs. visual information (e.g., Baddeley, 1996; Endress & Potter, 2012; Fougnie, Zughni, Godwin, & Marois, 2015), for agents vs. their actions (Wood, 2008), for view-dependent vs. view-invariant representations (Wood, 2009; see also Endress & Wood, 2011, for similar conclusions in sequence memory), and faces vs. other objects (e.g., Wong, Peterson, & Thompson, 2008). In many of these experiments, participants could remember more

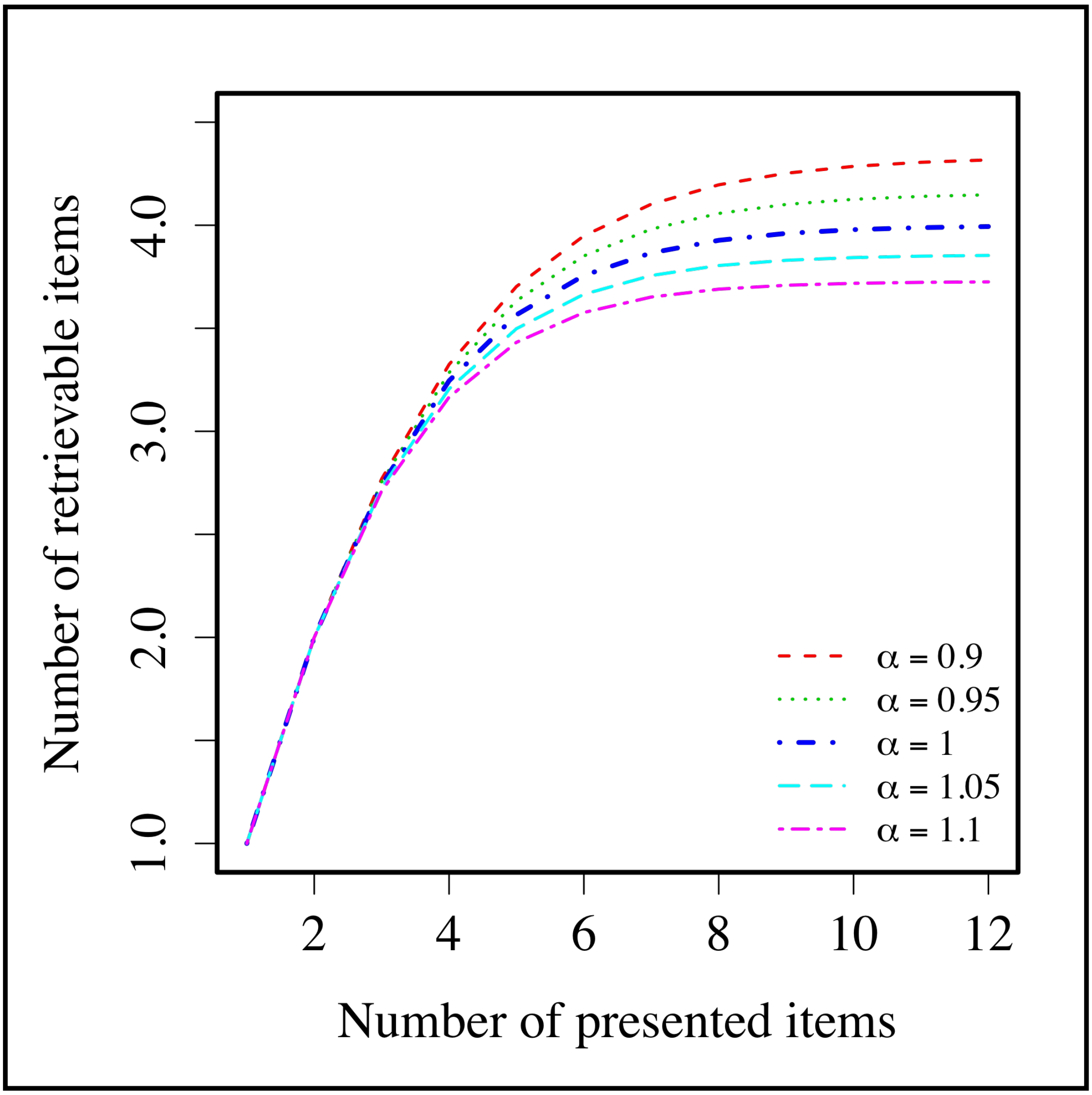

Figure 4. Approximations of a flexible resource-based memory encoding. By varying the strength of interference (parameter α), one can continuously modulate the observable memory capacity. The curves are based on models including the focus of attention.

memory items when they came from different categories as opposed to the same category. However, at least for the within-vision stores, an alternative interpretation is that items from different categories show less interference than items from the same category, especially if, as recent brain imaging studies suggest, memory items are stored in those brain regions where they are processed to begin with (e.g., Lee, Kravitz, & Baker, 2013; Riggall & Postle, 2012; Sreenivasan, Vytlacil, & D'Esposito, 2014; Sreenivasan, Curtis, & D'Esposito, 2014): if items are stored in areas that are separated anatomically in the brain, it is plausible that they also have a reduced tendency to interfere with each other psychologically. In line with this view, other research shows that, when memorizing visual information on top of verbal information (or vice versa), memorizing additional items from another modality only leads to a small decrease in memory performance compared to when only the items from the first modality have to be remembered (e.g., Cowan & Morey, 2007; Cowan, Saults, & Blume, 2014), though these studies also show that memory uses some central, amodal components as well. However, it should be noted that these experiments used stimuli from a limited set of items, and it is unclear how interference in different modalities interacts with each other.

While other memory phenomena might be explained with an interference account (e.g., Brown et al., 2007), our analysis is too general to make specific processing predictions. Importantly, this is not its goal either: rather, we see it as a meta-model with no processing assumptions at all (except that interference increases as a function of the number of interfering items, which is basically the definition of interference), showing that memory capacity limitations are largely inevitable as soon as interference is noticeable, even in the absence of active control mechanisms that are often assumed to be necessary to explain memory capacity limitations.

Relationship to earlier formal models

There is a wealth of formal models of memory, many of which account for interference phenomena (see, among many others, Anderson, 1983; Brown et al., 2007; Dennis & Humphreys, 2001; Lewandowsky & Murdock Jr, 1989; Oberauer et al., 2012; Oberauer & Lin, 2017; Raaijmakers & Shiffrin, 1981; Sengupta et al., 2014). While these models account for many aspects of memory functions about which our analysis is completely silent, they are special cases of our analysis when it comes to the relationship between interference and memory capacity. As we present necessary and sufficient conditions for interference to lead to fixed and finite memory capacities, we can make the following generalizations. Those models that yield fixed memory capacities use interference functions covered by our first case. Those that yield practically limited memory capacities use interference functions covered by our second case. Those models that do not show limited memory capacities as a function of interference use interference functions that do not rise to the memory strength of a new item.

We illustrate these generalizations with two well-known models of memory for which sufficient detail was given to calculate interference functions: Anderson's (1983) and Dennis and Humphreys's (2001) models. Finally, we will discuss Sengupta et al.'s (2014) and Knops, Piazza, Sengupta, Eger, and Melcher's (2014) model because it provides a computation illustration of how interference among items can limit WM.

Anderson (1983)

Anderson's (1983) model is concerned, among other phenomena, with (proactive and retroactive) interference in the paired-associates task. In this task, participants have to associate an item A with an item B, either after having associated A with D, or after having associated C with D. In the first case, A is associated with both B and D, which leads to proactive interference upon recall of B. In the second case, no such interference arises; hence, recall of B should be better compared to the first situation.

In Anderson's (1983) model, an item is retrieved if it receives sufficient activation from other items. For example, when A is presented, B can be retrieved by activation sent from A. Crucially, the activation A sends to B (called f_{AB} in Anderson's (1983) model) is proportional to the *relative* strength of association between A and B among all items to which A sends activation:

$$f_{AB} = \ell \frac{s_{AB}}{\sum_k s_{Ak}} \tag{13}$$

where ℓ is some constant, the s are strengths of associations, and the sum in the denominator is over all associations of A. By rewriting equation (13), we can turn it into a similar form as equation (1):

$$\begin{aligned}
f_{AB} &= \ell \frac{s_{AB} + \sum_{k \neq B} s_{Ak} - \sum_{k \neq B} s_{Ak}}{\sum_k s_{Ak}} \\
&= \ell \left(1 - \frac{\sum_{k \neq B} s_{Ak}}{\sum_k s_{Ak}}\right) \\
&\approx \ell \left(1 - \frac{N\hat{s}}{(N+1)\hat{s}}\right) \\
&= \ell - \ell \frac{N}{N+1} \\
f_{AB} &\equiv R - \mathcal{I}(N) \qquad (14)
\end{aligned}$$

In the third step, we made the simplifying assumption that all strengths of associations are approximately equal to $\hat{s}$, and that there are N interfering associations (excluding that with B). One can verify that $\mathcal{I}(0) = 0$, that the slope of $\mathcal{I}$ is strictly positive, and that $\mathcal{I}$ tends to R for large N without ever reaching it. Hence, to the extent that a pair-associate task is suitable for measuring memory capacity, capacity in Anderson's (1983) model would likely fall into the practically limited case.

Dennis and Humphreys (2001)

Dennis and Humphreys's (2001) model works by associating items (e.g., words) to their context. Specifically, a word is represented by a single (binary) node in the input array of nodes. Its context is represented by a set of binary nodes in the output array of nodes. If nodes in the input array and in the output array are active simultaneously, their connection is set to one with probability r. This is how learning occurs.

When presented with a test item, the model by Dennis and Humphreys (2001) decides whether it has seen the item in the following way. The model has two independent memories of the context, one in the pattern of associations between the input array and the output array (called the "retrieved" context by Dennis and Humphreys's (2001)), and one memory that is stored independently of the associations (called the "reinstated" context by Dennis and Humphreys (2001)). Upon presentation of a test item, the model activates the retrieved context. This is compared to the reinstated context. If the two memories of the context are sufficiently similar, the model concludes that the test item has been seen before. In Appendix C, we show that this model likely falls into our first category, where memory capacity reaches a fixed limit.[11]

[11] Dennis and Humphreys (2001) specifically argue that their model shows limited "cumulative proactive interference" as in Keppel and Underwood (1962). In such experiments, performance decreases over trials

Sengupta et al.'s (2014) and Knops et al.'s (2014)

Sengupta et al. (2014) and Knops et al. (2014) proposed that mutual inhibition among neurons can account for limited WM capacities. (They also investigated the relationship between WM and number processing; we will discuss this aspect of their work in the General Discussion.) Specifically, they described a network of neurons coding for spatial positions of memory items. Each neurons excites itself, and inhibits all other neurons. Similar "saliency maps" are thought to exist in the human posterior parietal cortex (e.g., Bays, Singh-Curry, Gorgoraptis, Driver, & Husain, 2010; Gottlieb, 2007; Roggeman, Fias, & Verguts, 2010).

If the i^{th} neuron has the activation x_i, the activation change is governed by the following differential equation:

$$\frac{dx_i}{dt} = -\lambda x_i + \alpha F(x_i) - \beta \sum_{j=1, j \neq i}^{N} F(x_j) + I_i + \text{noise} \tag{15}$$

The first term represents (exponential) decay, the second term self-excitation of the i^{th} neuron, the third term inhibition from all other neurons, and the fourth term the external input to the i^{th} neuron. $F(x)$ is the activation function $x/(1+x)$ for non-negative x, and zero otherwise.

Equation (15) shows that the inhibition a neuron receives is proportional to the activation of the other neurons (after application of the activation function). As a result, based on our analyses, we would expect the network's memory capacity to be limited for sufficiently high β (i.e., inhibition) values. In line with these expectations, Sengupta et al.

even if unrelated material is used across trials. In our analysis of the model, we consider their parameter corresponding to the probability that context items are activated in a trial due to prior experience. Increasing this parameter decreases performance.

(2014) and Knops et al. (2014) suggested that, with relatively high levels of inhibition (e.g., $\beta = .15$), the mean activation in the network rises up to a set-size of 4, and then remains relatively constant; in contrast, with lower levels of inhibition, the network's memory capacity was much larger.

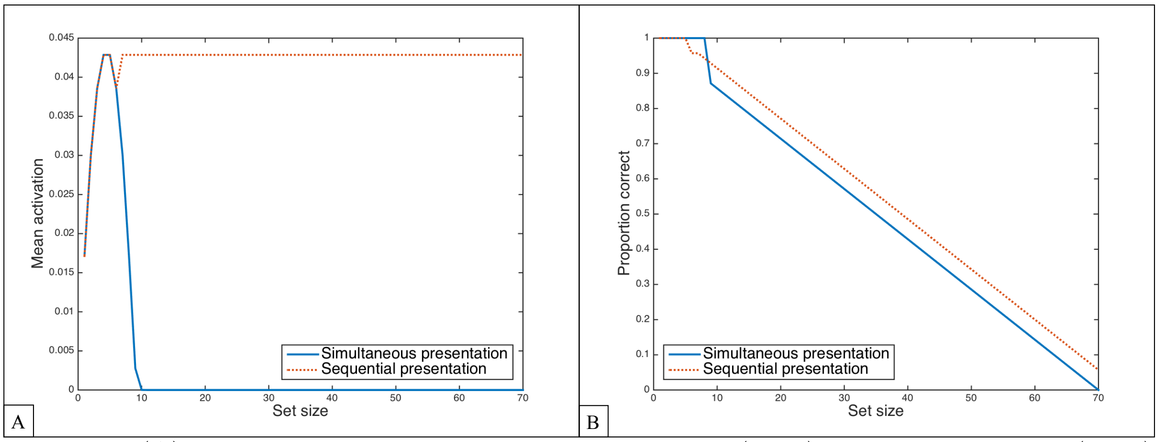

Figure 5. (A) Mean activation level in Sengupta et al.'s (2014) and Knops et al.'s (2014) network in the absence of noise for simultaneous vs. sequential presentation of memory items. The interference parameter is set to .15. As shown by Endress and Szabó (under review), when items are presented simultaneously, and the set-size exceeds the memory capacity, the network shows catastrophic interference, and becomes inactive. In contrast, when items are presented sequentially, the network maintains non-zero activation beyond the memory capacity. (B). Percent of neurons matching their input activation. When items are presented simultaneously, the proportion of neurons "remembering" their input goes to zero. In contrast, when items are presented sequentially, the network retains a memory of the most recent items.

However, there is an interesting complication. Endress and Szabó (under review) showed that, in the absence of noise, the network shows catastrophic interference when memory items are presented simultaneously (see Figure 5). That is, the network remembers items up the memory capacity perfectly; however, when more items are presented than fit into the memory capacity, the network forgets *all* of them, because all neurons become inactive. (In the presence of noise, the situation is similar, except that network activation does not completely die out.) In contrast, when items are presented sequentially one after the other, the network retains a buffer of the most recent items. These computational results fit well with the observation that participants in WM experiments tend to process items sequentially even when they are presented simultaneously (e.g., Liu & Becker, 2013; Vogel et al., 2006; but see Mance et al., 2012), as this might protect them from catastrophic interference (though infants might show catastrophic interference; Feigenson, 2005; Zosh & Feigenson, 2015). Be that as it might, Sengupta et al.'s (2014) and Knops et al.'s (2014) results show that mutual inhibition among neurons can lead to finite memory capacity limitations.

General Discussion

Our results suggest that fixed capacity limitations might be a natural consequence of interference among items, and that neither active, attentional processes, nor a finite resource that can be depleted are needed to explain capacity limitations. Further we show that an interference account can mimic patterns that have been attributed to either slot-based models or resource-based models, suggesting that these models might be less incompatible than currently believed, and might reflect different interference regimes. It is thus urgent to elucidate the psychological mechanisms that can produce either pattern of behavior.

These conclusions should not be interpreted to suggest that active maintenance plays no role in WM. Rather, they suggest that attentional processes are not *needed* to explain fixed and finite capacity limitations, and that such capacity limitations are thus not diagnostic of either the involvement of such processes in memorization, or of a slot-like memory architecture. Further, we speculate that a slot-like all-or-none pattern of WM performance might arise from crowding-like phenomena in memory. For example, a speculative account of such memory crowding effects is as follows. As in many other WM models, a number of LTM items is temporarily activated. During WM/STM retrieval, crowding-like phenomena might arise when a critical number of these activated items is too close in representational space, which would then impair retrieval. In the presence of interference, the relevant representations must include some sort of memory for context, similar to temporal distinctiveness models (e.g., Brown et al., 2007; Glenberg & Swanson, 1986; Neath & Crowder, 1990; Neath, 1993; Unsworth et al., 2008). For example, when we recall or recognize words, we do not need to retrieve all words we know or that are activated, but rather only those that appeared in the relevant trial. If this representation combining item information and context information becomes too crowded, retrieval is probably impaired. If so, a slot-like pattern of memory performance might not be a property of maintenance but rather reflect the effects of interference on retrieval, in contrast to slot-based models of memory where the slots are used for memory maintenance. Interestingly, this speculation that meshes well with earlier suggestions that the effectiveness of retrieval cues (and not only the quality of the memory representation or its match to a retrieval cue) is crucial for memory performance (e.g., Nairne, 2002a), and that memory performance might fundamentally reflect a decision problem, where participants have to evaluate the evidence provided by their memory representations (e.g., Pearson et al., 2014).

The role of active mechanisms

While our results show that active mechanisms are not required to explain memory capacity limitations, they should not be taken as evidence that such processes are *never* used in WM. In fact, in tasks such as complex span tasks (e.g., Daneman & Carpenter, 1980), it is difficult to imagine how participants could succeed without using active, attentional processes. In fact, the well documented correlation between performance on such tasks and IQ is mediated by the use of suitable strategies (Gonthier & Thomassin, 2015; though see Oberauer et al., 2012 for, an interference-based model of such tasks). Participants might thus well use active processes to reduce the effects of interference (Engle, 2002; see also Braver et al., 2008; Burgess et al., 2011), thereby increasing the apparent memory capacity.

However, in other tasks that revealed limited memory capacities, and that are, therefore, widely believed to tap into WM, such active, attentional maintenance mechanisms might not be recruited. Examples include Endress and Potter's (2014a) interference-rich conditions, or the change detection paradigm (Luck & Vogel, 1997) that has probably become the most prominent test case of visual WM in recent years (e.g., Alvarez & Cavanagh, 2004; van den Berg et al., 2012; Piazza et al., 2011; Rouder et al., 2008; Vogel et al., 2006; W. Zhang & Luck, 2008). In such tasks, we believe that the minimal assumptions of the present model provide a useful theoretical baseline against which more elaborate models of memory capacity limitations should be compared.

Given the striking variety of WM tasks, we propose that different WM tasks might recruit different mechanisms, some being active and some being passive and automatic, and that the label "WM" might thus reflect different psychological constructs in different experimental paradigms (even though they might share variance; see Engle et al., 1999).

If this view is correct, it raises the question of what should be called "WM." In fact, if WM is a form of memory that stores information over limited periods of time and keeps it accessible for cognitive processes, everyday cognition suggests that it does not necessarily have a limited capacity. As discussed by Endress and Potter (2014a), we can drive a car and monitor the traffic while remembering our destination, remembering to go to the gas station and to pick up some food on the way, while keeping in mind speed limits, having a conversation with a passenger, interrupting the conversation to listen to the news, resuming the conversation and so forth. If a limited capacity is a defining feature of WM, one would need to conclude that WM is not used in such everyday situations, but only in interference-rich situations such as remembering phone numbers or doing arithmetic. However, even this conclusion is problematic. If it turns out that we do not use active maintenance mechanisms to counter-act interference at least in some situations (which is unlikely to be the case in change detection experiments and those reported by Endress & Potter, 2014a), interference-rich situations and those with limited interference would recruit exactly the same memory mechanisms. If so, there are only two possible conclusions: either we use WM in both interference-rich and limited-interference situations, or in neither of these situations.

A similar conclusion follows from the view that WM (or at least its central feature) is some kind of active memory control mechanisms, but not a memory storage mechanism per se (e.g., Baddeley, 1996, 2003; Cowan, 2001, 2005). If so, WM would be deployed only in specific kinds of situations and tasks, but not in other situations that one would intuitively think to involve WM (e.g., the driving situation sketched above). We thus believe that, to understand what WM really is, tasks that are thought to tap into WM need to be decomposed into their underlying psychological mechanisms.

The role of interference in other cognitive capacity limitations

Our analyses show that interference can mimic different patterns of capacity limitations in memory, notably slot-based and resource-based memory performance. Interestingly, interference has also been proposed to account for the other two prominent case of cognitive capacity limitations: tracking multiple moving objects in a display, a task that has been called multiple object tracking, and quickly and accurately enumerating objects in a display without counting (i.e., subitizing; Kaufman & Lord, 1949; Trick & Pylyshyn, 1994). Traditionally, all three forms of capacities were thought to be limited to three or four items, and to rely on a system of parallel attention (e.g., Cowan, 2001; Piazza et al., 2011). However, recent evidence suggests that these capacities are not fixed and depend on the experimental situations. Accordingly, different authors have attributed these capacity limitations to different forms of interference among items. We will now discuss these data in turn.

Interference-based accounts of subitizing and multiple object tracking.

Interference-based accounts of subitizing. According to recent proposals, subitizing might be due spatial interference among items. As mentioned above, Sengupta et al. (2014) and Knops et al. (2014) implemented a saliency map where neurons excite themselves and inhibit each other.

Using this architecture, Sengupta et al. (2014) reported that, at high levels of inhibition, the network activation increased as a function of the set-size up to about four, and then reached an asymptote. Hence, if we have read-out mechanisms for the total activity in a brain network (e.g., as in the models by Dehaene & Changeux, 1993 and Verguts & Fias, 2004), the same architecture that explains WM limitations might also explain subitizing limitations, because the total activation in the network would give an estimate of the number of items. If so, the subitizing limitations would depend on the strength of inter-item inhibition.

Interference-based accounts of multiple object tracking. With respect to multiple object tracking, a traditional view held that observers can attentionally follow 3 or 4 moving objects on a display (e.g., Pylyshyn & Storm, 1988; Scholl & Pylyshyn, 1999), a capacity limitation that seems to coincide with traditional working memory and subitizing limitations. However, it is now clear that the number of objects that can be tracked depends on factors such as their speed and their spacing (Alvarez & Franconeri, 2007), and the main factor that limits how many item we can follow appears to be inter-item interference (Franconeri, Alvarez, & Cavanagh, 2013): objects interfere with each other when their receptive fields come too close. However, while such tracking abilities and WM might both be limited by interference, the current proofs do not apply to multiple object tracking, because memory interference depends on the number of objects in memory, while tracking interference depends on the number of objects present in a display, and not just those that are tracked (Bettencourt & Somers, 2009; see also Franconeri, Jonathan, & Scimeca, 2010), and, more generally, because tracking limitations are due to occasional interactions between objects (i.e., when they come to close), requiring a somewhat different model.

Do Working Memory, Subitizing, and Multiple Object tracking rely on a common mechanism? If capacity limitations in WM, subitizing and multiple object tracking are emergent properties of interference among items, does this suggest that all the capacities rely on the same underlying system? We will now suggest that the evidence is mixed in the case of the relationship between WM and subitizing, while there is converging evidence suggesting that WM and multiple object tracking rely on at least partially independent mechanisms.

The relationship between Working Memory and Subitizing. Piazza et al. (2011) demonstrated a correlation between the subitizing range and WM capacity (measured in a change detection experiment), suggesting that these cognitive abilities rely on a common system. Further, they showed that, in dual-task settings, WM load affects subitizing performance and vice versa, though Shimomura and Kumada (2011) did not find any dual-task costs between WM and subitizing. There is also evidence from brain imaging for a link between WM and subitizing (Knops et al., 2014), though the interpretation of these data is not entirely clear.[12]

In line with this adult data, the literature on object individuation in infancy also suggests that WM and small number processing might be partially linked, and partially dissociable. Around their first birthday, infants can individuate only up to three (identical) objects (e.g., Feigenson, Carey, & Hauser, 2002; Feigenson & Carey, 2005), a limitation that coincides with the subitizing range. For example, when infants witness identical objects being hidden in a box, and are allowed to retrieve only a subset of them, they will continue searching for the remaining objects only if the total number does not exceed 3 (e.g., Feigenson & Carey, 2005). However, when the objects can be combined into chunks,

[12]Knops et al. (2014) reported that voxels in the posterior parietal cortex could flexibly switch between a subitizing response profile in an enumeration task, and a WM profile in an WM task. Specifically, they defined the response profiles in terms of the brain activity as a function of the set-size. According to Knops et al. (2014), a subitizing profile should show relatively constant activation within the subitizing range, and increase thereafter, while a WM profile should increase up to the WM capacity, and plateau thereafter.

While Knops et al. (2014) concluded that the same brain area could flexibly switch between response profiles and sub-serve both WM and subitizing, there are two considerations that call for further experiments before accepting this conclusion. First, if the activity in this brain area is constant within the subitizing range, it is unclear how the set-size can be read from the network activity. Second, Knops et al. (2014) found that a classifier decoding the set-size performed better for numbers outside the subitizing range (i.e., 5 and 6) than for numbers within the subitizing range (i.e., 3 and 4) — although larger numbers might processed by different mechanisms from those involved in small-number processing (e.g., Feigenson & Carey, 2005; Hauser, Carey, & Hauser, 2000; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). As a result, it is unclear to what extent Knops et al.'s (2014) subitizing response profile reflects voxels that are involved in small number processing as opposed to voxels that are *not* involved in small number processing, and that only support large number processing instead.

or when they are made more distinctive, infants also succeed with a total of 4 objects (e.g., Feigenson & Halberda, 2008; Rosenberg & Feigenson, 2013; Zosh & Feigenson, 2015).

Crucially, looking time experiments suggest that the increased capacity does not necessarily imply increased WM for the objects' features. In fact, after objects are hidden, infants are sometimes surprised if their number changes — even while they do not notice massive changes in the objects' features (e.g., Kibbe & Leslie, 2011; Kibbe & Feigenson, 2016). A possible interpretation of such results is that the ability to keep track of cardinalities can be increased by some factors (e.g., chunking) that also increase WM capacity, suggesting again that there might be a link between the ability to keep items in memory and the ability to track their cardinality, though these results also suggest that small number processing is not isomorphic with WM.

In sum, while different strands of evidence suggest that WM and small number processing might be closely related, it is still unclear to what extent these cognitive abilities rely on a common mechanism.

The relationship between Working Memory and Multiple Object Tracking. In the case of the relationship between WM and multiple object tracking, the situation is somewhat clearer. First, and as mentioned above, there is very limited interference between parallel attention tasks and WM (e.g., Fougnie & Marois, 2006; Hollingworth & Maxcey-Richard, 2013; H. Zhang et al., 2010), suggesting that they rely on dissociable psychological mechanisms. Further, parallel attention and WM seem to have partially different properties. For example, Endress, Korjoukov, and Bonatti (under review) asked to what extent the category structure of memory items affects WM and multiple object tracking performance. Specifically, and as mentioned above, it is easier to memorize items from different categories (e.g., 2 cars and 2 faces) compared to items from the same category (e.g., 4 faces; e.g., Feigenson & Halberda, 2008; Wood, 2008; Wong et al., 2008). Endress et al. (under review) replicated this effect in a WM task, but showed that it is slightly *harder* to track objects from different categories than items from the same category. While these results were rather weak, they also suggest that WM and multiple object tracking rely on different mechanisms.

That being said, if our speculation is correct that a slot-like response pattern might be due to crowding-like phenomena in memory (retrieval), the underlying factors that lead to capacity limitations might be similar in WM, subitizing and multiple object tracking. In subitizing and multiple object tracking, performance might impaired if the receptive fields of the objects are too close in space (Franconeri et al., 2013); in memory, interference might arise when the "receptive fields" of the memory representations are too close together in representational space of the relevant features (that might include some form of context memory as in distinctiveness theories; e.g., Brown et al., 2007; Glenberg & Swanson, 1986; Neath & Crowder, 1990; Neath, 1993; Unsworth et al., 2008).

However, this crowding account does not explain relationships between different capacities, such as Piazza et al.'s (2011) correlation between subitizing ranges and WM capacities — unless a sensitivity to crowding is a trait that is stable in an individual across different tasks. There is some evidence that subitizing is vulnerable to crowding (Baron & Pelli, 2006), but it is unclear to what extent a general sensitivity to crowding in vision and in memory can explain the correlation found by Piazza et al. (2011). Some basic abilities certainly affect individual performance on a range of tasks. For example, an individual's ability to suppress irrelevant items seems to be used in tasks from perceptual decisions to

IQ tests and correlates across these tasks (Melnick, Harrison, Park, Bennetto, & Tadin, 2013). However, it is unknown whether a sensitivity to crowding has a similarly wide range of consequences.

Be that as it might, capacity limitations in at least WM, multiple object tracking and maybe subitizing might be due to a common cause, just as proposed by Miller (1956), Cowan (2001) and Piazza et al. (2011): performance might suffer from interference between nearby representations, where "nearby" refers to spatial locations in object tracking and subitizing, while it refers to some kind of representational space in memory.

However, contrary to earlier proposals that cognitive capacity limitations might be related through the use of a common mechanism, these limitations do not necassrily arise because the three tasks rely on the same underlying mechanism (i.e., parallel attention), but rather because each of the underlying mechanisms might be sensitive to similar constraints.

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Appendix A

A continuous resource interpretation of W. Zhang and Luck's (2008) model

According to W. Zhang and Luck's (2008) model, the probability density for the participants to pick a color γ is given by

$$p\Phi_\kappa(\gamma - \gamma_0) + \frac{1-p}{2\pi} \tag{16}$$

where p is the probability of an item to be in a slot (i.e., of being one of the items that are memorized), γ_0 is the target color, and Φ_κ is the von Mises distribution reflecting that, even within a slot, memory has only a finite resolution. The first term represents the probability of a response if participants have placed an item into a slot, while the second term reflects a uniform guessing distribution if the item has not been placed into a slot.

According to W. Zhang and Luck (2008), a flexible resource model can be modeled with the expression $\Phi_\kappa(\gamma-\gamma_0)$: participants remember the target color γ_0 with some spread around it. This, however, implies that even their original model can be interpreted as a flexible resource model. As pointed out by Bays et al. (2009), participants do not only have to memorize the colors, but also their location. If one interprets W. Zhang and Luck's (2008) p parameter not as the probability of placing a color into a slot, but rather as the probability of getting the color-location binding right, and if one assumes that the distribution of target colors is uniform, the probability distribution of a subject response is given by

$$p\Phi_\kappa(\gamma-\gamma_0)+(1-p)\int\Phi_\kappa(\gamma-\gamma')\frac{1}{2\pi}d\gamma' \tag{17}$$

The left summand reflects that participants will pick a color with some spread around γ_0 if they got the color-location binding right, while the right summand reflects that they will pick another color from the memory array if they don't remember the location of the color; given that the target colors are sampled uniformely, the colors picked on average correspond to the marginal distribution over all possible target directions. The integral is $1/(2\pi)$. Hence, equations (16) and (17) are identical, and W. Zhang and Luck's (2008) slot model is equivalent to a flexible resource model with uncertainty about location-color bindings.

It should be noted that van den Berg, Awh, and Ma (2014) showed that such binding errors are relatively rare. However, the equivalence between a slot-model and a resource model shows nonetheless that the psychological interpretations of such models are not always clear, and that psychological mechanisms might be easier to elucidate with psychological manipulations than by evaluating the fit to a model.

Appendix B

Some preliminary proofs

Lemma 1. *Below K, the sequence $\{M_P\}$ is monotonically increasing; above K, the sequence $\{M_P\}$ is monotonically decreasing.*

For $M_P < K$, this assertion follows from the observation that $M_{P+1} - M_P = R - \mathcal{I}(M_P) > 0$ for $M_P < K$. The last inequality follows from the monotonicity of $\mathcal{I}$, and the fact that $\mathcal{I}(K) = R$.

For $M_P > K$ (if such an M_P exists), M_{P+1} will necessarily be smaller than M_P:

$$\begin{aligned} M_{P+1} &= M_P + R - \mathcal{I}(M_P) && (18) \\ &< M_P + R - \mathcal{I}(K) && \textit{Since } \mathcal{I}'(M) > 0, \textit{ and since } K < M_P \\ &= M_P \end{aligned}$$

Lemma 2. *If the sequence $\{M_P\}$ converges, it converges to K.*

The proof proceeds by assuming that the sequence converges to some other number, and showing that this assumptions leads to a contradiction. Assume that $\{M_P\}$ converges to some limit $L_l < K$. If so, $|M_{P+1} - M_P|$ must converge to zero. However, $|M_{P+1} - M_P| = |R - \mathcal{I}(M_P)| \geq R - \mathcal{I}(L_l) > 0$. The penultimate inequality follows from the observation that $\mathcal{I}(M_P) \leq \mathcal{I}(L_l)$ for $M_P < L_l$. Hence, $\{M_P\}$ cannot converge to $L_l < K$.

Assume now that $\{M_P\}$ converges to some limit $L_u > K$. If so, $|M_{P+1} - M_P| = |R - \mathcal{I}(M_P)| = \mathcal{I}(M_P) - R \geq \mathcal{I}(L_u) - R > 0$. The penultimate inequality follows from the observations that $\mathcal{I}(M_P) > \mathcal{I}(L_u)$ for $M_P > L_u$, and that, due to the monotonicity of the *sequence, if there is convergence to L_u, it is necessarily from above L_u (i.e., after some P, M_P would be greater than L_u. Hence, $\{M_P\}$ cannot converge to $L_u > K$.*

Lemma 3. *For $\mathcal{I}'(K) > 2$, the sequence $\{M_P\}$ does not converge.*

To show that the sequence $\{M_P\}$ does not converge, we assume that it converges, and then show that this assumptions leads to a contradiction. If $\{M_P\}$ converges, it converges to K according to Lemma 2. Thus, if it converges, the difference between subsequent M_P's should converge to zero for some large P:

$$\begin{aligned} M_{P+1} - M_P &= g(M_P) - g(M_{P-1}) && (19) \\ &\approx \left(g(K) + g'(K)(M_P - K)\right) - \left(g(K) + g'(K)(M_{P-1} - K)\right) && (20) \\ &= g'(K)\left(M_P - M_{P-1}\right) && (21) \end{aligned}$$

This is simply the first order Taylor expansion around K. Given that we assume that the sequence converges, the higher order terms can be neglected. By taking the absolute value, we obtain

$$|M_{P+1} - M_P| = |g'(K)| \times |M_P - M_{P-1}|. \qquad (22)$$

$|M_{P+1} - M_P|$ is thus approximately a geometric sequence with a ratio of $|g'(K)|$ that converges if and only if $|g'(K)| < 1$. However, for $\mathcal{I}'(K) > 2$, $|g'(K)| > 1$, which contradicts the assumption that the sequence $\{M_P\}$ converges.[13] *As a result, the sequence M_P cannot*

[13] We further need to assume that $M_P \neq M_{P-1}$. For $I'(K) > 2$, this condition is necessarily true. Under this condition, g' is smaller than 0 in a neighborhood of K. g is thus locally decreasing and, therefore, injective. As a result, for M_P and M_{P-1} to be equal, both need to be equal to K. By recurrence, M_0 thus must be equal to K as well. However, this contradicts the assumptions that $M_0 = 0$ and that $g(0) = R > 0$. As a result, M_P cannot be equal to M_{P-1}.

converge to K, and by Lemma 2 to no other limit.

Appendix C

Interference in Dennis and Humphreys's (2001) model

Dennis and Humphreys's (2001) model has three sources of noise that impair old/new decisions. First, and as mentioned above, input and output nodes that are simultaneously active are associated with probability r. Second, the context representation in the output node have a probability p to be active due to prior experience; we assume that this probability represents the strength of proactive interference. Third, in the second, independent copy of the context, there is a probability d that the model "forgets" to turn on units that were active during study.

With these assumptions, Dennis and Humphreys's (2001) derive a formula for the likelihood ratio that a test item is old vs. new, given the evidence the model has seen (see Dennis and Humphreys (2001) for the derivation):

$$
\begin{aligned}
\Lambda &= \left(\frac{1-s+d(1-r)s}{1-s+ds}\right)^{n_{00}} \times \\
&\quad (1-r)^{n_{10}} \times \\
&\quad \left(\frac{p(1-s)+d(r+p-rp)s}{p(1-s)+dps}\right)^{n_{01}} \times \\
&\quad \left(\frac{r+p-rp}{p}\right)^{n_{11}} \\
&= \alpha^{n_{00}}\beta^{n_{10}}\gamma^{n_{01}}\delta^{n_{11}}
\end{aligned} \tag{23}
$$

In the last step, we just labeled the factors in Dennis and Humphreys's (2001) equation. n_{11} is the number of nodes that are active both in the reinstated and the retrieved context,

n_{10} is the number of nodes that are active in the reinstated but not the retrieved context, and so forth. As long as these exponents are not smaller than 1, they do not change the signs of the derivative below and the patterns of convergence. As such, they are irrelevant for the derivations below, and we will ignore them.

Importantly, chance performance is indicated by a likelihood ratio of 1. Hence, to compare the model behavior to our analysis, we need to ask whether performance converges to chance as a function of p, and whether it actually reaches chance.

In equation (23), only γ and δ depend on p, and can be rewritten as follows:

$$\gamma(p) = 1 + \frac{drs}{1-s+ds}\frac{1-p}{p} \tag{24}$$

$$\delta(p) = 1 - r + \frac{r}{p} \tag{25}$$

One can verify that $\gamma(1) = \delta(1) = 1$ and that the derivatives of both γ and δ with respect to p are strictly negative[14]. Further, both γ and δ are large for small p, i.e., $\lim_{p\to 0}\gamma(p) = \lim_{p\to 0}\delta(p) = \infty$.

Given that both α and β are strictly smaller than 1, that both γ and δ converge to 1 for $p \to 1$, and that Λ is a continuous function of p, Λ will reach 1 for some finite p and cross it. Hence, we would expect Dennis and Humphreys's (2001) model to fall into our first case, where memory capacity is fixed and finite.

[14] $\partial_p\gamma(p) = -\frac{drs}{1-s(1-d)}\frac{1}{p^2} < 0$; $\partial_p\delta(p) = -\frac{r}{p^2} < 0$.