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On the Meaning of Connectives (Apropos of a Non-Necessitarianist Challenge)

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Abstract. According to logical non-necessitarianism, every inference may fail in some situation. In his defense of logical monism, Graham Priest has put forward an argument against non-necessitarianism based on the meaning of connectives. According to him, as long as the meanings of connectives are fixed, some inferences have to hold in all situations. Hence, in order to accept the non-necessitarianist thesis one would have to dispose arbitrarily of those meanings. I want to show here that non-necessitarianism can stand, without disposing arbitrarily of the meanings of connectives, based on a minimalist view on the meanings of connectives.

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1. Introduction

Logical monism is the view that there is only one correct logic or, alternatively, the view that there is only one genuine consequence relation, only one right answer to the question on whether and why a given argument is valid, only one collection of valid inferences (or of logical truths), or only one right way of reasoning. Logic is at the center of philosophy and many theoretical and practical pursuits since they proceed by the way of argument, inference, and their evaluation. Thus, the problem of knowing whether there is only one correct logic is central in philosophy and of crucial importance to philosophy and other activities.

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Mortensen's logical *possibilism* or *non-necessitarianism* (cf. [14,15]) poses a particular challenge for logical monism.[1] Roughly, his arguments for "weakening the necessitarian intuitions and strengthening the possibilist intuitions" are based on the development of logic over the last one hundred years. Virtually every theorem or valid inference has been thrown out, in the sense that there are models thought of as "situations" where they no longer hold, and now nearly every structure resulting from dropping such theorems or valid inferences is accepted as a logic. But if every inference may fail in some situation, then there is no collection of inferences holding in all situations. Then either there is not logic at all, or logic is not to be thought of as being in the business of inferences holding in all situations, for there is no such a thing. Either option is devastating for a monist, so he must tackle the non-necessitarianist thesis that every inference may fail in some situation.[2]

In making a case for monism, Graham Priest [19] has put forward an argument against non-necessitarianism based on the meaning of connectives.[3] According to Priest, as long as the meanings of connectives are fixed, some inferences have to hold in all situations. Hence, in order to accept the non-necessitarianist thesis one would have to dispose arbitrarily of those meanings. I want to show here that it would be a good answer to non-necessitarianism if the conception of the meanings of connectives involved in the latter were maximalist, like Priest's. Non-necessitarianism can stand, without disposing arbitrarily of the meanings of connectives, based on a minimalist view of the meanings of connectives. However, Priest also rejects minimalism, so before I must put forward a case for it.

The plan of the paper is as follows. In Sect. 2, I present the non-necessitarianist challenge, which asks what if there were no inferences holding in all situations, and reconstruct Priest's reply to it. In Sects. 3 and 4, I expound my reasons for considering Priest's answer to be unsatisfactory. To the extent that my contentions are based on minimalism and Priest rejects it, I show in Sect. 3 why his arguments against minimalism do not work. In Sect. 4

[1] Mortensen's possibilism does not only concern logic, but for my present purposes I concentrate here solely on that aspect. He talks mainly of logical truths, but I think it does not affect the argument to speak rather in terms of inferences as I do. As a side remark, possibilism (every inference may hold in some situation) and non-necessitarianism (every inference may fail in some situation) are not equivalent. Mortensen defends both views, but possibilism does not necessarily pose the problem for logical monism I will discuss below, so I will speak exclusively in terms of non-necessitarianism.

[2] The non-necessitarianist thesis seems to be innocuous for logical pluralism: The latter only would say that logic is not in the business of inferences holding in all situations, but rather in inferences holding in all situations of a certain kind. However, possibilism and non-necessitarianism seem so outrageous that even pluralists of the sort just described reject them, cf. [27, p. 56]. Nonetheless, pluralists sometimes sail very close to the non-necessitarianist wind: "(...) we see no place to *stop* the process of generalisation and broadening of accounts of cases. For all we know, the only inference left in the intersection of (unrestricted) *all* logics might be the *identity* inference: From A to infer A." [2, p. 97, emphases in the original].

[3] Even though I will mention the original works by Priest as they were published, quotations and pagination are always done based on their reprints in [21].

I also show how, without meaning-variance involved, the inferences taken by Priest as secure by the very meaning of connectives can fail.

2. The Non-Necessitarianist Challenge and Priest's Reply

I am not going to reconstruct Priest's case for logical monism here, because it is not my intention now to go into that debate between logical monists and logical pluralists. What I am interested in are the following claims he, like the vast majority of logicians, endorses. The first claims are on validity (cf. [18, 19]); the last one on the existence of a logic (cf. [19]):

P1. An inference $X \vdash Y$ holds in a situation if and only if, in that situation, if the truth value of X is a designated one, then the value of Y is designated too (the pretheoretical notion of holding in a situation).

P2. An inference $X \vdash Y$ is valid if and only if it holds in all situations (the pretheoretical notion of validity.)

P2'. $X \vdash Y$ is not valid if and only if it does not hold in all situations (contraposition, P2):

> (...) logic is, in a nutshell, the study of validity. (...) validity is the relationship of truth-preservation-[from premises to conclusions]-in-all-situations. ([21, p. 176, my addition inside brackets])
>
> When we reason, we reason about various situations or states of affairs. These may be actual or hypothetical. We reason to establish what holds in these situations given what we know, or assume, about them. This is truth-preservation (forward), though it is not actually truth that is in question unless the situation we are reasoning about is itself actual. The point of deduction, then, is to give us a set of canons that preserve truth in this sense. A valid inference is therefore one such that in all the situations where the premises hold, the conclusion holds. ([21, p. 197])
>
> (...) it is only truth-preservation over all situations that is, strictly speaking, validity. One of the points about deductive logic is that it will work come what may: we do not have to worry about anything except the premises. As I have already observed, this is not to say that in practice one may not reason as if one were using a different, stronger, notion of validity, one appropriate to a more limited class of situations. But this is not because one has changed logical allegiances; it is because one is allowed to invoke contingent properties of the domain in question. ([21, p. 202])

Note that Priest interprets the phrase 'all situations' strictly, i.e. as *absolutely all situations*, unlike pluralists who relativize it to 'all situations *of a kind'*. I will call P1 and P2 (and P2') "the traditional notion of validity".

P3. There is at least one collection of inferences holding in all situations and this collection is large enough (existence of a logic):

> Is the same logical theory to be applied in all domains, or do different domains require different logics? [...] Even if modes of legitimate inference do vary from domain to domain, there must be a common core determined by the syntactic intersection of all these. In virtue of the tradition of logic as being domain-neutral, this has good reason to be called *the* correct logic. But if this claim is rejected, even the localist must recognise the significance of this core. Despite the fact that there are relatively independent domains about which we reason, given any two domains, it is always possible that we may be required to reason *across* domains. ([21, p. 204, emphases in the original])[4]

Thus, a logical monist typically insists on the existence of one true logic, claiming that the collection inferences holding across all situations of every kind are the real valid inferences, assuming that such collection is not empty. P1, P2 (which constitute the traditional pretheoretical notion of validity) and P3 (the assumption that there must be a non-empty logic) constitute an important part of the core of Priest's logical monism. However, P1 and P2 with the strict reading of the quantifier on one hand, and P2 on the other, may lead the monist perspective into trouble. What if P3 were false, i.e., what if there were no inferences holding in absolutely all situations, as trivialists and possibilists argue? To complicate things, P3 requires further a "large enough" number of valid inferences, for even though if the collection of valid inferences were not empty, if it consisted of, say, only one or just few inferences, it would be vacuous in practice to call 'logic' such a small number of valid inferences. However, the greater the collection of inferences, the more likely that they could not hold together in all situations. One possible answer is that there would be no logic at all. This is natural, for if a logic is thought of as being in the business of inferences holding in all situations, and no inference satisfies that, then there is no logic. However, even though there were no inferences valid in all of them, situations might need special inferences as inferential patterns ruling right reasoning in them. One can thought of a logic also in that way, but this lead easily into logical pluralism. Non-necessitarianism poses thus a challenge to logical monism.

Priest says that he has "(...) never seen any persuasive argument as to why one should suppose this [what non-necessitarianism says] to be the case." [21, p. 202] In effect, none of Mortensen's papers on the subject appears in the bibliography of *Doubt Truth to be a Liar*.[5] Fortunately, as in the case

[4] Priest says that there is one true logic, a logic whose inferences hold in all situations and that lacks principles depending on specific domains, but he works with a broad notion of logic in the sense that he is ready to accept that inferential tools for certain particular situations augmented with principles specific to those domains count also as logics, unlike other logical monists who say that the one true logic is also the only genuine logic.

[5] Not even on the chapter on trivialism, where a discussion of possibilism could take place naturally. Actually, [15] is a reply to one of the papers which that chapter is based on. Priest only makes a side remark on possibilism in [20, p. 343]. Neither non-necessitarianism nor possibilism in Mortensen's senses of the terms have been widely discussed. As far as I know,

of trivialism, Priest does not get rid of non-necessitarianism merely with an incredulous stare even if he has not heard of a good case for that position, but puts forward a very interesting argument against it. Priest rejects the idea that, in practice, every principle of inference –or at least a large number of them so as to make speaking of a logic vacuous– fails in some situation. His argument for this, P3, is that to the extent that the meanings of connectives are fixed, there are some principles of inference that cannot fail. For example, he says, in any case in which a conjunction holds, each conjunct holds, simply in virtue of the meaning of $\wedge$. In symbols, what Priest is saying is that the inferences

$$\alpha \wedge \beta \vdash \alpha$$
$$\alpha \wedge \beta \vdash \beta$$

cannot fail in some situations. Otherwise we would be using a different connective, not conjunction or, in a more charitable interpretation, we would be merely offering a different account of the meaning of conjunction (cf. [21, p. 203].

According to Priest, one could be wrong on what the truth conditions for a given connective are (conjunction in this case), but "[a]s long as meanings are fixed, one can't vary them to dispose of valid inferences. Priest is conjuring here the obvious reply that those inferences might fail because the meaning of conjunction is other than that embodied in them. Priest agrees that we might be wrong on what the meaning of conjunction is, but his counter-reply is that once the meaning of conjunction is given, in virtue of that very meaning there are some inferences that cannot fail. The inferences given serve just as examples of what we regard now as the meaning of conjunction.

3. Thoughts on Minimalism

Priest is implicitly endorsing a maximalist view, in a sense I will specify below, on the meanings of connectives. He says that the meaning of a connective is given by its truth conditions, but he is conflating on his view

- Truth conditions properly speaking, which presuppose
- a minimum number of truth values, usually two (true and false).
- An exact number of truth values.
- A very determinate notion of validity, which presupposes
- a very determinate way of separating truth values.

Roughly, a *maximalist* thinks that every element related to a connective, as those listed above, contributes to determine its meaning. A *minimalist* thinks

Footnote 5 continued
it has been mentioned in [8], as a "serious contender" on the issue of the existence of truths of the form $\Box\Box\alpha$. Humberstone mentions it again in [9], discussing a variation on a trivialist argument related to those put forward in [10], where non-necessitarianism and possibilism are also discussed.

that not every such element makes such a semantic contribution. The minimalism which I will endorse here is one where only truth conditions (presupposing only a minimum number of truth values) determine the meaning of connectives. The other elements would give shape to a logic, but not meaning to the connectives.[6]

The core of the disagreement between Priest's approach to the necessitarianist thesis and mine is minimalism, so let me address it first. Indeed, Priest has rejected a minimalist view endorsed "in personal discussion" by Greg Restall. Priest reconstructs Restall's minimalism as follows:

> Classical and intuitionist connectives do not have different meanings. Truth conditions are general and uniform. Apparently different truth conditions are just special cases. Thus, for example, the truth conditions for the connectives in a Kripke interpretation for intuitionist logic collapse into classical conditions at "classical" worlds, namely, those which access no worlds other than themselves. We do not, therefore, have connectives with different meanings. It is just that the intuitionist countenances more situations than their classical cousin. ([21, p. 204])[7]

Priest thinks this is not right. He says that for someone who takes classical truth-conditions to rule an account of the connectives, say negation, for example, what it means to say that $\neg\alpha$ holds in a situation is just that α fails to hold there. According to Priest, from that it follows that there can be no situations where neither α nor $\neg\alpha$ holds, that the situation cannot be "incomplete". In a footnote he claims that the case is even clearer with respect to inconsistent situations, for if one subscribes to classical negation, no situation can be such that α and $\neg\alpha$ hold at it. "This is a very part of what negation is taken to mean." He even suggests an analogy with the meaning of "to see":

> (...) to determine whether it is part of the meaning of 'to see' that the eyes must be employed, we might consider whether there are any (maybe hypothetical) situations where someone could be described as seeing, even though they had no (functioning) eyes. One who holds that seeing involves having eyes will deny the possibility of such situations.

The answer to these remarks of Priest is that only a maximalist would grant that there is an indissoluble tie between the meaning of a connective and all the (valid) formulas or inferences where such connective appears or any other alleged consequence of that meaning. In particular, not all classical features can be recovered from those truth conditions alone. Since we are talking about truth conditions, let us rephrase $\neg\alpha$ *holds iff* α *does not hold* as $v(\neg\alpha) = 1$ *iff* $v(\alpha) \neq 1$. As Priest says,

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[6] I take the labels from [7], and build upon his characterization of minimalism. For my present purposes, I have given model-theoretic characterizations of minimalism and maximalism, even though Hjortland originally was interested in proof-theoretic semantics.

[7] One can in fact read similar views in [1, Sect. 4], [2, pp. 51ff, 97ff]. Other minimalisms can be found in [16,17,22–24,26].

one can infer *Either* $v(\neg\alpha) = 1$ *or* $v(\alpha) = 1$ from this. We know that when $v(\alpha) \neq 1$, $v(\neg\alpha) = 1$, so at least one of them is equal to 1. When $v(\neg\alpha) \neq 1$, assuming contraposition, $v(\alpha) = 1$, so again at least one of them is equal to 1. By a similar reasoning one can infer *Either* $v(\neg\alpha) \neq 1$ *or* $v(\alpha) \neq 1$, and it is routine to verify that $\alpha \vee \neg\alpha$ and $\neg(\alpha \wedge \neg\alpha)$ are theorems (assuming that the truth conditions of conjunction and disjunction are $v(\alpha \wedge \beta) = min(\alpha, \beta)$ and $v(\alpha \vee \beta) = max(\alpha, \beta)$, respectively).

However, one cannot infer other classical claims, like *Either* $v(\neg\alpha) = 0$ *or* $v(\alpha) = 0$, unless one presupposes completely the classical constraints on truth values and their structure, that is to say, that there are only two, sharply separable truth values, corresponding one and only one to each proposition.[8] For suppose truth values do not obey the classical constraints; instead there are three truth values, $1, \mu$, and 0, with the order $0 <\mu <1$. First of all, one cannot say what the truth value of $\neg\alpha$ is only from the truth condition. The only thing one can say is that if $v(\alpha) = 1$ then $v(\neg\alpha) \neq 1$; it might be either μ or 0. It does not harm to any classical scruples to complete the truth condition as $v(\neg\alpha) = 1$ *iff* $v(\alpha) \neq 1$ *and* $v(\neg\alpha) = 0$ *otherwise*, so we will do it. Independently of this, there was a case when neither $v(\neg\alpha) = 0$ nor $v(\alpha) = 0$, namely when $v(\alpha) = \mu$ (since if $v(\alpha) \neq 1$ then $v(\neg\alpha) = 1$). Nor are all inferences judged as in classical logic. Assume that $\alpha \vdash \beta$ is a valid inference if and only if, if α holds, so β (and by contraposition, if β does not hold then α does not hold either), i.e. if $v(\beta) \neq 1$ then $v(\alpha) \neq 1$. It would not be wrong to infer β from $\alpha \wedge \neg\alpha$ when $v(\alpha) = \mu$ and $v(\beta) = 0$, and hence $v(\beta)<v(\alpha \wedge \neg\alpha)$, for the premise does not hold if the consequent does not hold. But given the structure of truth values in classical logic, it never happens that one could infer a conclusion such that its truth value is lesser than those of the premises. However, if one were to accept instead that $\alpha \vdash \beta$ if and only if $v(\alpha) \leq v(\beta)$, now $\alpha \wedge \neg\alpha \vdash \beta$ would not be a valid inference, unlike the classical case.

The truth conditions for negation given by Priest do not rule out by themselves a kind of paraconsistency as he thought. This is not completely surprising, though. The classical logician could recognize as a truth condition for negation the clause $v(\neg\alpha) = 0$ *if and only if* $v(\alpha) \neq 0$, which is "dual" to that given by Priest. We saw that, unless one assumes a classical structure on truth values, some valuations are not specified through this truth condition alone. For example, if $v(\alpha) = 0$ then $v(\neg\alpha) \neq 0$, but one cannot say whether $v(\neg\alpha) = 1$ or $v(\neg\alpha) = \mu$. "Dualizing" again what was done in the preceding paragraph, one can complete the truth condition in the following way: $v(\neg\alpha) = 0$ *if and only if* $v(\alpha) \neq 0$ *and* $v(\neg\alpha) = 1$ *otherwise*. But it will be easy to verify that, in spite of one is able to infer several "classical" features from this truth condition, now in general $\alpha \vee \neg\alpha$ is not a theorem, and that one should not accept as a valid inference $\beta \vdash \alpha \vee \neg\alpha$.

[8] By the *structure of truth values* I will mean, roughly and as suggested by the example, a maximum exact number of truth values, how do they relate to each other as well as how do they relate to propositions.

Technically, what I have just done is to give truth conditions for negation in Gödel logics, and this can be used to argue for a sameness of meaning between the connectives of classical, intuitionistic and superintuitionistic logics. What Priest gave were truth conditions dual to the Gödelian ones, so we found the compatibility of meanings of negation in classical logic and paraconsistent logic, or more exactly, between classical logic and a family of dual-intuitionistic logics. Thus, classical features result from truth conditions plus other restrictions, concerning truth values and their structure. More generally, a logic L can be seen as a result from general truth conditions, common to several logics, and particular desiderata concerning truth values and their structure.[9]

4. How Even the Most Secure Inferences Fail

Now, to show how those inferences thought by Priest as fixed in virtue of the meaning of connectives may fail too, suppose that the truth conditions of a conjunction were as usual: $v(\alpha \wedge \beta) = \min(v(\alpha), v(\beta))$, with respect to an order between two truth values, true (denoted '1') and false (denoted '0'). That would be the meaning of conjunction. Beyond the order between truth values, nothing else about them is presupposed; in particular, nothing is said on exactly how many of them there are.

Now, consider the notions of Malinowski's q-validity ([11,12]) or Frankowski's p-validity ([5,6]), instead of the traditional one stated above:

q-*validity*. A conclusion Y follows from premises X if and only if any case in which each premise in X is not antidesignated is also a case in which Y is designated. Or equivalently, there is no case in which each premise in X is antidesignated, but in which Y fails to be designated.

p-*validity*. A conclusion Y follows from premises X if and only if any case in which each premise in X is designated is also a case in which Y is not antidesignated. Or equivalently, there is no case in which each premise in X is designated, but in which Y fails to be antidesignated.

Under classical constraints on the structure of truth values, these notions of validity are indistinguishable from the traditional one. If there are only two truth values, 1 and 0 with their usual order, the collections of designated and antidesignated values exhaust all the possible values, designated = not antidesignated and not designated = antidesignated. But if this were a fault merely of classical logic, surely q-validity and p-validity would have arisen before they did. However, these notions of validity collapse if the collections of designated

[9] Field [4] defended the view that classical laws can be obtained from the mere meaning of connectives written in terms of (conditional) probability. To give a probabilistic semantics for intuitionistic logic only would reveal the difference of meaning of connectives. However, I think Beall and Restall's ([1, Sect. 4], [2, pp. 51ff, 97ff]) argument to show that the superficial difference in the possible-worlds truth conditions between the negation appearing in classical laws and the negation appearing in intuitionistic laws does not imply meaning-variance can be applied also for the probabilistic semantics. I hope to deal with this issue in further work.

and antidesignated values are disjoint and exhaustive with respect to the total collection of values given, as is assumed in almost every known logic.[10]

Let us look more closely these notions of validity at work. Given the truth values above, take 1 as designated value, 0 as antidesignated value and μ as neither designated nor antidesignated. An inference like $\alpha \wedge \beta \vdash \alpha$ fails even though the meaning of conjunction, its truth conditions, are the usual ones. Take for example $v(\alpha) = \mu$ and $v(\beta) = 1$. So $v(\alpha \wedge \beta) \neq 0$, i.e. $\alpha \wedge \beta$ is not antidesignated. The premise is not antidesignated here, but the conclusion is not designated, for $v(\alpha) = \mu$. Hence, $\alpha \wedge \beta \nvdash \alpha$, without changing the meaning of conjunction. We changed the logic (from logics validating those inferences to something else, for we changed the number of truth values, the notion of validity and the separation of truth values) without changing the meaning (the truth conditions were the usual ones).

An obvious worry at this point is whether invoking these strange notions of validity is legitimate. One of the non-Tarskian notions of consequence is well-known, non-monotonic logic, but non-reflexive and non-transitive consequence relations are not so popular. However, one can ask why it is possible doing without monotonicity but not without reflexivity or transitivity. The answer "Because preservation of designated values (from premises to conclusions) is a reflexive and transitive relation" begs the question, for non-Tarskian logics are precisely asking for ways of logically connecting premises and conclusions others than preservation of designated values. For example, q-validity deals with preservation of non-antidesignated values but in such a strong way that it rather forces passing from non-antidesignated values to designated values. Similarly, p-validity is preservation of designated values but in such a weak way that it allows passing from designated values to some non-designated values (but never from designated to antidesignated values!).

In the same way as inconsistency and triviality collapse under classical constraints, Tarskian consequence, Tarskian consequence without monotonicity, q-consequence and p-consequence also collapse under classical constraints. However, the identification occurs on a much deeper level than that between inconsistency and triviality. Indeed, the emergence of the study of q-validity and p-validity was due to an attempt to avoid some consequences of a theorem by Suszko which, if Tarskian logics were the only genuine examples of consequence relations, could be read as saying "Many-valued logics do not exist at all" or "All logics are bivalent".[11]

Thus, the easy answer is that, at least technically, non-Tarskian notions of consequence such as q-consequence and p-consequence are as legitimate as

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[10] A *Tarskian* consequence relation satisfies reflexivity, transitivity and monotonocity. The notion of consequence Priest or Beall and Restall think to be the traditional is not Tarskian, for it does not satisfy monotonicity. Notably, q-consequence fails to satisfy reflexivity, whereas p-consequence fails to satisfy transitivity. For more on q-consequence and p-consequence see [30].

[11] The literature on Suszko's theorem and its implications has rapidly grown over the last years. Some basic references are [3,13,28,29]. For a quick but very good survey of the state of the art see [30].

non-classical logics. More elaborate answers could be given along the lines of non-monotonic logics. For example, q-validity would serve to "jump" to conclusions more certain than the premises, and p-validity would allow to jump to conclusions less certain than the premises. Seen in that way, we are not really in face of totally new weirdness.

5. Conclusions

According to non-necessitarianism, every principle of inference fails in some situation. Priest thinks that some inferences cannot fail merely in virtue of the meaning of connectives involved. I have argued that a minimalist account of the meaning of connectives does not rule out the failure of those inferences. So the apple of discord is minimalism. I have tried to show that Priest's arguments against minimalism do not work, for clauses which he accepts as determining the meaning of connectives in general do not determine a unique logic. If my conclusions are correct, as I think they are, logical monism has lost a very important argument on its behalf and the non-necessitarianist challenge remains open.

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