COMPARATIVE OPINION LOSS

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Abstract

It is a consequence of the theory of imprecise credences that there exist situations in which rational agents inevitably become less opinionated toward some propositions as they gather more evidence. The fact that an agent's imprecise credal state can *dilate* in this way is often treated as a strike against the imprecise approach to inductive inference. Here, we show that dilation is not a mere artifact of this approach by demonstrating that opinion loss is countenanced as rational by a substantially broader class of normative theories than has been previously recognised. Specifically, we show that dilation-like phenomena arise even when one abandons the basic assumption that agents have (precise or imprecise) credences of any kind, and follows directly from bedrock norms for rational comparative confidence judgements of the form 'I am at least as confident in p as I am in q'. We then use the comparative confidence framework to develop a novel understanding of what exactly gives rise to dilation-like phenomena. By considering opinion loss in this more general setting, we are able to provide a novel assessment of the prospects for an account of inductive inference that is not saddled with the inevitability of rational opinion loss.

§1: Introduction

Suppose we present you with two coins. Coin 1 has one side head and one side tails, and we assure you that it is fair — i.e., it has a chance of 0.5 of landing heads, and a chance of 0.5 of landing tails. Coin 2 has one side black and one side grey. We tell you absolutely nothing about the bias of Coin 2. It could be completely biased towards landing black, or completely biased towards landing grey, or anywhere in

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between. You simply have no evidence that bears on the bias of this coin, or on how it will land when we toss it.[1]

Intuitively, there is a significant asymmetry between your epistemic situations with respect to the two coins. Regarding Coin 1, your credence that the coin will land heads when tossed should presumably equal your credence that it will land tails, and that judgement is based on unambiguous concrete evidence about the bias of the coin. Regarding Coin 2, you simply have no clue about whether the coin will land black or grey when tossed, since it is stipulated that you have absolutely no evidence that is relevant to the outcome. In the first case, your evidence of the coin's fairness licenses a definite judgement that the chance of heads equals the chance of tails. In the second case, because you have no evidence, it seems that you should simply suspend judgement. These two attitudes are not the same.

This simple observation is at the root of some of the most persistent and persuasive criticisms of the standard Bayesian approach to inductive inference and decision-making (see, e.g., Joyce (2010), Kaplan (1996), Levi (1974, 1985), White (2009)). According to the standard Bayesian picture, the norms of inductive inference require you to assign definite precise credences to all four of the propositions $H =$ 'Coin 1 will land heads when tossed', $T =$ 'Coin 1 will land tails when tossed', $B =$ 'Coin 2 will land black when tossed', and $G =$ 'Coin 2 will land grey when tossed'. Since you know that Coin 1 is fair, the Principal Principle requires your credences to satisfy $c(H) = \frac{1}{2} = c(T)$.[2] And since you have absolutely no evidence that yields reason to be more confident in B over G (or in G over B), it seems that your credences should satisfy $c(B) = c(G)$. Thus, given the Bayesian commitment to probabilism, it follows that $c(B) = \frac{1}{2} = c(G)$. This means that standard Bayesian norms require you to adopt the same credence distributions over the partitions $\{H, T\}$ and $\{B, G\}$,[3] even though your evidential situations with respect to these two partitions radically differ and thus seem to license distinct and non-equivalent doxastic attitudes.

Standard Bayesianism's apparent inability to respect the intuitive asymmetry between rational responses to Coin 1 and Coin 2 is widely cited as a central motivation for adopting a theory of *imprecise* or *indeterminate* rational credences.[4] Because this theory supplements the standard Bayesian picture by introducing

[1]This example is taken from White (2009). You might object that you could never be in this situation because your prior experience with coins disqualifies you from being in this evidential context (since, e.g., you have observed considerably more fair coins than unfair coins). If so, then just consider the case of a Martian who has never seen a coin.

[2]Roughly, David Lewis's (1980) Principal Principle requires that rational agents align their credences with their knowledge of objective chances (with some special exceptions, which don't seem to apply in this case).

[3]Of course, one could argue that Bayesian norms allow for a radical subjectivism on which any initial credence assignment over $\{B, G\}$ is permissible. Here, we assume that a rational agent's credences should reflect the evidential symmetry between B and G.

[4]There is substantive disagreement in the literature about whether one should think of this generalisation in terms of indeterminacy or imprecision. While Levi (1974, 1985) prefers to articulate the position in terms of indeterminacy, the

the possibility of representing an agent's credences as imprecise, it allows us to represent the attitudes towards the two coins differently — viz., the imprecise Bayesian can maintain that your credences toward the partition $\{H, T\}$ should be precise (so that you assign $\frac{1}{2}$ to both H and T) while your credences toward the partition $\{B, G\}$ should be imprecise (so that you adopt a maximally unopinionated *ranged* credal state towards both B and G).[5]

But while theories of imprecise credence improve on the standard Bayesian's inability to distinguish equal confidence and suspense of judgement, they also inherit a new problem that stems directly from having the resources to draw that distinction. Specifically, if one distinguishes between the rational doxastic attitudes that agents should hold toward the two coins by introducing imprecise/indeterminate credences, then core Bayesian norms imply that an agent's credences can *dilate*, where *dilation* occurs roughly when you inevitably become *less* opinionated as a result of learning *more*. For example, in the case of Coin 1 and Coin 2, it turns out that we can specify learning scenarios such that your credence that Coin 1 will land heads inevitably dilates to a maximally imprecise credence, despite your knowledge that the coin is fair. This not only *prima facie* seems counter-intuitive, but also conflicts with some imprecise generalisations of well established norms of inductive inference (e.g. van Fraassen's Reflection principle).

A number of authors follow White (2009) in maintaining that this consequence of imprecise Bayesianism is so undesirable that it constitutes a decisive refutation of the theory and therefore motivates a return to standard precise Bayesianism. In response, defenders of imprecise Bayesianism have characterised conditions under which dilation can be avoided, and have argued that even if dilation can't be avoided, its existence is not as problematic as it initially seems (and may even be a natural feature of rational inductive reasoning).[6]

In this paper, we develop a novel lens through which to view dilation that both generalises the phenomenon and allows us to more clearly and perspicuously diagnose its root causes and philosophical implications. We do this by temporarily abandoning the premise, common to both precise and imprecise Bayesianism, that the epistemic states of rational agents can be faithfully represented by precise or imprecise credences. Instead, we adopt a more coarse-grained representation of epistemic states in terms of *comparative confidence judgements*. We then examine whether and how an analogue of the dilation phe-

majority of contemporary authors follow Walley (1991) in adopting the 'imprecise' terminology. We remain ecumenical about this dispute, and use the notions interchangeably throughout this paper. This equivocation, though significant in other contexts, has no bearing on the present analysis.

[5]As we will see later, this amounts to adopting a *set* of probability functions that jointly output the entire range of possible probability estimates for both B and G.

[6]See, e.g., Bradley and Steele (2013), Joyce (2010), Stewart and Nielsen (forthcoming), Pedersen and Wheeler (2013), and Seidenfeld and Wassermann (1993).

nomenon follows directly from the more fundamental (and strictly weaker) norms of rational comparative confidence. Doing so allows us to precisely identify the weakest and most general inductive reasoning norms from which dilation-like phenomena emerge, and also demonstrates that dilation is a far more general and deep-lying phenomenon than has been previously recognised. Importantly, we demonstrate that rational opinion loss is not at all specific to imprecise Bayesianism, and that it will instead arise for any theory of inductive inference that coheres with some bedrock principles for rational comparative confidence. Then, in this general setting, we assess the prospects of abandoning one of these bedrock principle(s) so that rational comparative opinion loss is avoided. We do not take a stand on whether we *should* abandon any such principle, but we do survey the options and take stock of their implications.

The structure of the paper is as follows. §2 concisely recalls the most salient details of both an influential version of imprecise Bayesianism and a popular presentation of the problem of dilation. §3 introduces some compelling epistemic norms for comparative confidence judgements. §4 reexamines the problem of dilation through the lens of comparative confidence and presents a range of results demonstrating that comparative opinion loss follows directly from some very weak constraints on rational comparative confidence orderings. §5 then distills these results into a three-horned trilemma that reshapes the debate about how and whether dilation-like opinion loss can be avoided in light of the results from §4. §6 concludes.

§2 : **Imprecise Credences and Dilation**

§2.1 : **Imprecise Bayesianism**

On the standard precise Bayesian picture, the epistemic state of a rational agent that considers an algebra of propositions $\mathfrak{B}$ is given by a single probabilistic credence function $c : \mathfrak{B} \to [0, 1]$, which assigns to each proposition $p \in \mathfrak{B}$ the agent's 'degree of belief' or 'credence' in p. As we noted in the introduction, the assumption that the epistemic states of rational agents can always be faithfully represented in this way is a contentious one, and many authors have advocated for alternative representations that are (i) more psychologically realistic, (ii) more coarse-grained, and (iii) better able to distinguish between the kinds of epistemic states that are appropriate in different evidential scenarios. Perhaps the most influential alternative representation in the philosophical literature is the imprecise Bayesian representation, which represents the epistemic states of rational agents not with individual probabilistic credence functions, but rather with *sets* of such functions.[7] Specifically, the epistemic state of an agent at any time is represented by a set $\mathfrak{C}$ of probabilistic credence functions, called the agent's 'representor'. The individual epistemic judgements made by the agent can then be derived from $\mathfrak{C}$ via a supervaluationist semantics, according to which a judgement is (il)legitimately ascribed to an agent if and only if (no) every function in the representor instantiates that judgement. If the judgement is instantiated by some but not all the functions in the representor, then it is said to be indeterminate whether the agent makes that judgement.

The intuitive idea behind the imprecise representation is (roughly) that $\mathfrak{C}$ should contain all and only the probabilisitic credence functions that are compatible with the agent's evidence. For example, if the agent has decisive evidence that p is true, then every function in $\mathfrak{C}$ should assign p probability 1. However, if one has absolutely no evidence pertaining to the truth of p, then the evidence is compatible with assigning any credence to p (since there is no evidence), meaning that for any $x \in [0, 1]$, there should exist $c \in \mathfrak{C}$ such that $c(p) = x$. In this case, the agent's credence in p is completely indeterminate, since for any $x \in [0, 1]$, it is indeterminate whether the agent's credence in p is x.

In addition to the synchronic requirement that an agent's epistemic state should be accurately representable by a single probabilistic credence function, the standard Bayesian package also includes a diachronic norm, which specifies how an agent's credences should evolve over time as they gather new evidence. Specifically, it is typically argued that upon learning some evidence e, Bayesian agents should replace their initial credence function c with the function c_e defined by $c_e(p) = c(p|e) = \frac{c(e \wedge p)}{c(e)}$ for all $p \in \mathfrak{B}$. This rule is known as 'Bayesian conditionalisation' and is at the core of the Bayesian approach to inductive inference.[8] Just as imprecise Bayesianism generalises the central synchronic constraint of standard Bayesianism by requiring that a rational agent's epistemic state always be faithfully represented, not by a single probabilistic credence function, but rather by a set of such functions, it also generalises the central diachronic norm of standard Bayesianism by requiring that upon learning some new evidence e, an agent should replace their prior representor $\mathfrak{C}$ with the new representor $\mathfrak{C}_e = \{c_e | c \in \mathfrak{C}\}$—i.e., they should update every function in their representor by Bayesian conditionalisation and take the set of these updated functions as their new

[7]Here, it is important to acknowledge that there are multiple importantly distinct formalistations of the notion of 'imprecise credence', and that these different formalisations often yield importantly divergent epistemic norms. For the purposes of our exposition here, we focus on the set-based formalisation employed by e.g. van Fraassen (1990), Joyce (2005, 2010), Kaplan (1996), and Weatherson (2007), rather than e.g. the theories of coherent lower previsions, desirable gambles or probability filters (see e.g. Troffaes and de Cooman (2004), Campbell-Moore (2021), Campbell-Moore and Konek (2019), Walley (1991, 2000)). However, we also stress again that our main focus is on the relationship between the dilation-like phenomena and norms of rational comparative confidence, and hence that the main conclusions of our analysis are independent of the specific version of the imprecise Bayesianism that we deploy. We chose this version because we suspect that it is accessible and familiar to a wide philosophical readership.

[8]The other core tenet of the Bayesian approach is the assumption that rational agents have have probabilistic credences.

representor.

§2.2 : **Dilation**

We are now ready to see how imprecise Bayesianism leads directly to the dilation puzzle. Towards this end, consider the following thought experiment (due to White 2009).[9]

Recall the two coins from the introduction. Coin 1 is fair, and has an equal chance of landing heads (H) and tails (T) when tossed. Coin 2 has one side black (B) and one side grey (G), and you know *nothing* about its bias. We now tell you that we will perform one toss of each coin, and you are asked to predict the outcomes. Standard Bayesianism (combined with a typical principle of indifference) requires you to adopt the following initial credence function,

$$c(H \wedge B) = c(H \wedge G) = c(T \wedge B) = c(T \wedge G) = \tfrac{1}{4}$$

As we saw in the introduction, this approach has the problem that it yields identical credence assignments over the partitions $\{H, T\}$ and $\{B, G\}$, and therefore fails to respect the important epistemic asymmetry between the two coins. Happily, the imprecise Bayesian approach is able to avoid this shortcoming. To see this, note first that, since you have infallible evidence that the chance of H is the same as the chance of T, every credence function in your representor should satisfy the equality $c(H) = c(T) = \frac{1}{2}$, which implies that your credence in H (T) is determinately $\frac{1}{2}$. In contrast, you have absolutely no evidence pertaining to how likely Coin 2 is to land black (B) or grey (G). This means that any assignment of credence to B (G) is compatible with the evidence (since there is none), and hence that for every $x \in [0, 1]$, there should exist $c \in \mathfrak{C}$ such that $c(B) = x$. This implies that your credence in B (G) is completely indeterminate, and hence that your credences over the partition $\{B, G\}$ are also completely indeterminate. So in the imprecise picture, we *can* distinguish between the two coins in a way that respects the important evidential asymmetry between them. While the norms of imprecise Bayesianism demand that you adopt determinate and precise credences towards the outcomes for Coin 1, they say that it is rational to refrain from adopting any determinate or precise credences towards the outcomes for Coin 2. Thus, imprecise Bayesianism coheres with the natural intuition that the rational doxastic attitudes that one should adopt towards Coin 1 are not the same as the attitudes that one should adopt towards Coin 2.[10]

[9] Historically, the dilation phenomenon was already recognised and discussed by e.g. Good (1966, 1974), Seidenfeld (1981), and Walley (1991).

[10] Note that until now, we have not specified anything about how the outcomes of the two tosses relate to one another. In

Now, at some subsequent time t, we go on to actually flip the two coins. Depending on the outcomes, we will return one of two possible pieces of evidence. The first possible piece of evidence is that Coin 1 landed heads if and only if Coin 2 landed black, i.e. $H \equiv B$, i.e. $(H \wedge B) \vee (T \wedge G)$. The second possible piece of evidence is that Coin 1 landed heads if and only if Coin 2 landed grey, i.e., $H \equiv G$, i.e. $(H \wedge G) \vee (T \wedge B)$. Clearly, these two possible pieces of evidence form a partition, which implies that exactly one of them will turn out to be true. We will report whichever is true.

Suppose for now that after flipping the coins, we come back and tell you $H \equiv B$. You now need to update your epistemic state in order to incorporate this new evidence. On the precise Bayesian picture, updating the initial uniform credence function c on this evidence will yield the posterior credence function $c_{H\equiv B}$ defined by $c_{H\equiv B}(H \wedge B) = c_{H\equiv B}(T \wedge G) = \frac{1}{2}$. So with respect to the two partitions $\{H, T\}$ and $\{B, G\}$, nothing changes when we give you this evidence. Your posterior credence function continues to satisfy the equalities $c_{H\equiv B}(H) = c_{H\equiv B}(T) = c_{H\equiv B}(B) = c_{H\equiv B}(G) = \frac{1}{2}$. When it comes to predicting the outcomes of either of the two individual tosses, this evidence is essentially useless on the precise Bayesian picture. You initially assigned H/B and T/G equal credence, and you continue to do so after learning the evidence.

But things look very different on the imprecise picture. To see this, recall that imprecise Bayesianism requires you to replace your initial representor $\mathfrak{C}$ with the posterior representor $\mathfrak{C}_{H\equiv B} = \{c_{H\equiv B} | c \in \mathfrak{C}\}$. Since $\mathfrak{C}$ contains every probabilistic credence function that satisfies the constraint that $c(H) = c(T) = \frac{1}{2}$, it contains one function of the form $c_1(H \wedge B) = \frac{1}{2} = c_1(T \wedge B)$, and one of the form $c_2(H \wedge G) = \frac{1}{2} = c_2(T \wedge G)$. Since ${c_1}_{H\equiv B}(H) = 1$ and ${c_2}_{H\equiv B}(H) = 0$, it follows that, relative to the imprecise Bayesian's recommended posterior epistemic state, it will be indeterminate whether your credence in H is 0 or 1. So your credence in H has gone from having the precise value of $\frac{1}{2}$ to being completely indeterminate, in the sense that it is indeterminately located at both ends of the $[0, 1]$ interval.[11] So on the imprecise picture, the evidence makes a very substantial difference to your attitude regarding the outcomes for Coin 1. Your representor goes from being precise and determinate to completely imprecise and indeterminate. And since that means that you become *less* opinionated as a result of learning *more*, it seems that the imprecise Bayesian has some explaining to do.

Moreover, if we shift our focus to what would have been rational had you instead learned that $H \equiv$

particular, we have not specified whether you should consider the two tosses to be independent of one another, or whether you are allowed to view them as correlated. Nothing that we've said so far hinges on this, and one can additionally specify that the two tosses are stochastically independent at no cost.

[11] Because your credence in H spreads in this way, it is said to *dilate*.

G (rather than $H \equiv B$), then analogous reasoning again demonstrates that imprecise conditionalisation outputs a completely indeterminate posterior credence in H. This means that prior to any updating, you're in a position to know that no matter what you learn — i.e., $H \equiv G$ or $H \equiv B$ — it will be rational to adopt a maximally indeterminate credence towards H. So why not just adopt the indeterminate credence in H now, prior to learning anything about the outcomes of the tosses? After all, if the imprecise Bayesian is right, then you're in a position to know now that if you update rationally, then no matter what evidence you receive,[12] you'll be indeterminate towards H. The only salient difference between you and your future self is that your future self has strictly more evidence than you currently do. So when you know that your future self will adopt some credence no matter what they learn, doesn't it seem that you should, too? Doesn't the asymmetry between your evidential situations create grounds for deference? Of course, the problem with this line of reasoning is that if you defer to your future attitude towards H, then you'll be maximally indeterminate towards a coin that you know to be fair. Thus you'll effectively ignore your evidence that the coin is fair, and this is a price that few epistemologists are willing to pay.

Put differently, the problem here is that the core synchronic and diachronic constraints of imprecise Bayesianism force a conflict between the idea that we should defer to known chances and the idea that we should defer to our future credences when they're known. In the standard (precise) Bayesian context, these two ideas are captured by Lewis's (1980) Principal Principle (henceforth 'PP') and van Fraassen's (1984) Reflection Principle (henceforth 'REF'), respectively.[13] So we can sum the situation up as follows

$$\text{Imprecise Bayesianism} \Rightarrow \neg(PP \wedge REF)$$

Since both PP and REF are widely accepted as exceptionless norms of inductive inference, this observation should *prima facie* be regarded as a strike against imprecise Bayesianism. Of course, there are responses to this challenge,[14] but many philosophers have taken the existence of cases like this one to constitute a full-blown refutation of imprecise Bayesianism.[15] Are these philosophers right to be so worried about

[12] Remember that you know that you'll learn either $H \equiv G$ or $H \equiv B$.

[13] Strictly speaking, PP and REF, as standardly defined, do not straightforwardly extend beyond the confines of precise Bayesianism. But these principles can be generalised so that they apply to the imprecise context. In the case of PP, see Hájek and Smithson (2012) for some relevant discussion. In the case of REF, there are multiple ways to generalise the principle — some of which give rise to the conflict at issue, and some of which do not. But any formulation that requires deference to your future (more informed) representor's credal assignments when they are known will generate the conflict. See §5.4 for extended discussion of these issues.

[14] See e.g. Bradley (2018), Joyce (2010), and Pedersen and Wheeler (2014).

[15] Some philosophers believe that these cases are problematic for reasons that are distinct from those we mention here. For example, White (2009) suggests that the imprecise Bayesian is wrong to treat the biconditionals as evidentially relevant to the outcome of the fair coin's toss. We refrain from covering this concern in detail because we believe that it has already been adequately addressed by some defenders of imprecise Bayesianism (e.g., Hart and Titelbaum 2015 and Joyce 2010). We

imprecise Bayesianism? And if so, where should we turn as we try to model the difference between having reason to be equally confident in two events and having reason to suspend judgement about two events?

In the remainder of this paper, our first goal is to investigate the extent to which the puzzle of dilation is specific to imprecise Bayesianism by examining cases like this one through the more general lens of the comparative confidence modeling framework, where a modeling framework is more general than another when it assumes less structure than the other.[16] Then, because we find that the puzzle of dilation has a home in *any* framework that takes stock of the difference between the suspense of judgement and the judgement of equal confidence, we plot some novel responses to the puzzle and assess their plausibility.

§3 : **Comparative Confidence**

§3.1: Synchronic Norms

For the remainder of the paper, we will abandon the basic assumption, common to both precise and imprecise Bayesianism, that agents have credences of any kind, be they precise or imprecise. In its place, we will start from the far weaker (and thus more generally applicable) assumption that rational agents are capable of making *comparative confidence judgements* of the form 'I am at least as confident in p as I am in q'. Of course, if one assumes that agents have credences, then their comparative confidence judgements can be derived from those credences in a straightforward way. If the credences are precise, then the agent is at least as confident in p as they are in q if and only if their credence in p is at least as great as their credence in q. If their credences are imprecise, then (according to a popular view) they are at least as confident in p as they are in q if and only if every credence function in their representor assigns p a credence at least as great as that assigned to q — i.e., if and only if their credence in p is determinately at least as great as their credence in q. But it is clear that one cannot in general recover (precise or imprecise) credences just from an agent's comparative confidence judgements. Thus when we assume that agents have (precise

cover the incompatibility of imprecise Bayesianism with PP and REF in more detail because we are more sympathetic to the challenge that it poses.

[16]According to this conception of generality, the comparative confidence modeling framework is more general than, say, the precise Bayesian modeling framework because the latter models agents in terms of precise credences *and* comparative confidence judgements (comparative confidence judgements are generally taken to be entailed by precise credences), while the former models agents only in terms of comparative confidence judgments. It is impossible to compare the generality of our approach with other frameworks that lack a defined way to recover comparative confidence judgements from their preferred representations, since these frameworks cannot be said to represent agents in terms of *both* their own primary method of representation and comparative confidence judgments. But luckily the authors of other frameworks almost always specify a way of deriving comparative confidence rankings from their preferred representations. See Walley (2000) for helpful discussion.

or imprecise) credences, we are imposing strictly more structure on the epistemic states of rational agents than we do when we assume only that they make comparative confidence judgements.

Moreover, as we'll see in what follows, the comparative confidence framework allows for a strikingly direct representation of suspense of judgement insofar as it allows for representation of the agent as *abstaining* from making any comparative judgement. Thus it proves to be a nice setting for exploring whether there is some approach to inductive inference that avoids the dilation puzzle while capably treating the difference between the appropriate attitudes towards the two coins. On the one hand, if the puzzle were to disappear in this generalised setting, it would indicate that dilation is idiosyncratic to the imprecise Bayesian framework and that we should keep shopping for some other framework that capably treats the distinction in attitudes. But on the other hand, if the puzzle persists in this more general setting, it would show that dilation-like phenomena are here to stay (at least so long as we're committed to treating the appropriate attitudes toward the two coins as distinct) and that we'd better figure out some way of making peace with their presence.

We begin our investigations by recalling some basic structural coherence norms for comparative confidence.[17] Firstly, for a fixed agent A, let $\succsim$ denote the relation $\succsim \subseteq \mathfrak{B} \times \mathfrak{B}$ defined by $p \succsim q$ if and only if A is at least as confident in p as they are in q. We call $\succsim$ A's 'confidence ordering', since it reflects the order in which they rank propositions according to their plausibility. Our first basic assumption is that $\succsim$ is a *non-trivial monotonic partial preorder* over $\mathfrak{B}$, i.e. that it satisfies the following three conditions.

Partial Preordering: $\succsim$ is a partial preorder, i.e., it is transitive and reflexive.

Monotonicity: If $p \vdash q$ then $q \succsim p$.

Non-Triviality: $(\top \succsim \bot)$ and $\neg(\bot \succsim \top)$.

Partial Preordering condition requires only that (i) if A is at least as confident in r as they are in q and at least as confident in q as they are in p, then they should be at least as confident in r as they are in p, and (ii) A should always be at least as confident in p as they are in p. Monotonicity simply requires that A should always be at least as confident in p's logical consequences as they are in p itself. And Non-Triviality demands only that rational agents be strictly more confident in tautologies than they are in contradictions.

[17] For more extensive introductions to coherence norms for rational comparative confidence see e.g. Fine (1973), Fitelson and McCarthy (forthcoming), Halpern (2003), Konek (2019), and Koopman (1940).

We follow orthodoxy in accepting Partial Preordering, Monotonicity and Non-Triviality as fundamental coherence constraints for rational comparative confidence.

In addition to these bedrock principles, some authors have proposed a battery of further, more substantial normative constraints on rational comparative confidence. We will recall only those constraints that are pertinent to helping to diagnose the root causes of the dilation puzzle in the comparative setting. Firstly, it is important to stress that we will *not* assume the following constraint, which is almost ubiquitous in the extant literature on rational comparative confidence.

Opinionation: For all $p, q \in \mathfrak{B}$, either $p \succsim q$ or $q \succsim p$ (or both).

Opinionation requires that rational agents always make *some* comparative confidence judgements regarding any given pair of propositions. Informally, this means that $\succsim$ has no 'gaps', in the sense that all propositions can be determinately ordered in terms of their plausiblity. Formally, it means that $\succsim$ is a a 'total' or 'linear' preorder. While many authors are happy to assume Opinionation as a basic constraint on rational comparative confidence, we make no analogous assumption. There are two main reasons for this. First, we follow e.g. Forrest (1989), Kaplan (1983, 1996), Keynes (1921) and Eva (2019) in contending that there are many realistic epistemic scenarios in which Opinionation places implausible and unreasonable demands on rational agents, and precludes the proper formulation of other genuinely normative rationality constraints. Second, in the present setting, it is important to note that the theory of imprecise Bayesianism outlined in §2 implicitly rejects Opinionation. To see this, recall the two coin example discussed above. As we saw, imprecise Bayesianism requires your prior representor to contain one credence function c_1 satisfying $c_1(B) = 1, c_1(G) = 0$ and one function c_2 satisfying $c_2(B) = 0, c_2(G) = 1$. This means that neither of the inequalities $c(B) \geq c(G)$ or $c(G) \geq c(B)$ are satisfied by all the functions in your prior representor, and hence that neither of the comparative confidence judgements $B \succsim G$, $G \succsim B$ can be legitimately ascribed to you when you first consider the coins. So on the imprecise Bayesian view, you are required to view B and G as *incomparable*, which is a direct violation of Opinionation. Of course, this differs strongly from the advice given by the standard precise Bayesian picture, which implicitly assumes Opinionation and requires you to initially regard H, T, B and G as *equally plausible*. These observations allow us to characterise the different approaches that precise and imprecise Bayesianism take towards the coins in purely comparative terms. Towards this end, we introduce the following shorthand

Equal Confidence: Define the 'equal confidence' relation $\sim$ by

$$(p \sim q) \Leftrightarrow (p \succsim q) \wedge (q \succsim p)$$

Strictly More Confident: Define the 'strictly more confident' relation $\succ$ by

$$(p \succ q) \Leftrightarrow (p \succsim q) \wedge \neg(q \succsim p)$$

Incomparability: Define the 'incomparable' relation $\odot$ by

$$(p \odot q) \Leftrightarrow \neg(p \succsim q) \wedge \neg(q \succsim p)$$

It is easy to see that precise and imprecise Bayesianism require you to adopt the following initial comparative confidence judgements in White's two coin example.

Precise Bayesianism: $H \sim T \sim B \sim G$

Imprecise Bayesianism: $H \sim T, B \odot G$

Here, it should be stressed that $\odot$ does not denote an extra species of comparative confidence judgement, but rather the absence of any comparative judgement whatsoever. While precise Bayesians are always forced to make definite comparative confidence judgements regarding any pair of propositions, imprecise Bayesians are able to simply abstain from making any such judgement for the pair B, G. And this looks like an eminently natural response to the situation. Since we have no evidence pertaining to which of B/G is more likely to be true, we should not make any judgements about which proposition is more plausible. This is the content of the comparative reformulation of the principle of indifference defended by Eva (2019), which states.

Principle of Indifference $\odot$ (POI$_\odot$): Let $X = \{x_1, ..., x_n\}$ be a partition of the set W of possible worlds into n mutually exclusive and jointly exhaustive possibilities. In the absence of any relevant evidence pertaining to which cell of the partition is the true one, a rational agent should not make any comparative confidence judgements regarding the pairs (x_i, x_j) for $i, j \leq n$. Equivalently, for all $i, j \leq n$, $x_i \odot x_j$ should hold in the agent's confidence ordering.

$\text{POI}_{\odot}$ stipulates that, since you have no evidence pertaining to which cell of the $\{B, G\}$ partition is actual, you should not make any comparative confidence judgement for the pair (B, G). In contrast, since you know that Coin 1 is fair, it seems that you are required to make the judgement $H \sim T$. Thus, the approach to epistemic indifference codified in $\text{POI}_{\odot}$ allows us to distinguish between the rational doxastic attitudes that you should take towards the two coins in purely comparative terms. Crucially, the ability to make this distinction relies on the rejection of the Opinionation assumption, since Opinionation precludes the proper formulation of $\text{POI}_{\odot}$.

For the majority of this paper, we will assume $\text{POI}_{\odot}$ as the norm that specifies which comparative confidence judgements agents should make in the absence of relevant evidence (we review the possibility of abandoning $\text{POI}_{\odot}$ in section §5). We've seen that in the coin example, $\text{POI}_{\odot}$ coheres with the confidence ordering that is implied by the imprecise Bayesian approach. However, $\text{POI}_{\odot}$ is a more general norm that can be articulated and applied independently of the characteristic tenets of imprecise Bayesianism. Importantly, $\text{POI}_{\odot}$ does not assume any substantive synchronic or diachronic norms for rational comparative confidence, and is completely independent of the assumption that agents have credences of any kind. This means that when we distinguish between the two coins using $\text{POI}_{\odot}$, we are really assuming far less about the epistemic states of rational agents than what is assumed by imprecise Bayesianism. For while $\text{POI}_{\odot}$ is implicitly implied by imprecise Bayesianism in this example, it does not entail anything about the legitimacy of imprecise Bayesianism's core commitments. In what follows, we assess whether the dilation puzzle arises when $\text{POI}_{\odot}$ is packaged with other fundamental norms for rational comparative confidence, with the aim of exploring whether the puzzle really is idiosyncratic to imprecise Bayesianism.

So far, we've assumed only $\text{POI}_{\odot}$ and the requirement that $\succsim$ always be a partial pre order. At this stage, we will also make explicit the assumption (entailed by the Principal Principle) that when one knows that p and q have the same objective chance of being true, one should regard them as equally plausible and make the judgement $p \sim q$.[18] Call this assumption the 'Comparative Principal Principle (CPP)'. As with $\text{POI}_{\odot}$, we assume CPP as a background commitment until §5.

We turn now to reviewing some more of the most influential structural coherence norms for comparative confidence, many of which are articulated in terms of representability by functions from $\mathfrak{B}$ into $[0, 1]$. Given a comparative confidence ordering $\succsim$ over $\mathfrak{B}$ and a set S of functions $\mu : \mathfrak{B} \to [0, 1]$, say that $\succsim$ is 'fully

[18]Strictly speaking, this follows from the PP only if the agent has no "inadmissible" evidence. But this doesn't matter in the present context since the agent clearly has no inadmissible evidence when she forms her prior expectation about the flips.

represented' by S if and only if for every $p, q \in \mathfrak{B}$,[19]

$$\text{(i) } p \succsim q \Leftrightarrow (\forall \mu \in S)(\mu(p) \succsim \mu(q)),$$

$$\text{(ii) } p \odot q \Leftrightarrow (\exists \mu_1, \mu_2 \in S)((\mu_1(p) > \mu_1 q) \wedge (\mu_2(q) > \mu_2(p))).$$

If $\succsim$ is fully represented by the set $S = \{\mu\}$, say that S is fully represented by the function μ. It is easy to see that $\succsim$ is opinionated (satisfies Opinionation) if and only if there exists a function μ such that $\succsim$ is fully represented by μ. The most obvious and influential constraint that is articulated in terms of representability by a set quantitative functions is the following.

$\mathfrak{C}_1$: $\succsim$ should always be fully representable by a set of probability functions.

Of course, $\mathfrak{C}_1$ is directly entailed by both precise and imprecise Bayesianism, since both of these theories presuppose that an agent's comparative confidence judgements can be derived from the set of probability functions that represents their credences.[20] Importantly though, an agent's comparative confidence judgements might satisfy $\mathfrak{C}_1$ and be fully representable by many distinct sets of probability functions, none of which have any special claim to giving a true representation of the agent's epistemic state. So even if one accepts $\mathfrak{C}_1$ as a general norm for comparative confidence, this is still a much weaker requirement than imprecise probabilism.

Another substantially weaker coherence constraint is articulated in terms of *Dempster Shafer belief functions*. Call a function $\mu : \mathfrak{B} \to [0, 1]$ a 'mass function' if

$$(M1)\ \mu(\bot) = 0$$

$$(M2)\ \sum_{p \in \mathfrak{B}} \mu(p) = 1$$

[19]It is important to emphasise that the numerical functions we consider here are only allowed to range over propositions, and not over arbitrary gambles that depend on the truth values of those propositions. This is important, since when more general classes of functions than probability measures are considered, the notion of representability used here does not generalise in a manner that generates plausible preference orderings over arbitrary gambles (see e.g. Walley (2000)). This is why alternative notions of representability are sometimes used in the imprecise probability literature. But since we're concerned only with doxastic propositional attitudes in the present context, it makes sense to restrict the domain as we do here. Nevertheless, we do want to emphasise that we do not mean to take any firm stance on the normative status of any representability norm, but rather only intend to use prospective representability norms as we illustrate the logical weakness of the purely comparative norms from which dilation inevitably follows.

[20]In the opinonated setting, Scott (1964) famously identified a set of qualitative axioms for comparative confidence orderings that are jointly equivalent to $\mathfrak{C}_1$. Analogous results have been obtained in the unopinonated setting by e.g. Alon and Lehrer (2014), Rios (1992) and Harrison-Trainor et al (2016).

Any mass function $\mu : \mathfrak{B} \to [0,1]$ defines a corresponding 'Dempster Shafer belief function' $b_\mu : \mathfrak{B} \to [0,1]$ defined by $b_\mu(q) = \sum_{\{p \in \mathfrak{B} | p \vdash q\}} \mu(p)$. It has been extensively argued (see e.g. Shafer (1976)) that Dempster Shafer functions are preferable to probability functions when it comes to representing the credences of agents who properly attend to the available evidence. This view implicitly imposes the following constraint on the comparative confidence judgements of rational agents.

$$\mathfrak{C}_2 : \succsim \text{ should always be fully representable by a set of Dempster Shafer functions.}$$

Since the set of Dempster Shafer functions on $\mathfrak{B}$ is a proper superset of the set of probability functions on $\mathfrak{B}$, $\mathfrak{C}_2$ is strictly weaker than $\mathfrak{C}_1$. To give the reader an intuitive feeling for the way in which $\mathfrak{C}_2$ weakens $\mathfrak{C}_1$, note that a confidence ordering that satisfies $\mathfrak{C}_2$ can encode the judgement $p \succsim q$ whilst failing to encode the judgement $\neg q \succsim \neg p$, while any imprecise Bayesian confidence ordering that satisfies $\mathfrak{C}_1$ and encodes $p \succsim q$ will have to encode $\neg q \succsim \neg p$.[21]

Finally, the last well known synchronic coherence constraint that will play a role here is the following.

$$\mathfrak{C}_3 : \text{For any } p, q \in \mathfrak{B},\ (p \succsim q) \Leftrightarrow ((p \wedge \neg q) \succsim (\neg p \wedge q)).$$

$\mathfrak{C}_3$ is equivalent to de Finnetti's (1937) famous quasi-additivity axiom, which was conjectured to be equivalent to $\mathfrak{C}_1$.[22] However, it was subsequently shown (Kraft et al (1959)) that quasi-additivity (and hence $\mathfrak{C}_3$) is in fact strictly weaker than $\mathfrak{C}_1$. In the opinionated setting, it is possible to show that $\mathfrak{C}_3$ is strictly stronger than $\mathfrak{C}_2$ (see Wong et al (1991)), and hence that, in the presence of the Opinionation assumption, these three coherence constraints can be ordered by logical strength as follows,

$$\mathfrak{C}_1 \Rightarrow \mathfrak{C}_3 \Rightarrow \mathfrak{C}_2$$

Since $\mathfrak{C}_3$ is a biconditional, it can be weakened further by the following two conditionals.

$$\mathfrak{C}_{3_1} : \text{For any } p, q \in \mathfrak{B},\ (p \succsim q) \Rightarrow ((p \wedge \neg q) \succsim (\neg p \wedge q)).$$
$$\mathfrak{C}_{3_2} : \text{For any } p, q \in \mathfrak{B},\ ((p \wedge \neg q) \succsim (\neg p \wedge q)) \Rightarrow (p \succsim q).$$

In what follows, we will examine whether and how a comparative analogue of the dilation puzzle follows from (subsets of) the synchronic coherence constraints listed above, together with a single diachronic norm, to which we turn now.

[21]Proponents of the Dempster Shafer approach argue that any good theory of inductive inference should treat it as rationally permissible for an agent to assign low credence to both a proposition and its negation when the agent has no evidence supporting either possibility (of course, this requires a more radically evidentialist conception of rational credence than that which is presupposed by most subjective Bayesians). It is this basic commitment that leads the Dempster Shafer theory to allow for such a broad class of coherent confidence orderings.

[22]For opinionated non-trivial monotonic partial preorders.

§3.2: A Diachronic Norm

The rationality constraints described in §3.1 specify properties that a rational agent's comparative confidence judgements should satisfy at any given time. However, they are silent about how a rational agent should change their comparative confidence judgements over time as they gather new evidence. Here, we can follow Eva (2019, manuscript) in assuming the following the norm.[23]

Comparative Conditionalisation (CC): Upon learning the truth of any proposition e, a rational agent with a prior confidence ordering $\succsim$ should adopt a posterior $\succsim_e$ defined by

$$p \succsim_e q \Leftrightarrow (e \wedge p) \succsim (e \wedge q)$$

It is easy to see that in the special case in which $\succsim$ is defined by the credences of a precise or imprecise Bayesian, this updating rule will produce the same posterior confidence orderings as those implied by precise or imprecise Bayesian conditionalisation. In light of this observation, Eva dubs the rule 'comparative conditionalisation (CC)'. But it is important to note that while CC coheres with Bayesian updating methods in the special cases in which an agent's comparative confidence judgements are determined by (precise or imprecise) numerical credences, CC is actually a considerably more general updating method than any specific Bayesian updating rule for credences. This is because it applies to any confidence ordering that is a non-trivial monotonic partial preorder (including those that admit of no precise or imprecise probabilistic representation), and therefore assumes extremely little about the synchronic norms of comparative confidence.[24] One way to illustrate the fact that CC is intuitively compelling in a way that is independent of its relation to Bayesian methods is to note that CC is also implicitly assumed by a number of other competing theories of rational credence, all of which disagree about the specific synchronic and diachronic norms for rational credence. Specifically, ranking theory, possibility theory, and the theory of Dempster Shafer belief functions are three alternative, non-probabilistic theories of rational credence, all of which tell different stories about what kinds of structural properties should be instantiated by a rational agent's credences at a time, as well as about how these credences should evolve over time. However, rank conditionalisation

[23]This norm is a logical consequence of many extant diachronic updating principles from the literature—e.g., precise and imprecise Bayesian conditionalisation, rank conditionalisation (Spohn, 1988), possibility conditionalisation (Zadeh, 1978), some rules for updating Dempster Shafer belief functions (Fagin and Halpern, 1991), and Walley's (1991) principle for updating sets of desirable gambles—but it has only recently been considered by Eva as an autonomous general rule for updating comparative confidence judgements. See Joyce (1999, ch. 6) for discussion of a related rule in a different context.

[24]In contrast to the diachronic Bayesian norms, for example, which require at least $\mathfrak{C}_1$.

(Spohn, 1988), possibility conditionalisation (Zadeh, 1978), and Fagin and Halpern's (1991) rule for updating Dempster Shafer belief functions all implicitly identify CC as the unique rational rule for updating comparative confidence judgements. Furthermore, Eva (manuscript) has provided both a diachronic Dutch book argument and an evidentialist argument in favour of CC. Neither of these arguments assume anything close to $\mathfrak{C}_1$, which shows that the philosophical justification for CC extends well beyond the probabilistic setting in which Bayesian methods are normally applied.[25]

For the remainder of this paper, we will assume that rational agents update their comparative confidence judgements in accordance with CC. As we've stressed above, this norm is considerably weaker and more general than the probabilistic updating norms that are championed by both precise and imprecise Bayesians. CC does not require us to make substantive assumptions about coherence norms for comparative confidence, and is also implicitly assumed by many of the most influential non-probabilistic theories of rational credence.

§4 : **Dilation, Comparatively**

We are now ready to return to the dilation puzzle introduced in §2, and to diagnose its exact relation to the synchronic and diachronic norms for comparative confidence outlined in §3. Towards this end, let's recall White's (2009) two coin example. At the beginning of the game, you know that H and T have an equal chance of being true. By CPP, this requires you to adopt the initial judgement $H \sim T$. You also have absolutely no evidence pertaining to which cell of the partition $\{B, G\}$ is the true one. So $\mathrm{POI}_{\odot}$ requires you not to make any comparative confidence judgement regarding B and G, i.e. it requires that your initial confidence ordering should satisfy $B \odot G$. In the context of imprecise Bayesianism, the puzzle arises from the fact that learning one of the two possible pieces of evidence $H \equiv B$ or $H \equiv G$ inevitably destroys your initial judgement $H \sim T$. In order to avoid the puzzle, then, we need to ensure that after learning either $H \equiv B$ or $H \equiv G$, you continue to make the judgement $H \sim T$, i.e., $H \sim_{H \equiv B} T$ and $H \sim_{H \equiv G} T$ should always hold. Assuming that you abide by the diachronic norm CC, we have

$$\begin{aligned} H \sim_{H \equiv B} T &\Leftrightarrow (H \wedge (H \equiv B)) \sim (T \wedge (H \equiv B)) \\ &\Leftrightarrow (H \wedge B) \sim (T \wedge G) \\ &\Leftrightarrow (H \wedge \neg G) \sim (\neg H \wedge G) \end{aligned}$$

[25] For current purposes, it is especially important to remember that the the intuitive definition of CC given above does not depend at all on Opinionation. As Eva (2019) points out, CC can be straightforwardly and plausibly applied to non opinionated prior confidence orderings.

Now, suppose that your initial confidence ordering satisfies the weak synchronic constraint $\mathfrak{C}_{3_2}$ from §2. Then $H \sim_{H\equiv B} T$ entails $(H \wedge \neg G) \sim (\neg H \wedge G)$, which in turn entails (via $\mathfrak{C}_{3_2}$) $H \sim G$ and $T \sim B$. Since we've assumed $B \odot G$, we have $H \odot T$, which contradicts our assumption that you initially make the judgement $H \sim T$. So, by assuming only the synchronic norm $\mathfrak{C}_{3_2}$ and the diachronic norm CC, we have shown that the initial judgements $H \sim T$ and $B \odot G$ jointly entail that the desideratum $H \sim_{H\equiv B} T$ cannot be satisfied, and hence that learning $H \equiv B$ forces you to surrender your initial judgement $H \sim T$. Analogous reasoning shows that learning $H \equiv G$ likewise forces you to surrender the initial judgement $H \sim T$. So again, you begin with the initial judgement $H \sim T$ despite knowing that whatever happens, you will subsequently come to abandon that judgement and instead adopt $H \odot T$.

This demonstrates that a comparative analogue of the dilation puzzle follows directly from CC together with the very weak synchronic norm $\mathfrak{C}_{3_2}$.[26] Crucially, we have not assumed at any stage that rational agents are equipped with quantitative credences of any kind. We have assumed only two compelling norms for comparative confidence that are substantially weaker and more generally applicable than the corresponding norms of imprecise Bayesianism.

Given that this puzzling result is generated by the two norms $\mathfrak{C}_{3_2}$ and CC, one might hope to avoid it by dropping one of the two norms. Here, we show that dropping $\mathfrak{C}_{3_2}$ alone does not provide a plausible escape from the problem. To see this, note first that, in the absence of the norm $\mathfrak{C}_{3_2}$, it *is* possible to hold the initial judgements $H \sim T$ and $B \odot G$ whilst also ensuring that you continue to hold the judgement $H \sim T$ after learning $H \equiv B/H \equiv G$ and updating by CC. So strictly speaking, dropping $\mathfrak{C}_{3_2}$ does allow one to avoid the specific problem that is highlighted by White (2009). However, it is easy to generate analogous cases that arise completely independently of $\mathfrak{C}_{3_2}$ when one stipulates that $H \sim_{H\equiv B} T$ must hold.

By way of illustration, consider the propositions $H \wedge B$, $H \wedge G$. Since we have absolutely no evidence about the bias of Coin 2, and the two coin tosses are independent, it is natural to think that $(H \wedge B) \odot (H \wedge G)$ should hold in your prior confidence ordering. You certainly have no more evidence about the comparative plausibility of those two propositions than you do about B and G themselves. So it would be bizarre and arbitrary to regard $H \wedge B$ and $H \wedge G$ as equally plausible whilst also regarding B and G as incomparable. Furthermore, note that making the judgement $(H \wedge B) \sim (H \wedge G)$ entails (via CC) making the judgement $B \sim_H G$ after learning H, i.e., learning that Coin 1 landed H would suddenly cause you to think that B

[26]For this part of the analysis, we assume CPP and $\mathrm{POI}_\odot$ in the background and do not explicitly consider them as part of the puzzle. In the next section, we extensively consider the possibility of solving the puzzle by abandoning each of CPP and $\mathrm{POI}_\odot$.

and G are equally plausible, even though you still have absolutely no evidence pertaining to which of cell of the $\{B, G\}$ partition is the true one. So holding the initial judgement $(H \wedge B) \sim (H \wedge G)$ does not look like a reasonable option, which means that you are forced to view $H \wedge B$ and $H \wedge G$ as incomparable in your initial ordering. Now, assume that learning $H \equiv B$ does not force you to surrender your judgement $H \sim T$. As we saw above, this means that $(H \wedge B) \sim (T \wedge G)$ must hold in your initial ordering. But this implies

$$(T \wedge G) \sim (H \wedge B)$$
$$(H \wedge B) \odot (H \wedge G)$$

and thereby implies that $(T \wedge G) \odot (H \wedge G)$. This means that upon learning G, updating by CC will force you to make the posterior judgement $H \odot_G T$. In other words, learning that Coin 2 landed G forces you to give up your initial judgement $H \sim T$. Analogous reasoning shows that learning B will similarly force you to surrender the judgement $H \sim T$ and treat H and T as incomparable, thus again resulting in a loss of comparative opinion. Moreover, the way in which one inevitably loses the initial judgement $H \sim T$ in this case is at least as troubling as in the standard version of the puzzle. For in the standard version, the evidence that causes you to surrender that judgement at least refers in some way to the outcome of the toss of Coin 1 (it specifies how the outcome of the toss of Coin 1 is correlated with the outcome of the toss of Coin 2). But in this new version, the evidence that makes you give up the judgement $H \sim T$ does not depend in any way on the outcome of the toss of Coin 1. All that you learn is how Coin 2 landed when tossed. It is completely clear that this should not in any way affect what you think about the outcome of the toss of Coin 1. So even when we drop $\mathfrak{C}_{3_2}$, simply stipulating that you don't lose the judgement $H \sim T$ upon learning $H \equiv B$ and requiring that you view $(H \wedge B)$ and $(T \wedge G)$ as incomparable leads directly to another comparative analogue of the dilation puzzle that is at least as problematic as the original analogue.

§5 : **Trilemma**

In sum, then, we have seen that the generalised comparative version of the dilation puzzle is completely independent of all substantive coherence norms that go beyond the requirement that an agent's confidence ordering be a non-trivial monotonic partial preorder. Even $\mathfrak{C}_{3_2}$, which is far weaker than probabilistic representability, is not required to generate the puzzling results. Thus the puzzle is deeper, more general, and more resistant than has previously been recognised since it's not at all specific to the theory of imprecise Bayesianism. Moreover, if we try to weaken imprecise Bayesianism by adopting an analogous theory that represents an agent's imprecise credences by sets of Dempster Shafer functions/ranking functions/possibility functions and replaces imprecise conditionalisation by imprecise analogues of the best known updating rules for those functions, one would still run straight into the problem, since those representations all entail that an agent's confidence ordering is a monotonic partial preorder, and those updating rules all entail CC.

Having distilled the dilation puzzle into what we take to be its purest form, we turn now to surveying and tentatively evaluating the possible escape routes. Recall, first, that the puzzle can be understood as a demonstration that the following five conditions are jointly inconsistent.[27]

1: You initially judge $H \sim T$.

2: You initially judge $B \odot G$.

3: You update by CC.

4: You continue to judge $H \sim T$ after learning $H \equiv B/H \equiv T$.

5: Your confidence ordering is always a non-trivial monotonic partial preorder.

Thus, the possible routes out of the puzzle correspond to choices of which of these 5 conditions should be rejected. For present purposes, we ignore the possibility of rejecting 5, since it encodes what many take to be unassailable bedrock norms of rational inductive inference. If one were to reject 5, it's unclear whether any of the most natural justifications for the other four normative requirements would retain their force anyway, so we do not think that rejecting 5 is a pertinent possibility for the purposes of the current dialectic.

Similarly, we also disregard the possibility of rejecting 3, since that entails rejecting CC as a general diachronic norm, which would have radical consequences. CC is implicitly entailed by the diachronic norms posited by all of the most influential philosophical theories of rational credence, including precise and imprecise Bayesianism, ranking theory, possibility theory, and the theory of Dempster Shafer belief functions. Furthermore, CC has a number of autonomous epistemic and pragmatic justifications, including, e.g., Dutch book arguments and arguments from evidential relevance (see Eva (manuscript)). More generally, rejecting CC would require telling a plausible new story about diachronic norms for comparative confidence, and in particular would entail either rejecting the existence of any such diachronic norms or identifying alternative

[27] As we showed above, they can, strictly speaking, be reconciled if we drop $\mathfrak{C}_{3_2}$, but then one runs into an even more pernicious version of the puzzle.

independently plausible norms that allow one to avoid the puzzle.[28] While we acknowledge that these may be live dialectical possibilities, they do not strike us as especially plausible or promising ones, so we will not pursue them further here.

That leaves 1, 2, and 4 as possible escape routes. All three are entailed by comparative generalisations of prominent Bayesian norms. Condition 1 is entailed by the Comparative Principal Principle (CPP), which in turn is entailed by the standard Bayesian version of the Principal Principle.[29].

Comparative Principal Principle (CPP): If you are certain of the chances of p and q, then make the judgement $p \succsim q$ if and only if $Ch(p) \geq Ch(q)$ (unless you have inadmissible evidence).

Condition 2 is entailed by the comparative principle of indifference $\mathrm{POI}_{\odot}$, which is in turn entailed by representing complete epistemic indifference as maximally imprecise credence. Condition 4 is entailed by the following comparative reflection principle,[30]

Comparative Reflection (CR): If you are certain at time t_1 that your judgements will evolve rationally between t_1 and some future time t_2 and are likewise certain how you will order (or not order) p and q at t_2, then defer to that future order (or lack of order) at t_1.

CR is entailed by the standard precise Bayesian version of the reflection principle, and is the most straightforward generalisation of that principle to the comparative setting. It simply requires that when you are certain that your future self will have some comparative opinion (or lack of opinion) toward some pair of propositions as a result of updating by CC, then you should defer to your future self. Overall then, it looks like the only viable way out of the puzzle is to reject one of CPP, $\mathrm{POI}_{\odot}$, or CR.

[28]Even if there is reason to believe that diachronic norms do not apply universally (as e.g. Christensen (1991) and Easwaran and Stern (forthcoming) maintain), it seems like they should apply in some version of the puzzle discussed here, and that is all that's required to generate the problem.

[29]We do not want to dwell on the details of how best to formalise the Principal Principle in a comparative setting, but we take it that any plausible formulation will entail CPP. The notion of inadmissible evidence deployed here is analogous to the corresponding notion in the standard formulation of the Principal Principle. Specifically, we treat evidence e as inadmissible if learning e would make a difference to the agent's comparative confidence judgements regarding p and q even when the agent was already certain of the objective chances of p and q.

[30]This also isn't the place to dwell on the details of how best to formalise the Reflection Principle in a comparative setting, but it's worth noting that this statement is weaker than some possible candidates insofar as it applies only when the agent knows that the future judgements to which she defers will be the product of a rational updating procedure (i.e., CC). See Briggs (2009) for relevant discussion.

§5.1: Horn 1

First, rejecting condition 1, and with it CPP, amounts to requiring that you disregard what you know about the objective chance of the first coin's landing heads by refraining from making the judgement $H \sim T$. Of course, simply ignoring one's knowledge of the objective chances looks intuitively irrational. But it's true that CPP licenses rational agents' judgements to break from known chances when they possess what's known as *inadmissible evidence*. Might such evidence be at play here? The answer is 'no' because the agent has no inadmissible evidence at the start of the game. Though theories of inadmissibility differ, they agree that an agent has inadmissible evidence when the agent possesses some piece of information that defeats the chance estimate — e.g., when you use a crystal ball to see into the future and discover that the coin that you know to be fair *will* land heads. But at the beginning of the coin game, you don't have any evidence like this. All you know is that the coin is fair, and that you will learn either $H \equiv B$ or $H \equiv G$. There may be some reason to believe that each of these biconditionals should be regarded as inadmissible (since they're ultimately about the outcomes of the tosses),[31] but that doesn't solve things. For no matter whether the biconditionals are inadmissible, you don't have evidence of either at the beginning of the game, and it thus seems that your *initial* judgement should reflect your knowledge that the coin is fair.[32] Of course, by adhering to the CPP, you violate CR (since you can know now that you'll lose this opinion if you update by CC), but that seems to be precisely what you should do as far as the CPP is concerned.

Furthermore, if we reject condition 1, then we have to ask what comparative confidence judgements you *should* make about H and T at the beginning of the game. There are three options, namely $H \succ T$, $T \succ H$, and $H \odot T$. The first two can be immediately dismissed since there is no reason to be more confident in heads or tails when you know that the coin is fair. The only remaining option is to initially regard H and T as incomparable. This seems to lead us back to basic problem with the Bayesian approach to the coin game, namely that you end up adopting the same doxastic attitudes with respect to the two partitions $\{H, T\}$ and $\{B, G\}$, which seems incorrect since there is a fundamental asymmetry between your evidential situations with respect to those partitions.

But there may be a way to pursue this line of reasoning that successfully captures the difference between the two coins. For example, we could maintain that you should adhere to the CPP except when CR requires

[31] Joyce (2010) makes this point.

[32] The fact that you can be sure that you'll acquire some inadmissible piece of evidence does not itself generate reason to break from the known chance. As you call the outcome of a fair coin toss that flips through the air, you should be equally confident in heads and tails even if you're certain that you'll learn the final outcome of the toss. This shows that you should stick to the chances even when you're in a position to know that inadmissible evidence is coming.

otherwise. This would effectively amount to positing an additional way for rational opinions to break from known chances — i.e., not just when you have inadmissible evidence, but also when you know that your future (more informed) opinion will take some particular form other than that required by the chances. In this case, this means initially deferring to your future judgement that $H \odot T$ even when you know that the coin is fair. Thus, though any individual rational agent would initially have the same judgements towards both coins in this case — namely, $H \odot T$ — there is a subtle difference in the robustness of their permissibility. Counterfactually, were there no looming CR violation (e.g., because you were examining the fair coin in isolation), you would be required to be equally confident in H and T. But the same is not true of the other coin — i.e., you'd be required by $\text{POI}_{\odot}$ to judge B and G as incomparable no matter whether any CR violation looms.

The justification for this option is as strong as the justification for CR (since its plausibility clearly depends on whether there is some way to argue that CR is an unassailable criterion for rational confidence judgements, even when the resulting judgements conflict with CPP). Our opinion is that this demonstrates that CR deserves more attention than it has received in the literature thus far, though we remain wary of prospective violations of CPP and remain concerned that it does not seem rational for an agent to have symmetric epistemic attitudes towards the two coins, even if there is an asymmetry at the level of the norms that identify those judgements as rational. But as we'll see in what follows, no option is obviously compelling, and so this one should be treated as a serious possibility.

§5.2: Horn 2

The second option is to reject condition 2, and with it $\text{POI}_{\odot}$. This requires that you make some comparative confidence judgement about the pair (B, G). Again, regarding either element of this pair as strictly more plausible than the other looks arbitrary and unjustified. Assuming that you want to avoid such unjustified judgements, this leaves the option of making the judgement $B \sim G$. At first blush, one might think that this looks like a plausible option. It follows from the following alternative comparative formulation of the principle of indifference.

Principle of Indifference $\sim$ ($\text{POI}_{\sim}$): Let $X = \{x_1, ..., x_n\}$ be a partition of the set W of possible worlds into n mutually exclusive and jointly exhaustive possibilities. In the absence of any relevant evidence pertaining to which cell of the partition is the true one, a rational agent should regard each pair of proposition (x_i, x_j) as *equally plausible*, for $i, j \leq n$. Equivalently, for all $i, j \leq n$, $x_i \sim x_j$ should hold in the agent's confidence ordering.

POI$_\sim$ is entailed by the standard precise Bayesian version of the principle of indifference, and is perhaps the more obvious comparative formulation of the principle. However, as Eva (2019) shows, POI$_\sim$, while avoiding the inconsistencies inherent in the precise Bayesian formulation of the principle, is ultimately implausible. It implies that agents are required to be equally confident in a proposition and its logical consequences in situations where they have no evidence to go on, which is intuitively absurd.[33]

But there is another possible line of response here. Specifically, one could argue that while the agent is rationally obliged to judge $H \sim T$, they are merely *permitted* to judge $B \sim G$.[34] The idea then is that while the evidential asymmetry between the two coins may be obscured at the level of the doxastic judgements of individual rational agents, it is manifest at the level of the epistemic norms to which those agents are beholden.[35] On this view, an agent can make the initial judgement $B \sim G$ and be perfectly rational in doing so, but they are also rationally permitted to regard B and G as incomparable. By contrast, according to this line of reasoning, only a single doxastic stance is permissible with regards to the first coin, namely $H \sim T$. Unlike the first prospective approach to rejecting condition 2, this approach does not involve positing a norm (POI$_\sim$) that *requires* you to adopt the same doxastic stance towards the two coins. This is an improvement insofar as it at least allows for the possibility of your doxastic judgements reflecting the evidential asymmetry between the two coins. But this option shares a problem with our preferred way of rejecting condition 1 — namely, it again allows for the possibility that you rationally adopt doxastic judgements that completely ignore the asymmetry. If you are disinclined to take the first horn on the grounds that actual (rather

[33]To see this, suppose that we have absolutely no evidence about any of the propositions in the algebra generated by two atomic sentences p and q. Then POI$_\sim$ requires us to make the judgements $(p \wedge q) \sim (p \wedge \neg q) \sim (\neg p \wedge q) \sim (\neg p \wedge \neg q)$ over the partition $\{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\}$. It also requires that we make the judgements $p \sim (\neg p \wedge q) \sim (\neg p \wedge \neg q)$ over the partition $\{p, \neg p \wedge q, \neg p \wedge \neg q\}$. By transitivity of $\sim$, it follows that $p \sim (p \wedge q)$, which seems deeply implausible. If we have no evidence to go on, we should regard p as strictly more plausible than $p \wedge q$, since it is true in a proper superset of the possible worlds at which $p \wedge q$ is, and we have no evidence regarding which of those worlds is actual. Note that POI$_\odot$ can avoid this result, since incomparibility is not transitive. In fact, if we assume $\mathfrak{C}_3$, then POI$_\odot$ *entails* that we should be strictly more confident in p than we are in $p \wedge q$ in this situation. So we take POI$_\sim$ to be a less plausible comparative formulation of the principle of indifference, which undermines the idea that we can justify rejecting condition 2 on the grounds that doing so follows from POI$_\sim$.

[34]This is suggested by Bradley's (2018) version of imprecise Bayesianism in which further opinionation is rationally permissible. Though Bradley discusses dilation, he does not discuss the interplay between the dilation puzzle and this aspect of his imprecise Bayesianism. See Builes et al (forthcoming) for related discussion.

[35]This is similar to the way in which the asymmetry is respected when we reject condition 1 by maintaining that the requirements implied by CR trump the requirements implied by CPP when they conflict. In both cases, the individual rational agent is required to share the same judgements toward both coin flips, but the normative reasons for this common judgement are different for both coins.

than merely counterfactual) respect of the asymmetry is a non-negotiable rationality requirement, then you should be similarly disinclined here. But if you are considering which of these first two options is best, there are possible reasons to side with either. If you're especially impressed with both CR and $\text{POI}_\odot$ (and its ability to *require* that we avoid the indifference paradoxes), then the first horn is your best bet.[36] But if you share, e.g., Bradley's (2018) view that further opinionation is always permissible, then you might favor this second horn, albeit at the cost of *permitting* intuitively absurd epistemic states in the absence of evidence.

Either way, if we choose to drop condition 2 in this way, then given the other conditions that generate the puzzle, the rational agent will initially judge that $B \sim G$ and $H \sim T$ (because the conjunction of CR and CPP force this result). As with the last option, we are in no position to dismiss this out of hand, but it, too, has counterintuitive consequences.

§5.4: Horn 3

Rejecting condition 4 essentially amounts to embracing the comparative analogue of dilation (which we will refer to as 'opinion loss') and thereby violating CR. And violating CR may be a big cost. If you know what judgements your future self makes (under the supposition that you continue to be rational and don't forget anything in the future), then why wouldn't you defer to those judgements? It's at least *prima facie* hard to see how an agent that disregards evidence in this way could hope to make justified and reasonable judgements.

That said, Joyce (2010) has argued that in the imprecise setting, there is reason to doubt versions of the reflection principles that stick their necks out in the way that CR does here. Specifically, he has noticed that when dilation occurs, there can be important differences between the possible future epistemic states that the agent will inhabit with respect to the propositions at issue even when she knows that her future epistemic state will be maximally imprecise towards these propositions. For example, in the case of the coin game, he notices that when you learn one of the possible biconditionals, say $H \equiv G$, all of the functions in your representor will satisfy the equality $c(H) = c(G)$, but when you learn the other possible biconditional, $H \equiv B$, all of the functions in your representor will satisfy the complementary equality $c(H) = c(B)$. So even though you know that your future self will have maximally imprecise credence in H, there are important facts about your future self's attitude towards H that you are not privy to. As Joyce puts it,

[36] Though it's worth noting that there is no violation of CR when we take the second horn. This is because the biconditionals don't change our attitudes toward the coins when we're equally confident in every possible outcome in the prior.

From the perspective of your prior, the beliefs about H you will come to have upon learning $H \equiv B$ are complementary to the beliefs you will have upon learning $H \equiv \neg B$. There is, in fact, no single belief state for H that you will inhabit whatever you learn about $H \equiv B$. You will inhabit an imprecise state either way, but these states will differ depending on what you learn. (Joyce, 2010: 304)

In the comparative setting, something similar happens. That is, even when you know that your future self will maintain that $H \odot T$ no matter which biconditional is learned, you don't know whether you will maintain that $H \sim B$ or $H \sim G$ since this will depend on which biconditional you learn. This kind of consideration leads Joyce to prefer an imprecise version of Reflection that does not require the agent to defer to their future judgements in circumstances where this kind of discrepancy exists between the agent's possible future doxastic states. But importantly, Joyce's preferred imprecise reflection norm relies on resources that are unique to the imprecise framework and thus assumes more epistemic structure than we do in the comparative context. Specifically, it is couched in terms of expectations that are shared by all of the probability functions in the agent's prior representor, and these probability functions are not generally available when we assume only that agents are able to make comparative confidence judgements. Thus, the kind of formulation of reflection that Joyce prefers in the imprecise setting is not easily generalised to the comparative setting, and it's not clear how one should formulate a comparative reflection norm that captures the driving intuition behind Joyce's preferred formulation.

Moreover, we are genuinely unsure of whether there is reason to weaken CR in response to Joyce's concerns, simply because we still find it somewhat persuasive that we should defer to the *particular* judgements (or absence of judgements) that we know our future selves will make as a result of rational updating, regardless of whether we are uncertain of *other* judgements involving the propositions at issue. For example, if you know that your future self will maintain that $H \odot T$, that still strikes us a compelling reason to maintain that maintain that $H \odot T$ now, regardless of whether you know how you will compare H to G and B.[37]

Even if we remain undeterred by Joyce's discussion (and thereby remain sympathetic to reflection principles that recommend deference when confronted with cases like the coin game), there is yet another important subtlety to track as we evaluate the plausibility of CR. To see this, note first that CR can

[37]Since we are officially open to the possibility that we should take Horn 3, one might take the contribution of this paper to buttress Joyce's original criticism of CR style reflection norms by showing that if you don't weaken CR to something else, then some other revered norm of inductive inference will have to go. But even then, the problem of finding a comparative reflection norm that squares with Joyce's reasoning remains unsolved.

actually be broken down into the following two weaker principles.

Comparative Reflection $\succsim$ ($\mathbf{CR}_{\succsim}$): If you are certain at time t_1 that your judgements will evolve rationally between t_1 and some future time t_2 and are likewise certain that you will make the judgement $p \succsim q$ at future time t_2, then make that judgement at t_1.

Comparative Reflection $\odot$ ($\mathbf{CR}_{\odot}$): If you are certain at time t_1 that your judgements will evolve rationally between t_1 and some future time t_2 and are likewise certain that you will regard p and q as incomparable (that your will ordering satisfy $p \odot q$) at future time t_2, then regard them as incomparable (ensure $p \odot q$) at t_1.

Let's consider each of these principles in turn. First, suppose that an agent violates $\mathrm{CR}_{\succsim}$. Then there exist some p and q such that the agent knows that their future self will judge $p \succsim q$, but that they are happy to ignore this judgement and form their own opinions about p and q. In this case, the agent is actively distrusting the judgements of their future self. Their future self has effectively given them concrete epistemic advice — e.g., 'judge that $p \succsim q$' — and they've chosen to ignore it. That looks egregious.

Now suppose that an agent violates $\mathrm{CR}_{\odot}$. This means that there exist p, q such that the agent knows that their future self regards p and q as incomparable, but the agent makes some definite judgement regarding them anyway. In this case, the agent is not actually ignoring the judgements of their future self. For, by stipulation their future self regards p and q as incomparable, which means that *there is no judgement for the agent to ignore*. We can think about this as a case in which the agent asks their future self 'what should I think about the comparative plausibility of p and q?' and their future self replies 'I don't have a clue!'. If the agent now goes on to form a comparative confidence judgement about p and q, they haven't ignored the advice of their future self, for the simple reason that their future self didn't give them any real advice. This is very different to the situation where an agent violates $\mathrm{CR}_{\succsim}$. In that case, the agent receives definite meaningful advice from their future self (i.e. 'judge that $p \succsim q$'), and they ignore it. There is a clear sense in which the latter violation seems more problematic than the former. Of course, one should defer to a judgement when there is one to defer to, but it's less clear why one is normatively required to bind oneself to the cluelessness of one's future self. And note that the dilation puzzle only involves a violation of $\mathrm{CR}_{\odot}$ (the agent knows that they will regard B and G as incomparable in the future, but they initially judge $B \sim G$). So perhaps we can avoid the puzzle by sacrificing $\mathrm{CR}_{\odot}$. Importantly, this doesn't involve rejecting CR entirely, since we can still require that rational agents respect $\mathrm{CR}_{\succsim}$, which seems to be the more compelling principle.[38]

Like the other two horns, this strikes us as worth exploring. But also like the other two options, it has some counterintuitive implications that we are already in a position to identify. While it may be intuitive that one can rationally fail to defer to an unopinionated expert, it's important to remember that in cases when CR requires deference, your future self is unopinionated for a reason that intuitively lends grounds for deference — namely, your future self is unopinionated precisely because they have rationally updated on strictly more evidence than you have.[39] If you could choose your beliefs, and could choose either those that you have now, or those that you'd have were you to acquire more evidence and update rationally, isn't it obvious that you should choose the latter?[40] Maybe so, but if we opt to take this horn of the trilemma,

[38]One could insist here that there are at least some violations of $\mathrm{CR}_{\odot}$ that look problematic. For example, if an agent knows that they will come to regard p and q as incomparable in the future, it seems problematic for them to judge $p \succ q$, since knowing that they will come to regard p and q as incomparable in the future plausibly undermines any evidence they had that favoured p over q. In fact, we can further separate $CR_{\odot}$ into two weaker principles, namely

Comparative Reflection $\odot/\succ$ ($\mathbf{CR}_{\odot_{\succ}}$): If you are certain at time t_1 that you will regard p and q as incomparable (that your will ordering satisfy $p \odot q$) at future time t_2, then don't make the judgement $p \succ q$.

Comparative Reflection $\odot/\sim$ ($\mathbf{CR}_{\odot_{\sim}}$): If you are certain at time t_1 that you will regard p and q as incomparable (that your will ordering satisfy $p \odot q$) at future time t_2, then don't make the judgement $p \sim q$.

It is only $\mathrm{CR}_{\odot_{\sim}}$ that is violated in the coin game example. One can embrace the opinionation loss that occurs in that example as rational whilst still accepting $\mathrm{CR}_{\succsim}$ and $\mathrm{CR}_{\odot_{\succ}}$ as general epistemic norms. The only part of the comparative reflection principle that is violated in the example is $\mathrm{CR}_{\odot_{\sim}}$. However, one might suspect that an analogous example can be used to generate similar violations of $\mathrm{CR}_{\odot_{\succ}}$. To see this, imagine a case that is exactly like the original coin game, except that you are told that Coin 1 is not fair, but is more likely to land heads than tails. On the imprecise Bayesian model, for any $x \in (0.5, 1]$ your initial representor will then contain one function of the form $c_1(H \wedge B) = x$, $c_1(T \wedge B) = 1 - x$, and one of the form $c_2(H \wedge G) = x$, $c_2(T \wedge G) = 1 - x$. Since ${c_1}_{H \equiv B}(H) = 1$ and ${c_2}_{H \equiv B}(H) = 0$, it follows that imprecise Bayesianism recommends that upon learning $H \equiv B$ (or $H \equiv G$), you should regard H and T as incomparable. $\mathrm{CR}_{\odot_{\succ}}$ then requires that you should not initially judge $H \succ T$, but of course this conflicts with your knowledge that the coin is more likely to land heads than it is to land tails. So just as imprecise Bayesianism leads straight to violations of $\mathrm{CR}_{\odot_{\sim}}$, it also leads directly to violations of $\mathrm{CR}_{\odot_{\succ}}$. However, whereas violations of $\mathrm{CR}_{\odot_{\sim}}$ turn out to be pretty much unavoidable in the comparative setting, violations of $\mathrm{CR}_{\odot_{\succ}}$ can be avoided when one reconsiders this new example in a purely comparative context. In fact, there exist multiple confidence orderings that satisfy $\mathfrak{C}_3$, encode the desired judgements $H \succ T$, $B \odot G$, and ensure that updating by CC upon learning $H \equiv B$ allows one to retain the judgement $H \succ T$.

Thus, in the version of the coin game in which you are told that H is more likely to occur than T, it is possible to avoid opinion loss and any form of reflection violation whilst also satisfying all of the fundamental comparative confidence norms that we've considered here. In contrast, if one adopts the comparative confidence judgements that are mandated by imprecise Bayesianism, then opinion loss will inevitably follow and one will be unable to satisfy all of the salient comparative confidence norms. So while the original version of the dilation puzzle that arises from the standard coin game example is not specific to imprecise Bayesianism, and cannot be easily avoided in the more general comparative setting, the version of the puzzle that arises from this slightly amended coin game *is* specific to imprecise Bayesianism, and *can* be avoided in the generalised comparative setting, without surrendering any bedrock norms for rational comparative confidence.

[39]Remember that this is the only epistemically relevant difference between you and your future self when CR applies (since it's assumed that there is no forgetting, etc.).

[40]This rhetorical question may assume that your choice is driven by epistemic considerations, rather than pragmatic considerations (since it sometimes pays to have false beliefs).

then we must treat future suspense of judgement as an exception to this general principle. And it's hard to see why this is principled when the future suspense of judgement is more informed than the current opinionation.[41] That said, it should also be noted that this third horn of the trilemma is the only one that ensures that the asymmetry between the two coins is actually respected and directly reflected in the agent's comparative confidence judgements. Thus for anyone who views this property as non-negotiable, the third horn is the best horn.

§6 : **Conclusion**

Let's recap. Our first observation is that the dilation puzzle arises directly from a combination of fundamental norms for rational comparative confidence that are implicitly assumed by almost all extant influential theories of rational credence. Since these norms are not in any meaningful sense specific to the theory of imprecise Bayesianism, this shows that dilation is an issue that arises for a very broad class of theories of rational inductive inference.

Consequently, we've shown that the cost of avoiding the dilation puzzle may be somewhat higher than has previously been assumed. The norms that generate the puzzle all have very deep justifications that go well beyond the specific theoretical considerations that motivated the developement of imprecise Bayesianism. Even if one reimagines the foundations of inductive inference in a purely comparative setting, the puzzle doesn't simply go away.

What we're left with is a genuine trilemma — i.e., three options, all of which seem to have real faults. If we take the first horn, we can respect CR and $\text{POI}_{\odot}$, but only by making the CPP (and the PP) slave to CR (and REF), and there is not any precedent for this lexical priority in the literature. Moreover, if we take the first horn, then there are rational agents who adopt the same attitudes toward both coins at the outset, and some may regard this consequence as unacceptable. If we take the second horn, we can respect CR and CPP, but only by rejecting $\text{POI}_{\odot}$. As with the first horn, this results in symmetric treatment of the two coins at the level of comparative confidence judgements, and also conflicts with the attractive ideal that the appropriate response to complete epistemic indifference is suspense of judgement (i.e., incomparability). Finally, taking the third horn allows us to respect CPP and $\text{POI}_{\odot}$, and ensures that the asymmetry between the two coins is always reflected in a rational agent's comparative confidence judgements. But the price we

[41] To be clear, whenever we say that one agent is more informed than another, we mean that the former's evidence is a proper superset of the latter's.

pay for those advantages is that we can no longer advocate CR as a fully general rational deference norm (although we can still adhere to some analogous principles that are importantly weaker than CR).

The fact that the only three options left on the table involve rejecting weak comparative generalisations of what many take to constitute fundamental norms of rational credence shows that the dilation puzzle is both more pernicious and more wide-ranging than has hitherto been appreciated. Our treatment of the puzzle in terms of comparative confidence shows that we cannot properly distinguish between equal confidence and suspense of judgement without violating either the Principal Principle or the Reflection Principle. We have identified what we take to be the least problematic ways of violating these norms, but it's clear from our analysis that some canonical principle of rationality must go. Moreover, because we reached this conclusion in an extremely general setting (the comparative confidence framework), it is clear that we cannot avoid the puzzle simply by trading in imprecise Bayesianism for some alternative theory of rational credence that doesn't assume that credences are probabilistic. In sum, either we shouldn't suspend judgement in the absence of evidence, or one of our most revered epistemic deference principles is wrong.[42]

References

Alon, A and Lehrer, E. (2014). Subjective multi-prior probability: A representation of a partial likelihood relation. *Journal of Economic Theory*, 151: 476-492

Bradley, Richard. (2018). *Decision Theory With a Human Face.* Cambridge University Press.

Briggs, R. (2009). Distorted Reflections. *Philosophical Review*, 118(1): 59–85.

Builes, D, Horowitz, S, and Schoenfield, M. (Forthcoming). Dilating and Contracting Arbitrarily. *Nous.*

Campbell-Moore, C. (2021). Probability Filters as a Model of Belief. *ISIPTA.*

Campbell-Moore, C, and Konek, J. (2019). Believing Probabilistic Contents. *Analysis*, 80(2): 316–331.

Christensen, D. (1991). Clever Bookies and Coherent Beliefs. *Philosophical Review*, 100(2): 229–247.

de Finetti, Bruno. (1937). La prevision: ses lois logiques, ses sources subjectives. *Annales de l'Institut Henri Poincare*, 7: 1-68.

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Easwaran, K and Stern, R. (Forthcoming). Diachronic and Interpersonal Coherence. A. Flowerree and B. Reed (eds.) *Towards an Expansive Epistemology: Norms, Action, and the Social Sphere*. Routledge.

Eva, Benjamin. (2019). Principles of Indifference. *The Journal of Philosophy*, 116(7).

Eva, Benjamin. (manuscript). Comparative Learning.

Fagin, R and Halpern, J. (1991). A New Approach to Updating Belief Functions. Henrion, Kanal and Lemer (eds). *Uncertainty in Artificial Intelligence 6*. Elsevier.

Fine, Terrence. (1973). *Theories of Probability*. Academic Press.

Fishburn, Peter. (1986). The axioms of subjective probability. *Statistical Science* 1(3), 335-345.

Fitelson, B and McCarthy, D. (unpublished). *Toward an Epistemic Foundation for Comparative Confidence*. Manuscript.

Forrest, Peter. (1989). The Problem of Representing Incompletely Ordered Doxastic Systems. *Synthese*, 79:279-303.

van Fraassen, Bas. (1984). Belief and the Will. *The Journal of Philosophy* 81: 236-256.

Hajek, A and Smithson, M. (2012). Rationality and Indeterminate Probabilities. *Synthese*, 187:33–48.

Halpern, Joseph. (2003). *Reasoning About Uncertainty*. MIT Press.

Harrison-Trainor, M, Holliday, W, and Icard, T. (2016). A note on cancellation axioms for comparative probability. *Theory and Decision*, 80: 159-166.

Hart, C and Titelbaum, M. (2015). Intuitive Dilation?. *Thought*, 4: 252-262.

Joyce, James. (1999). *Foundations of Causal Decision Theory*, Cambridge University Press. Cambridge.

Joyce, James. (2005). How Probabilities Reflect Evidence. *Philosophical Perspectives*, 19: 153–178.

Joyce, James. (2010). A Defence of Imprecise Credences in Inference and Decision Making. *Philosophical Perspectives* 24(1): 281-323

Kaplan, Mark. (1996). *Decision Theory as Philosophy*, Cambridge University Press. Cambridge.

Keynes, John Maynard. (1921). *A Treatise on Probability.* Macmillan.

Konek, Jason. (2019). *Comparative Probabilities.* In J Weisberg and R Pettigrew (eds), *The Open Handbook of Formal Epistemology*: 267 – 348.

Koopman, Bernard. (1940). The axioms and algebra of intuitive probability. *Annals of Mathematics*, 41(2): 269-292.

Kraft, C, Pratt, J and Seidenberg, A. (1959). Intuitive probability on finite sets. *The Annals of Mathematical Statistics*, 30(2): 408-419.

Levi, Isaac. (1974). On Indeterminate Probabilities. *The Journal of Philosophy*, 71: 391-418.

Levi, Isaac. (1985). Imprecision and Indeterminacy in Probability Judgment. *Philosophy of Science*, 52: 390-409.

Lewis, David. (1980). A Subjectivist's Guide to Objective Chance. In *Studies in Inductive Logic and Probability*, ed. R. Jeffrey, 263–293. Berkeley: University of California Press.

Lewis, David (1999). *Papers in Metaphysics and Epistemology.* Cambridge: Cambridge University Press, 1999.

Pedersen, A and Wheeler, G. (2014). Demistifying Dilation.*Erkenntnis* 79(6): 1305–1342.

Rios, David. (1992). On the foundations of decision making under partial information. *Theory and Decision*, 33: 83–100.

Shafer, Glenn. (1976). *A Mathematical Theory of Evidence.* Princeton: Princeton University Press.

Scott, Dana. (1964). Measurement structures and linear inequalities. *Journal of Mathematical Psychology* 1(2): 233-247.

Seidenfeld, T and Wasserman, L. (1993). Dilation for Sets of Probabilities. *The Annals of Statistics*, 21(3): 1138–1154.

Spohn, Wolfgang (1988). Ordinal Conditional Functions: A Dynamic Theory of Epistemic States. In W.L. Harper and B. Skyrms (eds.), *Causation in Decision, Belief Change, and Statistics II.* Kluwer: Dordrecht, 105–134.

Stewart, R. and Nielsen, M. (forthcoming). Distension for Sets of Probabilities. *Philosophy of Science*.

van Fraassen, Bas. (1990). Figures in a Probability Landscape. In: Dunn J.M., Gupta A. (eds) *Truth or Consequences*. Springer, Dordrecht. https://doi.org/10.1007/978-94-009-0681-5_21.

Walley, Peter. (1991). *Statistical Reasoning with Imprecise Probabilities*. Chapman and Hall.

Walley, Peter. (2000). Towards a Unified Theory of Imprecise Probability. *International Journal of Approximate Reasoning* 24: 125-148.

Weatherson, Brian. (2007). The Bayesian and the Dogmatist. *Proceedings of the Aristotelian Society* 107: 169-185.

White, Roger. (2009). Evidential Symmetry and Mushy Credence. *Oxford Studies in Epistemology*: 161–186.

Wong, S. M., Y. Yao, P. Bollmann, and H. Burger (1991). Axiomatization of qualitative belief structure. *IEEE Transactions on Systems, Man and Cybernetics*, 21(4): 726-734.

Zadeh, Lotfi. (1978) Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, 1: 3–28.