On the Application of the Lorentz Transformation in O(3) Electrodynamics

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The correct method is given for application and interpretation of the Lorentz transformation in O(3) electrodynamics. This method relies on the invariance of the equations of motion in free space under Lorentz transformation, and on the correct interpretation of vector components.

Keywords: Lorentz transform; B Cyclic Theorem; O(3) electrodynamics

1. Introduction

In this short paper, the correct method is given of applying the Lorentz transform to O(3) vacuum electrodynamics by first demonstrating the Lorentz invariance in free space of the underlying equations of motion. The solutions therefore must also be Lorentz invariant, and the B Cyclic theorem [1-5] is also an invariant in the vacuum. The correct interpretation is given of the meaning of components of vectors with internal indices (1), (3) and (3) [1-5]. The O(3) electrodynamics remain valid under Lorentz transformation.

2. The B Cyclic Theorem

The B Cyclic theorem is the basic relation between the vector components $\mathbf{B}^{(1)}$, $\mathbf{B}^{(2)}$ and $\mathbf{B}^{(3)}$ of O(3) electrodynamics in the vacuum:

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*} \tag{1}$$

in cyclic permutation, where $B^{(0)}$ is the magnitude of $\mathbf{B}^{(3)}$. It is equivalent to the basic definition of $\mathbf{B}^{(3)}$ [1-5] in terms of the vector potential plane waves $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ in the basis ((1), (2), (3)), which are solutions of the vacuum d'Alembert wave equation in the vacuum:

$$\square A^{\mu} = 0 \tag{2}$$

It is well known that this is Lorentz invariant, *i.e.*:

$$\square' A^{\mu'} = \square A^{\mu} = 0 \tag{3}$$

so the solutions $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ can be the same in any observer frame *K*. It follows immediately that the B Cyclic theorem is a Lorentz invariant construct in the vacuum. It is the same in all observer frames because it is essentially a rotation generator relation of the O(3) group which generates the Jacobi identity:

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$$\left[\mathbf{B}_X^{(1)},\left[\mathbf{B}_Y^{(2)},\mathbf{B}_Z^{(3)}\right]\right]+\left[\mathbf{B}_Y^{(2)},\left[\mathbf{B}_Z^{(3)},\mathbf{B}_X^{(1)}\right]\right]+\left[\mathbf{B}_Z^{(3)},\left[\mathbf{B}_X^{(1)},\mathbf{B}_Y^{(2)}\right]\right]\equiv 0 \tag{4}$$

under all conditions. The Jacobi identity is clearly Lorentz invariant.

Another way of showing this invariance is that the decoupled equations of O(3) electrodynamics [1-5]:

$$\partial_\mu \tilde{G}^{\mu\nu(i)} = 0; \quad i = 1,2,3 \tag{5}$$

are also Lorentz invariants, whose solutions are also invariant.

3. Transformation of the Field Tensor

The field tensor is defined as

$$G^{\mu\nu} = G_{\mu\nu}^{(1)}\mathbf{e}^{(1)} + G_{\mu\nu}^{(2)}\mathbf{e}^{(2)} + G_{\mu\nu}^{(3)}\mathbf{e}^{(3)} \tag{6}$$

where $\mathbf{e}^{(1)}$, $\mathbf{e}^{(2)}$, and $\mathbf{e}^{(3)}$ are unit vectors in the basis ((1), (2), (3)). It is clear from the analysis in section 2 that the tensor is unaffected by Lorentz transformation in free space. It is therefore meaningless to attempt to apply a Lorentz transform in free space to the field tensor [1-5]. For example, it is well known that there is no physical $\mathbf{E}^{(3)}$ field in O(3) electrodynamics in the vacuum, and therefore the following quantity:

$$I = \mathbf{B}^{(3)} \bullet \mathbf{B}^{(3)} \tag{7}$$

is a Lorentz invariant. This follows from section 2. If, however, we attempt to apply, for example, an Y, X and Z boost to the matrix $G^{\mu\nu(3)}$, we obtain the results:

$$B_Z^{(3)'} = ?\gamma B_Z^{(3)} + \gamma BE_Z^{(3)} \tag{8}$$

$$B_Z^{(3)'} = ?\gamma B_Z^{(3)} - \gamma BE_Z^{(3)} \tag{9}$$

$$B_Z^{(3)'} = ?\,B_Z^{(3)} \tag{10}$$

There is no $\mathbf{E}^{(3)}$ field, but even if there were an $\mathbf{E}^{(3)}$ field it is defined as having only a Z component. Therefore $E_X^{(3)}$ and $E_Y^{(3)}$ are zero by definition. The X and Y boosts therefore produce:

$$B_Z^{(3)'} = ?\gamma B_Z^{(3)} \tag{11}$$

and the quantity $\mathbf{B}^{(3)} \bullet \mathbf{B}^{(3)}$ is not an invariant, contrary to section 2. The correct logic is to show that the equations of motion are Lorentz invariant, and therefore that the solutions are Lorentz invariant. The $\mathbf{B}^{(3)}$ field is an observable in several ways, but the radiated $\mathbf{E}^{(3)}$ has never been observed empirically.

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