# The Case for Incomparability

#### Abstract

According to influential arguments from several branches of philosophy, there exist some gradable natural language expressions that violate the following principle: if x and y are both F to some degree, then either x is at least as F as y or y is at least as F as x. Dorr, Nebel and Zuehl (2022) (DNZ), who refer to this principle as 'Comparability', respond to these arguments and offer a systematic case in support of Comparability. In this paper, I respond to DNZ and develop an opposing case against Comparability that (i) buttresses previous criticisms of the principle, (ii) identifies two new arguments against the principle, and (iii) rebuts DNZ's prospective arguments in favour of the principle.

## WORK IN PROGRESS, PLEASE DON'T CITE WITHOUT PERMISSION

### 1 Introduction

All natural languages (that we are familiar with) possess syntactic constructions that allow speakers to make comparisons between two objects with respect to the extent to which they exhibit some property (typically referred to by a corresponding adjective or adjectival phrase) (see e.g. Kennedy (2006)). For instance, consider the following sentences of English,

- (1) 'The dog is at least as heavy as the cat.'
- (2) 'Truro is closer to the border than Falmouth.'
- (3) 'I am fonder of the taste of chocolate than I am of the taste of soup.'<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>Note that this third example is unlike the first two in the sense that it involves the

In this paper, I will be interested in a specific aspect of the logic of comparative constructions like these. In particular, following Dorr, Nebel and Zuehl (2022) (henceforth 'DNZ'), I'll be focusing on the logic of comparative forms of adjectives ('F-er' or 'more F') and the equative form ('at least as F as'), and their adherence to the following general principle

**Comparability:** For any x and any y, if x and y are both F to some degree, then either x is at least as F as y or y is at least as F as x.

Before diving into the arguments surrounding Comparability, it will be helpful to clarify some of DNZ's background assumptions, commitments and conventions. Firstly, one might wonder why Comparability is given a conditional formulation, rather than the simpler non-conditional formulation presented below,

**Unconditional Comparability:** For any x and any y, either x is at least as F as y or y is at least as F as x.

The problem with Unconditional Comparability is that it admits of obvious counterexamples that are avoided by the conditional formulation. For instance, consider the sentence 'Either Beethoven's fifth symphony is at least as red as my car or my car is at least as red as Beethoven's fifth symphony', which is intuitively either false or meaningless, and certainly doesn't seem to be a logical truth of any kind. Plausibly enough, DNZ contend that the problem with an

expression of a speaker's preferences/values/attitudes, and so has a different grammatical form. Nonetheless, the content of (3) seems (in most ordinary contexts) to be equivalent to something like 'chocolate tastes better than soup' or 'my enjoyment of the taste of chocolate is greater than my enjoyment of the taste of soup', which might suggest that the logic of (3) should be equivalent to the logic of (1) and (2) (at least in most contexts). Importantly for present purposes, DNZ explicitly contend that first personal comparative constructions also obey an analogue of Comparability.

example like this is that Beethoven's fifth symphony is not red to any degree, and so can't meaningfully be compared to anything else in terms of its redness. By requiring that x and y both be F to some degree, the conditional formulation of Comparability allows us to sidestep cheap counterexamples like these. For the rest of this paper, I will always assume that the entities under consideration exhibit the relevant property to some degree. Given this assumption, the distinction between the conditional and unconditional forms of Comparability (and the other principles of comparative logic discussed here) seems unimportant, so I will often use the unconditional formulation for simplicity and brevity.

Secondly, DNZ also assume that any ambiguity or context sensitivity in the relevant comparative construction is always resolved uniformly. To illustrate, consider the sentence 'Either this seminar is at least as long as Fifth Avenue or Fifth Avenue is at least as long as this seminar', which seems highly dubious. The problem here is that the adjective 'long' is ambiguous between a spatial and a temporal interpretation. Once we choose to apply one of these interpretations uniformly throughout the sentence, it becomes clear that one of the entities under consideration will fail to exhibit the relevant property to any degree (Fifth avenue has no temporal length, a seminar has no spatial length), which then means that this comparative construction does not constitute a counterexample to the conditional formulation of Comparability (and is excluded from consideration if we ignore constructions involving entities to which the relevant adjective does not meaningfully apply, as I do here).

Thirdly, DNZ assume the law of excluded middle (which asserts the logical necessity of any sentence of the form 'P or not P') throughout their discussion. They note that Comparability, together with the intuitively valid scheme 'If y is F-er than x then x is not F-er than y' jointly imply the following principle.

**Exclusive Comparability:** If x and y are both F to some degree, then either x is F-er than y or x is not F-er than y.

They note that critics of the law of excluded middle may be tempted to reject Exclusive Comparability (which looks like a straightforward application of the excluded middle to comparative constructions), which may in turn lead them to reject Comparability in general (via modus tollens). This kind of argument is particularly salient in contexts where F is a vague predicate and the hypothetical critic of the principle of excluded middle is motivated by vagueness considerations. By assuming the law of excluded middle, DNZ effectively cut off this line of attack on Comparability. For present purposes, I'm happy to continue with all of the background assumptions described so far.

Returning to the issue at hand, DNZ present an array of arguments for Comparability (most of which appeal to the apparent validity of natural language inference patterns that seem to presuppose Comparability). They also rebut two influential arguments against the truth of Comparability, thereby rehabilitating the philosophical case for Comparability. In this paper, I rejuvenate the case against Comparability. I begin by reviewing two influential arguments against Comparability that are considered and rejected by DNZ, before (i) defending those arguments, and (ii) introducing two further arguments that are not considered by DNZ (§2). I then move on to reviewing DNZ's arguments in favour of Comparability, before responding to each in turn and showing that they provide little or no support for the principle (§3). I take the cumulative upshot of this analysis to be that the case for Comparability is weaker than the case against

 $<sup>\</sup>mathrm{it.}^2$ 

 $<sup>^{2}</sup>$ An important caveat: DNZ (2021) use their arguments against Comparability to support strong metaphysical theses about the nature of preference and credence. Given that DNZ (2022) present Comparability as a feature of the logic of natural language (rather than a metaphysical thesis about the comparative structure of real world properties), I think that the inference from Comparability to these strong metaphysical theses also warrants further

## 2 The Case For Incomparability

#### 2.1 The Tradeoff Argument

The first argument against Comparability that is considered by DNZ is known as the 'tradeoff argument'.<sup>3</sup> It begins with the observation that some gradable properties are plausibly multidimensional in the sense that they are a function of multiple other gradable properties. For instance, you might think that the quality of a novelist is a function of their originality, musicality, empathy, social insight, mastery of suspense etc. Furthermore, you might also think that there is no principled way to combine these dimensions of quality into a single overall metric that allows you to draw meaningful comparisons between all pairs of novelists. Maybe Mary Shelly is better than Jane Austen when it comes to creating suspense, but not as good when it comes to social insight, and since there's no principled way to weigh these two important qualities against eachother, there's likewise no meaningful way to compare the overall quality of the two authors. Similarly, consider e.g.

(i) a pair of lives, the first of which has more personal achievements, and the second of which involves deeper personal relationships. Which is better?,

#### (ii) the footballers Paolo Maldini and Paul Scholes. Which was more skilled?

In each of these cases, it seems that one of the items we're asked to compare wins along some dimensions and loses along others, and that there's no principled way to combine these multi dimensional variations into a single meaningful scrutiny. But for now, I concern myself with DNZ's analysis of Comparability itself, rather than the inferences they draw from it.

 $<sup>^{3}</sup>$ For influential examples and applications of the tradeoff argument, see e.g. Keynes (1921), Parfit (2016), Sen (1997).

comparison. It appears that the defender of Comparability is committed to the *prima facie* implausible assumption that for any gradable property that can vary along multiple dimensions, there is always a principled method for weighing the different dimensions to yield meaningful comparisons between any relevant pair of entities. That looks like a major problem.

Now, DNZ accept that in paradigmatic illustrations of the tradeoff argument, it can seem natural to make assertions that appear to be in conflict with Comparability. For instance, they accept that

(4) 'Neither George Carlin nor Richard Pryor was funnier than the other. But they were not equally funny.'

can sound natural when one attends to the multi dimensional character of the gradable adjective 'funny'. However, DNZ argue that we should interpret speeches like (4) in the same way we interpret e.g.

(5) 'George Constanza is not bald. But he is also not not bald.'

Although (5) might seem a natural thing to say in some contexts (e.g. when it's vague whether George Constanza is bald), we clearly don't take it to be literally true, i.e. we don't take it as a counterexample to the law of noncontradiction. The reason for this is that we acknowledge that baldness is a vague adjective, and that there can be borderline cases of baldness. When someone asserts (5), they are acknowledging that they are dealing with a borderline case. DNZ claim that we should think of (4) along similar lines, i.e. when someone asserts (4), they are simply acknowledging that there is extensional vagueness in some (at least two) of the sentences 'Carlin was funnier than Pryor', 'Pryor was funnier than Carlin', 'Pryor and Carlin were equally funny'. Just as there can be vagueness in predicates like 'bald' or 'funny', there can likewise be vagueness in predicates like 'funnier than Pryor' or 'equally funny to Carlin'. Apparent counterexamples to Comparability are really just symptoms of this fact.<sup>4</sup>

One might attempt to support this diagnosis by noting that alleged instances of incomparability seem to give rise to exactly the kind of sorites type paradoxes that are characteristic of vagueness. For instance, suppose you take a very good career x that is incomparably good to some career y and start removing one cent at a time from the salary of y until you reach a career  $y_1$  with a salary of 1 dollar. Plausibly, x is incomparably good to at least the first few careers in the sequence that begins with y. But by the time you get near to the end of the sequence, it's clear that x is better than  $y_1$  and its neighbours. But at what stage does the incomparability vanish? Is x better than y with \$10 dollars deducted? \$20? \$50,000? This lack of a principled boundary between incomparability and comparability looks analogous to the lack of a principled boundary between instances and non-instances of vague predicates. One might take this to support DNZ's diagnosis of alleged counterexamples to Comparability as mere instances of the vagueness of comparative predicates like 'F-er than x'. While I agree that chains of incomparability generate sorites-type phenomena that are reminiscent of vagueness, I do not agree that this observation supports DNZ's diagnosis of the counterexamples to Comparability identified by the tradeoff argument. In particular, I take it that the example conclusively establishes that the predicate 'incomparably good to x' is indeed vague. But that doesn't entail that there aren't determinate instances of incomparability any more than the fact that the predicate 'bald' is vague entails that there are no determinate cases of baldness.

<sup>&</sup>lt;sup>4</sup>Something resembling this approach to explaining away prospective counterexamples to Comparability is already familiar from e.g. Broome (1997), and has been vigorously resisted by e.g. Chang (2002).

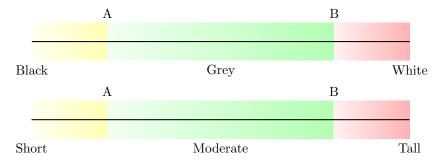
x is determinately incomparable to y, determinately incomparable to  $y_1$ , and indeterminately incomparable to (for argument's sake) the career we get by deducting \$10,000 from y's salary. The observation that comparative predicates (including incomparability) can themselves be vague does not entail or even directly confirm the claim that all instances of incomparability are instances of comparative vagueness.

Furthermore, I don't think that DNZ's response to the tradeoff argument does anything at all to undermine the basic insight that it's deeply implausible to think that there's always a privileged mechanism for weighing the different dimensions of multi dimensional predicates to produce meaningful comparisons between e.g. a career as an athlete and a career as a scientist. An adequate response to the tradeoff argument would need to show why the apparent implausibility of this idea is misleading, and give an account of how these tradeoffs can be made in a plausible and principled way in the kinds of cases that are used to illustrate the argument. Relatedly, it's worth pointing out that there is actually an important disanalogy between canonical examples of incomparability arising from tradeoffs and canonical examples of vagueness, namely the fact that the former always involve multidimensional variation while the latter usually involve variation along a single dimension. In the case of e.g. baldness, there is no one principled way to partition the spectrum of hairiness into 'bald' and 'not-bald', but it's always possible to compare any two points on the spectrum in terms of their hairiness. In the case of e.g. goodness of career, there are multiple relevant spectra of variation and not only is there no single principled partition of points in the multidimensional space into 'good' and 'not good' careers, there's also no single principled way of comparing all points in that space in terms of their goodness. This is a fundamental disanalogy between pure vagueness and incomparability arising from tradeoffs. DNZ's explanation of utterances like (4)

is that there is vagueness in the predicate 'F-er than x' in the same way that there's vagueness in the predicate 'bald'. But that seems to ignore this stark disanalogy. The intuition that Pryor is incomparably funny to Carlin isn't explained by him lying in a region of a single spectrum of funniness in comparison to Carlin that is on the borderline between the 'funnier than' and 'as funny as' regions. It's explained by the fact that he and Carlin simultaneously occupy multiple points across multiple orthogonal spectra that all contribute to funniness in complex ways that cannot be meaningfully compressed into a single comparison.

In fact, we can get a little more precise about the nature of the disanalogy between speeches like (4) and canonical instances of vagueness like (5). When we assert (5), we typically recognise both that Constanza occupies a fully determinate point on a linear spectrum of hairiness, i.e. he has a fully determinate number of hairs, and that that point/number lies on the borderline of the vague 'is bald' predicate. Similarly, when we take a shade to be on the borderline between white and grey, we acknowledge that the brightness value of the shade is fully determinate, it's just that the value lies on the borderline of the 'is white' predicate. And when we take someone to be on the borderline between tall and not-tall, we recognise that their height is fully determinate, even though it lies on the borderline of the 'is tall' predicate.

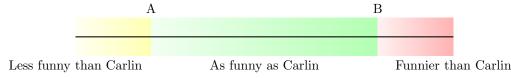
Now, DNZ interpret (4) as stating that Pryor lies on the borderline between the 'equally funny to Carlin', 'funnier than Carlin' and 'less funny than Carlin' predicates. If we hope to continue the analogy with canonically vague predicates, this seems to be like a shade being on the borderline of the 'white', 'grey' and 'black' predicates all at the same time, or like a person being simultaneously on the borderline of the 'tall', 'moderate height' and 'short' predicates. But of course vague predicates don't generally admit of possibilities like that. There are no shades that are both borderline grey/white and borderline grey/black, and there are no people that are both borderline moderate height/tall and borderline moderate height/short. This is because there is a linear spectrum of shades ordered by their darkness and the grey region *separates* the white region from the black region, and there is likewise a linear spectrum of heights such that the moderate height region *separates* the tall region from the short region, as depicted in the two figures below.



In both cases, it is impossible for something to simultaneously lie on the borderline denoted A whilst also lying on the borderline denoted B. If we had a shade that was both borderline white/grey and borderline black/grey, that would seem to preclude the possibility of any shade being determinately white, since it would suggest that there's no 'space' between A and B in the brightness spectrum. Similarly, if a person was both borderline short/moderate and borderline tall/moderate, that seems to rule out the possibility of anyone having determinately moderate height, for the same reason. But DNZ's interpretation of (4) commits the speaker to believing that Pryor is simultaneously on both the funnier than Carlin/equally funny to Carlin and the less funny than/equally funny to Carlin borderlines simultaneously.<sup>5</sup> Assuming a linear spectrum of de-

<sup>&</sup>lt;sup>5</sup>One might wonder whether we couldn't interpret (4) as saying that Pryor lies on just one of the borderlines, let's say A. But that implies that the speaker takes there to be an asymmetry in the funniness of Carline and Pryor – Pryor is on the less/equally funny to Carlin borderline, while Carlin is on the more/equally funny to Pryor borderline. But when people say things like (4), there is no evidence of there being any such asymmetry in their evaluation of the comedians, so we can safely disregard this possibility for now.

grees of funniness (the existence of which seems to be entailed by Comparability<sup>6</sup>), this amounts to saying that Pryor simultaneously occupies both borders A and B in the figure below.



But if he occupies both A and B simultaneously, that suggests that there's no space between A and B, i.e. that there are no instances of being determinately equally funny to Carlin, which is an absurd conclusion. So there must be something wrong with this picture. When somebody asserts (4), they are not asserting that Pryor occupies a point x on a linear spectrum of degrees of funniness such that x simultaneously lies on both the funnier than Carlin/equally funny to Carlin and the less funny than/equally funny to Carlin borderlines.

Here's another possible story. Maybe speeches like (4) are not assertions that Pryor inhabits a point that simultaneously occupies both the A and B borderlines on a linear funniness spectrum, but rather assert that there is some kind of indeterminacy regarding which point in the funniness spectrum is occupied by Pryor. It's not that there's some point (occupied by Pryor) on the spectrum that inhabits the intersection of the borderlines, but rather that there exist points x, y, z on the spectrum such that (i) it's indeterminate which of them Pryor occupies, and (ii) x/y/z inhabit the less/equally/more funny than Carlin regions, respectively. This story avoids the objection outlined above. But note – this story radically weakens the analogy between speeches like (4) and canonical examples of vagueness. As I stressed previously, in canonical vagueness cases, there is a linear spectrum of values and the indeterminacy arises not

 $<sup>^6 {\</sup>rm Since}$  Comparability entails that we can line all comedians up in a liner fashion based on their funniness, and we can assign them degrees of funniness in any way that respects this ordering

from an item's location on that spectrum, but rather from the vague boundaries of a region of that spectrum. When we assert (5), we generally think that Constanza has a determinate number of hairs, but we take that number to lie on the borderline of the bald/not-bald regions of the spectrum. On this new story, that is not what's going on in speeches like (4). It's not that Pryor occupies a definite point on a linear funniness spectrum that happens to be on the borderline of some vague comparative predicates. Rather, Pryor fails to determinately occupy any point on the funniness spectrum at all. So (4) cannot be straightforwardly interpreted as a recognition of the extensional vagueness of the comparative predicates. If we're going to explain it away in terms of indeterminacy, the root of the indeterminacy must lie deeper, and the defenders of such an explanation have a burden to plausibly identify that root.

Now, in the case of funniness, a paradigmatically multidimensional predicate, we might think that the indeterminacy arises from indeterminacy in the underlying dimensions like e.g. wit, irony, stage presence etc. If there is some indeterminacy in Pryor's score along those dimensions, then it's natural to think it would manifest in indeterminacy in his funniness overall. But this story won't work in general. Consider, for instance, the case of physical strength. We might plausibly think that physical strength is a multidimensional property that depends on e.g. leg strength, arm strength, core strength etc, where each of those underlying dimensions can be precisely operationalised in terms of e.g. weight lifting ability. I could plausibly take two people to be incomparably strong because one scores better along some dimensions but the other scores better along other dimensions. But it'd be implausible to interpret any assertion to this effect as evidence of indeterminacy in the individual dimensions of which overall strength is a function, since those are all fully determinate. So the root of the indeterminacy can't (always) be indeterminacy in the individual dimensions of which the relevant multidimensional property is a function.

The final possibility, it seems to me, is that the indeterminacy actually comes from the function that takes the scores along the relevant underlying dimensions and produces an overall score for the relevant multidimensional property, or, equivalently, takes the item's locations on the spectra of the underlying dimensions and returns its location on the linear spectrum of values for the property of interest. Returning to the strength example, the idea is that there is a linear spectrum of overall strength, and all of the underlying dimensions (leg strength, arm strength etc) are fully determinate. However, the function that takes scores for arm strength, leg strength etc and returns an overall strength score is itself somehow indeterminate, i.e. it doesn't return a determinate value for all vectors of values of the underlying dimensions. This sounds very much like an admission that there is in fact no linear spectrum of strength, and that there is in general no principled way to combine the underlying dimensions of a multidimensional property to yield determinate comparisons in all instances (which is the point of the tradeoff argument). If there are some people (possibly all people) who simply cannot be given a determinate location on a linear scale of strength, then it seems odd to insist that there is in fact a meaningful linear scale at all. But if there is no such linear scale, then Comparability must be false, since Comparability entails that it's possible to linearly order all people by their strength. Furthermore, as I've already argued, the defender of Comparability can't plausibly diagnose the indeterminacy in question as symptomatic of the fuzziness of some regions of a linear scale (as in canonical vagueness cases). If the claim was that all alleged instances of incomparability can be explained away as cases where the relevant items do in fact occupy definite points on a linear spectrum of F-ness, but where those points lie on the fuzzy borderlines of comparative predicates, that would be consistent with the idea that there really is a linear scale of F-ness, and hence with Comparability more generally. But since that story doesn't work, the Comparability advocate has to accept that the relevant items actually fail to determinately occupy any point on a linear scale of F-ness, which undermines the idea that there is a meaningful linear F-ness scale at all.

Now, one might simply insist that there always is a meaningful linear scale of F-ness even though there's rampant indeterminacy regarding where things lie on that scale. At this stage, I'm tempted to wonder what's really at stake in the disagreement. If the defender of Comparability accepts that it's often not the case that any comparison of two items is determinately true, and accepts that this indeterminacy is *not* a symptom of the items occupying borderline regions on any linear spectrum but is rather a brute fact about the the structure of the F predicate, then there doesn't seem to be much substantive disagreement remaining. The defender of this view agrees with the critic of Comparability that (i) it is often impossible to determinately rank items in terms of their Fness, and (ii) this impossibility cannot be attributed either to those items lying on the borderlines of any regions of a linear spectrum of F-ness, or (generally) to indeterminacy in the underlying properties of which F-ness is a function. If incomparability is indeterminacy, then the indeterminacy in question is a *sui*generis form of indeterminacy that is quite unlike the indeterminacy at play in sentences like (5). And the critic of Comparability can quite reasonably object that introducing a new species of brute indeterminacy to explain away natural speeches like (4) seems ad-hoc and unparsimonious, when one could simply take the speech at face value instead.

#### 2.2 The Small Improvements Argument

The second argument against Comparability canvased by DNZ is the argument from small improvements.<sup>7</sup> This argument begins by considering a case where it is plausible to deny both 'x if F-er than y' and 'y is F-er than x'. For instance, it might plausibly seem that a particular career as an athlete is neither better nor worse than a particular career as a scientist. This leaves two possibilities. Either the two careers are equally good, or they are incomparable, meaning that they constitute a counterexample to Comparability. If they are equally good, then any slight improvement to one should break the tie. In particular, if we increase the salary of the athletic career by \$1, that should lead us to prefer the career as an athlete to the career as a scientist. But that seems wrong. Increasing the salary by a dollar doesn't seem to plausibly break the tie. If we really didn't prefer that career before, then improving it by \$1 shouldn't be enough to generate a real preference. So the two careers must be incomparable.

DNZ do not commit to a specific response to the small improvements argument, although they do canvas a couple of possible diagnoses of what might have gone wrong in the argument, which they think can plausibly explain away the apparent truth of its premises. Firstly, they suggest that the vaguenesstheoretic diagnosis given for the tradeoff argument can be similarly applied to undermine the small improvements argument. I take my response to that diagnosis to be applicable here too, and will not rehash the details. Secondly, they propose that

[C] omparative expressions can in some contexts take on "coarsegrained" meanings, where the domain is "chunked" into some finite number of discrete sections. The relations expressed in a coarsegrained context by 'equally F' and 'at least as F as' will hold uni-

<sup>&</sup>lt;sup>7</sup>For influential examples and applications of the small improvements arguments, see e.g. Chang (1998), Raz (1985), Schoenfield (2012).

versally within each chunk, while the relation expressed by 'F-er than' holds only between things in different chunks. When we start paying attention to small differences in the relevant underlying factors, that generates pressure to change the context to a more fine grained one, which means the extension of 'more F than' will expand to include certain pairs of items that were previously in the same chunk, and the extensions of 'equally F' and 'at least as F as' correspondingly shrink. On this account, the small improvement argument can be diagnosed as turning on a shift in the context. When we initially consider the disparate items x and y the most natural resolution of context-sensitivity is a rather coarse-grained one that puts them in the same chunk, since, because of vagueness, there are no non-arbitrary grounds for choosing a resolution of contextsensitivity that makes one but not the other of 'x is more F than y' and 'y is more F than x' true. Premise P1 ('x is not more F than y and y is not more F than x') is thus true in the context in which it is uttered. (Given the validity of Trichotomy, it follows that 'x and y are equally F' is also true in this context.) Likewise for P2 (' $x^+$ is not more F than y'). P3 (' $x^+$  is more F than x'), by contrast, invites us to attend to the small but one-sided differences between xand  $x^+$ , thus pushing us into a different, more fine-grained, context in which P3 is true. But in no context are all three premises true. (Dorr et al (2022): 447)

The picture here (as I understand it) is that, when we make the judgement 'x is not F-er than y and y is not F-er than x', we are always operating in a coarse grained context where x, y and  $x^+$  (a small improvement on x) are all judged to be *equally* good. But when we are subsequently invited to pay

attention to the (small) difference between x and  $x^+$ , we move to a more fine grained context where we judge  $x^+$  to be better than both x and y.

The most immediate problem for this response to the small improvements argument is that it entails that there are contexts in which  $x^+$  is not considered to be F-er than x, which seems to be inconsistent with the stipulation that  $x^+$  is constructed to be uncontroversially better than x. It seems implausible to think that there's any context in which we would not consider a career  $x^+$  to be clearly better than an otherwise identical career x that comes with a small pay decrease. DNZ may reply that we are are supposed to ignore the small difference in salary between x and  $x^+$  in the coarse grained context, which justifies treating them as equally good. But that seems strange. Since they are stipulated to be identical apart from their salaries, we wouldn't be able to distinguish between them at all in a context that ignored small salary differences, i.e. we'd treat them as a single career, not as two distinct careers that are equally good. Furthermore, it seems clear that there can be some cases where the difference between x and  $x^+$  is too large to be reasonably ignored, but not large enough to break the tie between x and y. I think it's clear that both (i) a scientific career  $x^+$  that pays \$5,000 dollars more than an otherwise identical career x is clearly better than x(and that it would be bizarre to ignore that fact), and that (ii) the added \$5,000 dollars clearly isn't sufficient to break the 'tie' between x and an incomparably good artistic career.

To give another example: it seems that we can easily imagine a context in which we are considering three applicants for an academic position, where (i) applicants 1 and 2 are judged to be pretty much identical except for the fact that applicant 1 has one publication more than applicant 2, and is therefore deemed superior, and (ii) applicant 3 has less publications than both other applicants, but has a more impressive teaching dossier than either of them, and is therefore deemed to be neither better nor worse than either of the other applicants. It seems strange and ad-hoc to stipulate that there is always some contextual shift between the judgement that occurs in (i) (that applicant 1 is better than applicant 2) and the judgements that occur in (ii) (that applicant 3 is neither better nor worse than either of the other applicants). It feels possible to make those judgments simultaneously, without any shift in e.g. background knowledge/evidential standards/conversational background/the focus of our attention. Any story that appeals to such a contextual shift incurs the burden of explicating the details of its mechanics in a plausible way. The prospects for doing that in this kind of case look dim.

The second proposed response to the small improvements argument goes as follows,

[W]hen we say 'These things are equally F', we are very often speaking loosely (non-literally). Analogously, some philosophers argue that when we say 'The cities are 853 miles apart' or 'They arrived at 3pm' we are almost never speaking literally, since the literal truth conditions are ultra-demandingly exact (Hoek 2018). Given that view, it is plausible that the same kind of looseness is in play when, at the beginning of the small improvement argument, people are inclined to accept 'Neither of x and y is more F than the other.' Note that this seems to be a different kind of nonliteral speech from the kind exemplified by 'He is not bald, but he is also not not bald', since it doesn't have anything special to do with negation. However, the two sources of nonliteralness could sometimes both be in play. (Dorr et all (2022): 447-448)

It is unclear to me how this response could help critics of the small improvements argument. I agree that when we say 'John and Jane are equally tall', for instance, we are speaking loosely and really mean something like 'they are roughly the same height', since it's highly unlikely, for any pair of people we happen across, for them to be exactly the same height. If a similar kind of loose talk is going on when we say 'Neither of x and y is F-er than the other', then that utterance really means something like 'x and y are roughly as F as each other'. But it's just not plausible that that's what we mean when we say things like 'neither Maldini nor Scholes was more skilled than the other', 'neither Shelley nor Austen was a better novelist than the other', or 'neither the joy of love nor the pain of grief is more intense than the other'. We don't mean that they're roughly the same – we mean that they're *different*. In the example of height, it might seem initially plausible that we can interpret 'neither of you is taller' as something like 'you're roughly the same height' because we know that height is one dimensional and measured on a linearly ordered scale (the real numbers). But that's not true for multi dimensional concepts, where (as the tradeoff argument illustrates) we know that it's sometimes impossible to combine multiple dimensions to generate a principled all things considered comparison.

And even for one dimensional concepts like height, the story isn't as plausible as it initially seems. Suppose that it is common knowledge that John is 6 feet and one inch tall, and Jane is 6 feet and two inches tall. In many contexts, it will then be clearly true to say 'John and Jane are roughly the same height' and clearly false to say 'neither John nor Jane is taller than the other' (since it's common knowledge that Jane is slightly taller than John). So the truth conditions of 'Neither of x and y is F-er than the other' do not seem to generally line up with 'x and y are roughly as F as each other' in the way suggested by DNZ, even for one dimensional properties like height.

#### 2.3 The Argument from Temporal Relativity

I turn now to identifying two further arguments against Comparability that are not explicitly considered by DNZ. The first of these arguments draws inspriation from a famous scientific example, namely the *relativity of simultaneity*. Prior to this discovery, it was universally thought that all events were totally ordered with respect to their temporal location, i.e. that for any events a and b, either aoccurs earlier than b, b occurs earlier than a, or they occur simultaneously. We now know that this principle is fundamentally incorrect, since different inertial reference frames impose different temporal orderings, and there is no principled way to identify a single reference frame as being 'correct' in a meaningful way. To use the language of modern physics: the causal ordering of events is a partial order – some events really are prior to others (since they appear that way to all inertial observers), but there are many pairs of events for which there is no meaningful way to settle the temporal ordering independently of an arbitrary specification of the chosen reference frame.

At first blush, this looks like a clear counterexample to Comparability. If you present a physicist (or anyone that's taken a first course in special relativity) with a textbook illustration of the relativity of simultaneity and ask them to temporally order the relevant events, they'll say something like 'well a is earlier from the perspective of Alice, and b is earlier from the perspective of Bob, but neither of them is actually earlier, and they also don't occur at the same time'. That seems to show that the gradable adjective 'early' provides a direct refutation of Comparability.

In response to this, DNZ might reply that whenever somebody says 'a is earlier than b', they really mean 'a is earlier than b from my perspective'. But note that when physicists are playing around with spacetime diagrams on a blackboard and they point to an event b in the past lightcone of event a and say 'b is earlier than a' and then point to an event c outside that lightcone and say 'but a is neither earlier nor later than c', they clearly do not mean that bis earlier than a relative to their (or any other) inertial reference frame. They mean that b really is earlier than a, for all observers. Here, DNZ will be tempted to reply that the physicists' use of 'early' is not part of natural language, but is rather some stipulative technical use that doesn't tell us anything about the logic of natural language comparative constructions. This is the move they make when discussing the example of a logician saying that one proposition is 'logically stronger' than another exactly when the former entails the latter. Again, this notion of logical strength seems to be in tension with Comparability, since every Boolean algebra includes pairs of propositions that are not ordered with respect to logical strength (because neither entails the other). In response to this, I'd like to note first that the physicists' use of 'earlier than' seems to be more clearly continuous with natural language than the logician's use of 'logically stronger'. Physicists really seem to be talking about the same relation that ordinary folk are talking about when they say that one event is earlier than another. They haven't just invented a technical notion with arbitrarily stipulated properties. If they had, we wouldn't take special relativity to be telling us anything about 'time' at all – we'd say that the physicists were just playing around with weird technical notions.

Furthermore, even if one accepted that the physicists' use of 'earlier than' was in some sense stipulative and disconnected from natural language, there is still another problem for Comparability lurking here. DNZ argue that the fact that comparative constructions are sometimes used in stipulative ways that violate Comparability in technical contexts does not undermine the plausibility of the principle. They point out that one can also stipulatively use comparative constructions in ways that conflict with e.g. the transitivity and reflexivity of ordering relations, and that this possibility doesn't lead us to doubt that those principles are respected by our natural language use of comparative constructions. So the mere fact that one can construct stipulative uses of e.g. 'earlier than' and 'logically stronger than' that violate Comparability doesn't speak against the status of that principle as a feature of natural language, or so the argument goes. But here it should be emphasised that we really do seem to be disposed to use comparative constructions in ways that conflict with Comparability, even if those uses are written off as stipulative and artificial. Logicians and physicists find it natural to use comparative constructions to describe the partial orderings given by entailment relations and temporal relations, respectively.<sup>8</sup> In contrast, I cannot think of any examples where specialists have found it natural to use comparative language to describe relations that are uncontroversially irreflexive or intransitive.<sup>9</sup> This, it seems to me, suggests that Comparability is less tightly woven into the natural linguistic logic of comparative constructions than e.g. reflexivity and transitivity.

Finally, it's worth pointing out that DNZ's responses to the tradeoff and small improvement arguments are very clearly inapplicable to the examples of logical strength and temporal ordering. When a physicist says of two events that neither is earlier than the other, they don't mean that there's vagueness in the temporal order. They simply mean that one event occurs earlier in some inertial frames, and the other occurs earlier in other inertial frames. Similarly, when the logician says that neither  $p \lor q$  nor  $p \to q$  is logically stronger, she doesn't mean that there's vagueness in the entailment relations. She just means that neither entails the other. These are also clearly not examples of

<sup>&</sup>lt;sup>8</sup>Further examples abound. For instance, in causal inference, it is common to talk of 'causal priority', where we say that one variable is causally prior to the other if there exists a causal path from the first to the second. This notion of causal priority straightforwardly violates Comparability but satisfies transitivity and reflexivity.

 $<sup>^{9}</sup>$ And if I could, I'd take those examples to cast doubt on whether transitivity and reflexivity are really part of the logic of natural language.

loose speak, or contextual variation. Again, when the logician says that neither  $p \lor q$  nor  $p \to q$  is logically stronger, she doesn't mean 'they are roughly equally strong' (what would that even mean?), and she isn't ignoring any facts about logical relations to other propositions. She is just stating what she takes to be the literal, determinate, absolute truth.

#### 2.4 The Argument from Theoretical Utility

The fourth argument against Comparability is based on the prominent theoretical role that rejections of Comparability have come to play across numerous domains of philosophical inquiry. In value theory, the idea that states of the world can be incomparably valuable has played a crucial role in the most prominent responses to Parfit's mere addition paradox (see e.g. Chang (2016), Hajek and Rabinowicz (2022), Parfit (2016)), as well as a host of other moral dilemmas (see Sinnott-Armstrong (1985)). In epistemology, the idea that an agent's confidence in two propositions can be incomparable has been repeatedly employed to address the paradoxes of epistemic indifference (see e.g. Eva (2019), Joyce (2010), Levi (1985), White (2009)). In decision theory, incomplete preference orderings have been used to explain ambiguity aversion in the Ellsberg paradox (see e.g. Levi (1986)). Further examples abound.

The fact that rejections of Comparability have been thought to help resolve apparent paradoxes in so many philosophical contexts provides *prima facie* support for the negation of that principle. For, it suggests that Comparability is acting as a problematic implicit premise in all of those paradoxes, and that simply rejecting that one premise can kill multiple philosophical birds with one stone. Indeed, it suggests that the rejection of Comparability has the important theoretical virtue of *unification* (see e.g. Kitcher (1981)), i.e. that it provides a unified explanation of a broad and diverse range of relevant philosophical phenomena.

Naturally, it has to be acknowledged that the persuasiveness of this argument is directly proportional to that of the purported solutions to the relevant paradoxes. If one thinks that these paradoxes can all be resolved without rejecting Comparability then the argument won't have much force. But it's still important to acknowledge that critiques of Comparability need not be based entirely on observations of folk judgements and natural language. They can also be directly motivated by theoretical considerations surrounding e.g. the role of Comparability in derivations of paradoxical conclusions across a range of philosophical domains. If one is at all moved by any subset of the instances in which rejecting Comparability is alleged to solve philosophical problems, then that adds some *prima facie* weight to the case against Comparability. Although DNZ do not explicitly respond to this kind of argument against Comparability, they do acknowledge its possibility,

Many philosophers...either argue explicitly against certain instances of Comparability, or rely on such failures in accounts of other phenomena, in ways that could easily be retooled as abductive arguments against Comparability. (Dorr er al (2022): 423)

I'm simply noting here that an abductive argument of this sort will indeed be treated as forceful by the many philosophers that have identified Comparability as playing a pernicious role in generating problems that are most easily solved by abandoning that principle.

### 3 The Case for Comparability

At this stage, I take it that the four arguments discussed in section 2 put considerable weight behind the case against Comparability (and that DNZ's responses to the first two of these arguments do little to mitigate that weight). I turn now to reviewing DNZ's positive arguments in favour of Comparability to examine whether they do anything to tilt the scales in the other direction.

#### 3.1 Argument 1: 'Not As F As'

DNZ's first argument in support of Comparability takes the 'apparent validity' of the following inference scheme as a premise,

Not As F As: x is not as F as y. So, y is F-er than x.

To illustrate the appeal of this inference scheme, DNZ give a couple of examples where 'x is not as F as y' seems to imply (or even to be interchangeable with) 'y is F-er than x',

(6) 'Your favourite composer isn't as good as mine. So my favourite composer is better than yours.'

(7) 'Max's room isn't as tidy as Josh's. So Josh's room is tidier than Max's.'

If we accept the alleged validity of Not As F As, then we seem to be committed to Comparability as well. To see this, note first that 'as F as' plausibly seems to be truth-conditionally equivalent to 'at least as F as'. Next, note that the following principle is manifestly valid,

**Comparative/Equative:** x is F-er than y if and only if x is at least as F as y and y is not at least as F as x.

Now suppose the validity of Not As F As, and suppose that x is not at least as F as y, i.e. x is not as F as y. By Not As F As, it follows that y is F-er than x. Comparative/Equative then entails that y is at least as F as x. So assuming the validity of Not As F As and Comparative/Equative, together with the truth-conditional equivalence of 'as F as' and 'at least as F as', it is impossible for x and y to be incomparable. This in turn establishes the validity of Comparability. So critics of Comparability need to reject either (i) Comparative/Equative, (ii) the truth functional equivalence of 'as F as' and 'at least as F as', or (iii) the validity of Not As F As. For current purposes, I will accept the first two premises and focus rather on the validity of Not As F As.

To begin, consider the following inferences, which, given the truth conditional equivalence of 'as F as' and 'at least as F as' (which DNZ explicitly accept), should have the same logical status as (6) and (7) above,

(8): 'Your favourite composer is not at least as good as mine. So my favourite composer is better than yours.'

(9): 'Max's room is not at least as tidy as Josh's. So Josh's room is tidier than Max's.'

(8) and (9) do not strike me as compelling inferences. I would not readily accept that the conclusion of either inference follows necessarily from the corresponding premises. In both cases, I might be tempted to accept the premise, reject the conclusion, and say something like 'look, they're just different.' This is not (at least, not purely) a theoretically motivated intuition. It seems perfectly natural and reasonable for someone to accept (10)<sup>4</sup>Art Garfunkle was not at least as good a musician as Joni Mitchell.<sup>4</sup> whilst rejecting

(11) 'Joni Mitchell was a better musiscian than Art Garfunkle.' Similarly, I might think both that it seems wrong to say either

(12)'Paul Scholes was a better footballer than Paolo Maldini.'

or

(13) 'Paolo Maldini was at least as good a footballer as Paul Scholes.'

And in terms of first personal constructions, it strikes me as entirely plausible to reject both

(14) 'I'm more confident that there's life outside our galaxy than I am that the next Game of Thrones novel will be between 600 and 700 pages long.'

and

(15) 'I'm at least as confident that the next Game of Thrones novel will be between 600 and 700 pages long as I am that there's life outside our galaxy.'

These are not new insights – they're characteristic illustrations of why philosophers have been inclined to reject Comparability. But they do clearly indicate that the following inference scheme is far from obviously valid,

Not As F As\*: x is not at least as F as y. So, y is F-er than x.

And this raises a puzzle. Assuming (as we are) that 'as F as' and 'at least as F as' are truth conditionally equivalent, Not As F As and Not As F As\* must have the same logical status – either they are both valid or they are both invalid. At first blush, the first seems intuitively valid, while the second does not. DNZ resolve the tension by insisting that Not As F As is indeed valid, which implies that they both are. But one could equally argue in the other direction, and insist on the invalidity of Not As F As\*, which leads to the conclusion that neither scheme is valid. This is my preferred strategy. But simply insisting on either fork of the dilemma risks begging the question. Defenders of Not As F As have a dialectical burden to explain why Not As F As\* seems invalid, just as critics of Not As  $F^*$  As have a dialectical burden to explain why Not As F As\*

To explain why Not As F As might *seem* valid, begin by recalling Grice's (1975) famous observation that truth conditions and conditions of reasonable assertability do not always line up in the way that one might expect. For instance, it can be unreasonable to assert things that you believe with very high credence to be true. One example of this would be asserting 'either Shola's at the pub or he's at home' whilst knowing full well that he's at the pub.

Now suppose that you and your friend Margaret go to a wine tasting event. You try a bottle that Margaret hasn't tried yet and she asks you how it was. Suppose first that you reply 'It's not bad'. Your assertion here seems to be reasonable as long as you think the wine is okay, but not great. The assertion is clearly unreasonable if either (i) you think the wine is bad, or (ii) you think the wine is really good. Next, suppose that you reply 'It's not good' or 'it's not very good'. It seems like this is a reasonable assertion if and only if you actually think the wine is bad. If you think it's okay, then saying that it's not good/very good seems to be misleading – we usually reserve that type of evaluation for things to which we are negatively, rather than neutrally, disposed. In both cases, the assertability conditions fail to line up cleanly with the truth conditions. Interestingly, the disassociation between truth and assertability conditions in this example vanishes when we remove the negation from your reply. The responses 'It's good' or 'It's bad' seem to be reasonable to assert if and only if the wine is good or bad, respectively. Overall, this suggests that there are some adjectives (e.g. 'good' and 'bad') for which truth conditions and assertability conditions line up in ordinary unnegated contexts, but not in contexts where the adjective is negated. This phenomenon is known as 'negative strengthening' (Horn, 1989).

It seems plausible that the apparent validity of Not As F As might be explained by a comparative analogue of negative strengthening applying to phrases of the form 'as F as'. More specifically, the ideas is that the truth conditions of 'as F as' are the same as those of 'at least as F as', i.e. 'x is as F as y' is true if and only if x and y are equally F or x is F-er than y. Moreover, the conditions of reasonable assertability for 'as F as' seem to line up with its truth conditions – it seems reasonable to assert that x is as F as' construction shifts the conditions of reasonable assertability away from the truth conditions, in the same way that occurs when we negate 'good' or 'bad'. In particular, it seems to be reasonable to assert 'x is not as F as y' if and only if y is actually F-er than x. If this is right, then we have an explanation for the apparent validity of Not as F As, namely that it seems to be valid because it preserves reasonable assertability (but not truth) – it's reasonable to assert 'y is F-er than x' whenever it's reasonable to assert 'x is not as F as y'.

What are the advantages of this explanation? First, it successfully explains the apparent validity of Not As F As without committing to its actual validity (which would imply the validity of Not As F As\*). Second, it maintains the truth functional equivalence of 'as F as' and 'at least as F as'. Third, it maintains the compositionality of truth conditions – 'x is not as F as y' is true if and only if 'x is as F as y' is false. It's only the conditions of reasonable assertability that are not compositional. Fourth, we've seen that conditions of reasonable assertability fail to be compositional in exactly the way that I am claiming they are for 'as F as' in other linguistic contexts that do not involve comparatives (negative strengthening). Fifth, the idea that conditions of reasonable assertability can fail to line up with truth conditions is a familiar and uncontroversial one.

DNZ attempt to preempt this appeal to negative strengthening by raising the following concerns. Firstly, they contend that negative strengthening for comparative predicates would have to behave differently than it does for categorical predicates. For instance, they note that 'Cassidy was almost happy about the news' does not tend to suggest that Cassidy was rather unhappy about the news, but 'Cassidy was almost as happy as Riley' does seem to entail that Cassidy was less happy than Riley. It's not clear to me that this raises any kind of issue for the current proposal. The example illustrates that when applied to categorical predicates, 'almost' fails to induce an analogue of negative strengthening for categorical predicates but does induce such an analogue for comparative predicates. But it doesn't show that the kind of negative strengthening induced by 'not' (the kind at stake in this discussion) is importantly different in the categorical case compared to the comparative case.

Secondly, DNZ note that negative strengthening is usually asymmetric with respect to antonyms. 'I'm not opstimistic' suggests that one is rather pessimistic, but 'I'm not pessimistic' doesn't suggest that one is rather optimistic. If negative strengthening is supposed to apply to both 'no more F than' and 'no less F than', then that's in tension with this apparent property of negative strengthening. But note, the current proposal claims that negative strengthening applies specifically to the 'not as F as' construction, for which the symmetry concern does not apply. I'm not making any claims about how negative strengthening applies to the 'no more F than' or 'no less F than' constructions. Here, it's worth pointing out that negative strengthening applies to some but not all categorical predicates (it doesn't apply to 'black' for instance, since saying that a monochrome shade is not black need not suggest that it's white (it might be grey)). Similarly, it might be that negative strengthening applies to some but not all comparative constructions.

Thirdly, DNZ note that negative strengthening is defeasible in the sense that sentences like 'Riley is not happy, but she's not unhappy either' is a fine thing to say, while sentences like 'Riley is not as happy as Cassidy, but Cassidy isn't happier either' are not. Here, it's important to note first that negative strengthening for comparative is sometimes defeasible. For instance, 'I'm not more confident in p than q, but I'm also not at least as confident in p as I am in q' seems like a possibly reasonable thing to say, so it's not the case that categorical negative strengthening is always defeasible while it's comparative analogue is never defeasible. Furthermore, it also seems plausible to think that there may be some small differences in the way that negative strengthening works in the comparative compared to the categorical case, since the grammar and logic of comparative constructions is fundamentally different to that of categorical constructions. But it's clearly plausible that something very analogous to categorical negative strengthening also applies to comparatives, and taking this into account makes it possible to plausibly explain away the apparent validity of Not As F As without accepting Comparability or introducing any exotic new linguistic phenomena. The burden is on the defender of Comparability to provide a more compelling story that explains away the apparent invalidity of Not As F As\*.

#### 3.2 Argument 2: 'No *F*-er Than'

DNZ's second argument takes the validity of the following inference scheme as a premise,

No *F*-er Than: If x is no *F*-er than y, then y is at least as *F* as x.

To illustrate the apparent validity of this scheme, they cite the following example as an instance of good reasoning,

(16) 'Max's room is no tidier than Josh's, so Josh's room is at least as tidy as Max's.'

If we accept the alleged validity of No F-er Than, then we are led directly back to an endorsement of Comparability. To see this, suppose first (as DNZ do) that 'no F-er than' is truth conditionally equivalent to 'not F-er than'. Next, recall the principle Comparative/Equative introduced in the previous subsection, and suppose for reductio that Comparability is false, i.e. there exist x and y such that x is not at least as F as y and y is not at least as F as x (they are incomparable). Since x is not at least as F as y, it follows from Comparative/Equative that x is not F-er than y, i.e. x is no F-er than y. From No F-er Than, it follows that y is at least as F as x, which contradicts our assumption that x and y are incomparable, and completes the proof of Comparability by reductio from No F-er than' and 'no F-er than'. So the critic of Comparative/Equative of 'not F-er than' and 'no F-er than'. So the critic of Comparability is under significant pressure to reject the purported validity of No F-er Than. But as it turns out, I do not think that No F-er Than has any intuitive claim to validity. To see this, consider the following vignettes,

(17) Martha: 'You know, I really think that creme brulee is no more delicious than sticky toffee pudding.'

Lesley: 'Oh, so you think sticky toffee pudding is at least as delicious as creme brulee?'

Martha: 'No, I didn't say that. It's just that they're different. I really can't compare them.'

(18) Maxine: 'I've come to the conclusion that Scholes was no more effective than Maldini.'

Winston: 'Oh, so you think Maldini was at least as effective as Scholes.' Maxine: 'No, I didn't say that. I just don't think it's possible to meaningfully compare their efficacy.'

Again, these vignettes will feel familiar to critics of Comparability, who have long recognised that we are often prone to demure from making comparisons in exactly the way that Martha and Maxine demure in (17) and (18). Neither Martha nor Maxine seems to be making any kind of egregious reasoning error, or exhibiting any kind of incoherent preferences or attitudes. Indeed, their attitudes and judgements seem to be perfectly reasonable and natural. We aren't compelled to regard Martha or Maxine as wrong headed or inconsistent or deranged in any way on account of their preferences/judgements in (17) and (18). To illustrate, compare (17) and (18) to blatant instances of bad reasoning or incoherence, such as (19) Magnus: 'I think creme brulee tastes better than chocolate cake, even though chocolate cake tastes at least as good as creme brulee.'

(20) Mary: 'I think Scholes was better than Maldini, Maldini was better than Rivaldo, and Rivaldo was better than Scholes'.

It seems clear that assertions like (19) and (20) necessarily express some kind of confusion or incoherence, in a sense which is not true of (17) and (18). Martha and Maxine seem like reasonable, clear headed people. Magnus and Mary do not. So while some instances of No *F*-er than might seem like good pieces of reasoning, there are also counterexamples to the scheme that seem like perfectly reasonable pieces of reasoning.

It should also be emphasised here that simply finding some instances of an inference scheme that would be widely accepted as good pieces of reasoning is not very strong evidence for that inference scheme being part of the logic of our natural language in any interesting sense. We know, for instance, that people are prone to treating the relation of evidential relevance as transitive (see e.g. Sotos et al (2009)), and so accept many inferences of the form 'A is evidentially relevant to B and B is evidentially relevant to C, so A is evidentially relevant to C' as good pieces of reasoning. This doesn't lead us to think that transitivity is part of the logical structure of our natural language notion of 'evidence'. Part of the reason for this is that our best systematic formalisations of that folk notion are inherently non-transitive, and we can easily talk ourselves into seeing that, although inferences relying on transitivity might seem unproblematic, there are actually many cases where they are problematic. The case of Comparability is analogous. There are some natural language inferences, like e.g. (16), that are

instances of No F-er Than, and that many folks would naturally accept as good pieces of reasoning. But that doesn't imply that No F-er Than is part of the logic of natural language comparative constructions, since we can easily think of cases like e.g. (17) and (18) where No F-er Than condemns what would look to many like a perfectly good piece of reasoning.

### 3.3 F Things Are F-er Than Non-F Things

DNZ's third argument for Comparability takes the apparent validity of the following inference scheme as a premise,  $^{10}$ 

**Strong Monotonicity:** For any x and any y, if x if F and y is not F, then x is F-er than y.

To illustrate the apparent validity of Strong Monotonicity, consider the following examples,

(21) 'John is a talented musician, but Elizabeth is not a talented musician. So John is more talented than Elizabeth.'

(22) 'The Honda is fast. The Ford is not fast. So the Honda is faster than the Ford.'

(23) 'The cake tastes good. The soup does not. So the cake tastes better than the soup.'

 $<sup>^{10}</sup>$ DNZ use a qualified version of Strong Monotonicity to avoid counterexamples arising from cases where one of x and y fails to be F to any degree. As I mentioned in the article, I am always assuming that F applies meaningfully to the relevant items, and so am happy to work with the unqualified version of the principle.

These all seem like sound pieces of reasoning that collectively support the apparent validity of Strong Monotonicity. Similarly, one can also consider the degree-modified formulation of Strong Monotonicity stated below, where 'V' stands for any positive degree modifier such as e.g. 'very', 'somewhat', 'pretty', 'quite'.

**Strong Modified Monotonicity:** For any x and any y, if x if V F and y is not V F, then x is F-er than y.

The plausibility of Strong Modified Monotonicity is likewise illustrated by a range of examples, such as

(24) 'John is a very talented musician, but Elizabeth is not a very talented musician. So John is more talented than Elizabeth.'

(25) 'The Honda is extremely fast. The Ford is not extremely fast. So the Honda is faster than the Ford.'

(26) 'The cake tastes pretty good. The soup does not. So the cake tastes better than the soup.'

I am happy to accept the validity of both Strong Monotonicity and Strong Modified Monotonicity for current purposes. DNZ contend that both of these principles strongly support Comparability. I turn now to assessing their arguments for this claim. For brevity, I restrict my attention to Strong Modified Monotonicity, but the following discussion applies equally to ordinary Strong Monotonicity. The argument from Strong Modified Monotonicity to Comparability begins with the observation that the former entails the existence of 'bottlenecks' across which no incomparability can occur. For instance, Strong Modified Monotinicity entails that all very funny people are strictly funnier than, and hence comparable to, all people that are not very funny. In essence, degree-modified positive forms of gradable adjective divide populations into 'levels', and Strong Modified Monotonicity entails that there can be no inter-level incomparability, i.e. it entails that incomparability is a purely intra-level phenomenon. A somewhat funny person can only be incomparable (in their funniness) to other somewhat funny people, and similarly for extremely funny people, slightly funny people etc. DNZ take this consequence to be in tension with the idea that there can ever be any meaningful amount of incomparability.

If there is F-incomparability, it is best explained by the difficulty of making tradeoffs between dimensions of F e.g., two careers that are each better than the other in different respects, where our use doesn't privilege any particular weighting of these respects in such a way that one of them gets to count as better overall. One would expect to be able to find such tradeoffs anywhere in the domain: if two good clarinet careers differing only by \$10 in annual salary are each incomparable with some law career, because better in some ways and worse in others, it seems extremely implausible that subtracting the same amount from each salary, until one of the two clarinet careers is no longer good, should take us to a pair of clarinet careers that are not both incomparable with any law career. Each of the resulting clarinet careers, and in much the same pattern that generates incomparability everywhere else in the domain. (Dorr et al (2022): 439-440)

To further clarify the idea here, suppose we have three careers  $x, x^+$  and y such that (i)  $x^+$  is the same as x except that it carries a salary that is \$10 greater than that of x, (ii) x and  $x^+$  are both incomparably good to y, and (iii)  $x, x^+$  and y are all good careers. Now start subtracting money from the salaries of x and  $x^+$  until you obtain careers  $x'^+$  and x' (that also differ by only \$10) such that x' is no longer good but  $x'^+$  is. Stong Montonicity then implies that there does not exist any y' that is incomparable to both x' and  $x'^+$  since y' is either good or not, and if it is it's better than x' and if it's not it's worse than  $x'^+$ . And this looks implausible. If any pair of law careers differing by \$10 are both incomparable to some clarinet career.

To respond to this argument, I'll focus first on the premise that there exists a pair of careers, x' and  $x'^+$  differing by only \$10 such that one is good and the other is not. It's important to note in this regard that DNZ do not take themselves to be committed to any particular theory of vagueness. The only relevant explicit assumption that they make in this context is the law of excluded middle (LEM), so in evaluating this premise, we can make use of any approach to vagueness that respects LEM. Consider, for instance, Fine's (1975) supervaluationist theory of vagueness (which satisfies LEM). On this semantics, there will generally be a difference between the truth values of the sentences

(27) Determinately, there exists a pair of careers x' and  $x'^+$  differing by only \$10 such that one is good and the other is not good.

(28) There exists a pair of careers x' and  $x'^+$  differing by only \$10 such that one is determinately good and the other is determinately not good.

Specifically, (27) will be deemed true (because in every admissible valuation, there exists a pair of careers separated by \$10 such that exactly one of them is good) but (28) will be deemed false (because there's no pair of careers separated by \$10 such that every admissible valuation deems one of them to be good and the other to be bad). Recall, the premise in question states that there exists a pair of x' and  $x'^+$  differing by only \$10 such that one is good and the other is not. In Fine's semantics, this is ambiguous between (27) and (28), and only (27)is true. Furthermore, DNZ's argument seems to rely on (28). For, it assumes that if I take a pair of careers x and  $x^+$  that are both determinately good and that differ by only \$10, then it's possible to decrease the salaries of x and  $x^+$ until we obtain another pair of careers x' and  $x'^+$  that also differ by only \$10 and are such that  $x'^+$  is determinately good but x' is determinately not. But given the falsity of (28), that's actually not possible, since there do not exist any careers differing by \$10 such that one is determinately good and the other is determinately not good. In fact, since Fine's theory is explicitly constructed to allow for the possibility of higher order vagueness, we won't even be able to find a pair of careers x' and  $x'^+$  differing by \$10 such that one is determinately good and the other is borderline good/not good. So adopting something like Fine's supprvaluationist approach to vagueness allows one to deny that it will ever be possible to find a specific pair of careers differing by \$10 that lie on either side of the bottleneck induced by the modifier 'good'. This in turn makes it possible to avoid the conclusion that there exist specific careers  $x'^+$  and x'that differ by \$10 such that there does not exist any y' that is incomparable to both x' and  $x'^+$ , even while accepting both Strong Monotnicity and the truth of (27). More precisely, one can accept the truth of (29) below while rejecting  $(30)^{11}$ 

<sup>&</sup>lt;sup>11</sup>Note that supervaluationist theories of vagueness aren't unique in allowing us to distinguish between (27)/(28) and (29)/(30) whilst respecting LEM. Any semantics that assigns maximal truth values to all classical tautologies and allows disjunctions to have maximal

(29) Determinately, there exists a pair of careers x' and  $x'^+$  differing by only \$10 such that there does not exist any y' that is incomparable to both x' and  $x'^+$ .

(30) There exists a pair of careers x' and  $x'^+$  differing by only \$10 such that, determinately, there does not exist any y' that is incomparable to both x' and  $x'^+$ .

These observations also facilitate a response to the following objection to the small improvements argument,

The bottleneck model is also quite dialectically awkward for proponents of the small improvement argument, since the mode of thinking that leads them to accept the premises of that argument for particular trios of objects does not seem to control in any way for the possibility that one of the objects is on the opposite side of a bottleneck from the other two (and thus comparable with both of them). Positing the bottleneck would thus require conceding that the relevant mode of thinking is unreliable, leaving it unclear what could justify accepting it in any particular case. (Dorr et al 2022, 440)

Given any pair x' and  $x'^+$  that differ by only a very small improvement, we can in fact rule out the possibility that either of them are located on the opposite side of a bottleneck from the other, as long as we adopt a conception of vagueness that allows us to deny the relevant analogues of (28) and (30) while accepting the relevant analogues of (27) and (29). Once we fix a particular pair

truth values when none of their disjuncts do can in principle acheive the same effect. One example of such an alternative semantics would be e.g. a Boolean valued semantics (see e.g. Wu (2022)).

of careers x and  $x^+$  differing by only \$10, we can in fact be sure that they aren't separated by a bottleneck, as long as we reject (28). So we need not worry that any specific instance of the small improvements argument fails to control for the possibility of a bottleneck dividing the relevant items.

DNZ also contend that, given the truth of Strong Modified Montonicity, bottlenecks will be extremely widespread. They write,

Indeed, it is rather tempting to think that whenever 'x is more F than y' has a true interpretation, we can find some candidate interpretation of 'VF', for some positive degree modifier V (e.g., 'very'), that makes 'x is V F and y is not V F' true, and is compatible with the given interpretation of 'more F' (Dorr et al 2022, 441)

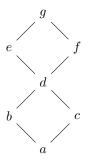
Applying this idea to our previous example, the worry is that even if \$10 is never enough to make the difference to whether any particular career is good or not, it will make the difference to whether a career is V good, where V is some positive modifier like 'very' or 'pretty' or 'extremely'. If that's right, then it seems to crowd out the possibility of incomparability since for any alleged instance of the small improvements argument, we can find a suitable modifier V such that V F induces a bottleneck between x and  $x^+$ , thereby precluding the alleged incomparability. However, I don't think it's plausible that there exists any natural language modifier V such that it'd be compelling to say that a law career x paying \$150,010 is V good but an identical career  $x^+$  paying \$150,000 is not. For instance, if someone were to say ' $x^+$  is extremely good but x is only very good' I'd be inclined to disagree and respond that the careers are basically the same – if one had to choose between them, they might as well choose  $x^+$  because it is ever so slightly better, but there's no meaningful sense in which  $x^+$  clears the threshold for being extremely good but x doesn't. One might respond here by saying that even if this is correct in most contexts, we just need to find one context where it seems right to say that  $x^+$  is V good by x isn't. But I can't think of any V or any context where that seems natural. Plausibly, the reason for this is that, just as the predicate 'good' is vague, so are all modified predicates of the form V good, where V is a natural language modifier. Now, one could try to cook up some highly artificial contexts in which it might seem more plausible to say that  $x^+$  is V good but x isn't, for instance by imagining a scenario where the value of money increases exponentially once one crosses \$150,000. But of course, in that kind of context, it's not just facts about which careers are V good that change, but also the comparative facts about how different careers compare to one another. So one can't use this kind of context to make inferences about our comparative judgements in more realistic situations, since it is so artificial that it completely changes the structure of those judgements. Overall then, it just doesn't seem at all plausible to me that there are so many bottlenecks that all or even most instances of x and  $x^+$  in the small improvements argument are separated by some positive modification of the relevant gradable predicate F.

Now, DNZ do have one last principle that they invoke to buttress a variant of the preceding arguments, namely

**Strong Superlative Motononicity:** x is one of the most F K's. y is not one of the most F K's, so x is F-er than y.

Strong Superlative Monotonicity may offer another avenue for the defender of Comparability to undermine the possibility of incomparability, via something like the following argument. Given an instance of the small improvements argument, let K be 'the items under consideration', namely  $x, x^+$  and y. Clearly, x is not one of the most F K's since  $x^+$  is F-er than x. Now, either y is one of the most F K's or it's not. If it is, then it must be F-er than x (by Strong Superlative Monotonicty) and if it's not then  $x^+$  must be the most F K (since there are no other K's remaining), in which case  $x^+$  is F-er than Y. Either way, y is comparable to one of x and  $x^+$ . But this argument presupposes that it's always possible to meaningfully divide the K's into those that are 'amongst the most F' and those that are not. Let's return to the case where x and  $x^+$  are two law careers differing by \$10 and y is some clarinet career that's ostensibly incomparable to both x and  $x^+$ . It seems perfectly plausible to contend that there's no way to meaningfully divide those careers into those that are 'among the best' and those that aren't. If somebody asked me which of the careers are the best, I'd simply say 'well  $x^+$  is better than x, but I can't compare y to either of them'. Now, one may respond that y being among the most F K's simply means that there is no K that is F-er than y. If that's true, then y being incomparable to both x and  $x^+$  entails that it's one of the best careers under consideration, which in turn entails that it's better than x, contradicting the assumption that it's incomparable to both x and  $x^+$ . But it's clear that when we say that y is among the most F K's, we typically don't mean that there's no other K that's F-er than it. For instance, I believe that Black Sabbath were one of the best rock bands ever, even though I think that Led Zeppelin were better, and chips are one of my favourite foods, even though I like ice cream slightly more. When we say that x is one of the most F K's, we typically mean that x is somewhere near the top of the ordering of K's when they are ranked by their F-ness. Clearly, this kind of assertion is vague and is only natural in contexts in which there are many items under consideration, which makes it useful to approximately locate an item within a broad region of the ordering. In contexts where we're only considering e.g. three items, as with the small improvements argument, it feels entirely artificial and misplaced to talk about the 'most F K's' – we don't naturally use that construction in those kinds of contexts. So while Strong Superlative Monotonicty seems to be plausibly valid in natural unconstrained contexts, it's unconvincing to appeal to claims about the implicit logic of these superlative constructions in contexts in which they are not naturally used. So again, the attempt to show that any pair x, y where one is F-er than the other must be seperated by an incomparability bottleneck is unsusceesful.

At this juncture, it is worth making the general observation that comparative language can naturally be employed to describe a vast array of different kinds of order structure, and that there's no obvious reason why these ordering structures can't involve incomparability bottlenecks. Consider, for instance, the order structure below, where elements are ordered according to their F-ness with the most F at the top and the least F at the bottom, and where elements on the same row are incomparable.



It seems that, in describing such a structure, we might naturally describe g as very F, e and f as F, d as somewhat F, b and c as slightly F and a as not at all F. We could use this language while accepting Strong Modified Monotonicity and acknowledging the existence of incomparability bottlenecks. There doesn't seem to be anything incoherent or unnatural about using comparative constructions and degree modified adjectives to describe this structure in this way.

Overall then, the picture that we're left with is that there are indeed incom-

parability bottlenecks scattered throughout the orderings induced by our natural language gradable predicates. These bottlenecks effectively stratify those orderings into levels such that every item is ranked higher than all the items in the lower levels. But within any given level, there can be a great deal of incomparability. Because the boundaries between these levels are always vague, and because the positive modifiers of natural language do not draw arbitrarily fine distinctions, this is all perfectly compatible with incomparability being rampant throughout natural language. Returning to the example of funniness: funny people are funnier than only slightly funny people. Many funny people are incomparably funny to other funny people and many slightly funny people are incomparably funny to other slightly funny people. Stewart Lee and Bill Hicks are both extremely funny, but one can't make a meaningful comparison of their comic prowess, even though one can definitively say that they are funnier than all but the very best comedians. This all sounds perfectly natural and correct. In sum then, I take myself to have shown that (i) the strongest arguments against Comparability survive unscathed in the face of DNZ's replies, (ii) DNZ's positive arguments in support of Comparability do little to countervail the arguments against it, and (iii) there are further powerful arguments against Comparability that go beyond those considered by DNZ. The case for Incomparability is stronger than the case for Comparability.

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