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A SIMPLE LOGIC FOR COMPARISONS AND VAGUENESS

ABSTRACT. I provide an intuitive, semantic account of a new logic for comparisons (CL), in which atomic statements are assigned both a classical truth-value and a "how much" value or *extension* in the range [0, 1]. The truth-value of each comparison is determined by the extensions of its component sentences; the truth-value of each atomic depends on whether its extension matches a separate *standard* for its predicate; everything else is computed classically. CL is less radical than Casari's comparative logics, in that it does not allow for the formation of comparative statements out of truth-functional molecules. I argue that CL provides a better analysis of comparisons and predicate vagueness than classical logic, fuzzy logic or supervaluation theory. CL provides a model for descriptions of the world in terms of comparisons only. The sorites paradox can be solved by the elimination of atomic sentences.

In his recent book on vagueness, Timothy Williamson (1994) attacks accounts of vagueness arising from either fuzzy logic or supervaluation theory, both of which have well-known problems, stemming in part from their denial of classical bivalence. He then gives his own, epistemic account of vagueness, according to which the vagueness of a predicate is fundamentally a matter of our not knowing whether or not it applies in every case. My main purpose in this paper is not to criticize Williamson's positive view, but to provide an alternative, non-epistemic account of predicate vagueness, based on a very simple logic for comparisons (CL), which preserves bivalence. Much more will remain to be said about comparisons and vagueness. In particular, I will not try to provide a thorough mathematical treatment of CL, a comprehensive semantics for comparatives in natural language, or an analysis of vague existence and identity.

1. COMPARISONS ARE NOT SIMPLE RELATIONAL SENTENCES

Look at sentences (1) and (2).

(1) Frank is more sincere than Suzanne.

(2) Frank is a brother of Suzanne.

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Introductory treatments of predicate logic treat comparative sentences like (1) and simple relational sentences like (2) as if they were logically alike. Both are symbolized as atomic two-place predications of the form Fab.

This is not a mistake, exactly. It is appropriate for the teaching of classical logic to treat most comparisons as simple relational sentences – and no textbook claims to provide a complete analysis of comparisons in doing so. But if anyone infers that a symbolic sentence like Fab represents the full logical form of a sentence like (1), then that is a mistake.

Here is an argument. It is not quite successful, but it points in the right direction. Look at sentence (3).

(3) Frank is more sincere than Frank.

Sentence (3) would normally be translated into a symbolic sentence of the form Faa. But (3) is plainly necessarily false, while the symbolic sentence Faa is not. Therefore, a string like Faa cannot be the complete, correct analysis of (3).

Here is an objection to my argument. Consider sentence (4).

(4) Frank is a brother of Frank.

(4) is a clear example of a simple relational sentence. Yet (4) would also normally be symbolized by something of the form Faa, and (4) is also necessarily false. By my reasoning, then, Faa is no better an analysis of (4) than of (3). So I have not yet successfully distinguished between the logical form of comparisons and that of simple relational sentences.

The situation is still worth exploring, however. A string like Faa clearly is a good translation of (4). What, then, accounts for (4) being necessarily false, while Faa is not? The usual (and plausible) answer is that some sentences are necessarily false, not in virtue of their logical form, but in virtue of the meanings of the non-logical terms they contain. For example, (4) is always false because the relation denoted by "brother" happens to be irreflexive – it is a function of the meaning of the word that no one can be his own brother. The same kind of thing might also be said of sentence (3): it is necessarily false, not because of its logical form, but because the phrase "more sincere" names an irreflexive relation.

There is an obvious difference, however. The meaning of more sincere is complex in a way that the meaning of "brother" is not. It is a function of the meanings of its two components, "more" and "sincere". Moreover, it is plainly the meaning of "more", rather than the meaning of "sincere", which works to make the relation named by "more sincere" an irreflexive one. If sentence (3) is closer to a purely logical falsehood than (4), this is why.

The term that *makes* it false, namely "more", is more of a logical term than either "brother" or "sincere", since prefixing "more" turns *any* adjective into an irreflexive relation. Less obviously, perhaps, any predicate at all, including sortals, even natural kind terms, can be irreflexivized in this way, if we allow as variants of "more" such things as "more of", "more a case of", and so on. Thus the sentences

(5) Frank is more of a liberal than Suzanne,

and

(6) The platypus is more of a mammal than the armadillo,

seem to be normal comparisons, while

(7) Frank is more of a liberal than Frank,

and

(8) The platypus is more of a mammal than the platypus,

seem just as self-contradictory as (3).[1]

We might then view the word "more" as a kind of operator that turns any one-place predicate into an irreflexive relation, since "a is more (of an) F than a" is always necessarily false.[2] It can also be shown that the "more F" relation is always transitive and asymmetric, since "a is more F than b, b is more F than c, and c is more F than a" and "a is more F than b, and b is more F than a", are also always necessarily false, for all normal substitutions of a, b, c, and F. Alternatively put, the "more F" relation is transitive because the argument

a is more F than b
b is more F than c
Therefore, a is more F than c

is always valid, and asymmetric because the argument

a is more F than b
Therefore, b is not more F than a

is always valid.

The algebraic properties of "less", applied to predicates, are just the same as those of "more". "At least" and "at most" each turn predicates into relations which are reflexive and transitive. "As" works reflexively, symmetrically, and transitively – any predicate becomes an equivalence relation. In addition, various entailments can be shown to obtain among appropriate instantiations of the five different types of comparison. For example,

a is more *F* than *b*
Therefore, *b* is less *F* than *a*

is always valid. So are

b is less *F* than *a*
Therefore, *a* is at least as *F* as *b*,

and

a is more *F* than *b*
b is as *F* as *c*
c is at least as *F* as *d*
Therefore, *d* is at most as *F* as *a*.

We can also look at the entailments which occur between comparisons and other, non-comparative sentences that contain the same non-logical terms. Consider the following argument.

(9) Frank is more sincere than Suzanne.
Suzanne is sincere.
Therefore, Frank is sincere.

This argument is plainly valid. But the usual scheme of translation would symbolize the argument in the form:

Fab
Gb
$\therefore Ga$

which is plainly not valid. Therefore, again, the usual scheme does not adequately represent the logical form of these sentences.

Someone could still object that such an argument really is not formally valid, that it is not just the logical forms of its premises and conclusions, but also the particular meanings of the terms “sincere” and “more sincere”, that make it appear to be valid. But again, *any* argument of the same form:

a is more *F* than *b*
b is *F*
Therefore, *a* is *F*

will be a valid one, regardless of what predicate F stands for.

We need, then, a new formal analysis of comparisons – one which accounts for the essential algebraic properties of comparative expressions, and also for the entailments which hold among comparisons, and between comparisons and other sentences.

2. COMPARISONS ARE MOLECULAR SENTENCES

At the deepest level, comparisons are best understood as relations between facts, not objects. The basic terms of comparison, "more than", "less than", and "as much as" should be seen not as predicate operators, but as something like sentential connectives. For example, Sentence (1) would be analyzed in the form:

(1′) Frank is sincere more than Suzanne is sincere,

or, quasi-formally:

$$Fa > Fb,$$

where Fa and Fb symbolize the two English sub-sentences in the usual way, and the undefined sign $>$ is intended to stand for their transformation into a single comparison.

That this is the general shape of a correct analysis cannot be proven until a full analysis is given. For the moment, it may help to note that the "analyzed" sentence (1′) above is also a normal English sentence, equivalent in meaning to (1). It would at least be odd if, at the deepest level of analysis, sentence (1) turned out to be a simple atomic while (1′) had a different, plainly molecular structure.

The difference between the two sentences is not essentially one of logical form, but just a matter of syntactic compression. After all, no one takes the fact that the word "not" usually occurs next to predicates, not in front of whole sentences, to entail that "not" is fundamentally a predicate operator. We just find it more convenient not to have to say "it is not the case that" every time we want to deny something. It is for similar, pragmatic reasons that we ordinarily attach terms of comparison to predicates in surface grammar: it allows us to avoid repeating the predicates, without creating any important ambiguities. This is especially useful because most of the comparisons we are make are similar in form to (1), i.e. involve a single predicate and different individual terms. Sometimes, however, we do make comparisons involving just one individual term and more than one predicate, and these cannot be compressed in the same way. For example:

(10) Suzanne is no more sincere than she is well-mannered,

and

(11) Frank listens to radio more than he watches TV.

No analysis in terms of predicate operators alone would seem to be possible for these cases.

In any event, some comparisons are irreducibly sentential:

(12) Frank is at least as wide around as Suzanne is tall.

(13) Duluth is as cold as Miami is hot.

(14) It rained on Friday less than it did on Thursday.

Unless we say that the terms of comparison have meanings in these sentences different from their usual ones, we have no alternative to a molecular analysis for all comparisons.[3]

3. COMPARISONS ARE NOT TRUTH-FUNCTIONS

Comparisons are not truth-functional. The truth of a sentence like (1′) is largely independent of the truth of its component sentences, although certain possibilities are ruled out. "ϕ more than ψ" must be false if ϕ is false and ψ is true, and true if ϕ is true and ψ is false, but what if ϕ and ψ are both true or both false? There may still be a determinate answer, but it cannot depend solely on *whether* ϕ and ψ are true. What it does depend on is *how much* ϕ and ψ are true. That is, whether Frank is more or less or as sincere as Suzanne depends on how much, not whether, each of them is sincere. If Frank is sincere to a certain degree or extent, and Suzanne is sincere to a lesser degree or extent, then Frank is more sincere than Suzanne.

To turn this commonsense idea into a workable formal analysis requires a semantic theory that includes some way to represent determinate answers to the question "how much", just as the two truth values represent answers to the question "whether". Presumably, the new semantic values can be given as numbers, or at least as members of an ordered set. A sentence "ϕ more than ψ" will then be counted as true just in case the new value assigned to ϕ is greater than the new value of ψ, and similarly for the other forms of comparison. This will entail a certain idealization, since in natural language all kinds of scales, some numeric and some not, are used to say how much one thing or another is so.

4. COMPARISONS ARE NOT EXACTLY "FUZZY"

Much of the philosophical discussion of fuzzy logic has concentrated on the question of its adequacy as a "logic of vagueness".[4] But it has also been considered as a tool for analyzing comparisons. Where traditional systems of logic often use 0 and 1 to represent the truth and falsity of sentences, fuzzy logic uses the whole range of real numbers from 0 to 1 to represent all possible "degrees of truth". The extensions assigned to predicates are not ordinary sets, but rather "fuzzy sets", in which each element is assigned a specific degree of membership. The degree of truth for an atomic sentence $\phi\alpha$ will be the same as the degree of membership α is assigned in the extension of ϕ. The valuation rules for negations and conjunctions are simple generalizations of the classical rules. To find the value of some sentence $\neg\alpha$ one just subtracts the value of ϕ from 1. For conjunctions, one takes the minimum of the values of the two conjuncts. The rule for universal sentences is analogous.[5]

A simple rule for comparisons can be added to this basic system. Just let $(\phi > \psi)$ take the value 1 whenever the value of ϕ is greater than the value of ψ, and 0 otherwise. The other forms of comparison can be given similar rules of their own, or defined in terms of the "more than" comparisons, as follows:

ϕ less than ψ: $(\phi < \psi) =_{\text{df}} (\psi > \phi)$,

ϕ at least as much as ψ: $(\phi \geq \psi) =_{\text{df}} \neg(\psi > \phi)$,

ϕ at most as much as ψ: $(\phi \leq \psi) =_{\text{df}} \neg(\phi > \psi)$,

ϕ as much as ψ: $(\phi = \psi) =_{\text{df}} (\neg(\phi > \psi)\ \&\ \neg(\psi > \phi))$,

Suppose that Frank is 0.9 degrees sincere, and Suzanne 0.3, on a scale of 0 to 1. The representative of the sentence "Frank is sincere" would then receive the value 0.9 (meaning something like "very true"), and the representative of "Suzanne is sincere" would get the value 0.3 (for something like "mostly false"). The image of "Frank is more sincere than Suzanne" would get the value 1 ("completely true"), as would the image of "Frank is at least as sincere as Suzanne", while the images of "Frank is as sincere as Suzanne", "Frank is at most as sincere as Suzanne", and "Frank is less sincere than Suzanne" would all receive the value 0 ("totally false").

These are not entirely implausible results. Fuzzy logic's notion of degrees of truth seems to answer the need for a "how much" semantics of comparisons in a correct and reasonably intuitive way. But the abundance of alleged truth-values creates immediate intuitive objections to fuzzy logic. While people sometimes say that some sentence is "very true" or "not entirely true", and while subjects in psychological experiments can be brought to assign numerical degrees of truth to sentences, this does not mean that we actually think or speak in a genuinely many-valued way. In fact, assignments of degrees of truth are not even meaningful unless *bi*valence, in the sense that every interpreted sentence must be either true or not true in the ordinary way, is taken for granted. For when we say that "the sky is blue" is very true, this is to say no more than that the sky is very blue, or that "the sky is very blue" is simply true. We can also say that "the sky is blue" is occasionally true, which means that the sky is occasionally blue, or that "the sky is blue" is true in California, which means that the sky is blue in California. These are all just ways of qualifying sentences in quotation. Far from necessitating whole new kinds of truth-values, they rely on the classical notion of truth to make sense.

What saves fuzzy logic from frank incoherence is the fact that it is really covertly bivalent. For the only way that the notions of validity arid consequence can be defined for such a system involves designating subsets of the range of degrees as sufficiently "truth-like" to play the formal role usually played by truth in those definitions. And the result can be understood only by thinking of the designated values as corresponding to ordinary truth: If "the sky is blue" is sufficiently truth-like to be used in inferences, then it must at least be true enough to be asserted. And if one can say (in ordinary English) that the sky is blue, then one can say that "the sky is blue" is just plain true.

Even if fuzzy logic were recast as an explicitly bivalent system, its basic math would still require a uniform definition of truth in terms of degrees of truth. But no such definition could be correct – the questions "how much" and "whether" are not tied together so rigidly. People can disagree or change their minds or withhold judgement as to *whether* some sentence is true, even while their beliefs about *how much* it is true are agreed upon or held fixed. For example, you and I might agree in assigning the sentence, "the moon is full" the values: 0 at the new moon, 0.5 at the half moon, and 1 when the moon is at its fullest, and we might also agree that the moon is full to degree 0.9 right now, and we still might disagree as to whether that sentence is true. We might also hold that different heights were necessary or sufficient for tallness, say, and still agree in every case about how tall someone is, and whether one person is taller than another. But in fuzzy logic, our sufficiency judgements would always have to be the same, and always have to follow automatically from our judgements of degree.

5. COMPARISONS ARE NOT COMPLETELY "GAPPY"

An alternative approach to comparisons relies on supervaluation theory.[6] The idea is that there are truth-value "gaps" for propositions involving ordinary, vague predicates. Thus, the extensions of these predicates are said to comprise three sets: a set of objects to which the predicate definitely applies, a set to which it definitely does not, and a third, intermediate set for which there is no determinate fact of the matter. Comparisons may easily be defined for cases where two objects fall in different of these sets. If Ralph is definitely tall, and Bill is intermediate, or definitely not tall, then Ralph is taller than Bill, etc. But what about the comparisons of objects which belong to the same set? Surely, 6′4″ Ralph can be taller than 6′3″ Bill, even though both are tall. Here, the technique is to refer to hypothetical valuations, which represent the different ways that the vague atomic statements in question could be made precise. Since every totally precise interpretation that made Bill tall must also make Ralph tall, but some such interpretations would make Ralph tall without making Bill tall, we can say that Ralph is taller than Bill. Thus the ordering of heights, which fuzzy logic represents in terms of different *degrees* of membership in a single extension for "tall", is represented (roughly speaking) as a fact about *possibilities* of membership in its ordinary, classical extension.

This approach avoids the problems that result from having infinitely many truth-values, and from ordinary truth and falsity being uniformly tied to this or that degree of truth. But it is ultimately no more satisfactory that the fuzzy-logical approach, due to the problem of "higher-order vagueness". If there is no definite border between tall and not-tall, why should there be a definite border between tall and neither-tall-nor-not-tall? The problem of vagueness is not that there are three determinate classes which define the extension of each (vague) predicate, rather than two. The problem is that the applicability of predicates increases and decreases more-or-less *continuously*. There are no simple, determinate classes at all that can be used to define their extensions. As the temperature rises on a spring day, we have less and less of an inclination to say that it is cold outside, until, perhaps, we finally have no inclination to say that it is cold at all. But nothing "clicks", either in the world or in the language, at the "point" where we lose all inclination to say that it is cold. The point exists only with reference to a kind of decision that one makes, and might make differently on another day, or that might have been made differently on this day by another person.

Here is a related question. Why is it that hypothetical valuations are able to impose an ordering on objects? The natural answer is that the objects already have the property in question to a greater or lesser degree. It seems very odd to try to represent comparative facts, such as the fact that one tall person is taller than another, as something metalogical or metalinguistic. Surely, these are ordinary, material facts about the world, existing at no higher a logical level than the fact that some person is tall *simpliciter.* In fact, as between a man of 5′10″ and a man of 5′11″, one would think that the comparative fact, i.e. that the second man is taller, has, if anything, *more* determinate reality than the simple fact, if it is one, that the second man is tall. Considerations of technical adequacy aside, one would hope for a theory of comparisons that accounted for this intuition.

What is basically right about the supervaluation approach is that there must be two simultaneous forms of valuation in a logic of comparisons. One is to establish a partial ordering of the objects – this alone will determine the truth of comparative statements. The other is to decide the classical extension, according to the principle that an object can be included in this set only if all objects at least as highly ordered relative to that predicate are also in the set. This suggests a relative, "how much" value for each object, plus a separate "cut-off" value attached to the predicate itself.

6. A MINIMAL COMPARATIVE LOGIC

CL is a minimal comparative logic based on the "how much" idea, but different from both fuzzy logic and supervaluation theory. In CL, the interval from 0 to 1 is used, not as a new set of truth values, but as an artificial scale of sub-values or *extensions* for atomic sentences. Every interpreted predicate letter in the language of CL is also assigned a minimum *standard* in the same range. The truth-value of each atomic sentence is then determined by whether its extension is at least as great as the standard for its predicate. The truth-values of comparisons depend only on the sameness or difference of the extensions of their component sentences. Everything else is computed classically, on the basis of truth-values alone.

CL has the same syntax as a classical logical language L, except that the symbol $>$ serves as a two-place logical connective for atomic sentences (the other comparisons are defined in the obvious way, as in fuzzy logic for comparisons). CL is thus less extensive than Casari's (1987) smallest system ("restricted" comparative logic), which allows comparisons to be formed between non-quantified molecular statements as well as atomics. I cannot make intuitive sense out of assigning degrees to conjoined, disjoined, and negated sentences – these really are just truth-functions, in my view. In any event, such an expansive system for comparisons is unnecessary for my purpose in this paper.[7]

The semantics of CL are more different. Instead of the usual formal definition of interpretations as ordered pairs, in CL an interpretation is an ordered triple I containing a domain D, an *extension* function e^*, and a *standard* function s^*. s^* takes predicate letters to members of E, the interval [0, 1]. e^* takes constants to members of D, and n-place predicate letters to functions from n-tuples of members of D into E.

The extensions of atomic sentences are defined by the following rule:

If ϕ is a sentence of the form $\psi t_1, \ldots, t_n$,

$$e(\phi) = e^*(\psi)(e^*(t_1), \ldots, e^*(t_n)).$$

(For example, if the domain includes Doris, and the extension function e^* assigns Doris to the constant a, and also assigns to the predicate letter F some set including the pair ⟨Doris, 0.84⟩, then $e(Fa) = 0.84$.)

This is the new rule for the truth-values of atomic sentences:

If ϕ is a sentence of the form $\psi t_1, \ldots, t_n$,

$$I(\phi) = \begin{cases} 1, & \text{if } e(\phi) \geq s^*(\psi) \\ 0, & \text{if } e(\phi) < s^*(\psi). \end{cases}$$ [8]

And this is the new rule for comparisons:

If ϕ is a sentence of the form $(\psi_1 > \psi_2)$,

$$I(\phi) = \begin{cases} 1, & \text{if } e(\psi_1) > e(\psi_2) \\ 0, & \text{if } e(\psi_1) \leq e(\psi_2). \end{cases}$$

Here is an illustration. Once again, let F stand for the predicate "sincere", and let a and b represent Frank and Suzanne, respectively. If Frank deserves a 0.9 for sincerity (on a scale of 0 to 1), and Suzanne gets a 0.7, then let the triples ⟨F, Frank, 0.9⟩ and ⟨F, Suzanne, 0.7⟩ be included in e^*. By the extension rule for atomic sentences, it follows that $e(Fa) = 0.9$. If we say that a person must rate a 0.75 or over to be properly called sincere, then the pair ⟨F, 0.75⟩ should be included in s^*. Since $e(Fa) > s^*(F)$, $I(Fa) =$ 1, thus the image of the sentence "Frank is sincere" gets evaluated as true in this interpretation. But the image of "Suzanne is sincere" is evaluated as false, since $e(Fb) < s^*(F)$, so $I(Fb) = 0$. The image of the comparison "Frank is more sincere than Suzanne" also comes out true, since $e(Fa) > e(Fb)$, so $I(Fa > Fb) = 1$. But the image of "Frank is as sincere as Suzanne" would be evaluated as false, because $e(Fa) \neq e(Fb)$, so $I(Fa = Fb) = 0$. If a different standard of sincerity were stipulated, then the truth-values of the two atomic sentences might be different, but those for the comparisons would be unchanged. If no standard were adopted, then the truth-values of the two atomics would be undefined, but again, the values of the comparisons would be unaffected.

Standards are not parts of the language in the way that the meanings of predicates are usually thought to be. Nor are they language-independent facts. Think of them, rather, as functions of speakers' dispositions to *judge sufficiency.* Thus, while it is part (maybe all) of the meaning of the word "tall" that tallness varies with height, and while the specific height of a person, say, is an ordinary empirical fact, the answer to the question whether that person is tall seems to require a decision about whether his height is sufficient for tallness, under the circumstances. I might normally call a man in his thirties tall only if he is at least six feet in height. Your standard for adult male tallness might be lower or higher than mine – if it is, there can be cases where we both know exactly how tall certain other people are, yet disagree (in good English) about whether they are tall.

There is a simple way of representing extensions and standards in the language of CL – that is, of saying explicitly how much some thing is some way, and how much a thing must be that way in order for it just to be that way. Simply include in the vocabulary a set of *metric constants*: i, j, k, i_1, etc, and let them function as atomic sentence letters. The extensions of these constants in the "how much" interval [0, 1] will be given by the extension function.[9] The formation rule for $>$ will apply indifferently to the constants and atomic sentences. Nothing else needs to be changed. For an example, the inference from "Ralph is 75 inches tall", "72 inches is just tall enough to be tall", and "75 inches is more than 72 inches", to the conclusion "Ralph is tall", could be validly represented as:

$$Fc = i$$

$$\forall x((Fx \geq j) \rightarrow Fx)$$

$$i > j$$

$$\therefore Fc$$

7. SORITES

The philosophical discussion of vagueness began with the sorites paradox, and much current analysis is couched in terms of it. Here is the problem. A man with no hairs on his head is plainly bald; a man whose head is entirely covered in hair is plainly not bald; but it seems that there is no number n such that a man with n hairs is bald, and a man with $n + 1$ is not.

Williamson's epistemic theory of vagueness makes sense as a reaction to this paradox. If one insists on bivalence, then there must be a fact of the matter as to whether a person with each number of hairs is bald. Since we plainly do not have access to such facts, the only apparent option is to say that the facts exist, but we simply cannot *know* what they are.

My view is different. I suggest that we eliminate simple positive sentences (like "So-and-so is bald") altogether, for purposes of the scientific description of the world. There is no ultimate need to use such sentences when one has comparisons available. If one can say *how much* a certain thing is a certain way, and how much everything else is that way, and how much every thing is every other way, then one has described the world completely in terms of its objects and their properties. One does not ever need to say *whether* any simple predication is the case.

Sorites springs from the assumption (implicit in classical logic) that our usual atomic judgements are matters of objective description. But they are not, since they contain the element of standards, which is subjective and variable. It is a fact about the world that Frank is 5 feet, $11\frac{1}{2}$ inches in height. It is not a further fact about the world that Frank is tall (or that he is not tall). It is, rather, in part a fact about a certain speaker that he *judges* someone of that height to be tall. Perhaps this speaker is normal, and his judgement is the same as most other people would make under the same conditions. But it does not need to be. The language allows for eccentric judgements as well. In any case, most such judgements are latent at best. Our standards are *evoked* by questioning or other special practical needs, not required for our understanding of the world. With respect to most of the people and things that one encounters, one simply has no beliefs (and needs none) as to whether they are large or small, young or old, bald or not bald, heaps or not heaps.

My view is deterministic in the relevant sense. There is no hole in the factual fabric of the universe through which the truth-values of atomic sentences escape. It is rather that these sentences are not to be taken entirely seriously in philosophical contexts, because they do not express complete propositions in the first place.[10]

As a practical matter, we use atomic statements because we need to speak and think compactly in a changing world. The attendant vagueness is a practical problem that we can ordinarily handle with little difficulty, because we are sufficiently alike in our perceptions and values. If a friend asks for a large glass of water on a hot day, I will usually give him something more than, say, ten ounces, because that is about how much I usually want when I ask someone for a large glass of water, and I believe that my desires are normal. But if I am worried for some reason that this will not please him (perhaps it is the King who is asking), then I am free to ask for more precise directions, and he is free to order a pint or a quart or whatever he wants, or to tell me that he does not care.

If classical logic is at fault for sorites, comparative logic shows us the way out. CL provides an intuitive model for complete descriptions of the world, without involving those subjective elements required for the interpretation of atomic sentences.

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NOTES

[1] One can also use the adjectival forms of natural kind terms. “More human”, for example, will do as well as “more of a human being”. For that matter, if one does not want to say that an *Australopithecus Africanus* is more human than something else (because one wants to exclude the possibility of its membership in humanity), then “more like a human” will serve. If one does not want to say that human fetuses are less human than, say, Einstein (because one wants to insist that they are included in humanity), then “less clear (or less paradigmatic) a case of humanity” will do. All of these locutions have essentially the same logical role.

[2] More generally, the word “more” turns any n-place predicate into an irreflexive n + 1-place predicate. “Suzanne is less friendly toward Frank than Suzanne is toward Frank’ should seem self-contradictory.

[3] Stalnaker (1977) describes a simple device for turning open sentences into predicates, similar to the lambda notation in Montague grammar. He uses the symbol ˆ, or “cap”, to form a non-quantifying variable binder: If F and H stand for “fat” and “happy”, respectively, then the open sentence $(Fx \ \& \ Hx)$ (“x is fat and x is happy”) can be transformed into the complex predicate $\hat{x}(Fx \ \& \ Hx)$ (“fat and happy”). The English sentence “John

is fat and happy" may then be translated either in the usual form (Fb & Hb), or, more perspicuously, as $\hat{x}(Fx \,\&\, Hx)b$. Similarly, "John is not fat" could be translated either as the usual $\neg Fb$, or as $\hat{x}\neg Fxb$, where the second represents the complex English predicate "not fat" correctly as a single thing.

This device applies straightforwardly to the analysis of comparisons. If an open sentence like "x is more sincere than y" is represented as $(Fx > Fy)$ – pending some definition of the symbol $>$, of course – then the English relational term "more sincere" can be represented separately as $\hat{x}\hat{y}(Fx > Fy)$. So, while sentence (1′) may best be symbolized as $(Fa > Fb)$, a more perspicuous image of sentence (1) would be $\hat{x}\hat{y}(Fx > Fy)ab$. Thus the general form of a relational sentence could be preserved the most common type of comparison (those involving a shared predicate), while the final analysis is still molecular.

[4] Contemporary discussion of fuzzy logic begins with Zadeh (1965, 1971) and Lakoff (1971, 1972) though its basic infinite-valued system was devised by Lukasiewicz (Lukasiewicz and Tarski 1930).

[5] Except that the greatest lower bound of the values of substitution-instances must be used, to make allowance for the fact that, given an infinite domain, there may be no minimum value for the substitution instances of some sentences.

[6] For an introduction, see Kamp 1975 or Fine 1975.

[7] By way of an intermediate system, it might be reasonable to include comparisons between comparisons themselves as well as atomic sentences (and metric constants; see below). Statements like "Suzanne is two feet taller than Frank", or "Suzanne is taller than Frank by more than Frank is taller than Ralph", do make sense. To allow for such things in CL, the following would be added to the extension rules below:

If ϕ is a sentence of the form $(\psi_1 > \psi_2)$,

$$e(\phi) = e(\psi_1) - e(\psi_2).$$

Classical truth-functions are unaffected.

[8] There is something arbitrary about assigning minimums as standards in all cases. In ordinary discourse, on might wish to say that something was some way just in case it was that way to a strictly *greater* extent than whatever was stipulated. If it were worth the trouble to incorporate this possibility into CL, interpretations would need to be redefined to permit each predicate ϕ either an inclusive standard $s^*(\phi)$ or an exclusive standard $sx^*(\phi)$, but not both. The relevant valuation rule would be replaced with:

If ϕ is a sentence of the form $\psi t_1, \ldots, t_n$,

$$I(\phi) = \begin{cases} 1, & \text{if } e(\phi) \geq s^*(\psi) \text{ or } e(\phi) > sx^*(\psi) \\ 0, & \text{if } e(\phi) < s^*(\psi) \text{ or } e(\phi) \leq sx^*(\psi). \end{cases}$$

[9] This simple system shares with fuzzy logic the artificial use of just one metric: all "how much" questions being answered on an arbitrary, uniform scale of 0 to 1. Therefore, CL cannot resolve the ambiguity of an English sentence like, "Sally is taller than Jack is wide around", which may be true if both sub-propositions are considered on a scale of 0 to 1, but false if both are to be measured in feet. A fuller treatment of comparisons would need to include units of measurement.

[10] I will not say that every predicate of English is imprecise. We use the term nonagenarian, for example, to mean between ninety and one hundred years old, exactly. I should say

rather that all predicates are either vague like "bald", or implicitly comparative like "nonagenarian". In either case, whatever can be said objectively turns out to have a comparative, not atomic, logical form.

REFERENCES

Casari, E.: 1987, 'Comparative Logics', *Synthese* **73**, 421–449.

Fine, K.: 1975, 'Vagueness, Truth, and Logic', *Synthese* **30**, 265–300.

Kamp, J. A. W.: 1975, 'Two Theories about Adjectives', in E. L. Keenan (ed.), *Formal Semantics of Natural Language*, Harvard University Press, Cambridge, MA.

Lakoff, G.: 1971, 'Linguistics and Natural Logic', in Davidson and Harmon (eds.), *Semantics of Natural Language*, Reidel, Dordrecht.

Lakoff, G.: 1972, 'Hedges: A Study in Meaning Criteria and the Logic of Fuzzy Concepts', *Chicago Linguistic Society* **8**, 183–228.

Lukasiewicz, J. and A. Tarski: 1930, 'Investigations into the Sentential Calculus', in Tarski, *Logic*, *Semantics*, *Metamathematics* (Oxford University, Oxford, 1956).

Stalnaker, R.: 1977, 'Complex Predicates', *Monist* **60**, 327–339.

Williamson, T.: 1994, *Vagueness*, Routledge, London.

Zadeh, L.: 1965, 'Fuzzy Sets', *Information Control* **8**, 000–000.

Zadeh, L.: 1971, 'Quantitative Fuzzy Semantics', *Information Sciences* **3**, 159–176.

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