#### Are generative models structural representations?

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#### Abstract:

Philosophers interested in the theoretical consequences of predictive processing often assume that predictive processing is an inferentialist and representationalist theory of cognition. More specifically, they assume that predictive processing revolves around approximated Bayesian inferences drawn by inverting a generative model. Generative models, in turn, are said to be structural representations: representational vehicles that represent their targets by being structurally similar to them.

Here, I challenge this assumption, claiming that, at present, it lacks an adequate justification. I examine the only argument offered to establish that generative models are structural representations, and argue that it does not substantiate the desired conclusion. Having so done, I consider a number of alternative arguments aimed at showing that the relevant structural similarity obtains, and argue that all these arguments are unconvincing for a variety of reasons. I then conclude the paper by briefly highlighting three themes that might be relevant for further investigation on the matter.

**Keywords**: Structural representations; Antirepresentationalism; Predictive processing; Representation wars; Generative models

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## 1 - Introduction

Predictive processing is a neurocomputational framework surrounded by a number

of philosophical disputes. Some of these disputes concern foundational matters: is

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realism about the theoretical posits of predictive processing warranted (e.g. Colombo, Elkin and Hartmann 2018)? And, if yes, are these posits representations (e.g. Kirchhoff and Roberston 2018)? Other controversies concern what predictive processing (henceforth PP) entails: does PP support an internalist or externalist view of the mind (Hohwy 2016; Clark 2017)? Does it provide a complete account of cognition, or is it unable to account for the systematic nature of human thought and the curiosity so many intelligent animals blatantly manifest (Sims 2017; Williams 2018a)? Is consciousness really just the brain's best guess (Hohwy 2013; Dolega and Dewhurst 2020)?

In this latter kind of disputes it is often assumed that PP is an inferentialist and representationalist theory of cognition. Rendered technically, the assumption is that PP revolves around approximated Bayesian inferences drawn by inverting a generative model operating under a predictive coding message-passing scheme (Hohwy 2013; Clark 2013). More mundanely, it is assumed that PP revolves around statistical inferences performed leveraging probabilistic models of the world. Models, in turn, are understood as *structural representations*: vehicles that represent their targets by mirroring the targets' relational structure (e.g. Williams 2017; 2018b). The tie between predictive processing and models runs so deep that some have suggested that predictive processing would not be *humanly intelligible* without them (Clark 2015).

Here, I challenge this assumption. I claim that the only<sup>1</sup> argument offered to identify generative models with structural representations (Gładziejewski 2016) is flawed, and that it cannot be easily ameliorated. By doing so, I hope to bring a small contribution to the disputes surrounding the philosophical foundations of PP.

The essay is structured as follows. In section 2, I introduce the theoretical apparatus of PP. In section 3, I introduce structural representations, and summarize the argument Gładziejewski offers to identify generative models with them. In section 4, I turn from exposition to criticism, showing a flaw in Gładziejewski's argument, and claiming that it cannot be easily adjusted. A brief concluding paragraph follows.

#### 2 - A brief introduction to predictive processing

Here, I provide a short introduction to PP. Since PP is now largely known, I will sketch only its most fundamental aspects.<sup>2</sup>

To successfully orchestrate behavior, an agent's brain must first determine in which environmental situation the agent is embedded; that is, what are the environmental causes of the energies impacting the agent's transducers. PP assumes that this task is burdened with uncertainty, as sensory states are under-determined

<sup>&</sup>lt;sup>1</sup> A reader might contest this, noting that numerous accounts of generative models as structural representations have been offered (e.g. Wiese 2018; Kiefer and Hohwy 2018; 2019). I am aware of the existence of such accounts. However, to me they all seem to presuppose the success of Gładziejewski's (2016) original argument, to then improve on it in various ways.

<sup>&</sup>lt;sup>2</sup> See (Hohwy 2013; Clark 2013; 2016; Tani 2016) for more introductory material.

To cope with this uncertainty, PP suggests that the brain resorts to a form of Bayesian inference, as it yields an optimal way to determine the *most likely* cause of a sensory state, given the incoming input and some prior knowledge of how environmental cause generate sensory states (Yuille and Kernsten 2006; Hohwy 2013, pp. 13-40). Importantly, since exact Bayesian inferences are often computationally intractable, PP suggests that the brain *approximates* their results by inverting a generative model operating under a predictive coding processing regime.<sup>4</sup>

Generative models are data structures capturing the relations holding between some observable data (here, sensory states) and their hidden causes (here, worldly objects). These models are said to be generative, as the knowledge they embody can be leveraged "from the top-down" to generate expected instances of data (Hinton 2007a; Danks 2014, p. 44). Since real sensory states are generated by the nested interaction of multiple causes operating at different spatiotemporal scales, generative models need to be *hierarchically organized* to capture these nested causal relations. Importantly, this only requires that each hierarchical level  $I_N$  treats the hierarchically lower level  $I_{N-1}$  as a data source, capturing the regularities it displays

<sup>&</sup>lt;sup>3</sup> Notice that this is a theoretical assumption, that can be theoretically contested (e.g. Orlandi 2016).

<sup>&</sup>lt;sup>4</sup> Importantly, model inversion is not *essentially* an approximated process. So, by saying that a generative model is inverted one has not yet shown *how* the intractability problem is solved. Since the technical details are fairly complex (see Bogacz 2017) and will not matter for my argument, I will not sketch them here. An anonymous referee has my gratitude for having noticed this issue.

(see Hinton 2007b). Lastly, these models must be *probabilistic*, embodying their causal knowledge formatted in terms of the probability density functions that are required to approximate Bayesian inferences (e.g. Knill and Pouget 2004).

Predictive coding is a message passing scheme which deploys generative models as follows (Rao and Ballard 1999; Huang and Rao 2011). With the exception of the bottommost level, each level  $\mathit{I}_{\scriptscriptstyle N}$  of the model generates a prediction signal: an "expected" pattern of activity of  $I_{N-1}$ , which is then conveyed to  $I_{N-1}$  through a set of descending connections. Hence, collectively, the levels of the model will generate a "downstream" flow of progressively spatiotemporally refined predictions about the incoming sensory inputs, ideally flowing from "higher" associative areas to primary sensory cortices (see Mesulam 2008). As this signal is received, each level contrasts it with its own actual activity (or the incoming input in the case of the bottommost level), and computes the mismatch between the two. The magnitude of the mismatch, known as prediction error, is then conveyed "upwards", from  $I_N$  to  $I_{N+1}$ , courtesy of a second set of ascending connections. As prediction error is received, each level changes the prediction signal conveyed downwards so as to minimize the incoming prediction error. This process is then iterated until the entire hierarchy reaches a global minimum of prediction error. Since the states of the generative model that best minimize the error correspond to the most likely causes of the incoming sensory signal (given the body of knowledge the generative model encodes), minimizing prediction error *inverts* the generative model, mapping the signal onto its most likely

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 7 causes, implicitly realizing a form of Bayesian inference (Clark 2013; 2016; Hohwy 2013; 2019; Kiefer and Hohwy 2019).

This crude sketch calls for significant amendments. Firstly, it is silent upon *lateral* connections, which allow different, explicitly coded, hypotheses to compete in the interpretation of incoming data (Friston 2005).<sup>5</sup> It is also silent on the *expected precision* of the incoming signals, which constantly modulates the message passing, determining the "impact" of error signals (Hohwy 2013, pp. 59-74; Clark 2016, pp. 53-82). Yet, the most significant amendment this sketch needs is the following: this mechanism is not *just* a mechanism of perception. It is also the engine of action (Hohwy 2013, pp. 75-96; Friston 2013a; Clark 2016, pp. 111-137). To see how, consider the following two points.

First: prediction error minimization can occur under *two* directions of fit (Shea 2013). One can change the predictions to make them fit the input, as sketched above. But one can also "keep the predictions still", and force *the input* to fit them. Secondly, given that the agent's body is, just like the external world, a source of sensory signals (and given that these two sources interact: moving towards an object will change the stimuli the object generates), the generative model must also model the agent's body (Hohwy 2015). Mashing these two observations together yields the gist of how PP accounts for action: agents act by predicting specific bodily signals, to then cancel out the error relative to these predictions through movement (Adams et

<sup>&</sup>lt;sup>5</sup> Many thanks to the anonymous reviewer who noticed that the original formulation of this point was too strong.

PP thus casts action and perception as two complementary sides of the same computational process of prediction error minimization. Given that processes of error minimization are inferential processes, as demonstrated by the brief analysis of the account of perception PP offers, this means that action is an inferential process too.<sup>6</sup> This is why, in the PP literature, action is referred to as *active inference*. More specifically, an agent engaged in active inference *tests* its model of the world, seeking sensory evidence to confirm the predictions licensed by that model (Hohwy 2015; 2016; 2017; 2018).

Importantly, however, active inference has a broader scope than action as usually understood (i.e. bodily movements fulfilling one's intentions). Nothing obliges the self-confirming expectations involved in active inference to be proprioceptive expectations, to be confirmed by bodily movements. They might be visual (i.e. exteroceptive) predictions, and elicit saccades (Friston *et al.* 2010). Or they might be interoceptive predictions, servicing an agent's homeostatic control (Seth 2015) and emotional regulation (Seth and Friston 2016). From this perspective, prediction error minimization is not *primarily* a tool for accurate perception and goal-directed action. Rather, it appears as a mean to the more fundamental end of maintaining an agent within its physiological bounds of viability. This line of reasoning connects PP to an ambitious framework in theoretical biology, namely the free energy principle (see

<sup>&</sup>lt;sup>6</sup> And in fact, according to PP, action too requires the inversion of a generative model (see Friston 2011).

Friston 2013b; 2019; Allen and Friston 2018). But the free energy principle will not be considered here<sup>7</sup>, and the sketch of PP just proposed seems a sufficient introduction, given the task at hand.

In the next section, I briefly introduce structural representations and then summarize Gładziejewski's (2016) argument to identify generative models with them.

### 3 - Structural representations and predictive processing

#### 3.1 - Structural representations

As hinted at in the introduction, structural representations are representations whose vehicles represent their targets by mimicking the inner relational structure of the targets. Consider, for instance, a cartographic map. It might depict a gulf being north of an isle by placing a certain element, corresponding to the gulf, above a second element, corresponding to the isle. Importantly, the PP literature on structural representations (e.g. Wiese 2017; 2018; Dolega 2017; Williams 2017; Kiefer and Hohwy 2018; 2019; Ramstead *et al.* 2019; Hohwy 2020) points to a single formalized account of structural representations; namely Gładziejewski's (2015; 2016) account. According to Gładziejewski:

A state R of a system S is a structural representation of a target T only if: (a) R is structurally similar to T; &

<sup>&</sup>lt;sup>7</sup> This is not *entirely* correct: some aspects of the free energy principle, namely the ones most related to neuroscience, will be considered here. But since these aspects tend to boil down to PP (see Friston 2009; 2010), I do not think I need to explicitly discuss the free energy principle here.

(b) R guides S's action aimed at T; &

(c) R can satisfy (b) when decoupled from T; &

(d) S can detect the representational error of R

Each point calls for clarification.

Point (a) clarifies that structural representations are iconic: their representational properties are (at least partially) grounded in the *similarity* holding between them and their targets. Yet notice that the relevant kind of similarity mentioned in (a) is *structural* similarity. The relevant<sup>8</sup> definition of structural similarity is provided in (O'Brien and Opie 2004, p. 11):

"Suppose  $S_V = (V, \Re_V)$  is a system comprising of a set V of objects, and a set  $\Re_V$  of relations defined on the members of V. The objects in V may be conceptual or concrete; the relations in  $\Re_V$  may be spatial, causal, structural, inferential, and so on. [...] We will say that there is a *second-order resemblance* between two systems  $S_V = (V, \Re_V)$  and  $S_O = (O, \Re_O)$ if, for at least *some* objects in V and *some* relations in  $\Re_V$ , there is a one-to-one mapping from V to O and a one-to-one mapping from  $\Re_V$  to  $\Re_O$ , such that when a relation  $\Re_V$  holds of objects in V, the corresponding relation  $\Re_O$  holds of the corresponding objects in O".

There are several important things to highlight about this definition. One is that it

<sup>&</sup>lt;sup>8</sup> Here, "relevant" means "the one adopted by Gładziejewski". Other definitions of structural similarity are surely possible (e.g. Shea 2018, p. 117). However, since my focus here is Gładziejewski's argument, I will stick to the definition Gładziejewski favors.

can be straightforwardly applied to point (a) assuming that R is  $S_v$  and T is  $S_o$ . Another is that structural resemblance does not require first order resemblance to obtain. In fact, nothing in the definition requires  $S_v$  and  $S_o$  to have any common property. All they need to share is a common pattern of relations among their elements. Thirdly, the definition of structural similarity is tripartite. For  $S_v$  to be structurally similar to S<sub>o</sub> it must be the case that: (i) at least some of the objects of which  $S_v$  and  $S_o$  are constituted map one-to-one onto each other; and (ii) at least some of the relations holding among these objects map one-to-one onto each other; and (iii) corresponding objects stand in corresponding relations in both  $S_v$  and  $S_o$ . Notice that (i) to (iii) need to obtain in conjunction. Notice, lastly, that (i) to (iii) obtaining in conjunction entails that  $S_v$  is semantically unambiguous in respect to  $S_o$ . By this I mean that once the mapping rule is known, it is always in principle possible to determine, for all elements of  $S_v$  mapping onto  $S_{o_i}$  to which element of  $S_o$  each element of  $S_v$  corresponds.

According to this definition, a structural similarity might hold among any two systems. However, the relevant structural similarity exhibited to satisfy point (a) must hold between a *representational vehicle* and the represented target (O'Brien 2015; Kiefer and Hohwy 2018). This is entailed by the *definition* of a structural representation: a representational *vehicle* that represents a target by being structurally similar to it.<sup>9</sup> It is thus immediately clear that the relevant structural

<sup>&</sup>lt;sup>9</sup> Alternatively, structural representations can be defined as: "A collection of representations in which a relation on representational vehicles represents a relation on the entities they represent" (Shea 2018, p. 118). This definition stresses

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 12 similarity holds between the vehicle (the concrete particular doing the representing) and the represented target. Therefore, R must be a representational vehicle: a concrete particular encoding content.

Point (b) establishes that structural representations are *causally responsible* for S's behavior (*G*ładziejewski and Miłkowski 2017). *G*ładziejewski unpacks point (b) in terms of *exploitable* structural similarity (*G*ładziejewski 2015; 2016; *G*ładziejewski and Miłkowski 2017). Hence, the relevant structural similarity in (a) must be exploitable<sup>10</sup> (Shea 2014; 2018, p. 120). Put simply:

R's structural similarity with T is exploitable by S only if:

(iv)  $\Re_v$  is a set of relations S's downstream computational processing is

systematically sensitive to; &

(v)  $\Re_0$  and O are of significance to S

Condition (iv) requires S to be sensitive, in its downstream processing, to the relevant relations  $\Re_v$  in virtue of which R structurally resembles T. Given that computational processing is a mechanical affair, this requires the relations in  $\Re_v$ , or at least the objects in V upon which  $\Re_v$  is defined, to causally impact the processing of S

the important fact that each element of the structural representation is also a representational vehicle, whose content is determined by the relevant structural similarity in which it participates. For instance, each object on a map stands for (i.e. represents) an environmental landmark, and spatial relations among objects on a map represent spatial relations holding among the corresponding landmarks. Notice that such a nesting of representational vehicles is entirely unproblematic: after all, both a sentence and the words forming it are representational vehicles in an entirely intelligible sense. Notice further that according to both Shea's and Gładziejewski's definition, the relevant structural representation is the *entire structure* of related objects, rather than any single part of that structure. That is, the elements (V and  $\Re_V$ ) of a structural representation need not be, on their own, structural representations.

<sup>&</sup>lt;sup>10</sup> This might or might not require a representational consumer. Gładziejewski asserts that a consumer is necessary in his (2015); but his (2016) does not mention consumers. Shea's definition of exploitable structural similarity (to which Gładziejewski adheres) does not require consumers, so I will skip them here. Notice that I adapted the notation in Shea's definition for the sake of orthographic consistency.

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 13 in some systematic way. Condition (v) requires R to structurally resemble a target that matters to S's purposes; that is, a target which matters to S's computational functioning.

Importantly, *exploitable* structural similarity is not a reflexive and symmetric relation (see Shea 2014; Williams and Collings 2017). For this reason, structural representations are immune to the objections that were fatal to iconic representations defined in terms of first order resemblance (see Goodman 1969, p. 3-4). Notice further that insofar as *exploitable* structural similarity determines representational content, R's content is not just causally efficacious (*Gł*adziejewski and Miłkowski 2017): it is also *intrinsic* to R's material constitution as a representational vehicle (O'Brien and Opie 2001; O'Brien 2015; Lee 2018), as it is literally inscribed in the physical form of R.

Condition (c) captures the idea that genuine representations function as *stand-ins* for their targets, enabling S to perform processes aimed at these targets even when they are absent (Grush 1997; Webb 2006; Pezzulo 2008). Gładziejewski unpacks this idea in terms of decouplability, defining it as follows (Gładziejewski 2015): R is *weakly decoupled* from T only if R and T are in no causal contact; R is instead *strongly decoupled* from T only if S and T are in no causal contact.<sup>11</sup>

Lastly, (d) captures the idea that representations can be semantically evaluated by the systems leveraging them (see Bickhard 1999). Importantly, representations are

<sup>&</sup>lt;sup>11</sup> In its original formulation, the definition of decouplability also mentions representational consumers (see Gładziejewski 2015). Here, I omit them for the reasons given in the previous footnote.

often semantically evaluated only *indirectly*, by assessing how successful they are in guiding action (*Gł*adziejewski 2015 pp. 78-79; 2016, p. 569). This indirect route of evaluation seems a natural outcome of exploitability. If, as (iv) entails, R's content determines downstream processing in S (and eventually S's behavior), then *successful* behaviors directly depend on R's content being *correct* (accurate and/or truthful). Pragmatic successes and failure thus appear as reliable indicators of the semantic status of a system's representational resources.

Before moving forward, it is important to clarify the scope of Gładziejewski's account of structural representations. Following Chemero (2009: 67-68), it is possible to distinguish between an *epistemic* representationalist claim and a *metaphysical* representationalist claim. Bluntly put, the epistemic representationalist claim is the claim that our *best explanations* of cognition need to posit representations. The metaphysical representationalist claim is instead the claim that cognitive systems contain components that *really* are representations. The two claims can in principle come apart. A fictionalist about representations, for instance, endorses the epistemic claim while denying the metaphysical one (Sprevak 2013; Ramsey 2020; see also Downey 2018 for a fictionalist interpretation of PP). *Gł*adziejewski's account of structural representations aims at vindicating both claims (*Gł*adziejewski 2015: 70).<sup>12</sup> Thus, his account of structural representations of PP (i.e. generative models) satisfy features (a) to (d) and

<sup>&</sup>lt;sup>12</sup> This commitment seems shared by the majority of accounts of generative models as structural representations (e.g. Williams 2017; Wiese 2018; Kiefer and Hohwy 2018; 2019).

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 15 these are the relevant sort of structures identified as representation by our best explanatory practices.

#### 3.2 - Generative models as structural representations.

Gładziejewski (2016) holds that his account of structural representations straightforwardly applies to generative models. The general outlook of his argument is as follows:

(P1) Items satisfying conditions (a) to (d) in conjunction are structural representations

(P2) Generative models satisfy conditions (a) to (d) in conjunction; therefore

(C) Generative models are structural representations.

The argument needs little clarification. (P1) follows directly from the definition of structural representations. Thus, (P2) carries alone the whole weight of the argument. Here, I sketch the reasoning Gładziejewski provides to substantiate (P2). I will focus in particular on point (a), as it will be central in the following discussion.

To claim that generative models satisfy condition (a), Gładziejewski (2016, pp. 571-573) reasons as follows. Generative models can formally be treated as Bayesian nets. Bayesian nets are directed acyclic graphs, and thus are *graphs*: sets of nodes connected by edges. And, in general, graphs are *always* structurally similar to their targets, because: (i) each node stands for one, and only one, environmental variable,

(ii) the edges connecting the nodes map onto some relation holding among environmental variables; and (iii) two nodes are connected by an edge if, and only if, the corresponding variables are in the relation of interest (see Danks 2014, pp. 39-41). Notice that graphs are not *just* structurally similar to their targets: they are *homomorphic* to them; and homomorphism is a special, *stronger*, case of structural similarity (see O'Brien and Opie 2004, p. 12).

Gładziejewski (2016, pp. 571-573) also suggests a specific way in which generative models structurally resemble their targets: the nodes of each hierarchical layer  $l_N$  in the model map onto the likelihoods of the corresponding sensory states. Interlevel connections between nodes mimic the temporal evolution of sensory states. Lastly, each node maps onto the prior probability of the corresponding worldly cause. The validity of the mapping on offer might be contested (e.g. Wiese 2017), but the general point Gładziejewski is trying to make is clear: generative models are graphs and, *as such*, they are structurally similar to their targets. Therefore, they easily satisfy condition (a).

Point (b) is satisfied because generative models can engage in active inference. Recall: during active inference, the model "purposefully" generates a false prediction, to then cancel the prediction error it generates through movement. Thus, generative models are the engines of action, and guide behavior exactly as (b) requires.

To show that generative models satisfy condition (c), Gładziejewski exhibits the following evidence. To begin with, prediction error minimization seems responsible for

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a variety of paradigmatically offline cognitive phenomena, such as memory and imagination (e.g. Clark 2016, pp. 84-107). Furthermore, early sensory cortices are involved in acts of imagination (e.g. Albers et al. 2013). Given that these cortices are commonly taken to be part of the neural machinery implementing our generative model, these studies suggest that generative models can in fact function when decoupled from their targets. Moreover, generative models must be *counterfactually deep* (Seth 2014): to function effectively, they need to be able to predict how the sensory signals would change, were the agent to move in such-and-such a way. Given that counterfactual scenarios have no causal powers, our generative models will always encode some information about targets from which we are strongly decoupled.

Lastly, generative models clearly satisfy condition (d). When a hypothesis about the incoming sensory flow is selected and tested in active inference, the *prediction error* that the hypothesis generated is a measure of its inaccuracy. For instance, when I drink coffee, I (subpersonally) expect a flow of gustatory sensations which I try to bring about through the ingestion of coffee. But if the ingestion of the liquid does not bring about these sensations, the ensuing error will inform me that I'm in fact not drinking a coffee, prompting my generative model to revise its expectations. In this way, the error ensuing from failed active inferences indicates the representational inaccuracy of the tested hypothesis.

This concludes the exposition of Gładziejewski's argument. In the next section, I will attack it, claiming that it does not succeed in equating generative models to

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 18 structural representations.

# 4 - Generative models and structural representations: an unjustified identification

I divide this section in two sub-sections. The first section examines Gładziejewski's argument for (a), and argues that the argument Gładziejewski offers fails in vindicating (a). The second section examines some alternative arguments for (a).

### 4.1 - A problem in Gładziejewski's argument for point (a)

Recall Gładziejewski's (2016) argument for (a). The argument is as follows: generative models can be thought of as Bayesian graphs, Bayesian graphs (and graphs in general) are structurally similar to their targets; therefore, generative models are structurally similar to their targets. *Therefore*, (a) obtains.

Yet, I believe that this line of reasoning cannot vindicate (a). This is because the relevant structural similarity exhibited to vindicate (a) must hold between a *representational vehicle* and a represented target. But graphical models *are not* representational vehicles. So the argument *Gł*adziejewski provides does not substantiate (a).<sup>13</sup> Let me unpack.

<sup>&</sup>lt;sup>13</sup> Notice that I'm not claiming that graphical models are not structurally similar to their targets. They are. As clarified above, a structural similarity might hold among any pair of entities. Yet, the relevant class of structural similarities that can be used to vindicate (a) is the class of structural similarities holding between representational vehicles and their targets; and graphs are not representational vehicles.

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If one is a realist about representations (as Gładziejewski surely is, see Gładziejewski 2015; 2016; Gładziejewski and Miłkowski 2017), then one is committed to the claim that representations are concrete particulars encoding content (e.g. Shea 2018, pp. 25-43). Accordingly, structural representations are defined as concrete particulars (i.e. representational vehicles) which carry content in virtue of a relevant exploitable structural similarity that holds between them and their representational targets. They are representational vehicles that do the representing by structurally resembling. Hence, the relevant structural similarity must hold between a representational vehicle (a concrete particular) and a represented target. This is why the content of structural representations is supposed to be intrinsic to their material constitution (e.g. O'Brien and Opie 2001; O'Brien 2015; Williams and Colling 2017; Lee 2018). Their content is intrinsic because it is inscribed in the physical form of the representational vehicle (the concrete particular that does the representing).

But Bayesian nets, and graphs in general, are mathematical objects (e.g. Danks 2014, p. 40; Leitgeb 2020). They are defined as a finite set of nodes connected by a finite set of edges (Koski and Noble 2009, p. 41) and sets, nodes and edges are mathematical objects. Mathematical objects might or might not be particulars (it is irrelevant for the purpose of the argument), but definitely are not concrete. So they cannot be representational vehicles, given that representational vehicles are concrete particulars. Hence, the structural similarity Gładziejewski exhibits cannot be used to

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 20 vindicate point (a). It just isn't what point (a) requires.

The same issue can also be appreciated from another point of view. Thus, consider condition (b). Recall that, in order to satisfy (b), the relevant structural similarity that satisfies (a) must be exploitable. Exploitability is partially defined by condition (iv):  $\Re_v$  is a relation S's downstream computational processing is systematically sensitive to. But, as clarified above, for a computation to be sensitive to  $\Re_{v}$ , either  $\Re_{v}$ or the objects V upon which  $\Re_{v}$  is defined, must possess the relevant causal powers needed to systematically orient S's processing. Computational processing is, at the end of day, a physical affair, which is ultimately responsive only to the physical properties of the computational states (e.g. Williams and Collings 2017, p. 1945). However, the only structural similarity Gładziejewski provides to vindicate (a) is defined over mathematical objects. But mathematical objects do not seem to have the causal powers needed to systematically influence S's downstream processing. Hence, the structural similarity that Gładziejewski exhibits to satisfy (a) cannot be exploitable.

This places Gładziejewski's line of argument in a dire situation: either the structural similarity he shows does not vindicate (a), or it does. But if it does, then (b) fails to obtain. Either way, the argument seems to fail, leaving the claim that generative models are structural representations unjustified.

As I understand it, the overall flaw in Gładziejewski's argument is the following. Structural representations are defined in terms of their vehicle properties. Hence Forthcoming on Minds and Machines. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 21 they should be identified at the level of the physical machinery doing the computation (what Marr would call the implementation level). Structural representations are bits of an information processing system literally resembling their target according to an appropriate mapping rule. But graphs, wherever they sit in the explanatory hierarchy of cognitive science, surely do not sit at the level of the physical machinery doing the computing (Danks 2014, pp. 13-37; 218-221). Hence, Gładziejewski's argument seems to be pitched at the wrong level of the relevant explanatory hierarchy. For this reason, I conclude that Gładziejewski's argument does not justify the claim that generative models are structural representations.

But there might be other means to justify that claim.

## 4.2 - Alternative arguments for (a).

Alternative argument 1: the claim that graph theoretic notions cannot sit at the level of the physical machinery might not be warranted. For instance, graph theoretic notions are used to map the connections between different neural regions (e.g. Sporns 2010). Thus, graph theoretic notions can sit at the level of the physical machinery. This suggests an alternative way to vindicate (a): if graphs are structurally similar to their targets, and these graphs can be transparently mapped onto cortical structures and/or neural activity patterns, then, since structural similarity is a transitive relation, the relevant cortical structures/activity patterns are structurally similar to the target of the graphs. Some (e.g. Kiefer 2017) defend

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 22 structural representations along precisely these lines.<sup>14</sup>

This alternative argument for (a) is fairly attractive, as it is maximally conservative over the structure of *Gł*adziejewski's original argument. It also nicely integrates with the existing *scientific* literature on PP, at least insofar some generative models, rendered as Bayesian nets, have been mapped onto cortical structures (e.g. Bastos *et al.* 2012; Friston *et al.* 2017a). Isn't this *sufficient* to show that at least these generative models are structurally similar to their targets?

I believe that a negative answer is warranted. This is because the graphs presented in (Bastos *et al.* 2012; Friston *et al.* 2017a) and a number of similar publications in the PP literature do not model any worldly target. There is thus no specific worldly target that they represent. So, even if the cortical machinery is in some relevant sense structurally similar to these graphs, there is no *third* element to which the cortical machinery can be structurally similar to by being structurally similar to these graphs. For this reason, it seems to me correct to conclude that the alternative argument for (a) provided above fails.

But what, then, is the purpose of the graphs in (Bastos et al. 2012; Friston et al. 2017a)? The answer, if I understand the literature correctly, is the following: these graphs are, in a sense, didactic tools, aimed at showing, with a fair degree of

<sup>&</sup>lt;sup>14</sup> Or, at least, so it *seems*. To be honest, I believe that Kiefer is no longer committed to the claim that generative models are structural representations. Rather, it seems to me that Kiefer is committed to some form of functional role semantics. To be precise, Kiefer (2017, p. 12) seems to endorse the claim that generative models are structural representations. However, he seems to have quickly changed his mind about this point, as, in numerous later publications (Kiefer and Hohwy 2018, p. 2393; 2019, p. 401-403; Kiefer 2020, footnote 19) he takes the content of generative models to be determined by internal functional roles rather than by the structural similarity holding between a generative model and its target. I will more directly confront this issue in the main text, when dealing with the fourth alternative argument for (a). Many thanks to an anonymous referee for having pressed me on this issue.

approximation, that the cortical machinery is arranged in a way such that it can easily perform the inferential processes PP revolves around (see Bastos *et al.* 2012, p. 703; Friston *et al* 2017a, p. 393). In fact, it seems to me that, within the PP literature, graphical models are often deployed to capture the *message passing* within the brain; that is, how inference is performed (see, for instance, de Vries and Friston 2017; Friston *et al.* 2017b; Friston *et al.* 2017c; Donnarumma *et al.* 2017; Matsumoto and Tani 2020; see also Hinton and Sejnowski 1983 for an earlier model).<sup>15</sup> I believe that this is an important point to notice for two distinct reasons.

First, if these graphical models are intended to be models of the relevant *message passing*, it seems more natural to suppose they will map onto the cortical machinery *performing* the inferences, rather than on the representational vehicles manipulated in inferential processes.<sup>16</sup> Secondly, and relatedly, if those graphical models are accurately characterized as portraying the inferential message passing in the brain, it seems to me that they *presuppose* the presence of some relevant representational vehicle, as inferences are defined over representations.<sup>17</sup> These representations might (but, as far as I can see, need not) be structural representations. However, as these graphical models seem to *presuppose* the presence of representations, it seems to me that they *presuppose* the presence of representations. However, as

<sup>&</sup>lt;sup>15</sup> Notice that the scope of my claim is restricted to PP and the usage of graphical models in the PP literature. I make no claim on how graphical models are used in the rest of cognitive neuroscience (and related disciplines). Many thanks to the reviewer who advised me to be more cautious on this point.

<sup>&</sup>lt;sup>16</sup> Importantly, this seems exactly how Kiefer interpreted these models, see (Kiefer 2017, pp. 12-16).

<sup>&</sup>lt;sup>17</sup> The same two points seems to apply whether these models are intended to capture computational processes more generally, given that computational processes are often defined in terms of representations (see Fodor 1981; Shagrir 2001; Ramsey 2007, pp. 68-77; Sprevak 2010; Rescorla 2012). This latter point, however, is not entirely uncontested (e.g. Piccinini 2008).

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 24 of circularity.

Importantly, I do not believe that the considerations offered above rule out in any way the *possibility* of using graphical models to justify (a). As far as I can see, one might still resort to a graphical model to argue that at least some representational vehicles in the brain are structurally similar to their targets using the argument by transitivity sketched above. However, to do so, one would need a graphical model depicting some specific worldly target. And, to the best of my knowledge of the PP literature, no such graphical model has yet been proposed.

Alternative Argument 2: artificial neural networks might provide a different way to leverage graph theoretic notions to vindicate (a). As formal objects, artificial neural networks are graphs. But they are also somewhat plausible sketches of the *physical machinery* implementing or realizing some given computational process of interest (see Haykin 2009, pp. 1-18; Rogers and McClelland 2014). Moreover, at least some artificial neural networks encoding generative models (such as Helmholtz machines) are Bayesian graphs (e.g. Dayan and Hinton 1996). Therefore, even if these artificial neural networks cannot prove that generative models in the brain are structural representations, they can show that generative models can be structural representations, thereby providing circumstantial evidence in favor of (a) obtaining. If our plausible sketches of the physical machinery encoding generative models are graphs (or at least graph-like), then we have a solid reason to believe the *real* physical machinery encoding generative models is graph-like. And, given that graphs Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 25 are structurally similar to their targets, we have a solid reason to believe that (a) obtains. However, I think such a belief would be misplaced. Indeed, it seems to me that a closer consideration of artificial neural networks provides a reason to believe that (a) *does not* obtain.

To see why, consider first that artificial neural networks are often said to encode generative models in their weighted connections (e.g. Dayan and Hinton 1996; Hinton 2014; Spratling 2016, p. 3).<sup>18</sup> But weighted connections (or, more precisely, weight matrices) are typically considered to be *superposed* representations. And the vehicles of superposed representations are *not* structurally similar to their targets. As a consequence, if considering artificial neural networks provides evidence regarding the status of (a), the evidence they provide is *not* in favor of (a) obtaining.

To elaborate a little, consider first the notion of a superposed representation. A representation R is said to be a superposed representation of two targets T and T' when R encodes information about T and T' using the same set of physical resources. When applied to weight matrices, the idea is that weight matrices superpositionally represent their targets when each individual weight is assigned a value such that the network can exhibit the functionality needed to operate on all its targets (Clark 1993 17-19, see Van Gelder 1991, 1992 for further discussion). For instance, a single net can be first trained to recognize (or generate) instances of T. If the network is then trained so as to recognize (or generate) both instances of T and T', then the weights

<sup>&</sup>lt;sup>18</sup> A similar claim is sometimes made in the neuroscientific literature on PP (ad es. Friston 2005, p. 820; Shipp 2016, p. 3). The claim, however, might not be entirely correct, as I will soon clarify in the main text.

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 26 of the net will encode information about *both* representational targets, and the weight matrix will represent both in a superposed fashion.

However, in weight matrices: "Each memory trace is distributed over many different connections, and each connection participates in many different memory traces" (McClelland and Rumelhart 1986, p. 176). So, it seems each individual weight maps onto many different representational targets (or aspects thereof). But if this is the case, then either condition (i) or (ii)<sup>19</sup> of structural similarity are blatantly violated, since they require a one-to-one mapping. As a further proof of their violation, recall that the obtaining of (i) to (iii) in conjunction entails semantic unambiguity (see section 3.1). That is, if (i) to (iii) jointly obtain, it is always in principle possible to tell, for each "bit" of the representational vehicle, which "bit" of the represented target it corresponds to. However, in superposed representations: "It is impossible to point to a particular place where the memory of a particular item is stored" (Rumelhart and McClelland 1986, p. 70). Superposed representations are thus not semantically unambiguous. Therefore, at least one condition among (i) and (iii) is not met. As a consequence, superposed representations do not support the claim that (a) obtains.<sup>20</sup>

<sup>&</sup>lt;sup>19</sup> Or both. The formulation in terms of "either (i) or (ii)" is due to the fact that it seems to me that one might interpret weighted connections either as parts of a structural representation (i.e. as members of V) or as *relations* among parts (i.e. as relations in  $\Re_V$ ).

<sup>&</sup>lt;sup>20</sup> Notice that I'm not denying that weight matrices encode the invariant relations that hold among the elements of the domain upon which the network has been trained to operate. I am only denying that there is a mapping from weight matrices (that is, from individual weights or sets of weights) to relations such that the mapping satisfies (i) or (ii). In simpler terms, I'm not denying that weight matrices represent invariant relations, I'm only denying that weight matrices represent invariant relations by being structurally similar to the target domain (or by participating in some relevant structural similarity with the target domain). Notice, importantly, that not all invariant relations need to be encoded in a vehicle that is structurally similar to its target. We might, for instance, stipulate that the sign "§" represents the fact that my father is *n* years older than me. If we do so, then "§" encodes an invariant relation holding between me and my

The argument outlined above can be challenged in two ways.<sup>21</sup> First, generative models are not encoded in connections alone; they are *jointly* encoded by connections and activity vectors (e.g. Buckley et al 2017, p. 57). Secondly, the definition of structural similarity relevant to the obtaining of (a) quantifies only over *some*. Thus, noticing that connections do not participate in any one-to-one mapping does not provide an argument to the effect that (a) does not obtain: connections might simply be excluded from the objects V or relations  $\Re_V$  of R participating in the structural similarity. I address these challenges in turn.

Are generative models really jointly encoded by activity vectors and weighted connections as the first challenge suggests? As far as I can see, the answer is positive; and focusing only on weighted connections (as I did above) is a mistake. But, to my excuse, it is a mistake the that the PP literature encourages:

"We allowed the network to learn a hierarchical internal model of its natural image inputs by maximizing the posterior probability of generating the observed data. The internal model is encoded in a distributed manner within the synapses of the model at each level". (Rao and Ballard 1999, p. 80, emphasis added)

"The representation at any given level attempts to predict the representation at the level below; at the lowest level this amounts to a

father, and yet there just seems to be no structural similarity holding between "§" and the target it represents. Many thanks to an anonymous reviewer for having pressed me on this point.

<sup>&</sup>lt;sup>21</sup> Many thanks to an anonymous referee for having raised these objections.

prediction of the raw sensory input. It is the backward connections, therefore, that instantiate the generative model." (Shipp 2016, p. 3, emphasis added)

"The prediction error minimization (PEM) framework in cognitive science is an approach to cognition and perception centered on a simple idea: organisms represent the world by constantly predicting their own internal states. [...] Cascades of predictions are matched against the incoming sensory signals, which act as negative feedback to correct a generative model encoded in the top-down and lateral connections." (Kiefer and Hohwy 2019, p. 384, emphasis added)

"The generative model, which in theories such as hierarchical predictive coding is hypothesized to be *implemented in top-down cortical connections*, specifies the *Umwelt* of the organism, the kinds of things and situations it believes in independently of the current sensory data

[...]" (Kiefer 2020, p. 2, emphasis added)

Sadly, this excusation does not address the first challenge. Importantly, however, it seems to me that the first challenge is really no challenge at all. Allowing (so to speak) activity patterns to participate in the relevant structural similarity alongside weighted connections does not change the fact that, at least *prima facie*, weighted connections do not appear to map *one-to-one* onto any target. Thus, simply counting activity patterns as elements of V does not vindicate (a). This is because weighted connections are still considered elements of either V or  $\Re_V$ , and, as a result, at least some elements of the vehicle do not map one-to-one onto elements of the target as required by (a). Counting activity vectors in, on its own, is not enough: one must also be able to exclude that weighted connections participate in the relevant structural similarity.

This brings me to the second challenge. Is it possible to define some relevant vehicle target structural similarity without involving weighted connections? I think that the correct answer is negative.

To start, it is surely *possible* to define some relevant network-target structural similarity without appealing to weighted connections. There is nothing particularly new in this claim: Paul Churchland's state-space semantics is the most obvious example of a network-target structural similarity that does not involve connections (see Churchland 1986; 2012; see also O'Brien and Opie 2004). In his view, the entire *activation space* of the hidden layers of a network structurally resembles the target domain upon which the network has been trained to operate. And I'm fairly confident that a similar structural similarity can be found by considering artificial neural networks encoding generative models.<sup>22</sup>

Isn't this just conceding that (a) obtains? I do not think so. For activation spaces (the first relevant *relatum* of the structural similarity) are not vehicles, because they are not concrete particulars. They are abstract mathematical spaces that are

<sup>&</sup>lt;sup>22</sup> And even if my confidence were misplaced, I would concede the point for the sake of discussion.

Notice that I'm not claiming that the activation space-target domain structural similarity cannot determine the content of each individual activity vector. The relevant structural similarity holding between the activation space and the target might be enough to determine the content of each individual vector (i.e. of each individual element of V, using O'Brien and Opie's notation). However, the fact that each individual vector acquires content in virtue of the relevant structural similarity holding between the entire activation space and the target domain does not entail that each individual vector is a structural representation. This is because the elements of the vehicle (i.e. the objects of V and relations of  $\Re_{v}$ ) involved in a structural similarity need not be structurally similar to elements of the target (i.e. the members of O and  $\Re_0$ ) they correspond to. Given that, to my knowledge, only individual vectors are tokened in connectionist systems, the relevant structural similarity holding between the state space of a network and the network's target domain is insufficient to substantiate the claim that structural representations are tokened within the system.

Moreover, I honestly doubt that it is possible to *rightfully* exclude weighted connections from the relevant structural similarity. To see why, consider the following: if a vehicle represents in virtue of the structural similarity it bears to a target, then the more the vehicle and the target are structurally similar, the more the representation will be accurate. The accuracy of a structural representation non-accidentally increases when (and, at least *prima facie*, only when) the elements of the vehicle are rearranged in a way that increases the extent to which the vehicle is structurally similar to the target.

If this is correct, then there seems to be a solid reason to deny that we can exclude connections from the relevant network-target structural similarity, for modifications of weighted connections made in accordance to the learning algorithm *do improve* the representational accuracy of connectionist systems. Thus, if these systems represent by means of structural similarity, it seems that weighted connections *must* be counted among the elements participating in the similarity. Surely, the relevant definition of structural similarity provided when unpacking condition (a) quantifies only over some, but that "some" seems to include weighted connections. However, if my argument based on superpositionality is correct, then weighted connections do not map one-to-one on their targets as (a) requires. In short: if artificial neural networks deploys structural representations, connections must be involved. Yet, their involvement seems to prevent the obtaining of (a).<sup>23</sup>

<sup>&</sup>lt;sup>23</sup> At this point, it might be tempting to wonder whether the relevant definition of structural similarity could be relaxed, so as to allow connections to be elements in the structural similarity in spite of the lack of any intelligible *one-to-one* mapping holding between them and the elements of the target domains. As an anonymous reviewer aptly noticed, O'Brien and Opie's (2004) definition of structural similarity is not the only one on the market, and at least some alternative formulations do not require a one-to-one mapping (e.g. Kiefer and Hohwy 2019, p. 400; Shea 2018, p. 117). As far as I can see, the mapping can be relaxed so as to allow *many* elements of the vehicle to map onto *one* element of the target. However, I believe the mapping *cannot* be relaxed so as to allow *one* element of the vehicle to map onto *many* elements of the target. To see why this is the case, consider a minimal structural representation constituted by two objects *a*\*and *b*\* in a relation *R*\*. Suppose that *R*\* corresponds to a relation *R*, that *a*\* corresponds to an element *a* and that *b*\* maps onto two elements *b* and *c*. Now, given this mapping, the representation is accurate when *aRb* is the case. It is also accurate when *aRc* is the case. Hence, misrepresentation occurs only when *both aRb* and *aRc* are not the case.

*Alternative argument 3*: maybe one does not need to look at artificial neural networks to vindicate the claim that generative models are structurally similar to their targets. In fact, PP theorists often point to a relevant structural similarity one might leverage to vindicate (a). Friston (2013a, p. 133) provides one clear example, worth quoting at length:

"[...] every aspect of our brain can be predicted from our environment.

[...] A nice example is the anatomical division into what and where pathways in the visual cortex. Could this have been predicted from the free-energy principle? Yes - if the anatomical structure of the brain recapitulates the causal structure in the environment, then one would expect independent causes to be encoded in functionally segregated neuronal structures".

Since points (i) and (ii) in the definition of structural similarity quantify only over some, this quote by Friston provides us a structural similarity sufficient to vindicate (a): if Friston is right, there is a structure-preserving mapping form some cerebral regions onto some environmental targets. Furthermore, examples like the one

But, if this is correct, then the representation represents (aRb or aRc), and its content is disjunctive and thus indeterminate. Yet, it is widely assumed that a successful theory of content *must* deliver us determinate content. So, it seems to me that, in order for a structural-resemblance based theory of content to be successful, it must exclude *one-to-many* mappings. Now, the issue with weights in connectionist systems is that they seem to map one-to-many: each weight encodes information about many targets (see Clark 1993, pp. 13-17; Van Gelder 1991, pp. 42-47; Ramsey, Stich and Garon 1991, pp. 215-217 for early renditions of this point). Hence, it seems that each weight is bound to map onto *many* targets, generating the problem with content determinacy. Notice, importantly, that the same line of reasoning holds even when the *relations* map onto many. To see why, consider a modified version of the minimal structural representation considered above, in which  $a^*$  maps onto  $a, b^*$  maps onto b and  $R^*$  maps onto two relations R and F. Again, given this mapping, misrepresentation occurs only when *both* aRb and aFb are not the case, and so the representation represents (aRb or aFb). In both cases, the disjunction problem is brought about by the claim that one-to-many mappings might constitute structural similarities, so as to circumvent the problems raised by superspositionality. Hence, we should *not* allow one-to-many to constitute structural similarities. Thanks to an anonymous referee for having pressed me on this point.

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 33 highlighted in the quote seem fairly easy to multiply. It might be pointed out, for instance, that the anatomical segregation of visual and auditory cortices reflects the fact that visual and sensory input can have different worldly causes. So there *is*, I submit, a relevant brain-world structural similarity. Therefore, if *the whole brain* is the generative model (a claim that is not uncommon in the PP literature<sup>24</sup>, e.g. Bastos *et al.* 2012, p. 702), then condition (a) is met.

I must confess that, to me, this way of vindicating (a) seems to lead to a Pyrrhic victory at best. To begin with, claiming that the whole brain is a structural representation seems to prevent Gładziejewski's account from vindicating the epistemic representationalist claim. This is because, in our best explanatory practices, "representation" typically denotes states of information processing systems (e.g. Kandel *et al.* 2012 p. 372), rather than entire information processing systems.<sup>25</sup> The structural similarity presented above seems to enable us to vindicate *only* metaphysical representationalism about the whole brain (i.e. the claim that the whole brain *really* is a "big" representation). Given that Gładziejewski's account of structural representations aims at vindicating both metaphysical and epistemic representationalism, this way of vindicating (a) seems to lead to a partial failure of his account.<sup>26</sup>

<sup>&</sup>lt;sup>24</sup> More precisely, it is common in the PP literature most heavily influenced by Friston's free energy principle. Many thanks to an anonymous referee for having noticed this imprecision.

<sup>&</sup>lt;sup>25</sup> Notice that the former usage of "representation" is consistent with the PP literature (e.g. Friston 2005, p. 819; Kiefer and Hohwy 2018, p. 2396).

<sup>&</sup>lt;sup>26</sup> One might contend this verdict is premature. For the elements (i.e. objects of V and relations of  $\Re_V$ ) of structural representations are representational vehicles in their own right (e.g. Shea 2018, p.118; Ramsey 2007, p. 79, footnote 3). Thus, claiming that the brain as a whole is a structural representation might in principle justify the claim that the relevant elements of the structural similarity (i.e. patterns of activation) are representations too, leading to a vindication

Secondly, a complaint about content. What would such a "whole brain" structural representation represent? If I understand Friston correctly, the brain is supposed to recapitulate the causal structure of the world. Thus, the relevant structural similarity holds between the anatomical structure of the brain and the causal structure of the world. But a structural representation represents the target whose structure is mirrored in the structure of the vehicle, and here such a target is *the world* (see Wiese 2018, p. 219; Williams 2018, p. 154-155). This is not the kind of content naturalistic theories of content are supposed to deliver, for *the world* is not the kind of content invoked in the scientific explanations of our cognitive capacity, nor the kind of content relevant to our personal-level mental states. This isn't a knockdown objection against alternative argument 3. But it surely shows that the argument has some very undesirable consequences.

Lastly, and, I believe, most importantly, this way of vindicating (a) seems to prevent (c) from obtaining. If the entire brain is a single gigantic representation representing the world, it is very hard to see how decouplability might obtain. There is always *some* sort of causal contact between brains and worlds. Since point (c) spells out decouplability in terms of causal contact, this way of vindicating (a) seems to prevent the obtaining of (c).<sup>27</sup>

of epistemic representationalism. I believe that the problem with this line of reasoning is the following: the brain-world structural similarity Friston envisages is *not* defined over patterns of activation in the brain. Rather, it is defined over the anatomical structure of the brain. The relevant elements in the structural similarity are not patterns of activation. Hence, this way of vindicating (a) fails to properly vindicate the epistemic representationalist claim.

<sup>&</sup>lt;sup>27</sup> Here, one might be tempted to simply reject condition (c) and accept that entire brains are structural representations of the environment. As far as I can see, this is a legitimate move. However, it seems quite an *ad hoc* move. There are good independent reasons to hold that representations are necessarily decouplable from their targets (see Grush 1997; Webb 2006; Pezzulo 2008: Orlandi 2014, pp. 120-134). Moreover, abandoning (c) would likely make Gładziejewski's account

Alternative argument 4: perhaps there is a way to make "whole brain" representations work. Thus, consider Kiefer and Hohwy's (2018; 2019) defense of generative models as structural representations.<sup>28</sup>

On the view Kiefer and Hohwy favor, we should conceive the brain as a complex causal network. If I understand them correctly, we should interpret each node in such a network as a definite pattern of neuronal activity, and the arrows connecting the nodes as causal relations between patterns (i.e. if node *a* is connected to node *b*, then pattern *a* causes pattern *b*). This network of causal relations, on the account Kiefer and Hohwy propose, structurally resembles the causal structures of the world as captured by "material inferences"; that is, inferences such as that from "It's raining" one infers "The street is wet" (see Kiefer and Hohwy 2018, pp. 2392-2393). In this way, the entire brain (which instantiates the causal network), comes to reflect, and hence to represent, the causal structure of the world.

Kiefer and Hohwy's account of "whole brain" structural representations seems to me a significant improvement from the previously scrutinized one. For one thing, given that in this view the relevant elements of the structural representation are patterns of activation, and given that the elements of a structural representations can be

of structural representations far too liberal, as Gładziejewski himself acknowledges (Gładziejewski 2016, p. 571).

<sup>&</sup>lt;sup>28</sup> To be clear, Kiefer and Hohwy do not *explicitly* set out to defend "whole brain" representations. However, it seems to me that their account entails that the whole brain is a structural representation, at least insofar they take the entire causal network *instantiated by the brain* to be the relevant structural representation. A reviewer noticed that this characterization of Kiefer and Hohwy's position might be too ungenerous, since, strictly speaking, Kiefer and Howhy speak only of connections among *cortical* regions. Hence, their position is best described as a form of "whole cortex", rather than "whole brain" representationalism. However (and the reviewer seems to agree) noticing this does not substantially alter the dialectical situation. So, I will continue to speak of Kiefer and Hohwy as endorsing a form of "whole brain" representationalism, mainly for the sake of simplicity.

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 36 counted as representations in their own right, Kiefer and Hohwy's proposal would allow one to substantiate the epistemic representationalist claim. Moreover, it can also assign each individual pattern of activation a determinate content, depending on its causal embedding within the network. However, it seems to me that relying on Kiefer and Hohwy's proposal to vindicate (a) has serious drawbacks.

To start, the problem with (c) is not solved by Kiefer and Hohwy's account.<sup>29</sup> If the brain is a complex causal network mirroring the causal structure of the world, it is correct to say that the relevant structural representation (i.e. the brain) represents the world. And I simply do not see how one could sever the constant brain-world causal contact so as to vindicate (c).<sup>30</sup>

Secondly, Kiefer and Hohwy's account raises a puzzle about the inferential status of brain processes. If causal relations holding among patterns of activation are members of  $\Re_{v}$ , it seems to me that it logically follows that they are part of the vehicle. After all, according to O'Brien and Opie's (2004) definition of structural similarity, both the objects of V and the relations in  $\Re_{v}$  are parts of  $S_{v}$ . But in order for the relevant structural similarity to satisfy (a),  $S_{v}$  must be a vehicle. Hence, Kiefer and Hohwy's proposal seems to imply that causal relations among patterns of

<sup>&</sup>lt;sup>29</sup> Notice, importantly, that Kiefer and Hohwy *seem* to consider decouplability a necessary feature of representations, see (Kiefer and Hohwy 2019, p. 400).

<sup>&</sup>lt;sup>30</sup> Of course, individual patterns of activation can be decoupled from the individual target they represent in virtue of the overall brain-world structural similarity. However, to be satisfied, point (c) requires that the *entire vehicle* of structural representation (in this case, the whole brain) is decoupled from its target (in this case, the world). Thus, noticing that in some cases (e.g. during dreaming) certain patterns of activation are tokened in a way that is functionally independent from the incoming sensory stimulation is not sufficient to vindicate point (c). This is because individual patterns of activations are not the *entire* vehicle of the structural representations, but rather elements of that vehicle. Thanks to an anonymous referee for having pressed me to clarify this point.

activations are part of the vehicle. But if this is the case, then it seems to me that these causal relations cannot be inferential processes, for inferential processes seem to be distinct from the representational vehicles upon which they operate. So, it seems that if Kiefer and Hohwy's (2018; 2019) account of structural similarity is accepted, causal interactions among neural activity patterns cannot be rightfully called inferences. And this seems a problem, given that the inferentialist reading of PP tends to go hand in hand with the claim that generative models are structural representations (e.g. Kiefer 2017; *G*ładziejewski 2017; Hohwy 2018).

Lastly, a wholesale acceptance of Kiefer an Hohwy's (2018; 2019) account might, paradoxically, force one to abandon the claim that generative models are structural representations. The point is subtle but important. According to Kiefer and Hohwy:

"The contents of parts of a structural representation are (at least in the case of causal generative models of an environment) in effect determined by their internal functional roles ". (Kiefer and Hohwy 2018, p. 2393; see also Kiefer and Hohwy 2019, p. 402; Kiefer 2020, endnote 17)

But this is not how the parts (i.e. objects and relations) of a structural representation acquire their contents. The content of a structural representation is determined by the relevant structural similarity it bears to a target; and the content of the *parts* (i.e. the elements of V and  $\Re_V$ ) of a structural representation is determined by the way in which they participate in the relevant structural similarity;

that is, by the way in which they map onto a corresponding element of the target. The relevant relation determining the contents of the elements of a structural representation is the structural similarity holding between the vehicle and the target; not the relations  $\Re_{v}$  holding among the various members of V. Surely, since structural similarity is *structural*, it must, in some relevant sense, be *sensitive to* the relevant members of  $\Re_{v}$ . But this does not entail that the content of the elements of a structural representation is determined by their relations  $\Re_{v}$ .

Another, perhaps more perspicuous, way to put the same point is this: were the content of the elements of V determined by the relations  $\Re_{v}$  holding between them, then the elements of V would have content whether the entire vehicle is structurally similar to something or not. Moreover, even in cases in which the whole vehicle  $S_v$  is structurally similar to some target  $S_{0}$ , there is, as far as I can see, no prior guarantee that the content assigned to each element in V in virtue of the relations in  $\Re_v$  would match the content each element of V would bear, were their contents determined by the relevant mapping from  $S_v$  to  $S_o$  constituting the structural similarity. To put it bluntly, what I'm trying to point out is this: Kiefer and Hohwy espouse a form of functional role semantics. But functional role semantics and structural similarity have no essential connections, pace Kiefer and Hohwy. It thus seems to me that a wholesale adoption of Kiefer and Hohwy's proposal ends up undermining the broader structural-representationalist claim. Kiefer and Hohwy might provide a way to vindicate (a); but a wholesale acceptance of their proposal

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 39 seems to make such a vindication redundant. If one adheres to functional role semantics, one does not need a structural similarity.

This is not to deny that Kiefer and Hohwy (2018, p. 2393; 2019, p. 402) stress that the relevant (i.e. content constituting) functional relations in  $\Re_{v}$  mirror, in the relevant sense, the causal structure of the world (i.e.  $\Re_0$ ): in their view, functional role semantics entails a relevant structural similarity. But this surely does not allow us to count Kiefer and Hohwy as defenders of structural representations. For, as many have stressed, causal/informational theories of content entail a relevant vehicle-target structural similarity too (Morgan 2014; Nirshberg and Shapiro 2020; Facchin 2021). But surely causal/informational theories of content are not structural resemblance-based theories of content, for at least three reasons. First, the claim that content is determined by causal/informational factors is logically distinct from the claim that content is determined by a relevant vehicle-target structural similarity. Secondly, it is not the case that the obtaining of all vehicle-target structural similarities hinges upon some appropriate causal/informational relation holding the vehicle and the target (see Shea 2018, p. 139-140). Lastly, in the case of genuine structural representations, the relevant structural similarity is what determines the relevant content. Hence, it should not be a "side effect" of some other content-determining factor (see Gładziejewski and Miłkowski 2017 for further discussion).<sup>31</sup> It seems to me that one needs only to substitute "causal/informational

<sup>&</sup>lt;sup>31</sup> One might object that Kiefer and Hohwy (2018, 2019) should be counted as defending structural representations because they stress that the relevant structural similarity is relevant for the system's success. As I understand it, the

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 40 relations" with "functional relations" to make the same remarks about Kiefer and Hohwy's proposal.

Alternative argument 5: one might further try to vindicate (a) by claiming that, since generative models can be rendered as Bayesian nets, and that Bayesian nets are computationally useful because they are structurally similar to their target (Danks 2014 p. 39), whatever piece of machinery is instantiating the relevant generative models must also be structurally similar to the target to be computationally useful. This way of vindicating (a), however, seems flawed. Generative models can be run by everyday personal computers: Von Neumann architectures computing over arbitrary symbols. And symbols surely aren't structural representations: in fact, the two are typically contrasted (O'Brien and Opie 2001; Williams and Collings 2017).

There might be other ways to vindicate (a). But I'm not claiming that the proposition "generative models are structural representations" is *false*. I'm only claiming that it is presently *unjustified*. And the discussion above seems sufficient to substantiate that claim. If the arguments provided above are correct, *Gł*adziejewski's original argument fails to substantiate the claim that generative models are structural representations; and, as far as I can see, there is no "rough and ready" way to vindicate that claim on offer in the current philosophical market.

## 5 - Concluding remarks

problem with this line of argument is that the same holds true also for causal theories of content (see Nirshberg and Shapiro 2020, pp. 6-7; Facchin 2021, pp. 9-12)

In this essay, I have tried to argue that the identification of generative models with structural representations is, at present, unjustified. Here, I wish to single out some recurrent themes that emerged in the discussion above, as they are likely to be important to understand the theoretical commitments of PP.

One issue that repeatedly emerged from the discussion above concerns the vehicles of generative models. The PP literature, I believe, is fairly confusing on this point. The word "model" is in fact applied to a variety of quite distinct things, including: (i) the whole brain (e.g. Bastos et al. 2012, p. 702), (ii) axonal connections (e.g. Shipps 2016, p. 3), (iii) functionally specialized networks of neural areas (e.g. the mirror system as a model of bodily kinematics, see Kilner et al. 2007), (iv) neuronal responses and connections (Buckley et al. 2017, p. 57), (v) the spinal cord<sup>32</sup> (Friston 2011, p. 491), (vi) single hierarchical levels<sup>33</sup> (e.g. Kiefer and Hohwy 2019, p. 387) and I would not be surprised if this list is not complete. This liberal, almost casual, usage of "model" makes it very difficult to understand what piece of neural machinery should be taken as the vehicle of the relevant representation. It also makes unclear what sort of structural similarity would be appropriate to vindicate a structural representationalist claim. I believe that clarifying this point should be a priority for philosophers (and empirical scientists) interested in defending the claim that generative models are structural representations. This is because structural

<sup>&</sup>lt;sup>32</sup> To be precise, Friston suggests that the *dorsal horn* of the spinal cord embodies an *inverse* model. But an inverse model still seems to me to count as a model.

<sup>&</sup>lt;sup>33</sup> Presumably, single, well identified, regions of the cortical hierarchy.

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 42 representations are defined in terms of a (relational) vehicle property; namely, structural similarity. Hence, determining what counts as the relevant vehicle is essential in order to vindicate the claim that the relevant vehicle is the vehicle of a structural representation.

A related problem is that it is unclear whether the candidate vehicle (that is, the candidate piece of neural circuitry) is supposed to be (or embody) a model or merely to encode a model.<sup>34</sup> As far as I can see, "being/embodying a model" and "encoding a model" are used roughly as synonyms in the PP literature. Yet, there seems to be an obvious difference between the two claims: the hard drive of my PC surely encodes numerous early drafts of this very essay, but my hard drive is not an early draft of this essay. If the core claim PP makes is that the relevant candidate vehicle *is* a model, and models really are structural representations, then *some* structural similarity *must* be found; otherwise, PP would be in trouble. But if the core claim PP makes is just that the relevant candidate vehicle simply *encodes* a model, then the absence of any relevant vehicle-target structural similarity might be entirely unproblematic (supposing that not each and every form of encoding entails a structural similarity).

This latter point also suggests that, where the core claim of PP that the brain only *encodes* a generative model, the reading of generative models as structural

<sup>&</sup>lt;sup>34</sup> This issue seems to me importantly related to the "having VS. being" a model in the literature on the free energy principle (see van Es 2020; Baltieri *et al.* 2020; see also Bruineberg *et al.* 2020). I must confess, however, that I'm unsure about how to properly articulate such a relation.

Forthcoming on *Minds and Machines*. Please refer to the published version. Doi: 10.1007/s11023-021-09559-6 43 representations would not be mandatory. For this reason, I believe that philosophers willing to defend a representationalist account of PP need not *necessarily* commit themselves to a "structuralist" account of representations. Representations in cognitive science need not necessarily be structural representations; and it would be interesting to explore whether a representationalist account of PP not based on structural representations is viable.<sup>35</sup>

Lastly, a point about the rhetoric of the philosophical discussion surrounding PP. Many philosophers defending a representational reading of PP based on structural representations seem to hold that the "representation wars" are over, since PP has conclusively resolved the issue in favor of (structural) representationalism (e.g. Clark 2015; *Gt*adziejewski 2016; Williams 2017). Now, I find it sincerely hard to deny that structural representations are increasingly popular in cognitive neuroscience (e.g. Williams and Colling 2017; Poldrack 2020).<sup>36</sup> However, if the argument I have provided here is correct, the newfound popularity of structural representations might have very little to do with predictive processing. Identifying the factors that actually contribute to the popularity of structural representations might prove useful to fully understand the strength and merits of both representationalism and

<sup>&</sup>lt;sup>35</sup> Arguably, Kiefer and Hohwy's (2018; 2019) account is one such account, given Kiefer and Hohwy's commitment to functional role semantics. However, given that they seem to take (wrongly, in my opinion) functional role semantics as a kind of structural resemblance, it is very hard to evaluate their proposal as an *alternative* to structural representations-based accounts of PP.

<sup>&</sup>lt;sup>36</sup> A reviewer noticed that structural representations are less popular in the philosophy of mind, where teleosemantic theories of content still appear to dominate. It might be worth noticing, at this point, that teleosemanticists are increasingly willing to incorporate *some* forms of structural similarity in their accounts (e.g. Neander 2017; Millikan 2020). Moreover, the standard notion of *exploitable* structural similarity has been elaborated within a roughly teleosemantic framework (Shea 2018). Yet, nothing, in my argument, hinges over this.

antirepresentationalism in cognitive science.

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