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Research Article

A Novel Approach to Fuzzy Soft Set-Based Group Decision-Making

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There are many uncertain problems in practical life which need decision-making with soft sets and fuzzy soft sets. The purpose of this paper is to develop an approach to effectively solve the group decision-making problem based on fuzzy soft sets. Firstly, we present an adjustable approach to solve the decision-making problems based on fuzzy soft sets. Then, we introduce knowledge measure and divergence degree based on α-similarity relation to determine the experts' weights. Further, we develop an effective group decision-making approach with unknown experts' weights. Finally, sensitivity analysis about the parameters and comparison analysis with other existing methods are given.

1. Introduction

The mathematical modelling of vagueness and uncertainty has become an increasingly important issue in diverse research areas. In recent years, uncertain theories such as rough set theory [1], fuzzy set theory [2], and intuitionistic fuzzy set theory [3] and other mathematical tools have been widely applied in lots of social fields. But all these theories have their own difficulties as pointed out in [4]. To overcome these difficulties, Molodtsov [4] proposed the soft set theory for modeling uncertainty.

Recently, works on soft set theory are progressing rapidly. Many efforts have been devoted to further generalizations and extensions of Molodtsov's soft sets. Maji et al. [5] defined fuzzy soft sets by combining soft sets with fuzzy sets. Yang et al. [6] initiated the notion of interval-valued fuzzy soft set by combining the interval-valued fuzzy sets and soft sets. Maji et al. [7, 8] introduced the concept of the intuitionistic fuzzy soft set which is a combination of the soft set and the intuitionistic fuzzy set. Xu et al. [9] defined a concept of vague soft set. Moreover, they also studied its basic properties and applications. By integrating the interval-valued intuitionistic fuzzy sets with soft sets, Jiang et al. [10] proposed a more general soft set model called interval-valued intuitionistic fuzzy soft sets.

Applications of fuzzy soft sets have made great progress, especially in decision-making. Feng et al. [11] applied level soft sets to discuss fuzzy soft set-based decision-making. Based on Feng et al.'s works, Basu et al. [12] further investigated the previous methods to fuzzy soft set-based decision-making and introduced the mean potentiality approach, which was showing more efficiency and more accuracy than the previous methods. Alcantud [13] presented two innovations that produced a novel approach to the problem of fuzzy soft set-based decision-making in the presence of multiobserver input parameter data sets. Tang [14] proposed a novel fuzzy soft set approach in decision-making based on grey relational analysis and Dempster-Shafer theory of evidence. Li et al. [15] introduced an approach to fuzzy soft set-based decision-making by combining grey relational analysis with Dempster-Shafer theory of evidence and given a practical application to medical diagnosis problems. Liu et al. [16] proposed a decision model based on fuzzy soft set and ideal solution. Alcantud et al. [17] put forward an algorithmic solution for the diagnosis of glaucoma through a hybrid model of fuzzy and soft set-based decision-making techniques.

Group decision-making is an important research topic in decision theory. In recent years, a lot of methods have been developed for solving group decision-making problems in existing literatures. Yue [18] presented a multiple-attribute group decision-making model based on aggregating crisp values into intuitionistic fuzzy numbers. Xu and Shen [19] proposed an outranking method aimed at solving multicriteria group decision-making problems under Atanassov's interval-valued intuitionistic fuzzy environment. Wan and Dong [20] developed some power geometric operators of trapezoidal intuitionistic fuzzy numbers and applied them to multiattribute group decision-making with trapezoidal intuitionistic fuzzy numbers. Sun and Ma [21] studied the group decision-making problem with linguistic preference relations. Qin and Liu [22] investigated a new method to handle multiple-attribute group decision-making problems based on a combined ranking value under interval type-2 fuzzy environment. Wan et al. [23] developed a new method for solving multiple-attribute group decision-making (MAGDM) problems with Atanassov's interval-valued intuitionistic fuzzy values (AIVIFVs) and incomplete attribute weight information. In [24], Wan et al. investigated the group decision-making (GDM) problems with interval-valued Atanassov intuitionistic fuzzy preference relations (IV-AIFPRs) and developed a novel method for solving such problems. In [25], Wan et al. investigated a group decision-making (GDM) method based on additive consistent interval-valued Atanassov intuitionistic fuzzy (IVAIF) preference relations (IVAIFPRs) and likelihood comparison algorithm.

In dealing with multiexpert group decision-making problems, experts have their own characteristics and structure of knowledge; normally, each expert should have different weights. So experts' weights play an important role in group decision-making. In this paper, we suppose that the weights of experts are different and unknown. How to measure the experts' weights? Up to now, some methods have been developed to do this. Yue and Jia [26] used an extended TOPSIS method and an optimistic coefficient to obtain the weights of decision-makers. Mao et al. [27] introduced a method for determining the weights of experts by using the distance between intuitionistic fuzzy soft sets. Wan et al. [28] constructed an intuitionistic fuzzy linear programming model to derive experts' weights. Based on the generalized cross-entropy measure, Qi et al. [29] developed a method to determine unknown experts' weights by considering divergence of decision matrices from positive or negative ideal decision matrix and similarity degree between individual decision matrices. Zhang and Xu [30] proposed the consensus index from the perspective of the ranking of decision information and constructed an optimal model based on the maximizing consensus in order to derive the experts' weights.

In this paper, we present an adjustable approach to fuzzy soft set-based decision-making problems using the distance measure. Moreover, we introduce a new knowledge measure and α-similarity relation over fuzzy soft sets. Based on the proposed knowledge measure and α-similarity relation, we develop two methods for obtaining appropriate experts' weights. Then, an effective group decision-making approach is constructed by integrating the aforepresented methods, from which we can find the optimal object with minor risk by tuning the value of parameters.

The rest of this paper is organized as follows. In Section 2, some basic notions of soft sets and fuzzy soft sets are reviewed. In Section 3, a new method based on the distance is proposed to solve the problems of decision-making. In Section 4, a knowledge measure and α-similarity relation are proposed for obtaining the experts' weights. Then, an approach integrating the above methods for group decision-making is developed. In Section 5, an example and a comparative analysis are given to illustrate effectiveness and practicality of presented methods. Finally, conclusions are stated in Section 6.

2. Preliminaries

In this section, some basic notions of soft sets and fuzzy soft sets are reviewed, which will be required in the later sections. Let U be an initial universe set and E be a set of parameters.

Definition 2.1 (see [4]). A pair (F, E) is called a soft set over U, if F is a mapping of E into the set of all subsets of U. In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(e)$ $(e \in E)$ from this family may be considered as the set of e-elements of the soft set or as the set of e-approximate elements of the soft set.
In [5], Maji et al. introduced the definition of fuzzy soft set by combining fuzzy set and soft set, which can be shown as follows.

Definition 2.2 (see [5]). Let U be the universe and A be the parameter set. $P(U)$ denotes the set of all fuzzy subsets of U; a pair (F, A) is called a fuzzy soft set over U, where $F : A \to P(U)$ is a mapping from A into $P(U)$.

Definition 2.3 (see [5]). Let U be the universe and (F, A) and (G, B) be two fuzzy soft sets over U; (F, A) is called a fuzzy soft subset of (G, B), denoted by $(F, A) \tilde{\subseteq} (G, B)$; if $A \subseteq B$ and $\forall e \in A$, $F(e) \subseteq G(e)$.

Example 2.1. Consider a fuzzy soft set (F, E), which describes the "attractiveness of houses" that Mr. X is considering for purchase. Suppose that there are four houses under consideration, namely, the universes $U = \{h_1, h_2, h_3, h_4\}$ and the parameter set $E = \{e_1, e_2, e_3, e_4\}$, where e_i stands for "beautiful," "large," "modern," and "cheap," respectively. Let

$$\begin{aligned} F(e_1) &= \left\{\frac{0.3}{h_1}, \frac{0.4}{h_2}, \frac{0.6}{h_3}, \frac{0.3}{h_4}\right\}, \\ F(e_2) &= \left\{\frac{0.3}{h_1}, \frac{0.5}{h_2}, \frac{0.3}{h_3}, \frac{0.5}{h_4}\right\}, \\ F(e_3) &= \left\{\frac{0.2}{h_1}, \frac{0.8}{h_2}, \frac{0.6}{h_3}, \frac{0.3}{h_4}\right\}, \\ F(e_4) &= \left\{\frac{0.7}{h_1}, \frac{0.3}{h_2}, \frac{0.2}{h_3}, \frac{0.1}{h_4}\right\}. \end{aligned} \tag{1}$$

The tabular form of such fuzzy soft set is represented in Table 1.

Definition 2.4 (see [31]). Let (F, E) be a fuzzy soft set over U and E be the parameter set, $|U| = m, |E| = n$. Fuzzy soft matrix $\tilde{\mathbf{F}} = (f_{ij})_{m\times n}$, where $f_{ij} = \mu_{F(e_j)}(x_i)$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$. That is,

$$\tilde{\mathbf{F}} = \begin{pmatrix} \mu_{F(e_1)}(x_1) & \mu_{F(e_2)}(x_1) & \cdots & \mu_{F(e_n)}(x_1) \\ \mu_{F(e_1)}(x_2) & \mu_{F(e_2)}(x_2) & \cdots & \mu_{F(e_n)}(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{F(e_1)}(x_m) & \mu_{F(e_2)}(x_m) & \cdots & \mu_{F(e_n)}(x_m) \end{pmatrix}. \quad (2)$$

From this definition, we can see that there is a one-to-one correspondence between the fuzzy soft set and fuzzy soft matrix, so we will use fuzzy soft set and fuzzy soft matrix without distinction in the following.

Definition 2.5 (see [31]). Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{G}}$ be two fuzzy soft matrices over U and $\lambda > 0$. Then,

$$\begin{aligned} \tilde{\mathbf{F}} \oplus \tilde{\mathbf{G}} &= \left(\mu_{\tilde{F}(e_j)}(x_i) + \mu_{\tilde{G}(e_j)}(x_i) - \mu_{\tilde{F}(e_j)}(x_i) \times \mu_{\tilde{G}(e_j)}(x_i)\right)_{m\times n}, \\ \lambda\tilde{\mathbf{F}} &= \left(1 - \left(1 - \mu_{\tilde{F}(e_j)}(x_i)\right)^{\lambda}\right)_{m\times n}. \end{aligned} \quad (3)$$

Theorem 2.1 (see [31]). *Let $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{G}}$ be two fuzzy soft matrices over U and $\lambda, \lambda_1, \lambda_2 > 0$, then we have*

$$\begin{aligned} \tilde{\mathbf{F}} \oplus \tilde{\mathbf{G}} &= \tilde{\mathbf{G}} \oplus \tilde{\mathbf{F}}; \\ \lambda\left(\tilde{\mathbf{F}} \oplus \tilde{\mathbf{G}}\right) &= \lambda\tilde{\mathbf{F}} \oplus \lambda\tilde{\mathbf{G}}; \\ \lambda_1\tilde{\mathbf{F}} \oplus \lambda_2\tilde{\mathbf{F}} &= (\lambda_1 + \lambda_2)\tilde{\mathbf{F}}. \end{aligned} \quad (4)$$

Theorem 2.2 (see [31]). *Let $\tilde{\mathbf{F}}_k (k = 1, 2, \dots, K)$ be fuzzy soft matrices over U and $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_K)$ be a given weight vector, where $\lambda_k > 0$ $(k = 1, 2, \dots, K)$, $\sum_{k=1}^{K}\lambda_k = 1$, then*

$$\begin{aligned} f_{\lambda}\left(\tilde{F}_1, \tilde{F}_2, \dots, \tilde{F}_K\right) &= \oplus_{k=1}^{K}\lambda_k\tilde{F}_k \\ &= \left(1 - \prod_{k=1}^{K}\left(1 - \mu_{\tilde{F}_k(e_j)}(x_i)\right)^{\lambda_k}\right)_{m\times n}. \end{aligned} \quad (5)$$

From this theorem, we know how to integrate multiple fuzzy soft sets into one fuzzy soft set.

3. An Adjustable Approach to Fuzzy Soft Set-Based Decision-Making

Generally, the existing approaches to fuzzy soft set based-decision-making are mainly based on different kinds of level soft sets. However, it is hard for decision-makers to select a suitable level soft set. In dealing with decision-making problems, it is obvious that the smaller the distance between the alternative and the decision-maker's ideal object, the better the alternative is. So in this section, we present an adjustable approach to fuzzy soft set-based decision-making problems using the distance of fuzzy soft sets in [32]. This approach is effective and reasonable under uncertain conditions. It not only allows us to avoid the problem of selecting the suitable level soft set but also helps reduce the complexity of computations in the process of decision-making.

Table 1: Fuzzy soft set (F, E).

(F, E)	e_1	e_2	e_3	e_4
h_1	0.3	0.3	0.2	0.7
h_2	0.4	0.5	0.8	0.3
h_3	0.6	0.3	0.6	0.2
h_4	0.3	0.5	0.3	0.1

Definition 3.1 (see [32]). Suppose (F, E) and (G, E) be two fuzzy soft sets over U, the distance between (F, E) and (G, E) can be defined as

$$d((F, E), (G, E)) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left|F(e_i)(x_j) - G(e_i)(x_j)\right|. \quad (6)$$

Next, to cope with weighted fuzzy soft set-based decision-making problems, we introduce a new distance between the two fuzzy soft sets as follows.

Definition 3.2. Suppose (F, E) and (G, E) be two fuzzy soft sets over U, the distance between (F, E) and (G, E) can be defined as

$$\tilde{d}((F, E), (G, E)) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}p_i\left|F(e_i)(x_j) - G(e_i)(x_j)\right|, \quad (7)$$

where $p_i = 1 - w_i$ and $w_i \in (0, 1]$ is the weight of the ith parameter e_i.
It is easy to prove that (6) and (7) satisfy the following properties:

$$\begin{aligned} & 0 \le d((F, E), (G, E)) \le 1, \\ & d((F, E), (G, E)) = 0 \Leftrightarrow (F, E) = (G, E), \\ & d((F, E), (G, E)) = d((G, E), (F, E)), \\ & (F, E) \tilde{\subseteq} (G, E) \tilde{\subseteq} (P, E), \Leftrightarrow d((F, E), (G, E)) \\ & \quad \le d((F, E), (P, E)), d((G, E), (P, E)) \\ & \quad \le d((F, E), (P, E)). \end{aligned} \quad (8)$$

In the following, we develop an algorithm to deal with decision-making problems based on fuzzy soft sets.

Algorithm 1 (decision-making based on fuzzy soft sets).
Input: A fuzzy soft set $(\tilde{F}, E)$ over U as given in Table 2, where E is a set of parameters denoted by $E = \{e_1, e_2, \cdots, e_m\}$ and U is an initial universe set denoted by $U = \{x_1, x_2, \cdots, x_n\}$.
Output: The order relation of all the alternatives.

Step 1. Construct an ideal fuzzy soft set $(\tilde{F}_0, E)$ with a single object x as given in Table 3.

Step 2. Obtain a fuzzy soft set $(\tilde{F}_k, E)$ $(k = 1, 2, \ldots, n)$ with respect to a single alternative x_k as given in Table 4.

Step 3. Calculate the distance between fuzzy soft sets $(\tilde{F}_0, E)$ and $(\tilde{F}_k, E)$ $(k = 1, 2, \ldots, n)$ using (6). If $(\tilde{F}, E)$ is a weighted fuzzy soft set and $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_m)$ is a weighting vector of parameters, we will use (7) to calculate the distance, where $p_i = 1 - \omega_i (i = 1, 2, \ldots, m)$.
It is clear that the smaller the $d((\tilde{F}_0, E), (\tilde{F}_k, E))$ is, the closer the $(\tilde{F}_k, E)$ approaches to the ideal fuzzy soft set $(\tilde{F}_0, E)$. Thus, x_k is better.

Step 4. Rank all alternatives according to $d((\tilde{F}_0, E), (\tilde{F}_k, E))$ or $\tilde{d}((\tilde{F}_0, E), (\tilde{F}_k, E))$.
In order to better understand the above idea, let us consider the following example.

Example 3.1 (see [14]). Let (F, A) be the fuzzy soft set given in Table 5.

Then, we can use the proposed decision-making method to get the ranking of the alternatives.

(1) Construct an ideal fuzzy soft set (F_0, A) with a single alternative x as given in Table 6.

(2) Based on the above fuzzy soft set (F, A), we get the fuzzy soft set $(F_k, A)(k = 1,2,3)$ with respect to a single alternative x_k as given in Tables 7–9.

(3) Utilize (6) to get the distance $d((F_0, A), (F_k, A))$ $(k = 1,2,3)$:

$$\begin{aligned} d((F_0, A), (F_1, A)) &= 0.398, \\ d((F_0, A), (F_2, A)) &= 0.358, \\ d((F_0, A), (F_3, A)) &= 0.366. \end{aligned} \tag{9}$$

(4) Rank all alternatives according to the distance $d((F_0, A), (F_k, A))$ $(k = 1,2,3)$.

Then, the order relation among all the alternatives is $x_2 \succ x_3 \succ x_1$, and the best alternative is the alternative x_2.

For comparison with different decision-making methods, we give the ranking orders of the proposed method and other methods [12, 14, 15] as shown in Table 10. Therefore, from Table 10 we can see that the three ranking orders of them are the same, and the alternative x_2 is the best choice in the proposed method and the other two methods [14, 15]. But Basu et al.'s method [12] is another ranking order, and the alternative x_3 is the best choice.

Table 2: Fuzzy soft set $(\tilde{F}, E)$.

$(\tilde{F}, E)$	e_1	e_2	e_3	$\cdots$	e_m
x_1	a_{11}	a_{12}	a_{13}	$\cdots$	a_{1m}
x_2	a_{21}	a_{22}	a_{23}	$\cdots$	a_{2m}
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
x_n	a_{n1}	a_{n2}	a_{n3}	$\cdots$	a_{nm}

Table 3: Ideal fuzzy soft set $(\tilde{F}_0, E)$.

$(\tilde{F}_0, E)$	e_1	e_2	e_3	$\cdots$	e_m
x	1	1	1	$\cdots$	1

Table 4: Fuzzy soft set $(\tilde{F}_k, E)$.

$(\tilde{F}_k, E)$	e_1	e_2	e_3	$\cdots$	e_m
x_k	a_{k1}	a_{k2}	a_{k3}	$\cdots$	a_{km}

Table 5: Tabular representation of the fuzzy soft set (F, A).

(F, E)	e_1	e_2	e_3	e_4	e_5
x_1	0.85	0.71	0.38	0.32	0.75
x_2	0.56	0.82	0.76	0.64	0.43
x_3	0.84	0.51	0.82	0.53	0.47

Table 6: Ideal fuzzy soft set (F_0, A).

(F, A)	e_1	e_2	e_3	e_4	e_5
x	1	1	1	1	1

Table 7: Fuzzy soft set (F_1, A).

(F_1, E)	e_1	e_2	e_3	e_4	e_5
x_1	0.85	0.71	0.38	0.32	0.75

Table 8: Fuzzy soft set (F_2, A).

(F_2, A)	e_1	e_2	e_3	e_4	e_5
x_2	0.56	0.82	0.76	0.64	0.43

Table 9: Fuzzy soft set (F_3, A).

(F_3, A)	e_1	e_2	e_3	e_4	e_5
x_3	0.84	0.51	0.82	0.53	0.47

Table 10: A comparison of the preference orders of the alternatives for different methods.

Decision approach	Reference	Preference order
Basu et al.'s method	[12]	$x_3 \succ x_2 \succ x_1$
Tang's method	[14]	$x_2 \succ x_3 \succ x_1$
Li et al.'s method	[15]	$x_2 \succ x_3 \succ x_1$
Proposed method		$x_2 \succ x_3 \succ x_1$

From above, we can see that the proposed method is reliable and reasonable. Moreover, the complexity of the proposed method of decision-making in this paper is lower than that of Tang and Li et al.'s methods.

4. An Approach to Fuzzy Soft Set-Based Group Decision-Making

As we all know, experts' weights play an important role in integrating individual fuzzy soft sets into a collective one for group decision-making problems. In [23], the authors comprehensively considered the similarity and proximity degrees and employed a control parameter to construct the combined weight. In [24], considering different knowledge, experiences, and preferences of diverse decision-makers, the authors seek a weight vector such that the deviations between the individual preferences and the group opinion are minimized. In [25], to derive decision-makers' weights, the authors constructed an optimization model by maximizing the group consensus. In [33], the authors proposed a method based on maximizing consensus for determining experts' weight for interval-valued intuitionistic fuzzy group decision-making problems. Inspired by these works, we introduce a new approach to determine experts' weight for group decision-making based on fuzzy soft sets.

In the practical group decision-making process, the experts usually come from different research fields, and each expert has his unique characteristics with regard to knowledge, skills, and practical experience. Thus, they are familiar with some of the attributes, but not others. In other words, there usually exists the fuzziness of the information provided for decision-makers by the experts and the divergence among the individual experts' opinions. Therefore, we consider the group decision-making problem from the perspective of the group and from the perspective of the individual. That is to say, we consider not only the consistency between the individual expert and the group but also how useful he can provide information for decision-makers as the individual expert.

4.1. Method Based on Knowledge Measure of Fuzzy Soft Set for Determining Experts' Weights. In the practical multiexpert group decision-making process, the weights of experts usually play an important role in determining the final decision results and should be taken fully into account. Generally, the existing methods for determining the experts' weights are mainly based on the relation between each expert and the other experts or the ideal expert. Few methods are proposed to obtain the individual expert's weight by considering the fuzziness of the information provided for decision-makers by the expert. So in this subsection, we propose a knowledge measure to measure the degree of fuzziness of fuzzy soft set. Then, we develop a method for obtaining appropriate experts' weights by using the proposed knowledge measure. In other words, the weights of experts can be obtained from the perspective of the individual.

Definition 4.1.1. Let (F, E) be a fuzzy soft set over U, the knowledge measure is defined as

$$K(F,E) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\sqrt{F(e_i)(x_j)^2 + \left(1 - F(e_i)(x_j)\right)^2}. \quad (10)$$

It is easy to prove that the presented knowledge measure satisfies the following properties:

(1) $K(F,E) = 1 \Leftrightarrow \forall e_i \in E, \forall x_j \in U,\ F(e_i)(x_j) = 0$, or $F(e_i)(x_j) = 1$.

(2) $K(F,E) = K((F,E)^c)$.

(3) $K(F,E) \geq K(G,E) \Leftrightarrow (F,E)$ is less fuzzy than (G,E), that is, $\forall e_i \in E, \forall x_j \in U,\ F(e_i)(x_j) \leq G(e_i)(x_j) \leq 0.5$, or $F(e_i)(x_j) \geq G(e_i)(x_j) \geq 0.5$.

(4) $K(F,E) = \sqrt{2}/2 \Leftrightarrow (F,E)$ is the fuzziest, that is, $\forall e_i \in E, \forall x_j \in U,\ F(e_i)(x_j) = 0.5$.

These properties show that the smaller the knowledge measure is, the fuzzier the available information becomes. During multiexpert group decision-making process, if the knowledge measure of a fuzzy soft set given by an expert is larger, he can provide decision-makers with more useful information, then the expert plays a relatively more important role in the group decision-making process. Therefore, the expert should be assigned a bigger weight. Otherwise, such an expert will be judged unimportant by most decision-makers. In other words, such an expert should be assigned a smaller weight.
Suppose there are P experts and they give evaluation values in the form of fuzzy soft sets, respectively, namely, $(F_k, E)(1 \leq k \leq P)$. We can get the weight of the expert k as follows:

$$\lambda_k^{(1)} = \frac{K(F_k,E)}{\sum_{k=1}^{P} K(F_k,E)}. \quad (11)$$

4.2. Method Based on Divergence Degree for Determining the Experts' Weights. In order to solve group decision-making problems, flexible and agile α-similarity relation is introduced in fuzzy soft sets. Similar to the ideas in [24, 25, 33], to measure the deviations between the individual preferences and the groups' opinion, the concept of divergence degree based on the α-similarity relation is first proposed to determine the experts' weights.

Definition 4.2.1. Let (F, E) be a fuzzy soft set over U, $A \subseteq E$ and $\alpha \in [0, 1]$, a α-similarity relation over fuzzy soft set (F, E) can be defined as

$$(S_A)^\alpha = \left\{ (x, y) \in U \times U \mid \frac{F(e_i)(x) \wedge F(e_i)(y)}{F(e_i)(x) \vee F(e_i)(y)} \geq \alpha, \forall e_i \in A \right\}. \tag{12}$$

From this definition, it can be observed that if a pair of objects (x, y) from $U \times U$ belongs to $(S_A)^\alpha$, then they are perceived as similar. It is easy to test that the α-similarity relation $(S_A)^\alpha$ is a tolerance relation (a relation that is reflexive and symmetric, but not necessarily transitive).

Let (F, E) be a fuzzy soft set over U; for $x \in U, A \subseteq E$; then, the α-similarity class of object x with respect to A is defined as

$$\left([x]_A\right)^\alpha = \{ y \in U \mid (y, x) \in (S_A)^\alpha \}. \tag{13}$$

In dealing with multiexpert group decision-making problems, the α-similarity relation under the expert k can be expressed as

$$\left(S_A^k\right)^\alpha = \left\{ (x, y) \in U \times U \mid \frac{F_k(e_i)(x) \wedge F_k(e_i)(y)}{F_k(e_i)(x) \vee F_k(e_i)(y)} \geq \alpha, \forall e_i \in A \right\}. \tag{14}$$

Then, the α-similarity classes of object x with respect to A under the expert k can be expressed as

$$\left([x]_A^k\right)^\alpha = \left\{ y \in U \mid (y, x) \in \left(S_A^k\right)^\alpha \right\}. \tag{15}$$

Thus, the family set of the α-similarity class under the expert k for the alternative set U with respect to the parameter set A can be obtained as follows:

$$\frac{U}{\left(S_A^k\right)^\alpha} = \left\{ \left([x]_A^k\right)^\alpha \mid x \in U \right\}. \tag{16}$$

In the following, we introduce the concept of divergence degree between the experts k and l for all alternatives with respect to the parameter set A, which is defined as

$$D_{kl} = \frac{1}{|U|} \sum_{x \in U} \frac{\left| \left([x]_A^k\right)^\alpha \cup \left([x]_A^l\right)^\alpha \right| - \left| \left([x]_A^k\right)^\alpha \cap \left([x]_A^l\right)^\alpha \right|}{|U|}, \quad k, l \in P. \tag{17}$$

Equation (17) expresses the divergence degree between two experts for all alternatives with respect to parameter set A. It is easy to verify that $0 \leq D_{kl} \leq 1 - (1/|U|)$. The closer the D_{kl} is to 0, the poorer the divergence. That is to say, the smaller the D_{kl} is, the more similar the experts k and l are. Further, it is easily seen that the above definition of divergence degree is effective and reasonable because it avoids the use of distance or similarity functions to measure the divergence in group decision-making and thus reduces the effect of the application of some different distance or similarity functions for measuring divergence in group decision-making.

Then, according to (17), we can get the divergence degree of expert k and all the other experts $l (l = 1, 2, \ldots, P, l \neq k)$ as follows:

$$M_k = \sum_{l=1, l \neq k}^{P} D_{kl}. \tag{18}$$

Based on the above analyses, we can get a simple and exact formula for determining the weight of the expert k as follows:

$$\lambda_k^{(2)} = \frac{M_k^{-1}}{\sum_{k=1}^{P} M_k^{-1}}. \tag{19}$$

In practice, (11) and (19) can be integrated in accordance to attitudinal characteristics of decision-makers as the following (20) for the determination of experts' weights in group decision-making based on fuzzy soft sets.

By integrating $\lambda_k^{(1)}$ and $\lambda_k^{(2)}$, the ultimate weight of expert k can be obtained:

$$\lambda_k = \rho \lambda_k^{(1)} + (1 - \rho) \lambda_k^{(2)} \quad (k = 1, 2, \ldots, P), \tag{20}$$

where ρ is the parameter that reflects attitudinal characteristics of decision-makers, $\rho \in [0, 1]$.

Finally, we could develop an algorithm to deal with multiexpert group decision-making problems with unknown experts' weights.

Algorithm 2 (group decision-making based on fuzzy soft sets).
Input: The fuzzy soft sets $(F_k, A) (1 \leq k \leq P)$ over a finite initial universe U and a finite parameter set A.
Output: The order relation of all the alternatives.

Step 1. Calculate the knowledge measures of all individual fuzzy soft set (F_k, E) of expert k $(k = 1, 2, \ldots, P)$, and then get experts' weighting vector $\boldsymbol{\lambda}^{(1)} = (\lambda_1^{(1)}, \lambda_2^{(1)}, \ldots, \lambda_P^{(1)})$, through (10) and (11).

Step 2. Set the value of α, and calculate the divergence degree between each expert and the other experts, and then ensure the experts' weighting vector $\boldsymbol{\lambda}^{(2)} = (\lambda_1^{(2)}, \lambda_2^{(2)}, \ldots, \lambda_P^{(2)})$ using (17), (18), and (19).

Step 3. Set the value of ρ, and let $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_P)$ be the ultimate weighting vector of experts, which can be formed through (20).

Step 4. Calculate the integrated fuzzy soft set $(\tilde{F}, E)$ according to (5).

Step 5. Apply Algorithm 1 to the integrated fuzzy soft set $(\tilde{F}, E)$, then get the optimal alternatives.

5. Comparison Analysis

In this section, we consider an illustrative example and comparison analyses to demonstrate the practicability, feasibility, and effectiveness of the proposed method.

5.1. Illustrative Example. In this subsection, to demonstrate the applicability of the proposed approach for effectively solving the group decision-making problem based on fuzzy soft sets, we present an example modified from [33] as follows.

Example 5.1. Suppose a company is considering 4 short-listed candidates $U = \{x_1, x_2, x_3, x_4\}$ for a job position. An interview session is held where three experts are to evaluate each candidate over four criteria, namely, good attitude e_1, pleasant personality e_2, good command in English e_3, and competent communication skills e_4. $E = \{e_1, e_2, e_3, e_4\}$ is the set of parameters. Here, we assume that the weighting vector of the parameters is determined as $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3, \omega_4) = (0.1, 0.2, 0.6, 0.1)$. And the experts' weights is unknown. (F_k, E) $(k = 1,2,3)$ is a fuzzy soft set given by expert k (Tables 11–13).

Then, we utilize the developed approach to get the ranking of the alternatives.

(1) Calculate the knowledge measure of fuzzy soft set (F_k, E) $(k = 1,2,3)$.

According to Definition 4.1.1, we can get the knowledge measure $K(F_k, E)$ $(k = 1,2,3)$ as the following:

$$\begin{aligned} K(F_1, E) &= 0.7522, \\ K(F_2, E) &= 0.7393, \\ K(F_3, E) &= 0.7489. \end{aligned} \tag{21}$$

(2) Determine the weights of experts.

By utilizing (11), the weighting vector of experts can be obtained as the following:

$$\boldsymbol{\lambda}^{(1)} = \left(\lambda_1^{(1)}, \lambda_2^{(1)}, \lambda_3^{(1)}\right) = (0.3357, 0.3300, 0.3343). \tag{22}$$

(3) Set $\alpha = 0.6$, and obtain the family set of the α-similarity classes under the expert k $(k = 1,2,3)$ for the alternative set U with respect to the parameter set E.

Through (14), (15), and (16), we can get the family set of α-similarity class $([x]_E^k)^\alpha$ $(k = 1,2,3)$ as the following:

$$\begin{aligned} \frac{U}{(S_A^1)^\alpha} &= \{\{x_1, x_2, x_3, x_4\}, \{x_1, x_2, x_4\}, \{x_1, x_3\}, \{x_1, x_2, x_4\}\}, \\ \frac{U}{(S_A^2)^\alpha} &= \{\{x_1, x_2, x_4\}, \{x_1, x_2, x_4\}, \{x_3\}, \{x_1, x_2, x_4\}\}, \\ \frac{U}{(S_A^3)^\alpha} &= \{\{x_1, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}, \{x_2, x_3, x_4\}\}. \end{aligned} \tag{23}$$

Table 11: The evaluation value of expert 1.

(F_1, E)	e_1	e_2	e_3	e_4
x_1	0.6	0.7	0.5	0.8
x_2	0.7	0.6	0.5	0.7
x_3	0.4	0.9	0.6	0.7
x_4	0.5	0.5	0.8	0.6

Table 12: The evaluation value of expert 2.

(F_2, E)	e_1	e_2	e_3	e_4
x_1	0.5	0.7	0.6	0.6
x_2	0.5	0.6	0.7	0.7
x_3	0.6	0.3	0.5	0.8
x_4	0.7	0.6	0.7	0.5

Table 13: The evaluation value of expert 3.

(F_3, E)	e_1	e_2	e_3	e_4
x_1	0.6	0.7	0.6	0.5
x_2	0.7	0.4	0.7	0.8
x_3	0.7	0.6	0.6	0.5
x_4	0.8	0.4	0.8	0.6

(4) Calculate the divergence degree between the two experts.

According to (17), the divergence degree between the two experts can be calculated based on the α-similarity class:

$$\begin{aligned} D_{12} &= 0.125, \\ D_{13} &= 0.5, \\ D_{23} &= 0.625. \end{aligned} \tag{24}$$

(5) Obtain the weights of experts.

By utilizing the (18) and (19), the weighting vector of experts can be obtained as the following:

$$\boldsymbol{\lambda}^{(2)} = \left(\lambda_1^{(2)}, \lambda_2^{(2)}, \lambda_3^{(2)}\right) = (0.4186, 0.3488, 0.2326). \tag{25}$$

(6) Set $\rho = 0.5$, and determine the ultimate weight vector of experts.

According to (20), the ultimate weight vector of experts can be obtained as the following:

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3) = (0.3772, 0.3394, 0.2834). \tag{26}$$

(7) Calculate the integrated fuzzy soft set.

Through the (5), we can aggregate the overall individual fuzzy soft set (F_k, E) $(k = 1,2,3)$ to get the integrated fuzzy soft set (F, E) as given in Table 14.

(8) Rank the alternatives.

By utilizing the Algorithm 1, the ranking order of all the alternatives x_i $(i = 1,2,3,4)$ can be obtained as the following:

$$x_3 \succ x_2 \succ x_1 \succ x_4. \tag{27}$$

Thus, the optimal alternative is x_3.

Table 14: Collective fuzzy soft set.

(F, E)	e_1	e_2	e_3	e_4
x_1	0.5685	0.7000	0.5649	0.6720
x_2	0.6432	0.5513	0.6362	0.7326
x_3	0.5704	0.7132	0.5685	0.6979
x_4	0.6757	0.5119	0.7705	0.5685

5.2. Sensitivity Analysis on the Parameters. In the above example, the computation results are obtained by given the parameters a priori in (14) and (20). Normally, the different values of parameters can lead to different weights of experts, then different ranking order of alternatives. To inspect the influence of different parameters on the experts' weights, it is necessary to do sensitivity analysis of the parameters.

From Example 5.1, we can see that if $\alpha \leq 0.43$ or $\alpha \geq 0.86$, the divergence degree between any two experts is 0, which is meaningless. So it is significant for decision-makers to know the range of the parameter α. Let

$$a_{jk} = \min \left\{ \frac{F_k(e_i)(x_j) \wedge F_k(e_i)\left(y_j\right)}{F_k(e_i)(x_j) \vee F_k(e_i)\left(y_j\right)} \mid j = 1, 2, \ldots, |U|, k = 1, 2, \ldots, P, e_i \in E, x_j \neq y_j \right\}, \tag{28}$$

and

$$\begin{aligned} a &= \min \left\{ a_{jk} \mid j = 1, 2, \ldots, |U|, k = 1, 2, \ldots, P \right\}, \\ b &= \max \left\{ a_{jk} \mid j = 1, 2, \ldots, |U|, k = 1, 2, \ldots, P \right\}. \end{aligned} \tag{29}$$

We have $\alpha \in [a, b]$, where $|U|$ and P are the number of all alternatives and experts, respectively, and E is the set of parameters. It is clear that the divergence degree between any two experts is 0 when $\alpha \leq a$ or $\alpha \geq b$. So decision-makers can choose suitable parameter α according to the practical situation.

As can be seen from Table 15, experts' weights vary along the parameters α and ρ sensitively. When decision-makers' attitudinal characteristic parameters ρ are not changed, we find that the experts' weights change as α changes. And when the parameter α remains unchanged, the experts' weights are not same for different values of parameters ρ. In other words, every expert's weight changes when the value of any one of parameters takes different values. Moreover, it is observed that the ranking results may be not identical with respect to different values of parameters α and ρ.

According to the above analysis, it can be observed that the group decision-making approach proposed in this paper enables decision-makers to express their preference information more comprehensively during decision processes. In other words, the proposed method can yield proper ranking results in accordance with decision-makers' opinions by choosing suitable parameters. Furthermore, the proposed approach can provide decision-makers with more choices in solving the problems of group decision-making and thus has better flexibility and agility.

5.3. Comparison with Existing Methods for Deriving the Experts' Weights. To illustrate the advantage of our proposed approach of determining experts' weights, we make a comparative analysis with other previous methods including Mao et al.'s method [27], Zhang and Xu's method [30] and Wan et al.'s methods [23–25].

The detailed comparisons with the methods [23–25, 27, 30] are listed in Table 16.

As shown in Table 16, we can conclude the following.

Compared with Mao et al.'s method in [27], the proposed method is based on divergence degree and knowledge measure. Whereas, Mao et al.'s method is based on distance. Considering the consistency between the individual expert and the group, Mao et al. obtain the experts' weights by using the distance. However, different distance functions can produce different results. In order to avoid the use of distance functions to determine the experts' weights, we propose the concept of the divergence degree for determining the experts' weights. Moreover, we also consider the experts' weights from the perspective of the individual. That is to say, we can obtain the experts' weights by considering the fuzziness of the information provided by the individual expert, so we introduce the knowledge measure for determining the experts' weights. Therefore, the proposed method can determine the weights of the experts more objectively.

Compared with methods [24, 25, 30], our method is based on divergence degree and knowledge measure, and the methods in [24, 25, 30] are based on consensus degree.

Table 15: Experts' weights and ranking results with different parameters α and ρ.

Parameters		Expert weights	Ranking results	Optimal
$\alpha = 0.60$	$\rho = 0.3$	{0.3937, 0.3432, 0.2631}	$x_3 \succ x_2 \succ x_1 \succ x_4$	x_3
	$\rho = 0.5$	{0.3772, 0.3394, 0.2834}	$x_3 \succ x_2 \succ x_1 \succ x_4$	x_3
	$\rho = 0.8$	{0.3523, 0.3338, 0.3139}	$x_2 \succ x_3 \succ x_1 \succ x_4$	x_2
$\alpha = 0.65$	$\rho = 0.3$	{0.3586, 0.3201, 0.3213}	$x_2 \succ x_3 \succ x_1 \succ x_4$	x_2
	$\rho = 0.5$	{0.3521, 0.3229, 0.3250}	$x_2 \succ x_3 \succ x_1 \succ x_4$	x_2
	$\rho = 0.8$	{0.3422, 0.3272, 0.3306}	$x_3 \succ x_2 \succ x_1 \succ x_4$	x_3
$\alpha = 0.70$	$\rho = 0.3$	{0.3478, 0.3460, 0.3062}	$x_2 \succ x_3 \succ x_1 \succ x_4$	x_2
	$\rho = 0.5$	{0.3443, 0.3415, 0.3142}	$x_2 \succ x_3 \succ x_1 \succ x_4$	x_2
	$\rho = 0.8$	0.3391, 0.3346, 0.3263	$x_3 \succ x_2 \succ x_1 \succ x_4$	x_2
$\alpha = 0.80$	$\rho = 0.3$	{0.3807, 0.3790, 0.2403}	$x_2 \succ x_3 \succ x_1 \succ x_4$	x_2
	$\rho = 0.5$	{0.3679, 0.3650, 0.2671}	$x_3 \succ x_2 \succ x_1 \succ x_4$	x_3
	$\rho = 0.8$	{0.3486, 0.3440, 0.3074}	$x_3 \succ x_2 \succ x_1 \succ x_4$	x_2

Table 16: Comparison with other existing methods for deriving the experts' weights.

Method	Measurement tool	The final results
Method in [27]	Distance	The consistency between the individual expert and the group
Methods in [24, 25, 30]	Consensus degree	The consistency between the individual expert and the group
Method in [23]	Distance	Similarity
	Distance	Proximity
Proposed method	Divergence degree	The consistency between the individual expert and the group
	Knowledge measure	The fuzziness of the information provided by the individual expert

In [24, 25, 30], the authors only consider the consistency between the individual expert and the group for determining the experts' weights; they fail to consider the fuzziness of the information provided by the individual expert. To derive the weight of each expert more objectively, we consider the problem not only from the perspective of the group but also from the perspective of the individual. That is to say, we consider not only the consistency between the individual expert and the group but also how useful he can provide information for decision-makers as the individual expert. So the method in this paper is more suitable for determining the experts' weights.

Compared with the method in [23], experts' weights are determined by similarity and proximity degree, where the similarity degree in [23] is the knowledge measure in our method in essence. The proximity degree defined by distance in [23] is similar to the divergence degree in our method. However, different distance functions can produce different results. In order to avoid the use of distance functions to determine the experts' weights, we propose the concept of the divergence degree for determining the experts' weights.

According to the above analysis, we can see that the proposed method for determining the experts' weights is reasonable and effective. And it can determine the weights of the experts more objectively. Furthermore, the weights of the experts may be changed by changing the parameters (see (14) and (20)), which can give greater flexibility to the decision-makers in solving the problems of group decision-making.

5.4. Comparison with the Method in [33]. In this subsection, we compare the developed approach in this paper with another method proposed by Sulaiman and Mohamad [33] to explain the superiorities of the proposed method.

The detailed comparison with the method in [33] is listed in Table 17.

From Table 17, we can see that the ranking of the alternatives obtained by the proposed method is not the same as that obtained by Sulaiman and Mohamad's method. The chief reason for different ranking results is that the methods used to obtain the weights of experts in the proposed approach and in [33] are different. The method in [33] only considers the consistency between the individual expert and the group, and it ignores the fuzziness of the information provided for decision-makers by the individual expert. To derive the weight of each expert more objectively, the proposed method considers not only how close the expert's opinion is to other experts' but also how useful he can provide information for decision-makers as an expert in the process of group decision-making. So we introduce a knowledge measure and divergence degree between two experts based on α similarity relation for obtaining the experts' weights. In [33], Sulaiman and Mohamad proposed a method based on

Table 17: Comparison with the method in [33].

Method	Measurement tool	Parameters	Experts' weights	Ranking results
Proposed method	Divergence degree and knowledge measure	$\alpha = 0.60, \rho = 0.3$	$\{0.3937, 0.3432, 0.2631\}$	$x_3 \succ x_2 \succ x_1 \succ x_4$
		$\alpha = 0.60, \rho = 0.5$	$\{0.3772, 0.3394, 0.2834\}$	$x_3 \succ x_2 \succ x_1 \succ x_4$
		$\alpha = 0.60, \rho = 0.8$	$\{0.3523, 0.3338, 0.3139\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
		$\alpha = 0.65, \rho = 0.3$	$\{0.3586, 0.3201, 0.3213\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
		$\alpha = 0.65, \rho = 0.5$	$\{0.3521, 0.3229, 0.3250\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
		$\alpha = 0.65, \rho = 0.8$	$\{0.3422, 0.3272, 0.3306\}$	$x_3 \succ x_2 \succ x_1 \succ x_4$
		$\alpha = 0.70, \rho = 0.3$	$\{0.3478, 0.3460, 0.3062\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
		$\alpha = 0.70, \rho = 0.5$	$\{0.3443, 0.3415, 0.3142\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
		$\alpha = 0.70, \rho = 0.8$	$\{0.3391, 0.3346, 0.3263\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
		$\alpha = 0.80, \rho = 0.3$	$\{0.3807, 0.3790, 0.2403\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
		$\alpha = 0.80, \rho = 0.5$	$\{0.3679, 0.3650, 0.2671\}$	$x_3 \succ x_2 \succ x_1 \succ x_4$
		$\alpha = 0.80, \rho = 0.8$	$\{0.3486, 0.3440, 0.3074\}$	$x_2 \succ x_3 \succ x_1 \succ x_4$
Method [33]	Similarity measure	$\alpha = 0.50$	$\{0.3308, 0.3322, 0.3370\}$	$x_4 \succ x_2 \succ x_1 \succ x_3$

Note: "α" in [33] is different from that in this paper.

similarity measures for determining the experts' weights in multiexpert group decision-making. However, different similarity functions can produce different results. In order to avoid the use of similarity functions to determine the experts' weights, we introduce the concept of divergence degree for obtaining the experts' weights. It is easily seen that the definition is simple and reasonable. Moreover, if [33] uses the decision-making method proposed in Section 3 to rank the alternatives, the ranking of the alternatives is $x_2 \succ x_1 \succ x_3 \succ x_4$. From Table 15, we can see that the ranking results are different from those obtained by the decision-making method in [33]. Therefore, the different decision-making approaches given in our approach and in [33] are another reason of different ranking results. The former uses the proposed distance between two fuzzy soft sets to rank the alternatives, while the latter ranks the alternatives based on the score indexes. And the costs of computation of decision-making method in this paper are lower than those in [33].

Although different values of parameters will result in different ranking order of alternatives, we can see from Table 17 that the desirable ranking order may be $x_2 \succ x_3 \succ_1 x \succ x_4$ or $x_3 \succ x_2 \succ_1 x \succ x_4$. But how to determine which ranking order will be the desirable ranking order? We observe that $x_2 \succ x_3 \succ_1 x \succ x_4$ occurs eight times and $x_3 \succ x_2 \succ_1 x \succ x_4$ occurs four times in Table 17, so we can choose the desirable ranking order according to their frequency of occurrence; thus, $x_2 \succ x_3 \succ_1 x \succ x_4$ will most likely be the desirable ranking order. So it will have minor risk when we choose x_2 as the optimal object.

Because of the parameters α, ρ involved in Algorithm 2, we know that different values of parameters will result in different ranking order of alternatives. So we can do repeated experiments through choosing the parameter values of α, ρ randomly in Algorithm 2, which will output multiple ranking results. Counting the occurrence times of every ranking order among all of ranking orders, then the desirable ranking order is obtained, which is the one repeated most often among all of the ranking orders.

Compared with Sulaiman and Mohamad's method [33], our proposed approach has some advantages:

(1) The proposed approach has capability to deal with the involvement of multiple experts and the presence of subjectiveness and imprecision in the multiexpert group decision-making problem.

(2) The introduced technology can determine objectively the weights of the experts, which avoids the subjective randomness of determining the weights.

(3) The weights of the experts may be changed by changing the parameters, which can give greater flexibility to the decision-makers in choosing the ranking order which will most likely be the desirable ranking order.

6. Conclusion

In this paper, we develop a new method based on distance to solve the fuzzy soft set decision-making problem. In order to determine the weight of each expert objectively, we introduce the concepts of knowledge measure and divergence degree. Based on the concepts, we develop two methods for obtaining appropriate experts' weights. The ultimate weights of experts are obtained by integrating the two methods. Then, we develop an effective group decision-making approach by integrating the aforepresented methods. Finally, we give an example and a sensitivity analysis about the parameters to illustrate the proposed method and compare this method with other existing methods, which demonstrates the reasonability and efficiency of the new group decision-making method proposed in our paper.

Further study could be required to extend developed approaches to other practical decision-making environments such as interval-valued intuitionistic fuzzy environment.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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