Stable regularities without governing laws?

Forthcoming in *Studies in History and Philosophy of Modern Physics*

Aldo Filomeno*

Abstract

Can stable regularities be explained without appealing to governing laws or any other modal notion? In this paper, I consider what I will call a 'Humean system'—a generic dynamical system without guiding laws—and assess whether it could display stable regularities. First, I present what can be interpreted as an account of the rise of stable regularities, following from Strevens [2003], which has been applied to explain the patterns of complex systems (such as those from meteorology and statistical mechanics). Second, since this account presupposes that the underlying dynamics displays deterministic chaos, I assess whether it can be adapted to cases where the underlying dynamics is not chaotic but truly random—that is, cases where there is no dynamics guiding the time evolution of the system. If this is so, the resulting stable, apparently non-accidental regularities are the fruit of what can be called statistical necessity rather than of a primitive physical necessity.

Keywords

Laws of nature; physical necessity; non-accidental regularities; dynamical systems; method of arbitrary functions

Highlights:

- It is studied the applicability and the significance of results from dynamical systems theory to philosophical debates on laws of nature and physical necessity.

*Czech Academy of Sciences, Institute of Philosophy. **Acknowledgments:** I would like to thank Michael Strevens, Carl Hoefer, and Robert Batterman for their unvaluable support and comments. Many thanks also to Axel Barceló, Claudio Calosi, Eduardo García, John Horden, Mairym Llorens, Ricardo Mena, Stephen Mumford, Elías Okon, Carlos Romero, Alessandro Torza, Jos Uffink, anonymous reviewers at *Studies in History and Philosophy of Modern Physics*, and audiences at Utrecht, Barcelona, Krakow, and Valparaíso. This work was supported by the Instituto de Investigaciones Filosóficas (Universidad Nacional Autónoma de México) through a fellowship from the postdoctoral fellowship program DGAPA-UNAM, and by the grant Future Synthesis of the program Praemium Academicum of the Czech Academy of Sciences.

- The application of the method of arbitrary functions to explain the patterns of complex systems is extended to process-random, lawless, systems.
- On the one hand, the difficulty of avoiding the assumption of dynamical conditions remarks a serious difficulty in Humean accounts (such as the Best System Account) mostly neglected in the literature.
- On the other hand, a proposal to meet this difficulty is offered, based on the method of arbitrary functions and a non-dynamical justification of the sufficient conditions for its applicability.
- A Humean may welcome the results presented, even if dynamical conditions are assumed, as she would regard them as more intelligible and of less modal import than a standard set of governing laws.

Contents

1 Introduction

Some regularities in the world do not seem to be accidental. Their traditional explanation, then, appeals to the existence of laws of nature. However, under philosophical scrutiny the notion of law of nature turns out to be mysterious. Philosophers have tried to reduce it to other notions (for example, necessity relations, propensities, or causation). Still, some of us felt that little was gained, since these are still modal notions that postulate a mysterious primitive physical necessity in our ontology. Thus, some have sought a reductionist account of laws of nature to something non-modal. Associated with David Hume's skepticism towards necessary connections, such an account is known as the 'Best System Account' (Mill, 1884; Lewis, 1999; Earman and Roberts, 2005; Cohen and Callender, 2009). According to it, the laws of nature (and any modal notion) supervene on the Humean mosaic, that is, on the vast non-modal spatio-temporal mosaic of local matters of particular fact. The Humean does not confer physical necessity on the Humean mosaic and thus neither onto the mosaic's time evolution; in other words, laws do not *govern* the mosaic's trajectory through state-space.

Now, some Humeans consider that there is no need to explain the abundance of the apparently non-accidental regular behaviour ubiquitous in our Humean mosaic, such as that described by the current laws of physics. Yet, others among us find this too quick, to say the least. Our Humean mosaic displays some regularities that are extremely stable, even exceptionless. Thus for instance Foster [1983, 89] finds that something is missing in Humean or 'deflationary' accounts (see also Carroll, 2008, 2016):

> "The past consistency of gravitational behavior calls for some explanation. For given the infinite variety of ways in which bodies might have behaved non-gravitationally and, more importantly, the innumerable occasions on which some form of non-gravitational behavior might have occurred and been detected, the consistency would be an astonishing coincidence if it were merely accidental – so astonishing as to make the accident-hypothesis quite literally incredible."

In this paper I share the Humean spirit in that I do not commit myself to any genuine physical necessity; however, I consider that the "cosmic coincidence" of the empirically acknowledged extremely stable regularities demands, or at least welcomes, an explanation.[1]

Accordingly, in this paper I address the following question: *can stable regularities be explained without appealing to governing laws?* A positive answer—not presupposing primitive causation, propensities, or any such modal notion—would complement deflationary accounts of physical necessity.

I wish to start by suggesting that an answer to the question, be it positive or negative, could be sought in the fields of dynamical systems theory and probability theory, in particular in the *stability results of dynamical systems theory* and the *convergence results of the theory of stochastic processes*, both fields which have barely been explored in philosophy.

In this paper, in fact, I study the formation of stable regularities through one such mathematical result, the so-called 'method of arbitrary functions' (MAF). I assess whether the results of MAF—coarse-grained stable patterns—could hold in scenarios lacking a guiding dynamics.

Accordingly, in Section 3 I reconstruct how MAF accounts for coarse-grained stable patterns in deterministic chaotic systems. I refer to work by Strevens [2003] that generalizes the application conditions of the method and applies it to the domain of complex systems sciences, such as meteorology and statistical mechanics.

Before that, in Section 2, I characterize a physical system that lacks guiding laws. Drawing from dynamical systems theory, I characterize a physical system that lacks guiding laws in terms of *all its possible state-space trajectories.*[2] Such a 'Humean system', as I will call it, can be described in terms of

[1] For a defense that even the Humean should demand an explanation of the extremely specific arrangement of the Humean mosaic, see Filomeno [20xxa,x].

[2] The state-space of a physical system is the set of its possible states, where a state is a set of values for all of the system's

the mathematical notion of *randomness* of a process. In Appendix A I complement this characterization with further technical details.

In Section 4, then, I assess whether the results of MAF could also obtain in such a system without guiding laws.

I first argue that in MAF the particular form of the assumed laws—the "governing" differential equations—is irrelevant as long as the dynamics is chaotic. More specifically, I propose a way to meet the two conditions required by MAF without assuming any dynamical property (in §4.1 and §4.2 respectively). These two conditions are those that the MAF literature assumes in standard examples.

This leaves, then, the required dynamical property of chaos. Yet as I spell out in §4.3 (and Appendix B), this property is unproblematic. The underlying reason is that chaos is required for the explanation of stable patterns in complex systems sciences *precisely because* a chaotic guiding dynamics produces (relative to relevant variables and a sufficiently coarse-grained description) a random-looking distribution among the possible values. And now, instead of a chaotic guiding dynamics, we explicitly postulate the desired randomness. We just postulate a randomly generated state-space trajectory which, as I spell out below, typically is random-looking.

Finally, in the Conclusion (Section 5) I discuss whether the results presented could be significant for metaphysical debates on laws of nature. On the one hand, this paper allows us to see how difficult is it for deflationist accounts of laws to provide an explanation of lawful behaviour. This upshot aims to highlight a serious difficulty in deflationist accounts mostly neglected in the literature. On the other hand, a proposal to meet this difficulty is offered: the results presented aim to provide a possible explanation of the rise of stable regularities from non-dynamical conditions.

Now, the significance of this proposal is proportional to the extent that the conditions assumed are completely non-dynamical. In the best case, the conditions are completely non-dynamical, in that they can be justified without presupposing any governing dynamics—for instance along the lines proposed in Section 4. However, it is unclear that my proposal could be completely non-dynamical, I object in Section 5. So, while I think that it would not be cautious to discard the best case since the objections might be addressed in future research (pursuing another non-dynamical justification of the conditions, e.g. seeking another reductive explanation or justifying them a privileged status), I consider that the conditions are not completely non-dynamical. This is a worse, yet less controversial, case, in which we have arrived to two conditions, dynamical to some extent, which suffice to account for stable regularities. Even in this case, I argue, the resulting proposal would be of interest to Humean (or deflationist, more generally) accounts of physical necessity. For as we will see, a Humean would regard as more intelligible, and of

variables. A *state* is represented as a point in the state-space, and a *trajectory* of successive points represents the system's time evolution. A *system* is to be understood as is standard in physics, that is, a model of a portion of the universe, which can also represent the whole universe. Hence, the Humean mosaic (as it includes a temporal dimension) corresponds to the trajectory in state-space of a point representing the state of the whole universe. More details in Section 2 and Appendix A.

less modal import, the postulation of such conditions rather than the postulation of a fully determinate set of specific governing laws.

2 Preliminary Definitions

Contemporary Humean metaphysics does not address how to model a scenario without governing laws—it assumes that the whole Humean mosaic is a brute fact. Here instead I address such a scenario, which I call 'Humean' because of the stipulated lack of any physical necessity. It is convenient throughout what follows to keep in mind the Humean view on laws and to express the scenario under study in those terms. To characterize a scenario without guiding laws we have to consider all the possible *trajectories* that a generic physical system can display through its state-space. From a Humean point of view, that is to say that for any initial condition each of the trajectories has a corresponding best induction which results in a set of axioms—the *Humean laws*—that best *describe* such a trajectory.

There is in principle no restriction on how the trajectory evolves. It is not generated or governed by any dynamics. It is, we will say, *randomly* generated. Accordingly, a physical system without governing laws can be described in terms of the mathematical notion of randomness. We must specify now which specific notion of randomness we refer to.

2.1 Lack of Guiding Laws in Terms of Process Randomness

Two senses of randomness are distinguished in the mathematical literature. One is the so-called *process* (or genesis) sense; the other, the *product* (or performance) sense. The product sense refers to a feature of a sequence that has occurred previously; it is a judgement as to whether the resulting outcomes, as they appear, present certain characteristics that warrant the sequence meriting the qualification of random. This sense has been historically formalized by mathematicians in terms of: 1) *complexity*; 2) *disorder*; and 3) *typicality*.[3] Additionally, Schnorr's theorem proves that the three definitions turn out to be equivalent [Eagle, 2012, §2.3].

The process sense, meanwhile, is in principle orthogonal to the appearance of the outcomes of a sequence. It is related to the process that generates an outcome: with its generation. Earman [1986, 137] describes the process sense of randomness as describing a process that is *"without principle, without guidance of laws"* (my italics), whereas the product sense describes an output that is "disordered or lacking in pattern" (see also Berkovitz et al. 2006, 666).

Hence, a scenario that lacks governing laws is to be described by the process sense. The resulting domain of possible trajectories of a randomly generated (i.e. process-random) dynamics is the entire

[3] For details on how these 3 notions are mathematically defined see, e.g., Earman [1986, Ch. VIII], Dasgupta [2011], Volchan [2002], or Eagle [2012, §2].

possibility space of trajectories, thus coinciding with the characterization laid out above in terms of all possible state-space trajectories.

One distinctive property which a process-random system possesses is that the state at time t is *independent* of any states at any other time. In a model with discrete time instants, the previous states (including the present) $x_t, x_{t-1}, \ldots x_0$ are irrelevant for determining the successive state x_{t+1}. So-called *Bernoulli* processes are commonly used to model real systems that are governed by laws but that their states at different times are independent. In our model, we will explicitly *postulate* that each state is independent of the others. Throughout the paper we will come to know more about what a system without governing laws amounts to; especially in §4.1, where we will see the close relationship between process and product randomness, and in §4.3, where we will see the relationship of both notions of randomness with chaos. More specifically, I include in Appendix A a formal model of a generic dynamical system whose time evolution follows according to the Bernoulli property: the so-called Bernoulli scheme.

The present characterization, which considers that the fundamental time evolution is randomly generated and thus considers all possible trajectories, shares similarities with characterizations found in physics, worth exploring elsewhere.[4]

A caveat on the lack of constraints in the state-space trajectory is worth mentioning now. Given that many interesting results in support and against the rise of coarse-grained patterns hold assuming certain constraints on the dynamics space (for example in the so-called Hamiltonian dynamical systems), it might be worth exploring also such scenarios, which I briefly survey in the appendix B. Then, however, their philosophical significance for accounts of laws of nature should be reassessed, because such scenarios assume what seems to be some dynamical condition. For instance, Hamiltonian dynamical systems, extensively studied and showing several potentially significant results of stability, assume the principle of conservation of energy. To what extent it would be worth exploring an account of lawful behaviour which presupposes the principle of conservation of energy? This constraint would in fact threaten to limit the scope of the project.

[4] In physics, certain projects model the lawless scenario following a similar strategy as well as seeking a similar goal: projects seeking the derivation of laws from what they call a "random dynamics" and projects concerning entropic forces. According to the former, *all complex* Lagrangians lead in the low-energy limit to the laws of particle physics [Froggatt and Nielsen, 1991, Froggatt and Nielsen, 2002, Chadha and Nielsen, 1983, Chkareuli et al., 2011]. See also Mukohyama and Uzan [2013], Jacobson and Wall [2010]. Similarly, according to the (speculative) projects of entropic forces, allegedly fundamental forces (such as gravity) are explained not as fundamental but as emergent, arising from the statistical behaviour of lower-level degrees of freedom; see Verlinde [2011, 2017] or the more elaborated derivation of the Einstein field equations from thermodynamic assumptions of Jacobson [1995]. Previously, Wheeler proposed the idea of "law without law" [Wheeler, 1983, Deutsch, 1986]. According to him, chaos and unknown regulating principle(s) lead to approximate laws. Cf. also the research on chaotic cosmologies [Linde, 1983, Misner, 1969, Barrow, 1977], which assumes an undetermined fundamental chaotic dynamics. Finally, despite substantial differences, a proposal for the emergence of laws from a lawless level can be traced back to the (metaphysical) 'evolutionary cosmology' of C. S. Peirce [Peirce, 1867, Reynolds, 2002].

Still, let me suggest two lines of response here. (1) Such conservation principle could be in turn justified by other reasons, as in fact has been historically attempted (for a survey of (disputable) historical attempts see Darrigol, 2014, ch.3). For instance, one could (1.a) confer a privileged status to this conservation principle, sometimes considered a *meta-law*, or (1.b) seek a non-dynamical interpretation of the principle along the lines suggested in §4.2.3, where it is remarked that the spacetime structure leads to certain symmetries, and the specific symmetry of time-translation leads to the conservation of energy (by Noether's theorem). (2) Otherwise, one can just assume the principle and argue why the account still is philosophically significant for the metaphysical debates on laws of nature. Still in this case, as I argue in Section 5 and appendix A, the results can be significant for deflationist accounts of physical necessity and worth exploring in future research.

2.2 Chaos

The application of MAF requires certain conditions to hold. Drawing from Strevens [2003, 2013], I appeal to sufficient conditions. As we will see in the next section, one of these conditions is chaos. There is no common shared definition of chaos, but fortunately we can leave aside these debates by specifying the feature that we will need: *sensitive dependence on initial conditions* [Strogatz, 1994, Hasselblatt and Katok, 2003]. That is, in a chaotic dynamical system arbitrarily small variations in the initial conditions become magnified over time—what is also known as *divergence of nearby trajectories* (we will not need the divergence to be exponential).[5]

We will therefore refer to chaos, although future research might show that this condition for MAF can be replaced by the weaker condition of *ergodicity*: the condition that the system's trajectory through state-space visits regions of the same volume with the same frequency.[6]

[5] Other characterizations further demand aperiodic long-term behaviour [Strogatz, 1994, 323]; topological mixing; the system exhibiting dense periodic orbits [Hasselblatt and Katok, 2003]; autocorrelations vanishing in the infinite-time limit; continuous power spectra; positive Liapunov exponents; and the presence of strange attractors [Smith, 1998, ch. 10]. Aperiodic long-term behaviour means that there are trajectories which do not settle down to fixed points, periodic orbits, or quasiperiodic orbits as $t \to \infty$. For a dynamical system to exhibit topological mixing means that the system evolves over time such that any given region of its state-space will eventually overlap with any other given region. That its periodic orbits are dense means that every point in the space is approached arbitrarily closely by periodic orbits. Alternatively, within the so-called ergodic hierarchy, which I will introduce in §4.3, Werndl [2009] defines chaos in terms of *mixing* (following on from a stronger characterization in terms of the Kolmogorov property by Belot and Earman [1997]). Finally, Berkovitz et al. [2006] define chaos as a matter of degree within the ergodic hierarchy.

[6] Chaos implies ergodicity, through the 'chaotic hypothesis' [Gallavotti, 2008], or as seen in the ergodic hierarchy explained in §4.3.

3 The Method of Arbitrary Functions

MAF dates back to Von Kries [1886] and Poincaré [1896]; yet I focus on the recent treatment carried out by Strevens [2003].[7] As we will see, MAF resembles approaches found in the foundations of statistical mechanics: (1) ergodic theory; (2) what Uffink [2006, 135] calls 'coarse-graining stochastic dynamics'; and (3) 'typicality' approaches to equilibrium, especially when the typicality *of dynamics* is stressed, as in [Frigg, 2009]. Besides, there are results similar to those that MAF yields, whose significance could be explored elsewhere:

1. in dynamical systems theory, results of stability (and instability) and periodicity (and aperiodicity) of trajectories.[8]

2. in the theory of probability and random processes, results of convergence of random sequences. The most representative is the law of large numbers.[9]

Regarding the philosophical significance of these results, Batterman [1992] is the first to point out that in some physical systems certain phenomena emerge whose explanation is irreducibly statistical.[10] Among those who have explored MAF, Suppes [1987] explicitly contended that it has metaphysical implications, arguing that this result could explain the metaphysical notion of *propensity*. Hence, although Suppes talks of propensities and I talk of stable regularities, our aims do not seem to genuinely differ.

Besides the approaches cited above and those in footnote 4 from physics, it is also worth citing a different and venerable body of literature that has sought the same goal—an explanation of lawful behaviour—by trying to vindicate a (logically) necessary status to laws of nature. See for instance Darrigol's (2014) proposal, which also discusses previous proposals in the history of physics. Rather than seeking the emergence of coarse-grained regularities from the *free* time-evolution of the system, this approach seeks to vindicate the logical necessity of the laws by deriving them from allegedly more basic assumptions (see Darrigol [2007] for the case of the laws of classical mechanics). It is worth keeping in mind this different line of thought insofar as it could be useful to justify the dynamical assumptions that one might need in the other approaches. As I have advanced in the last paragraph of §2.1, although in this paper the results presented allegedly dispense with dynamical assumptions, other convergence results (outlined in appendix B) hold only in systems with certain dynamical assumptions.

[7] Others to have explored this technique include Hopf [1934], Keller [1986], Von Plato [1983], Suppes [1987], Engel [1992] and Myrvold [2014, Forthcoming].

[8] See e.g. Ruelle [1989, ch. 4-8], Wiggins [2003, ch. 1-12], Strogatz [1994, ch. 2, 7], and Sklar [1993, ch. 5.III]. Some of these results are cited in Appendix B.

[9] See e.g. Grimmett and Stirzaker [1985, ch. 7, 9, 10, 13] and Bhattacharya and Majumdar [2007, ch. 2, 3].

[10] Although it has been surprisingly neglected in the literature, this paper advanced many other subjects later discussed: deterministic chances, probabilities from symmetries, an indifference principle justified by the dynamics, and the existence of an irreducibly statistical explanation of emergent phenomena in chaotic ('dynamically unstable') systems.

In what follows, the plan is to rely on Strevens' (2003) proposal—whose validity is, of course, disputable, see Werndl [2010]—and study in Section 4 its applicability to a scenario that lacks guiding laws.

3.1 Outline of the method

The goal of Strevens [2003, 2005] is to explain the fact that higher-level laws are simple, whereas they are assumed to be reducible to lower-level laws that are instead complex.[11] Clear examples of simple laws are the laws of thermodynamics and the laws of the rate of increase of a population of rabbits in a certain ecosystem. In each case, there is an assumed complex microlevel dynamics to which the macrolevel laws reduce. The complexity at the microlevel is due to the large number of degrees of freedom (e.g., the large number of particles) as well as the non-linear interactions between them. In the example of thermodynamics, the microlevel is modeled by kinetic theory (or statistical mechanics), which presupposes a large number of particles ruled by chaotic deterministic Newtonian dynamics. Strevens explains how, in certain cases, the simplicity of the higher-level laws is in part (surprisingly) due to the complexity of the lower-level laws. It is the microlevel chaotic behaviour, together with some other properties of the systems, that leads to the macrolevel simple laws. This explanation is grounded in MAF.

To convey the core results of MAF it will help to refer to an example. Consider a roulette wheel, as analyzed by both Poincaré and Strevens. Other games of chance could be chosen, as well as more "exotic" examples such as Poincaré's application of MAF to prove the equidistribution of planets across the sky [Poincaré, 1896, 129]. Strevens [2003, 48] says of the roulette wheel:

> "The complex probability of the ball's ending up in a red section is determined, like all complex probabilities, by two things: the physics of the wheel, represented by an evolution function, and the distribution of the initial conditions, represented by an ic-density. The initial condition distribution will be determined by facts about the croupier who is spinning the wheel. Because the croupier changes from time to time, the relevant ic-density presumably changes from time to time as well. But, as everyone knows, the probability of obtaining red remains the same".

This fragment states a key idea that leads to the conclusion that almost any probabilistic distribution of the initial conditions will determine approximately the same probability of the outcome. Let me note that in the quote, what Strevens calls the "physics of the system" is fixed; later on, I will consider the possibility of variation in that physics.

Let us analyse how it is that the roulette wheel tends to exhibit, in the long run, an approximate 50/50 frequency of red and black outcomes, and does so irrespective of the croupier (irrespective of the way

[11] 'Simple' as opposed to 'complex', where 'complex' is understood as in complex systems theory and 'simple' means that the simple regularities can be described by equations with few variables (usually linear equations but not necessarily). Simple behaviour is characterized by convergence to an equilibrium fixed-point, periodic cycles, and similar variants.

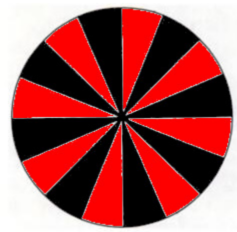

Figure 1: A roulette wheel

the croupier launches the ball). The outcome of a spin of the wheel is determined, among other things, by the initial speed with which the wheel is spun, which can be modeled by a random variable v. While this initial condition v can take the form of different probability distributions, the outcome probability, as we know, is always the same: approximately 0.5 for red and 0.5 for black. We have *stability* of the outcome probability, independently of the initial probability distribution. A way of understanding this (remarkable) fact is as follows.

The outcome red, equidistributed between the black slots, is represented as a function of the variable 'speed of the wheel' v as seen in Figure 2. Now take two different initial probability distributions, one

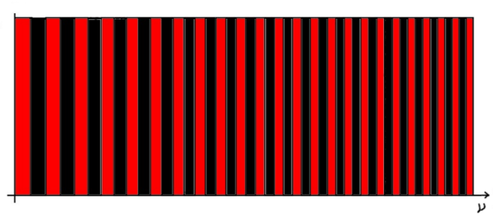

Figure 2: Red and black roulette outcomes as a function of the initial speed

corresponding to an enthusiastic croupier who spins the wheel faster than a second mellower croupier; both are represented in Figure 3. We already have the intuition—and the casino already knows—that changing the croupier does not affect the final outcome probability. The reason for this is that the contributions of each slice of red over the entire graph will be approximately the same proportion as those of the black slices. Crucially, this will also be the case if we examine a small enough region of possible values of the speed (i.e., it will be so in any small region of the domain of the function of Fig. 2). Thus, for each croupier the proportion of red (or black) outcomes will be approximately 0.5. For this to occur, there must be *quick* alternation between red and black (i.e., the alternation must be *sensitive to the value of the initial condition*—speed in this case) and it must be *constant* (i.e., its pattern must be constantly repeated). Strevens uses a single term to label these two crucial properties as the property of '*microconstancy*'. It is easy to visualize graphically (see Figure 2).

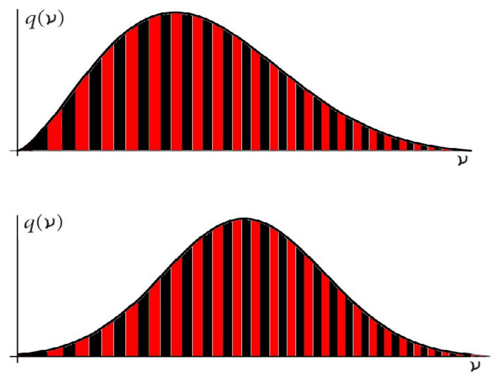

Figure 3: The frequency distribution of speeds for the spins of two different croupiers, superimposed over the outcome in function of the initial speed

Together with this property, the other condition is that the probability distribution over the initial conditions must be *smooth*. That is, however strong or mellow the croupiers, they cannot have a highly peaked distribution that does not cover a whole pattern (i.e., the variance of the distribution cannot be too small). Not covering a whole pattern will be difficult when the patterns are constantly repeated, that is, when the aforementioned 'microconstancy' obtains. Then,

> "a smooth density will be approximately flat over any neighbouring pair of red and black areas in the evolution function, for which reason the contribution made by that part of the ic-density to the probability of red will be approximately equal to the contribution made to the probability of black" [Strevens, 2003, 50].

In sum, Strevens generalizes the case presented showing that, in general:

> (S-MAF): *If the evolution function for an outcome 'e' is microconstant, then any smooth distribution of initial conditions determines, in the long run and with probability 1, the same probability for 'e'*; where the value of this probability is equal to the ratio of outcome *e* to the other outcomes (in this case of red to black).

This is the result of 'Theorem 2.2', stated and proved in [Strevens, 2003, 2.C, pp. 127–138]; for an informal defense see also [Strevens, 2003, 2.23].[12]

[12] For a comparison with previous formulations of MAF (cited in footnote 7) see Strevens [2003, 2.A]. For theorems and applications of previous formulations I recommend Engel [1992]. To give a glimpse, Theorem 5.3 of [Engel, 1992] proves that: being X a random variable and $t \in \mathbb{R}$ large, the random variable (tX)(mod1) converges in the variation distance to a uniform

I have explained MAF with the classic example of the roulette wheel, yet many others have been studied. Case-by-case study is required to prove that stable simple behaviour emerges in a particular context. This is what Strevens [2003, 4.8 and 4.9] pursues for statistical mechanics and population ecology respectively.[13]

4 Can a Humean System Meet the Conditions?

In the previous section, I outlined Strevens' generalization of MAF. In his terms, the system is to give rise to stable behaviour if three conditions are met:

1. Smoothness in the distribution of relevant initial conditions (§4.1);

2. Microconstancy (§4.2);

3. Chaos, understood as sensitivity to initial conditions (§4.3).

In this section, I argue that the first two conditions can be met independently of modal notions, and then that chaos, even if it is a dynamical condition, is typically met by a Humean system.

In other words, in the previous section the rise of simple behaviour has been explained in standard cases in which "the physics of the wheel" is ruled by the actual deterministic Newtonian dynamics. In this section, I instead elaborate and assess the following claim by Strevens [1998, 19, italics added]:

> "The value of a microconstant probability may come out the same *on many different, competing stories about fundamental physics*. The probability of heads on a tossed coin, for example, is one half in Newtonian physics, quantum physics, and the physics of medieval impetus theory."

4.1 Smoothness Condition

First, recall that the kind of physical system under consideration is one in which a large number of trials is repeated (for more examples besides those cited in Section 3, see §4.2.2). The method is then called 'the

distribution on the unit interval if and only if X has a density. See also Theorems 3.1, 3.9, 4.1, and 5.3. With these results, cases such as the roulette wheel are treated. Being X a random variable and n a positive integer, it was proved that equation 1 holds for any random variable X with an *absolutely continuous* density (an extension by Fréchet [1921] upon Poincaré [1896]):

$$\lim_{n\to\infty} P\{(nX)(mod 1) \leq 1/2\} = \frac{1}{2} \tag{1}$$

These results are for a physical system with one degree of freedom. The generalization to higher dimensions can be found in [Engel, 1992, ch. 4], and is analogous to the one-dimensional case (compare the necessary and sufficient conditions for the two cases as summarized in [Engel, 1992, p.35 and p.72]).

[13] For more on statistical mechanics, see Engel [1992, 4.2.4, 5.6], Myrvold [2014, Forthcoming], and Strevens [2013, ch. 8]. For other applications, see [Engel, 1992, 3.2, 4.2] reconstructing Poincaré's law of small planets, billiards, etc.

method of arbitrary functions' because, regardless of the distribution of initial values of the variables, in the long term the same output pattern obtains. Strictly speaking, though, this does not work for any distribution: the distribution of initial values of the relevant variables (the initial conditions, or 'IC') has to be *smooth*.

Smoothness means, in a standard analysis of MAF (such as those cited in fn 7), that the distribution of the IC is *absolutely continuous* with respect to the variable. One of the refinements in [Strevens, 2003] is that in order to hold for finite but large values of the variable at stake, the smoothness condition must be stronger than usually understood. The stronger degree of smoothness (called 'macroperiodicity' in [Strevens, 2003] and 'microequiprobability' in [Strevens, 2013]) demands that the initial probability distribution be approximately uniform over almost all micro-sized regions. This definition of smoothness suffices for our purposes, yet a more formal definition can be found in [Strevens, 2003, §2.C 'definition 2.5' and 'approximation 2.2'].[14]

4.1.1 Obtaining the condition

We have, then, to require such a smoothness condition. Its reasonableness and ubiquity is defended in [Strevens, 2013, 12.3], reworking what is thoroughly argued in [Strevens, 2003, 2.53]. In short, Strevens proposes an argument based on the idea that random perturbations or its surrogates (such as environmental noise) provoke a general tendency in the IC-distributions to smooth out or even become uniform. Likewise, North [2010, p. 22 fn 48, p. 35] defends something stronger than this smoothness, namely a uniform distribution, but restricted to holding only for a narrow set of variables, the "canonical variables" of fundamental physics. See also Poincaré [1905, 222] defending the reasonableness of assuming smoothness as continuity, and the justification of absolute continuity seen in Von Plato [1983, 45].

Aside from these arguments, in the particular setting that concerns us—a lawless scenario—we have a more straightforward justification of smoothness. In each of the cases studied in the literature there is some causal explanation for the IC distribution being as it is: for instance, the croupier's being a human with certain characteristics explains the distribution in Figure 3 and its smoothness. Instead, in our lawless scenario the value of the IC for each trial is chosen at random. That is to say, in a lawless scenario it is not only the time evolution that is randomly generated, but also the values of the IC in each trial. To be clear, here the probability function over IC is not to be interpreted as modally loaded—that is, it is not describing any genuine *propensity* such as the propensity of the croupier to launch the ball. On the contrary, the probability function over IC is to be understood merely as the statistics describing

[14] What is micro-sized in one context may be macro-sized in another. 'Micro' and 'macro' are thus relative to the case in point. This should not be problematic; see Strevens [2013, 5.6] for discussion. MAF is also relative to the outcome and the way of measuring the IC. The latter might be more problematic: it is analogous to the relativity of choosing measures in statistical mechanics. An attempt to deal with this relativity is found in [Strevens, 2013, 5.6, 12.1] and in more detail in [Strevens, 2003, 2.5].

the frequencies of each IC after a large number of trials. So, what can we say about the smoothness of the IC distribution resulting from a large number of trials?

We can say that *a distribution resulting from a process-random process is typically approximately product-random*. This automatically guarantees a smooth IC distribution.

It is true that there is no *conceptual* connection between process and product randomness. In fact, Eagle [2012, §4] proposes counterexamples to the following conceptual connection (labeled by him 'RCT'):

> An outcome happens by chance [i.e. process-randomness] iff, were the trial which generated that outcome repeated often enough under the same conditions, we would obtain a [product-]random sequence including the outcome (or of which the outcome is a subsequence).

However, the counterexamples *only* affect a conceptual connection between process-randomness and product-randomness. As Eagle [2012, §8] concludes, we can safely accept "an evidential and epistemic connection".

In more detail, an evidential and epistemic connection can be justified by appealing to Brudno's theorem. Following Frigg [2004, 432], the theorem states that for almost all trajectories of a system, its algorithmic complexity—which is equated with product-randomness—is equivalent to communication-theoretic entropy—which is equated with process-randomness. The conclusion is then that "whenever a dynamical system behaves randomly in a process sense [...], almost all of its trajectories exhibit product randomness (in the sense of algorithmic complexity), and vice versa". Thus, if we know that a process is process-random, we should expect (eventually, with high and increasing probability) a product-random sequence of outcomes [Eagle, 2012, §8].

Hence, in the long run the IC distribution will tend not only to a smooth distribution but to one that is very smooth: a uniform distribution. [15]

4.1.2 The role of the dynamics

Now we have to ask, is the smoothness condition non-dynamical? In other words, is no dynamical property required for it to obtain?

This can be clearly seen by considering the classic system studied in statistical mechanics: a system of ideal gas particles in a closed container with an initial distribution of their positions and velocities. Here, the distribution of initial conditions is independent of the Newtonian dynamics: the dynamics determines the motion and collisions *once* particular initial conditions are provided (and there is no causal mechanism providing the values of the initial conditions).

[15] One might wonder whether we need a *physical* probability distribution over initial conditions. Strevens [2013, 12.5, 13.2] addresses this issue and argues that we do not: we can "infer a tendency to a stable distribution over outcomes (...) without the oversight of a governing physical probability distribution."

One might be tempted to think of the IC distribution as a Humean law, something along the lines of David Albert's proposal that the 'Past Hypothesis' (the postulation of the initial conditions of the universe in a very low entropy state) should be considered a Humean law. However, even if endorsing this line of thought, it would be misleading to believe that the IC distribution, interpreted as a Humean law, is a dynamical condition, for a *Humean* law is, by definition, far from being a rule that guides the time evolution of a system, and does not involve any modal notion.

4.2 Microconstancy condition

The other condition is so-called microconstancy, so now we ask: Is there a way to obtain microconstancy without requiring any dynamical property?

We will see that there is a way, imposing certain non-dynamical conditions (*viz.*, symmetries), such that the only dynamical property needed is chaos. (Then in §4.3 we will see that chaos typically obtains in a Humean system.)

4.2.1 Obtaining the condition

Microconstancy is defined as that property we ascribe to a system if and only if within any small neighbourhood of the evolution function the proportion of initial conditions producing a given outcome is the same (see Fig. 2 p. 10; regarding the relativity of notions such as 'small' see fn 14). Thus, to satisfy this condition there must be a *constant* ratio of the outcomes and they must alternate *frequently*. So, upon what does the constant stable form of the evolution function and its frequent alternation depend?[16]

We can draw upon a "non-dynamical" account of microconstancy proposed by Strevens [2003, 2013]. He opts for another "dynamical" account, yet the reasons for his choice are irrelevant to our purpose. More specifically, the following account of microconstancy is upheld by the analysis carried out by Strevens both in his article "Inferring probabilities from symmetries" [Strevens, 1998] and in his two books (see especially Strevens 2013, 64-5, 90-1, 119 and Strevens 2003, 62).

The underlying idea of the non-dynamical account of microconstancy is that given a chaotic dynamics, certain physical constraints suffice for having microconstancy. The physical constraints are relevant *stable properties* of the system under consideration—in the examples presented, properties of *symmetry*, such as a symmetry of the spatial structure of the system, as illustrated by the three examples below.

Thus, the non-dynamical account of microconstancy can be summarized as:

[16] In the case of the roulette outcomes represented in Fig. 2 the ratio is 50:50, but this would be equally true for any other ratio. Had the wheel been painted so that one third of its slots were red and two thirds black, the evolution function would be different but it would still display a constant ratio of outcomes: 1/3 for red and 2/3 for black.

(MicroConst): A variable's evolution function is microconstant if the system displays sensitive dependence on initial conditions and relevant physical symmetries.

For the sake of simplicity and in keeping with the examples, I formulate (MicroConst) appealing to symmetry properties; notwithstanding, the results can be generalized to other relevant stable properties, as shall later be addressed.

In other words, we can see that the underlying idea behind (MicroConst) is that the quick alternation (the 'micro' of 'microconstancy') is achieved by the random-looking behaviour due to the chaotic trajectories, and the constancy (of 'microconstancy') is achieved through the symmetry of the physical configuration of the system.

In the following, I illustrate with three examples how (MicroConst) yields microconstancy, while also showing that *the specific form of the underlying dynamics is irrelevant.*

4.2.2 The role of the dynamics

First, let us return to the roulette wheel. We explained in §3.1 how it is that microconstancy obtains in such a setup. Would the roulette wheel still be microconstant if we modified the actual dynamics, for instance by considering a wheel that wobbled as it rotated on its axis? Yes—microconstancy is ensured by the symmetric colour scheme of the wheel and the rotational symmetry of its dynamics. The latter, in turn, is related to the symmetric circular shape of the wheel and with the periodic (hence symmetric) cycles it makes. This is suggested by Strevens [2003, 62]:

> "The physical details underlying these facts are unimportant in themselves. In a wheel that comes slowly to a halt, for example, the precise facts about the frictional forces that slow the wheel do not matter. Only one fact about these forces matters, the rather abstract fact of the rotational symmetry of their combined effect."

In more detail, the constant 50:50 ratio that we see in Fig. 2 is a consequence of the fact that, at any moment during a spin, the wheel takes approximately equal time to rotate through a red segment as it does to rotate through a black segment. Likewise, due to the rotational symmetry of the wheel, the whole pattern is repeated in the evolution function each time the wheel performs a whole cycle.

The same is true for the example of a tossed coin: only the symmetrical distribution of mass in the coin matters. Again, we can infer the value of the coin's probability "from few facts about physical symmetries, even if one knows very little about physics" (ibidem, 62). In general, what is needed is whatever guarantees the existence of relevant symmetries in the operation of the mechanism, for example "whatever entails that a spinning coin takes about the same time for each half-revolution, or that a spinning roulette wheel takes about the same time for each 1/36th of a revolution" [Strevens, 1998, 19]. The symmetry in the coin case results from the coin having two equal sides. The time one side takes to flip is the same time the other side takes. There is no physically relevant difference between the two sides of the coin: as such, no law whatsoever could possibly differentiate one side from the other.

An analogous story goes for the hard-sphere model of classical statistical mechanics. Here, the spherical shape of the gas particles is involved: the spherical symmetry makes it logically impossible to have a dynamics that discriminates one particular side of the particle as reacting differently to a collision.

On the whole, notice that the account of microconstancy (MicroConst) does not appeal to the details of the actual dynamics; besides the sensitive dependence, the antecedent in (MicroConst) does not appeal to how the dynamics has to be; for instance, it does not appeal to how collisions between objects have to be or, say, what rate of decrease some repulsive force has to obey.

4.2.3 A candidate for meeting the condition

It depends on each particular case what the relevant symmetries are; a general account cannot be more specific here. Case-by-case analysis is required, as Strevens [2003, 4.8 and 4.9] performs for the cases of statistical mechanics and population ecology. In addition to the examples given above, I wish to conclude this subsection 4.2 by proposing another allegedly non-dynamical candidate that would satisfy the microconstancy condition.

We need something that plays the role of the stable non-dynamical properties. Some of the symmetries found in physics might play this role. The global, continuous symmetry principles of modern physics are commonly interpreted as *meta-laws*. There is a *priority* of the symmetries of the laws over the laws themselves. In fact, the symmetries "(...) can (...) be realized (...) by any number of distinct sets of fundamental laws of physics (...)" [Albert, 2015, 13]. [17]

Aside from this alleged priority, symmetries can be instantiated by something non-dynamical: symmetries of many sorts can be exhibited *by the geometrical structure of space* in physical theories (by its topological, affine, and metric structure). In fact, Eugene Wigner highlighted that *symmetry principles are grounded by the stable properties of the defining structure of spacetime* [Martin, 2003, 50]. In more detail, according to the so-called geometrical interpretation, the symmetries of the laws are interpreted as symmetries of spacetime itself; they codify "the geometrical structure of the physical world" [Brading and Castellani, 2013, §5]. To give an example, think of the homogeneity of space, assumed "in the physical description of the world since the beginning of modern science" (ibidem, §2.1).

Furthermore, a symmetry of a law can result from the presence of a scalar or vectorial field—that is, again from something non-dynamical. A certain field can lead to symmetric properties of the space that it permeates.

[17] Lange [2007] studies why symmetry principles are prior to the associated conservation laws—it is not obvious why the priority should be in that order and not the other way around. In short, he argues that this is the case because symmetry principles remain true under a larger set of counterfactual suppositions than all other laws do (including conservation laws).

To sum up this subsection, the account of microconstancy (MicroConst) appeals to sensitivity to initial conditions (SIC) and stable properties such as the aforementioned symmetries. In the next subsection, I assess how we can tolerate the dynamical condition of SIC.

4.3 Substituting chaos by randomness

We have arrived at a result which can be naturally interpreted as an account of the rise of stable coarse-grained behaviour in the setups displaying the relevant symmetries and for any chaotic dynamics. Previously in §2.1 we have characterized a scenario with no governing dynamics as one whose state-space evolution is randomly generated, in the process sense of randomness. So, can we obtain the same results—coarse-grained stable patterns—substituting a chaotic dynamics by a process-random dynamics?

Yes, we can. In a nutshell, we can because process-randomness is a stronger condition than chaos (or any condition that has been proposed as being required by MAF): hence, if we merely need chaos, we will *a fortiori* obtain the results with process-randomness.

In more detail, a chaotic dynamics is required in MAF *precisely because* it produces (relative to relevant variables and a sufficiently coarse-grained description) an approximately product-random evolution of the system's state-space trajectory. (This yields, as explained in Section 3, a visitation rate among the outcomes proportional to the patterns in the evolution function, which in turn in the long-term yields a stable output probability distribution.) In fact, it is known that the resulting outcome of a sufficiently coarse-grained chaotic system is indistinguishable from a product-random sequence [Smith, 1998, ch. 9]. We have also seen that process-randomness typically yields a product-random sequence (§4.1.1). Thus, if instead of chaos we directly postulate process-randomness, we will be able to obtain the same results.[18]

We can see this in more detail by framing the notions in the so-called ergodic hierarchy: this allows us to clearly contrast the outputs of chaotic and process-random systems. The ergodic hierarchy consists of five dynamical properties, which are used to classify the degree of product-randomness of deterministic (i.e. law-governed) systems, and which we can also use to classify the output of our not-law-governed process-random system. The ergodic hierarchy is this:

$$\text{Bernoulli} \subset \text{Kolmogorov} \subset \text{strong mixing} \subset \text{weak mixing} \subset \text{ergodic}$$

[18] Which particular dynamical property is required by MAF is a contentious matter; moreover, there is also general discussion in dynamical systems theory regarding the definitions of the possible candidate properties. For instance Brown and Chua [1998] present dozens of counterexamples to a variety of theses and definitions of chaos and related notions (SIC, ergodic, non-linear, inflationary, complex, etc.). See also above §2.2 and fn 5. In any case, in this paper we are following an account by Strevens [1998, 2003, 2005, 2013] in which the only dynamical condition needed is chaos, and in particular the feature of SIC. (SIC can also be achieved by non-linear systems that are not chaotic, but this is irrelevant because we are simply not interested in such systems!) Strevens himself considers stronger or weaker conditions along with other approaches. Caution has driven me to choose chaos, given that it is a condition sufficiently strong to deliver the results, while at the same time it is still weaker than process-randomness.

where $A \subset B$ is to be read as systems with property A also possess property B. Following Berkovitz et al. [2006], we can interpret the five levels of the hierarchy as corresponding to *different degrees of unpredictability*, which in turn correspond to different patterns of *decay of correlations between their past states and present states.*[19] Berkovitz et al. [2006] understand product randomness as unpredictability (along the lines of Eagle [2005], which traces back to Von Mises), and is defined as a matter of degree. The lower the degree, the more the correlations arise, yielding a weaker notion of randomness. At its highest degree, randomness is instead associated with the Bernoulli level, at which there is a total lack of correlation between the present state and the past states.

We are now focusing on the output generated by a process-random system. This output, we have seen in §4.1.1, typically coincides with the maximum degree, the Bernoulli level (recall the approximate extensional equivalence between process and product randomness).

Now consider the following account of chaos introduced in §2.2. Following Werndl [2009], define chaos in terms of strong mixing. (Likewise, the present argument holds equally for the previous characterization by Belot and Earman [1997], where chaos is defined in terms of the higher Kolmogorov level, and for the characterization by Berkovitz et al. [2006], who advocate seeing chaos as a matter of degree, quantified according to the position within the hierarchy: whatever the specific degree chosen, the present argument holds.)

As we can see in the hierarchy, it follows that a Bernoulli system possesses the property of mixing; hence, it possesses the properties of a chaotic system. No matter what the degree of chaos we consider within the hierarchy, the Bernoulli level is included in any other set. Hence, if we obtain some results in virtue of the property of chaos, we can obtain the same results in a Bernoulli, i.e. Humean, system.

Measures of the dynamics space. Alternatively, an interesting way to assess whether chaos can be substituted by process-randomness is by analysing the dynamics space, measuring the proportion of chaotic trajectories. This can be done appealing to results from the fields known as measure theory and topological dynamics. I outline some results in Appendix B aimed at ascertaining whether chaotic trajectories are typical (or 'generic'), which means that most of all the trajectories are chaotic trajectories. However, this task would require such a technical background and discussion of controversial decisions as, for reasons of space, goes beyond the scope of this paper.

5 Conclusion

Strevens' aim was the obtaining of a stable probability distribution from something non-probabilistic. My aim instead has been the obtaining of a stable probability distribution from something non-probabilistic

[19] For our purposes it suffices to follow this interpretation of each level's meaning, but for more details see Frigg et al. [2016] and references therein.

and non-dynamical. In doing so, I have assessed whether certain results support an explanation of stable regularities in a lawless system; an explanation that presupposes neither the (suspicious) existence of laws, nor the entire (suspiciously) regular Humean mosaic. Instead, the proposal assessed assumes two conditions, arguably non-dynamical (i.e. conditions that can be obtained without presupposing any dynamical property).

This stable behaviour is, then, the result of what can be called *statistical necessity*. In the type of contexts described, the apparent physical necessity *is just* statistical necessity—there is no need for physical necessity between pure contingency and logical necessity. That is, the resulting non-accidental regularities are just *robustly stable contingent regularities*, arising not because some pre-existing rules dictate them, but because of a statistical explanation. For such an explanation, grounded in the results presented in Section 3, does not need to postulate any modal notion, as has been argued in Section 4. Thus, by statistical necessity I refer to the resulting (metaphysically innocent) *stability* of those regularities yielded by the results presented here. The term 'statistical necessity' aims to convey that the purported explanation of stable behaviour does not deterministically deduce the explanandum (the stable patterns) from the explanans (the applicability of MAF) but rather provides, in the long run, convergence in probability results of approximate statistical patterns. The suggestion, then, is that the appearance of physical necessity in these cases simply corresponds to such statistical necessity.

While much obviously remains unsettled, I have assessed what I consider one promising result which would account for such statistical necessity: the method of arbitrary functions, spelled out in Section 3. MAF has already been applied to complex systems sciences (where, crucially, the underlying dynamics is chaotic) to explain coarse-grained stable patterns. I have assessed whether it could also be applied to a lawless "Humean system". Thus, rather than assuming as a brute fact the extremely stable, apparently non-accidental, contingent regularities of the Humean mosaic, I have at least proposed a potential explanation.

In a nutshell, MAF explains the rise of stable behaviour for a large number of IC distributions. I have argued that MAF explains the rise of stable behaviour for a large number of IC distributions *and* a large number of state-space trajectories. Given the way I model a system that lacks governing laws (in Section 2 and Appendix A) this could be significant, as long as 'large' is large enough, for an explanation of stable behaviour in such a lawless system.

First and foremost, the discussion has revealed the difficulties faced by this thesis—most seriously, the difficulty of avoiding dynamical properties.[20] However, we have at least clarified what this thesis requires.

More specifically, I have argued that as long as we assume two conditions (that the MAF literature

[20] Also, the proposal is just a candidate explanation (i.e., stable behaviour might arise due to other reasons); the results provided obtain in the long run (and nothing has been said about relaxation times); and the actual verification of the abstract conditions is of course a tricky issue.

assumes in standard examples), in the long run almost any state-space trajectory of a system that is not guided by any dynamics will display, with probability 1, stable behaviour. First, as explained in Section 3 (see 'S-MAF' on p.11), MAF accounts for the said stable behaviour—a *stable* probabilistic distribution of the coarse-grained values of an outcome variable. Then, in Section 4, I assessed whether the conditions required by MAF could be met without appealing to dynamical properties.

One condition is a certain degree of smoothness in the distribution of the initial conditions (§4.1); the other is that the system displays certain relevant physical symmetries—or, more generally, stable (non-dispositional) properties (§4.2). Besides standard examples, I proposed a candidate for these physical symmetries: spacetime symmetries from modern physics (such as the homogeneity of space). As I explained, such spacetime symmetries can be accounted for, in turn, by the structure of spacetime or by postulating scalar or vectorial fields.

This leaves the dynamical condition of chaos, defined as sensitivity to initial conditions (§4.3). A chaotic guiding dynamics was required in MAF precisely because it produces (relative to relevant variables and a sufficiently coarse-grained description) an approximately product-random evolution of the system's state-space trajectory. Hence, *a fortiori* we can obtain the same results if instead of chaos we directly postulate process-randomness.

The application of the results at a local scale would be in tune with an anti-realist view of laws, in the sense of something like oases of order emerging among the "chaos", *à la* Cartwright. That is, MAF applying not globally but to local set-ups over which the proper conditions hold—sort of "nomological machines" without propensities. Otherwise, the conjecture that the results apply universally at a fundamental scale would be in tune with the projects in physics cited in footnote 4, which aim to derive the current laws of particle physics from a complex or random underlying dynamics or explain them statistically (as 'entropic forces'). Future work could in fact analyze the applicability of the conditions of the present proposal in contexts of physics (Filomeno [2014, Ch.3,5] and Filomeno [20xxc] point in this direction).

Finally, in spite of the non-dynamical justifications of the conditions, one might still be skeptical that the two conditions can be completely non-dynamical (in particular, the stable non-dispositional properties). Even so, as I argue in §2.1 and Appendix A, the present proposal's significance would not be seriously undermined, for in any case the ontological status of the two conditions is closer to that of meta-laws than to that of standard governing laws. That is to say, there is a difference of ontological and modal import in postulating, as metaphysically primitive, (I) non-dispositional properties of the matter-content (such as the spacetime structure proposed in §4.2.3), as opposed to (II) a set of governing laws (or primitive propensities, or primitive causation). A Humean, at least, would regard as more intelligible, and of less modal import, the first kind of postulation.

A Appendix. Dynamical Systems and Randomness

In this Appendix I put forward (1) a mathematical characterization of a generic dynamical system endowed with a set of possible laws describing its time evolution, capturing the idea of describing all state-space trajectories (as explained in Section 2); and (2) a 'Bernoulli scheme', from the field known as 'random dynamical systems', which models the notion of random state-space evolution (also explained in Section 2). I include and discuss constraints that might be imposed, which restrict the logically possible trajectories to the kinematically possible trajectories.

Dynamical systems with a set of laws. A generic dynamical system [X, Σ, μ, T] is defined as a probability space [X, Σ, μ] and a transformation T of it.[21] X is a set of elements, interpreted as a state-space (e.g., phase space in classical mechanics); Σ is a σ-algebra of measurable subsets of X; and there is a probability measure μ on Σ as usually defined. In the standard approach, T consists of a dynamical law over the probability space [X, Σ, μ]. T has the generic form of an ordinary differential equation: $\frac{d}{dt}x = f(x,t)$; which is autonomous of the independent variable t which represents time, where x is the position of the system in the n-dimensional state-space (where n is the number of degrees of freedom). In footnote 2 it has been explained what a state-space, a state, a system, and a trajectory are. Here it is worth remarking only that a trajectory can be continuous as well as discontinuous. The trajectory of a state x is the time-ordered *collection* of states, that usually follow from x using the evolution rule [Meiss, 2007].

There is a generalization in which T is not a single transformation that is iterated to give a unique dynamics of the system, but instead is a monoid or a group of transformations $T_s : X \to X$ parametrized by $s \in \mathbb{R}$, where each transformation of T_s satisfies the same requirements as T above.[22]

The range of kinematical possibility. Throughout the paper we have considered all the possible trajectories, insofar as the results predicted by MAF obtain without imposing any constraint on the details of the dynamics (besides the SIC). One might be willing to assume some constraints on the space of all possible trajectories for some reasons, but the legitimacy of these reasons should be discussed. Consider the following reasons:

(i) one casts doubts on the arguments presented in Section 4, in particular on the completely non-dynamical character of the conditions, therefore on the complete lack of constraints to the dynamics space; or

(ii) one considers that the whole space of logical possibilities is too wide since it tolerates, for instance,

[21] For a clear introduction to dynamical systems see, e.g., [Strogatz, 1994], or [Wiggins, 2003].

[22] In topological dynamics, it is considered a space of r-differentiable ($0 \leq r \leq \infty$) maps of $\mathbb{R}^n$ into $\mathbb{R}^n$. This denotes the dynamics space and is usually labelled as $C^r(\mathbb{R}^n, \mathbb{R}^n)$. Its elements are to be thought of as trajectories, as defined above [Wiggins, 2003, ch.1]

that any discontinuous function from points in space to points in space qualifies as a possible trajectory—in this case, T_s would follow assumptions aimed at excluding certain conceptual possibilities not ruled out by logic; or

(iii) one decides to pursue other approaches at our disposition, such as those cited in Section 3, and their results hold only in a constrained class of dynamical systems. For instance, as explained in appendix B, interesting results in support and against the rise of coarse-grained patterns hold in so-called Hamiltonian dynamical systems.

However, following any of these reasons is problematic and limits the significance of the results presented. Reason (ii) is weak in that it is motivated by a subjective intuition of ruling out "strange" discontinuous trajectories, and we already have empirical evidence to object to this intuition, since orthodox quantum mechanics exhibits discontinuous particle trajectories (discontinuous spectra for most of the observables). Regarding (i), the arguments in Section 4 are in fact far from uncontroversial, as pointed out in Section 5, and in any case they should be elaborated (hopefully in further research). In general, regarding any of these reasons, one might object that constraining the space of trajectories undermines the philosophical significance of the project.

Still, I have argued (last paragraphs of §2.1 and Section 5) that such a "constrained" project, if successful, is significant insofar as it is able to account for all sort of regular and potentially complex behaviour from few and simple dynamical assumptions. That is, merely by imposing few and simple dynamical assumptions, a state-space point that is assumed to evolve free displays, in the long run, coarse-grained stable patterns. From a Humean point of view, the ontological and modal import of these few assumptions seems preferable than the standard set of governing laws. Besides, it remains of course open the possibility of justifying elsewhere the dynamical assumptions, for of course not justify here these assumptions does not mean that they cannot be justified.

Be that as it may, in the case that we follow any of these reasons, we would consider a dynamics space that can be said to correspond to the space of the *kinematically possible*. The kinematically possible is a space of functions that represent histories of the system. Define T_s as the class of all the kinematically possible trajectories in state-space. The idea is that T_s delineates the subspace of "metaphysical possibilities consistent with the theory's basic ontological assumptions" [Pooley, 2013, 12].

To illustrate the range of kinematical possibility, consider the framework of classical mechanics: consider a model of gravitating point particles with distinct masses, as in [Belot, 2011, 7]. A point in the space of the kinematically possible models of the theory assigns to each of the particles a worldline in spacetime, without worrying about whether the worldlines of each particle jointly satisfy the Newtonian laws of motion. There is then a subset of the kinematically possible models called the set of dynamically possible models. This is a space of solutions which is a 6N-dimensional submanifold whose points correspond to the motions of particles that obey Newton's laws. Here, such a submanifold will be ignored for it would obviously constrain us too much, to a single set of laws—the actual ones.[23]

The Bernoulli scheme from random dynamical systems. Having laid out the model of a dynamical system endowed with a set of kinematically possible equations of motion, I include now the characterization of a lawless scenario stemming from the field of 'random dynamical systems'. Random dynamical systems are characterized by:
(1) a set of maps T_s from X into itself that can be thought of as the set of all possible equations of motion; and
(2) a *stationary* probability distribution P on the set T_s that represents the random choice of map.
Then, the motion of state $x \in X$ evolves according to a succession of maps "randomly" (probabilistically) chosen according to the distribution P. We can then characterize the randomly generated dynamics as the resulting evolution of a Bernoulli system. A dynamical system $[X, \Sigma, \mu, T]$ as defined in this appendix is a *Bernoulli system* if T is a Bernoulli automorphism. A Bernoulli automorphism is an automorphism $T : X \to X$ of a probability space [X, Σ, μ] where there exists a partition $\alpha = \{\alpha_1, \ldots \alpha_n\}$ such that $T^i \alpha(-\infty < i < +\infty)$ are *independent* of each other. The partitions are said to be independent of each other iff for any two partitions α and β, $\mu(\alpha_i \cap \beta_j) = \mu(\alpha_i)\mu(\beta_j)$ for all atoms $\alpha_i \in \alpha$ and $\beta_j \in \beta$.

The 'Humean' process-random system studied in this paper can then be identified as a *Bernoulli scheme*, which is a generalization of the Bernoulli process to more than two possible outcomes.

Last but not least, it could be objected to this model that the distribution P is in tension with a truly process-random system, since P is stationary. However, in light of the properties of process randomness that we have surveyed throughout this paper—in particular, the approximate extensional equivalence of process and product randomness—it could be replied that if P is chosen to be a *uniform distribution*, then the stationarity of P would not be problematic because the model would faithfully capture the expected behaviour of a Humean *process-random* system.

B Appendix. Measures of the Dynamics Space

As mentioned in §4.3, obtaining the same results (*viz*, coarse-grained stable patterns) by substituting a chaotic system by a random (Bernoulli) system is further supported if it turns out that chaotic trajectories are generic over the whole of the dynamics space. This has been one of the main tasks throughout the history of dynamical systems—in particular, in the area known as topological dynamics.

Here I first review some classic results for and against the genericity of chaos, which I discard in that

[23] The range of the kinematically possible can be also formulated in the lagrangian formalism, where the kinematically possible models are monotonically rising curves in the product space formed from the configuration space (whose points represent the possible instantaneous states of the system) and a one-dimensional space representing time [Pooley, 2013, 40]. Then, the dynamically possible models are the curves that extremize a particular functional of such histories: the action.

their domain is too constrained. Then I outline further results for the genericity of chaos relative to an arguably appropriate domain of possible trajectories. A proper discussion of the significance of these results, though, goes beyond the scope of this paper (see Filomeno [20xxc] and references therein for more details).

Classic but excessively constrained results. There are methods that allow us to rule out the possibility of closed orbits, thus of periodic motion: the methods of index theory, the existence of Lyapunov functions, and gradient systems (see any textbook on dynamical systems, e.g., Strogatz 1994). However, these desirable results have to be studied case by case, analysing each dynamical equation, and say nothing about how frequently such periodic orbits are ruled out. On the negative side, the Poincaré-Bendixson theorem proves the existence of periodic orbits, which is incompatible with the existence of chaotic motion [Strogatz, 1994, 210]. However, this result has only been proven for vector fields on a plane, that is, for 2 degrees of freedom.

Similarly, literature in statistical mechanics has been critical of ergodicity, mainly on the grounds of the KAM theorem. Yet not only is the KAM theorem narrowed down to Hamiltonians of fixed kinetic energy, $T = p^2/2$, but a careful look at the theorem shows that it can hardly be taken to represent the general behaviour of classical mechanical systems; see Frigg [2009, 1005-6], Frigg and Werndl [2011, sec.5], Berkovitz et al. [2006, sec. 4.1], and Sklar [1993, 174-5].

Ergodicity as a generic property. Since in the approach of topological dynamics we are measuring trajectories, we have to use a topological counterpart of the standard Lebesgue measure, namely a topological measure based on Baire categories. The topological counterpart of a set of measure zero is a set that is said to be of first Baire category, aka scarce. A set is scarce iff it is a countable union of nowhere dense sets. The topological counterpart of a set of non-zero measure is a set that is said to be of second Baire category, aka *generic*. A set is generic iff its complement is scarce.[24]

It is crucial that the class is appropriate. For our purpose, in order to cover all the kinematically possible worlds (defined in Appendix A), it seems appropriate to select the class of measure-preserving homeomorphisms and automorphisms on all compact manifolds. However, there is a possible objection to this choice: measure-preserving dynamical systems are those in which energy is conserved, and this new assumption can be considered a dynamical condition. To resolve this issue, we could concede a privileged status to this conservation principle, sometimes considered a *meta-law*. Otherwise, a non-dynamical interpretation of this principle could be given along the lines suggested in §4.2.3. Still, even

[24] For a little more detail, cf. [Wiggins, 2003, 162]: a property of a vector field is said to be C^k *generic* if the set of vector fields possessing that property contains a residual subset in the C^k topology, where a residual subset contains the intersection of a countable number of sets, each of which is open and dense in the topological space. A property is defined as *generic* (or typical) if it holds for a countable intersection of open dense subsets.

if taking the principle as dynamical, the results might be of philosophical interest for the reasons given in Section 5.

That being said, the first result worth mentioning comes from Oxtoby and Ulam [1941], who discovered that ergodicity is generic for measure-preserving homeomorphisms on all compact manifolds. More specifically, the discovery was that the set of dynamical flows that are not ergodic is of the first category in the set of measure-preserving generalized dynamical flows.

Remember from §2.2 that ergodicity might be a sufficient condition (not chaos) and that the class of Oxtoby and Ulam's result is (arguably) appropriate, as it covers not only the actual physical possibilities but the whole set of kinematical possibilities.

Extension of the results for ergodicity and weak mixing. Furthermore, the results have since been extended. The genericity of ergodicity is extended to automorphisms: Halmos [1944a] shows that ergodicity is generic to the weak topology in the space of all automorphisms.

Finally, the genericity of the stronger condition of weak mixing has also been proved, both in homeomorphisms and in automorphisms [Katok and Stepin, 1970, Halmos, 1944b]. A discussion of these and other results can be found in [Filomeno, 20xxc] and [Alpern and Prasad, 2001].

References

David Z. Albert. *After Physics*. Harvard University Press, 2015.

S. Alpern and V.S. Prasad. *Typical Dynamics of Volume Preserving Homeomorphisms*. Cambridge Tracts in Mathematics. Cambridge University Press, 2001. ISBN 9781139433204.

J. D. Barrow. A chaotic cosmology. *Nature*, 267:117–120, May 1977. doi: 10.1038/267117a0.

Robert W. Batterman. Explanatory instability. *Noûs*, 26(3):325–348, 1992.

Gordon Belot. Symmetry and equivalence. In Robert Batterman, editor, *The Oxford Handbook of Philosophy of Physics*. Oxford University Press, 2011.

Gordon Belot and John Earman. Chaos out of order: Quantum mechanics, the correspondence principle and chaos. *Studies in History and Philosophy of Science Part B*, 28(2):147–182, 1997.

Joseph Berkovitz, Roman Frigg, and Fred Kronz. The ergodic hierarchy, randomness and hamiltonian chaos. *Studies in History and Philosophy of Science Part B*, 37(4):661–691, 2006.

R. Bhattacharya and M. Majumdar. *Random Dynamical Systems: Theory and Applications*. Cambridge University Press, 2007. ISBN 9781139461627.

Katherine Brading and Elena Castellani. Symmetry and symmetry breaking. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Spring 2013 edition, 2013.

Ray Brown and Leon Chua. Clarifying chaos ii: Bernoulli chaos, zero lyapunov exponents and strange attractors. *International Journal of Bifurcation and Chaos*, 08(01):1–32, 1998. doi: 10.1142/S0218127498000024.

John W. Carroll. Nailed to hume's cross? In Theodore Sider, John Hawthorne, and Dean W. Zimmerman, editors, *Contemporary Debates in Metaphysics*, pages 67–81. Blackwell, 2008.

John W. Carroll. Laws of nature. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Fall 2016 edition, 2016.

S Chadha and Holger Bech Nielsen. Lorentz invariance as a low energy phenomenon. *Nuclear Physics B*, 217(1): 125–144, 1983.

J.L. Chkareuli, C.D. Froggatt, and H.B. Nielsen. Spontaneously Generated Tensor Field Gravity. *Nucl.Phys.*, B848: 498–522, 2011. doi: 10.1016/j.nuclphysb.2011.03.009.

Jonathan Cohen and Craig Callender. A better best system account of lawhood. *Philosophical Studies*, 145(1):1–34, 2009.

Olivier Darrigol. On the necessary truth of the laws of classical mechanics. *Studies in History and Philosophy of Science Part B*, 38(4):757–800, 2007.

Olivier Darrigol. *Physics and Necessity: Rationalist Pursuits From the Cartesian Past to the Quantum Present.* Oxford University Press, 2014.

Abhijit Dasgupta. Mathematical foundations of randomness. In Prasanta Bandyopadhyay and Malcolm Forster, editors, *Philosophy of Statistics (Handbook of the Philosophy of Science: Volume 7)*. Elsevier, 2011.

David Deutsch. On wheeler's notion of "law without law" in physics. *Foundations of Physics*, 16(6):565–572, 1986. ISSN 0015-9018. doi: 10.1007/BF01886521.

Antony Eagle. Randomness is unpredictability. *British Journal for the Philosophy of Science*, 56(4):749–790, 2005.

Antony Eagle. Chance versus randomness. In *Stanford Encyclopedia of Philosophy*. 2012.

John Earman. *A Primer on Determinism.* D. Reidel, 1986.

John Earman and John Roberts. Contact with the nomic: A challenge for deniers of humean supervenience about laws of nature part i: Humean supervenience. *Philosophy and Phenomenological Research*, 71(1):1–22, 2005.

E. Engel. *A Road to Randomness in Physical Systems*, vol. 71 of lecture notes in statistics. springer-verlag, heidelberg edition, 1992.

Aldo Filomeno. *On the Possibility of Stable Regularities Without Fundamental Laws.* PhD thesis, Autonomous University of Barcelona, 2014. URL https://dialnet.unirioja.es/servlet/tesis?codigo=91884. ISBN:9788449047107.

Aldo Filomeno. Are non-accidental regularities a cosmic coincidence? Revisiting a central threat to Humean laws. Manuscript, 20xxa. URL https://philpeople.org/profiles/aldo-filomeno/publications.

Aldo Filomeno. Suspend judgment on the Humean account of laws. Manuscript, January 20xxb.

Aldo Filomeno. Emergence of stable patterns for typical dynamics. Manuscript for a special issue from the conference 'Modality in physics' held in June 2018 in Krakow, 20xxc.

John Foster. Induction, explanation, and natural necessity. *Proceedings of the Aristotelian Society*, 83:87–101, 1983.

Roman Frigg. In what sense is the kolmogorov-sinai entropy a measure for chaotic behaviour?—bridging the gap between dynamical systems theory and communication theory. *British Journal for the Philosophy of Science*, 55 (3):411–434, 2004.

Roman Frigg. Typicality and the approach to equilibrium in boltzmannian statistical mechanics. *Philosophy of Science*, 76(5), 2009.

Roman Frigg and Charlotte Werndl. Explaining thermodynamic-like behavior in terms of epsilon-ergodicity. *Philosophy of Science*, 78(4):628–652, 2011.

Roman Frigg, Joseph Berkovitz, and Fred Kronz. The ergodic hierarchy. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Summer 2016 edition, 2016.

C. D. Froggatt and H. B. Nielsen. Derivation of Lorentz Invariance and Three Space Dimensions in Generic Field Theory. *ArXiv High Energy Physics - Phenomenology e-prints*, November 2002.

C.D. Froggatt and H.B. Nielsen. *Origin of symmetries*. World Scientific, 1991. ISBN 9789971966300.

M. Fréchet. Remarque sur les probabilités continues. *Bulletin des Sciences Mathematiques*, 46:87–88, 1921.

G. Gallavotti. Chaotic hypothesis. 3(1):5906, 2008. revision 126984.

G. Grimmett and D. Stirzaker. *Probability and random processes*. Clarendon Press, 1985.

Paul R. Halmos. Approximation theories for measure preserving transformations. *Trans. Amer. Math. Soc.*, 55: 1–18, 1944a.

Paul R. Halmos. In general a measure preserving transformation is mixing. *Ann. of Math. (2)*, 45:786–792, 1944b.

Boris Hasselblatt and Anatole Katok. *A First Course in Dynamics: With a Panorama of Recent Developments*. Cambridge University Press, 2003.

E. Hopf. On causality, statistics, and probability. *Journal of Mathematics and Physics*, 13:51–102, 1934.

Ted Jacobson. Thermodynamics of space-time: The Einstein equation of state. *Phys.Rev.Lett.*, 75:1260–1263, 1995. doi: 10.1103/PhysRevLett.75.1260.

Ted Jacobson and A. C. Wall. Black hole thermodynamics and lorentz symmetry. *Found.Phys.*, 40:1076–1080, 2010.

A. B. Katok and A. M. Stepin. Metric properties of homeomorphisms that preserve measure. *Uspehi Mat. Nauk*, 25:193–220, 1970. English translation in Russian Math. Surveys, 25 (1970), 191–220.

Joseph B. Keller. The probability of heads. *The American Mathematical Monthly*, 93(3):pp. 191–197, 1986.

J. Von Kries. *Die Principien Der Wahrscheinlichkeitsrechnung: Eine Logische Untersuchung*. Frieburg: Mohr, 1886.

Marc Lange. Laws and Meta-Laws of Nature: Conservation Laws and Symmetries. *Studies in History and Philosophy of Modern Physics*, 38(3):457–481, 2007.

D. Lewis. *Papers in Metaphysics and Epistemology*. Cambridge Studies in Philosophy. Cambridge University Press, 1999. ISBN 9780521587877.

A.D. Linde. Chaotic inflation. *Physics Letters B*, 129(3):177 – 181, 1983. ISSN 0370-2693.

Christopher Martin. On continuous symmetries and the foundations of modern physics. In *Symmetries in Physics: Philosophical Reflections*, pages 29–60. Cambridge University Press, 2003.

J. Meiss. Dynamical systems. *Scholarpedia*, 2(2), 2007. doi: 10.4249/scholarpedia.1629.

John Stuart Mill. *A System of Logic*. Longman, 1884.

Charles W. Misner. Mixmaster universe. *Phys. Rev. Lett.*, 22:1071–1074, May 1969. doi: 10.1103/PhysRevLett.22.1071.

Shinji Mukohyama and Jean-Philippe Uzan. From configuration to dynamics: Emergence of Lorentz signature in classical field theory. *Phys.Rev.*, D87(6):065020, 2013. doi: 10.1103/PhysRevD.87.065020.

Wayne C. Myrvold. Probabilities in statistical mechanics. In Christopher Hitchcock and Alan Hajek, editors, *The Oxford Handbook of Probability and Philosophy*. Oxford University Press, 2014.

Wayne C. Myrvold. Steps on the way to equilibrium, August Forthcoming. To be published in Daniel Bedingham, Owen Maroney, and Christopher Timpson, eds., Quantum Foundations of Statistical Mechanics (Oxford University Press).

Jill North. An empirical approach to symmetry and probability. *Studies in History and Philosophy of Science Part B*, 41(1):27–40, 2010.

J. C. Oxtoby and S. M. Ulam. Measure-preserving homeomorphisms and metrical transitivity. *Ann. of Math. (2)*, 42:874–920, 1941.

C. S. Peirce. *The Essential Peirce, Volume 1: Selected Philosophical Writings (1867-1893)*. Indiana University Press, 1992 edition, 1867.

Henri Poincaré. *Calcul des probabilités*. 1st ed. Gauthier-Villars, Paris, 1896.

Henri Poincaré. *Science and Hypothesis*. Library of philosophy, psychology and scientific methods. Science Press, 1905.

Oliver Pooley. Substantivalist and relationalist approaches to spacetime. In Robert Batterman, editor, *The Oxford Handbook of Philosophy of Physics*. Oxford University Press, 2013.

A. Reynolds. *Peirce's Scientific Metaphysics: The Philosophy of Chance, Law, and Evolution*. The Vanderbilt Library of American Philosophy Series. Vanderbilt University Press, 2002. ISBN 9780826591517.

D. Ruelle. *Elements of Differentiable Dynamics and Bifurcation Theory*. Academic Press, 1989. ISBN 9780126017106.

Lawrence Sklar. *Physics and Chance: Philosophical Issues in the Foundations of Statistical Mechanics*. Cambridge University Press, 1993.

Peter Smith. *Explaining Chaos*. Cambridge University Press, 1998.

Michael Strevens. Inferring probabilities from symmetries. *Noûs*, 32(2):231–246, 1998.

Michael Strevens. *Bigger than chaos. Understanding complexity through probability*. Harvard University Press, 2003.

Michael Strevens. How are the sciences of complex systems possible? *Philosophy of Science*, 72(4):531–556, 2005.

Michael Strevens. *Tychomancy: Inferring Probability from Causal Structure*. Harvard University Press, July 2013.

S.H. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Addison-Wesley studies in nonlinearity. Westview Press, 1994. ISBN 9780738204536.

Patrick Suppes. Propensity representations of probability. *Erkenntnis*, 26(3):335–358, 1987.

Jos Uffink. Compendium of the foundations of classical statistical physics, March 2006. Chapter for "Handbook for Philsophy of Physics", J. Butterfield and J. Earman (eds) to appear.

Erik P. Verlinde. On the Origin of Gravity and the Laws of Newton. *JHEP*, 1104:029, 2011. doi: 10.1007/JHEP04(2011)029.

Erik P. Verlinde. Emergent Gravity and the Dark Universe. *SciPost Phys.*, 2(3):016, 2017. doi: 10.21468/SciPostPhys.2.3.016.

Sérgio B. Volchan. What is a random sequence? *The Mathematical Association of America, Monthly*, 109:46–63, 2002.

J. Von Plato. The method of arbitrary functions. *British Journal for the Philosophy of Science*, 34(1):37–47, 1983.

Charlotte Werndl. What are the new implications of chaos for unpredictability? *British Journal for the Philosophy of Science*, 60(1):195–220, 2009.

Charlotte Werndl. The simple behaviour of complex systems explained? review: "bigger than chaos, understanding complexity through probability". *The British Journal for the Philosophy of Science*, 61 (4):875–882, 2010.

John Archibald Wheeler. On recognizing 'law without law', oersted medal response at the joint aps–aapt meeting, new york, 25 january 1983. *American Journal of Physics*, 51:398, 1983.

S. Wiggins. *Introduction to Applied Nonlinear Dynamical Systems and Chaos*. Texts in Applied Mathematics. Springer, 2003. ISBN 9780387001777.