- This talk is (mainly) about the relationship two types of epistemic norms: accuracy norms and coherence norms.
- A simple example that everyone will be familiar with:
  - **The Truth Norm for Belief** (TB). Epistemically rational agents should only believe propositions that are true.
  - **The Consistency Norm for Belief** (CB). Epistemically rational agents should have logically consistent belief sets.
- **Fact.** (TB) entails (CB). Suppose S violates (CB). Then, some of S's beliefs are false. Therefore, S violates (TB). □
- I don't think (TB) is a very interesting "norm". It's of little use in *guiding* our epistemic lives. The fact that (TB) entails (CB) suggests that (CB) isn't a very interesting "norm" either.
- I think (some) *preface cases* are counterexamples to (CB) [and ∴ (TB)], as *norms*. If I'm in a (suitably bad) preface case, you won't be able to show me a consistent belief set that *seems "epistemically better"* to me than my own.

- I'll explore the relationship between *different sorts* of accuracy & coherence norms (which seem more interesting).
- For simplicity, I'll talk about *finite*, *logically omniscient*, *opinionated* agents who make *definite judgments* regarding all propositions (or pairs thereof) in some algebra $\mathcal{B}$.
- I will consider three kinds of judgments:
  - **Qualitative**. S believes p [$B_S(p)$]. S disbelieves p [$D_S(p)$].
  - **Comparative**. S is strictly more confident in p than q [$p \succ_S q$]. S is strictly more confident in q than p [$q \succ_S p$]. S is "doxastically indifferent" between p and q [$p \sim_S q$].
  - **Quantitative**. S's degree of confidence/credence in p is r.
- Time permitting, for each of these, I'll discuss relationships between (analogous) accuracy & coherence norms.
- The first step is to *define inaccuracy* for each of the three types of judgments. Once we've done that, we'll examine some new relationships between (in)accuracy & coherence.

- The *inaccuracies* of S's three types of *judgment sets* are (these get increasingly controversial — more on this below).
  - **Qualitative**. Let $\mathfrak{B}$ be the full set of S's qualitative judgments over $\mathcal{B}$. The *innaccuracy* of $\mathfrak{B}$ at a world w is given by the number of incorrect judgments in $\mathfrak{B}$ at w.
    - $B_S(p)$ is (in)correct in w iff p is true (false) at w.
    - $D_S(p)$ is (in)correct in w iff p is false (true) at w.
  - **Comparative**. Let $\mathfrak{C}$ be the full set of S's comparative judgments over $\mathcal{B} \times \mathcal{B}$. The *innaccuracy* of $\mathfrak{C}$ at a world w is given by the number of incorrect judgments in $\mathfrak{C}$ at w.
    - $p \sim_S q$ is (in)correct at w iff $p \equiv q$ is true (false) at w.
    - $p \succ_S q$ is (in)correct at w iff $p \,\&\, {\sim}q$ is true (false) at w.
  - **Quantitative**. Let b be S's credence function (b is a *function from $\mathcal{B}$ to the real numbers*). The *degree of inaccuracy* of b at a world w [$I(b, w)$] will be given by some *scoring-rule*.
    - There are various scoring rules that have been proposed in the literature. I will not delve into this controversy here.
    - In my lecture on Friday, I will return to the quantitative case.

- Consider this notion of (qualitative) *accuracy-dominance*:
  - One set of qualitative judgments $\mathfrak{B}'$ *accuracy-dominates* another $\mathfrak{B}$ iff (i) $\mathfrak{B}'$ has *strictly fewer* incorrect judgments at *some* possible worlds, and (ii) $\mathfrak{B}'$ contains *at most as many* incorrect judgments as $\mathfrak{B}$ at *every* possible world.
- Next, consider the following qualitative coherence norm:

(QC) S should not have a qualitative judgment set $\mathfrak{B}$ that is (*a priori*) *accuracy-dominated* by some alternative set $\mathfrak{B}'$.

- Why is (QC) compelling? For one thing, it is immune from one analogue of preface cases. Allow me to explain.
- In a (sufficiently bad) preface case, S has a judgment set $\mathfrak{B}$ which is inconsistent, but which is such that no consistent alternative $\mathfrak{B}'$ "looks better" to them, *given their evidence*.
- If we show S an alternative, consistent set $\mathfrak{B}'$, their evidence will suggest — *perhaps non-misleadingly!* — that $\mathfrak{B}'$ contains *more incorrect judgments* than their own set $\mathfrak{B}$.

- However, there can be no analogous cases when it comes to violations of (QC). Such cases would be *paradoxical*. Why?
    1. Suppose S violates (QC). Then, there exists a $\mathfrak{B}'$ which S can know *a priori* accuracy-dominates their judgment set $\mathfrak{B}$.
    2. Suppose I show S such an alternative set $\mathfrak{B}'$. If this were analogous to a preface case, then S's evidence would suggest that $\mathfrak{B}'$ has more incorrect judgments than $\mathfrak{B}$.
    3. But, then, I can run S through the (*a priori*) argument which shows that $\mathfrak{B}'$ *cannot* have more incorrect judgments than $\mathfrak{B}$ — in *any* possible world. This is a new kind of trouble for S.
- So, (QC) is more appropriate for those who want to maintain that there *are some* "coherence"/rationality requirements. [That debate has been "stacked" in Kolodny's [8] favor!]
- But, what in the world is this (QC) norm *like*? Are there independent ways to understand it or get a grip on it? Yes.
- I'll give two characterizations of (QC) — in terms of violation and satisfaction — and then I'll mention some applications.

- The following theorem gives *one* way to understand (QC):
    - **Theorem**. *S violates* (QC) *iff $\mathfrak{B}$ contains a subset B which has a majority of incorrect judgments in every possible world.*
- Of course, (QC) is *strictly weaker* than (CB). That (QC) does *not* entail (CB) can be seen *via* a simple counterexample.
    - S can believe *both P and $\sim P$ without* violating (QC).

	P	$\sim P$	$B_S(P)$	$B_S(\sim P)$	$B_S(P)$	$D_S(\sim P)$	$D_S(P)$	$B_S(\sim P)$
w_1	**F**	**T**	incorrect	correct	incorrect	incorrect	correct	correct
w_2	**T**	**F**	correct	incorrect	correct	correct	incorrect	incorrect

- Here's a revealing conjecture about *satisfying* (QC).
    - **Conjecture**. S *satisfies* (QC) *iff* their $\mathfrak{B}$ can be numerically Pr-represented in the following precise, "Lockean" way:

($\mathcal{L}_{\mathfrak{B}}$) There exists a probability function Pr such that, $\forall p \in \mathfrak{B}$:

$$B_S(p) \text{ iff } \Pr(p) > \tfrac{1}{2}, \text{ and } D_S(p) \text{ iff } \Pr(p) \le \tfrac{1}{2}$$
OR
$$B_S(p) \text{ iff } \Pr(p) \ge \tfrac{1}{2}, \text{ and } D_S(p) \text{ iff } \Pr(p) < \tfrac{1}{2}.$$

- Next, a concrete example in which (QC) *is* violated.

	$\mathfrak{B}$	$\mathfrak{B}'$	$\mathcal{L}_{\mathfrak{B}'}$
$\sim X \,\&\, \sim Y$	B	D	$\frac{11}{32}$
$X \,\&\, \sim Y$	B	D	$\frac{7}{32}$
$X \,\&\, Y$	B	D	$\frac{13}{32}$
$\sim X \,\&\, Y$	D	D	$\frac{1}{32}$
$\sim Y$	B	B	$\frac{18}{32}$
$X \equiv Y$	B	B	$\frac{24}{32}$
$\sim X$	D	D	$\frac{12}{32}$
X	B	B	$\frac{20}{32}$
$\sim(X \equiv Y)$	D	D	$\frac{8}{32}$
Y	D	D	$\frac{14}{32}$
$X \vee \sim Y$	B	B	$\frac{31}{32}$
$\sim X \vee \sim Y$	B	B	$\frac{19}{32}$
$\sim X \vee Y$	B	B	$\frac{25}{32}$
$X \vee Y$	B	B	$\frac{21}{32}$
$X \vee \sim X$	B	B	1
$X \,\&\, \sim X$	D	D	0

- S's $\mathfrak{B}$ isn't dominated by any *consistent* set, but $\mathfrak{B}$ is — *uniquely* — dominated by the "coherent" $\mathfrak{B}'$.
- As I mentioned, it is *impossible* for S's evidence to *non-misleadingly* make it appear to S that $\mathfrak{B}'$ contains more incorrect judgments than $\mathfrak{B}$.
- But, it is still possible for there to be a weaker sense in which S's evidence non-misleadingly suggests that her violation of (QC) may be "OK".
- Suppose S's evidence *non-misleadingly* supports the truth of the conjunction $X \,\&\, \sim Y$. Then, S may reason in the following way, when they see $\mathfrak{B}'$.
    - Look, I realize that $\mathfrak{B}'$ cannot have more incorrect judgments than my $\mathfrak{B}$ does.
    - But, *I have good evidence for/know $X \,\&\, \sim Y$*, which (if true) *rules-out* $\mathfrak{B}'$. Since *my* violation of (QC) is *equivalent* to my being dominated by $\mathfrak{B}'$, why should I be *moved* by my violation of (QC)? [*Kolodny's revenge*!?]

- Rachael Briggs, Kenny Easwaran and I are investigating various applications of this accuracy-dominance approach.
- One interesting application is to judgment aggregation. *E.g.*,
    - Majority rule aggregations of the judgments of a bunch of agents — each of whom satisfy (CB) — need not satisfy (CB).
- Question: does majority rule preserve *our* "qualitative coherence", *viz.*, is (QC) preserved by MR? Answer: No!
    - There are sets of judges (minimum # = 5) that (severally) satisfy (QC), while their majority profile *violates* (QC).
    - But, if a set of judges is (severally) *consistent* (in the classical sense), then their majority profile *must* satisfy (QC).
- Another application: define an "entailment" relation ($\Vdash$), as a relation between a set of judgments Γ and a p-judgment:
    - $\Gamma \Vdash B_S(p)$ iff $\Gamma \cup \{D_S(p)\}$ is "*incoherent*" [in the (QC) sense].
    - $\Gamma \Vdash D_S(p)$ iff $\Gamma \cup \{B_S(p)\}$ is "*incoherent*" [in the (QC) sense].
- Next: accuracy & coherence of *comparative* judgments.

- Once upon a time, *comparative* confidence judgments were thought to be more "secure" or "basic" or "fundamental" than *numerical* confidence/credence judgments [7].
- In his watershed essay, de Finetti [2] begins his story about the foundation of subjective probability theory, as follows:

  > Let us consider a well-defined event and suppose that we do not know in advance whether it will occur or not; the doubt about its occurrence to which we are subject lends itself to comparison, and, consequently, to gradation. If we acknowledge only, first, that one uncertain event can only appear to us (a) equally probable, (b) more probable, or (c) less probable than another; second, that an uncertain event always seems to us more probable than an impossible event and less probable than a necessary event; and finally, third, that when we judge an event E more probable than an event E', which is itself judged more probable than an event E'', the event E can only appear more probable than E'' (transitive property), it will suffice to add to these three evidently trivial axioms a fourth, itself of a purely qualitative nature, in order to construct rigorously the whole theory of probability.

- What de Finetti is describing here is a "foundationalist" conception of subjective probability, where the *foundation* consists of *relations of comparative confidence* ($\succ_S$).

- First, de Finetti thought that the following two "axioms" for the "strictly more confident in" relation $\succ_S$ were *self-evident*:
  - (1) $\succ_S$ is transitive, asymmetric, and irreflexive.
    - More precisely, that $\succ_S$ imposes a *strict total order* on $\mathcal{B}$. [Note: de Finetti *assumes*: $p \sim_S q$ *iff* $p \nsucc_S q$ & $q \nsucc_S p$.]
  - (2) For all p, $p \nsucc_S \top$ and $\bot \nsucc_S p$.
    - (2.1) S should never be strictly more confident in any p than $\top$.
    - (2.2) S should never be strictly more confident in $\bot$ than any p.
- Second, de Finetti thought that the following "additivity" axiom — together with axioms (1) and (2) — *suffices* to ensure *numerical probabilistic representatability* of $\succ_S$.
  - (3) If $\langle p, q\rangle$ and $\langle p, r\rangle$ are both mutually exclusive pairs, then:

    $$q \succ_S r \textit{ only if } (p \vee q) \succ_S (p \vee r).$$
- That is, de Finetti believed that the following was true:
  - (†) If the relation $\succ_S$ satisfies (1)–(3), then there exists a *numerical probability function* b such that:

    $$p \succ_S q \textit{ if and only if } b(p) > b(q).$$

- A few decades later, it was discovered [9] that de Finetti was *wrong* about (†). This was a *crack in the foundation*.
- Scott [11] gave *a much stronger* "additivity" axiom (3′), such that (1)–(3′) *does suffice* for numerical Pr-representability of $\succ_S$. I won't discuss Scott's (rather complex) axiom here.
- Instead, I'll focus on *justifying de Finetti*'s "intuitive" axioms for $\succ_S$. *Nobody* seems to offer *any* justifications for (1)–(3) [3, 4]. But, some *do* offer (dominance) justifications of the *numerical* Pr-axioms (see below). d.F.'s overall strategy was:
  - (i) First, lay down some "*intuitive*" axioms for $\succ_S$-orderings.
  - (ii) Then, show that these axioms suffice to ensure numerical probabilistic representability of any "intuitive" $\succ_S$-ordering.
  - (iii) Finally, justify the axioms of numerical probability theory (*via* the *Brier-dominance approach*, to be discussed Friday).
- If this could be achieved, then one could ground the desired "foundationalist" conception of subjective probability. This project failed at step (ii). But, I think even (i) is dubious…

- Recall our definition of "inaccuracy" of a set of comparative judgments. [Note: other "scoring schemes" are possible for comparative judgments. I will return to this issue, below.]
- We can use an analogous notion of "accuracy dominance" to *justify* (some) "axioms" of comparative probability. To wit:
  - $\mathfrak{C}'$ *accuracy-dominates* $\mathfrak{C}$ iff $\mathfrak{C}'$ contains *strictly fewer* incorrect judgments than $\mathfrak{C}$ at *some* w's, and $\mathfrak{C}'$ contains *at most as many* incorrect judgments as $\mathfrak{C}$ at *every* w.

☞ de Finetti's axioms (1) & (2) *can* then (*modulo* a small *caveat* — see below) be given an accuracy-dominance justification.
  - **Theorem.** If S's $\succ_S$-ordering (*viz.*, S's comparison set $\mathfrak{C}$) violates *either* (1) *or* (2), then (*modulo* a small *caveat* — see below) there exists a $\mathfrak{C}'$ which *accuracy-dominates* $\mathfrak{C}$.
- As far as I know, we (this is joint work with David McCarthy) are the first to offer *any* justification of (1) and/or (2).

  [And, ours is *natural*, given de Finetti's *numerical* approach [1]. I'll explain what I mean by this in my lecture on Friday.]

- But, *all is not beer and skittles...*
- de Finetti's axiom (3) does *not* have an accuracy-dominance justification [$\therefore$ the Scott Axiom (3′), which is required for numerical Pr-representation of $\succ_S$ doesn't have one either].
- This suggests that a de Finetti-style "foundationalist" approach to epistemic probability contains a *logical gap*.
  ☞ *Why* should we think $\succ_S$ has a numerical Pr-representation?
- It is very interesting to note that the principles of comparative probability that have been uncontroversial (or "self-evident") in the literature [4] are (basically) those with an accuracy-dominance justification in our sense. *E.g.*,
  (4) If p entails q, then $p \not\succ_S q$. ["*Monotonicity*" of $\succ_S$.]
- And, those which have been seen as controversial *fail* to have an accuracy-dominance justification. *E.g.*, "additivity" (3), the Scott Axiom (3′), and other principles, such as:
  (5) If $p \succ_S q$, then $\sim p \not\succ_S \sim q$. ["*Complementarity*" of $\succ_S$.]

- Consider the following weak form of transitivity.
  (WT) If $p \succ_S q$ and $q \succ_S r$, then $r \not\succ_S p$.
- If S violates (WT), then S is accuracy-dominated. Proof:

	P	Q	R	$P \succ_S Q$	$Q \succ_S R$	$R \succ_S P$	$P \sim_S Q$	$Q \sim_S R$	$P \sim_S R$
w_1	**T**	**T**	**T**	incorrect	incorrect	incorrect	correct	correct	correct
w_2	**T**	**T**	**F**	incorrect	correct	incorrect	correct	incorrect	incorrect
w_3	**T**	**F**	**T**	correct	incorrect	incorrect	incorrect	incorrect	correct
w_4	**T**	**F**	**F**	correct	incorrect	incorrect	incorrect	correct	incorrect
w_5	**F**	**T**	**T**	incorrect	incorrect	correct	incorrect	correct	incorrect
w_6	**F**	**T**	**F**	incorrect	correct	incorrect	incorrect	incorrect	correct
w_7	**F**	**F**	**T**	incorrect	incorrect	correct	correct	incorrect	incorrect
w_8	**F**	**F**	**F**	incorrect	incorrect	incorrect	correct	correct	correct

- In fact, this is the *unique* dominating $\mathfrak{C}'$. [*Kolodny's revenge* applies — suppose S *has good reason for/knows* $P \succ_S Q$.]
- The "small caveat" is that *not all* violations of transitivity are dominated! For instance, S can be such that $p \sim_S q$, $q \sim_S r$, and $p \succ_S r$ *without* being accuracy-dominated.
  - This is welcome (to me), since I think there are permissible examples of this kind (*e.g.*, perceptual indiscriminability).

- Various 2-valued or 3-valued schemes are possible. *But*,

Theorem. *No* 2 *or* 3-valued scoring scheme is such that:

(0) S entails (at least *some* instances of) both transitivity and additivity as (weak) dominance norms.

and, the the following eight (8) scoring *desiderata* are met:

(1) Having a subset of judgments $\{p \succ_S q, p \succ_S r, q \sim_S r\}$ should not (in and of itself) ensure "incoherence".

(2) Ditto for subsets of the form $\{p \succ_S q, p \succ_S r, q \succ_S r\}$.

(3) $p \succ_S q$ should get a "worst" score when p is **F** and q is **T**.

(4) $p \succ_S q$ should get the same score when p and q are both **T** as it does when p and q are both **F**.

(5) $p \sim_S q$ should get the same score when p and q are both **T** as it does when p and q are both **F**.

(6) $p \sim_S q$ should get the same score when p is **T** and q is **F** as it does when p is **F** and q is **T**.

(7) The score of $p \succ_S q$ when p is **T** and q is **F** should not be strictly worse than the score of $p \succ_S q$ when p, q are both **T**.

(8) The score of $p \succ_S q$ when p is **T** and q is **F** should be strictly better than the score of $p \succ_S q$ when p is **F** and q is **T**.

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