A Historical Introduction to Confirmation Theory

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- Here's what Nicod [23] *said* about instantial confirmation:

  > *Consider the formula or the law: A entails B. How can a particular proposition, or more briefly, a fact, affect its probability? If this fact consists of the presence of B in a case of A, it is favourable to the law ... on the contrary, if it consists of the absence of B in a case of A, it is unfavourable to this law.*

- By "is (un)favorable to", Nicod meant "raises (lowers) the inductive probability of". Here, Nicod has in mind *Keynesian* inductive probability (more on *that* later).
- While Nicod is not very clear on the logical details (stay tuned for Hempel!), some things are clear in what he says:
  - Instantial confirmation is a relation between singular and general propositions/statements ("facts" and "laws").
  - $\pm$Confirmation consists in $\pm$*probabilistic relevance*.
  - Positive instances confirm; negative instances disconfirm.

- When Hempel [16] logically reconstructs Nicod's inchoate remarks about instantial confirmation, some things get lost.
- For instance, *probability* completely falls out of Hempel's reconstruction of Nicod. And, he reconstructs Nicod as describing a relation between *objects* and propositions.
- Hempel's initial, formal reconstruction of "Nicod's Criterion" for instantial confirmation is as follows:

(NC_0) An object a confirms a universal generalization of the form $\ulcorner(\forall y)(\phi y \supset \psi y)\urcorner$ iff a exemplifies both ϕ and ψ.

- Rendering "A entails B" as $\ulcorner(\forall y)(\phi y \supset \psi y)\urcorner$ *is* charitable.
- But, the rest of (NC_0) just *can't* be what Nicod had in mind (*e.g.*, probability is *non*-monotonic — more on that below). Also, (NC_0) is clearly absurd, as Hempel makes very clear.
- According to (NC_0), if "$Ra \,\&\, Ba$" is true, then a will confirm "$(\forall y)(Ry \supset By)$", but *nothing* can confirm the *logically equivalent* claim "$(\forall y)[(Ry \,\&\, {\sim}By) \supset (Ry \,\&\, {\sim}Ry)]$"!
- I'll say a lot more about "Nicod's Criterion" tomorrow.

- Hempel proceeded to formulate a precise, logical theory of confirmation as a relation between sentences in first-order languages. He was inspired by Nicod's *instantial idea*.
- Hempel begins by making precise the informal Nicodian idea of a "conforming instance". He starts with the notion of *the E-development of a hypothesis H*: $\mathrm{dev}_E(H)$.
- $\mathrm{dev}_E(H)$ is constructed from the instances of H with respect to the individual constants appearing in E. *E.g.*:
  - Let $E = Ra \,\&\, Ba$, and $H = (\forall y)(Ry \supset By)$. The set of E-instances of H is the singleton $\{Ra \supset Ba\}$. And, $\mathrm{dev}_E(H)$ is the *conjunction* of H's E instances, which is just $Ra \supset Ba$.
  - Generally, $\mathrm{dev}_E(H)$ will be the conjunction (disjunction) of the E-instances of H if H is a universal (existential) claim. And, if H is neither a $\forall$ nor a $\exists$ sentence, then $\mathrm{dev}_E(H) = H$.
- Hempel's basic idea: E (directly) confirms H if $E \vdash \mathrm{dev}_E(H)$.
- On this account, $Ra \,\&\, Ba$ confirms $(\forall y)(Ry \supset By)$. But, so does ${\sim}Ra \,\&\, {\sim}Ba$ (more tomorrow!). Indeed, the only claim that *dis*confirms H is $Ra \,\&\, {\sim}Ba$. Very Nicodian (*sans* Pr)!

- Hempel's confirmation relation has various properties. Most notably, his conception satisfies the *equivalence condition*:

(EQC) If E confirms H and $H \dashv\vdash H'$, then E confirms H'.

- Recall, Hempel's reconstruction of Nicod (NC_0) did not satisfy the (analogous) equivalence condition (absurd?).
- Hempel's relation also has the following properties:

(EC) If $E \vdash H$, then E confirms H.
(SCC) If E confirms H and $H \vdash H'$, then E confirms H'.
(M) If E confirms H and $E' \vdash E$, then E' confirms H.
(CC) If E confirms both H and H', then H and H' are consistent.

- (M) plays a key role in the ravens paradox (more on that tomorrow). We'll talk more about these properties below.
- Hempel's theory *lacks* certain other properties, such as:

(CCC) If E confirms H and $H' \vdash H$, then E confirms H'.

- Before we get to *probabilistic* accounts of confirmation, we'll look briefly at "Hypothetico–Deductive" (HD) confirmation.

- The (naïve) idea behind (HD) seems to be that science works by *deducing predictions* (E) from hypotheses (H).
- Thus, in a case where $H \vdash E$, if we observe that E obtains, then this ("correct prediction") *confirms* H, and if we observe that E fails to obtain, then this *dis*confirms H.
- *I.e.*, if $H \vdash E$, then E HD-confirms H [$\sim E$ HD-disconfirms H].
- (HD)-confirmation *satisfies* (CCC), but *violates* (EC), (SCC), (M), and (CC). Very *non*-Hempelian! [They agree on (EQC).]
- (HD)-confirmation has other problems of its own:
  - **Duhem–Quine**. *Auxiliary assumptions* (A) are always needed for the *deduction* of predictions (E). How do we *apportion* praise/blame between H and A, when $E/\sim E$ is observed?
  - **Irrelevant Conjunction**. If E HD-confirms H, then E also HD-confirms $H \,\&\, X$, even if X is utterly irrelevant to H, E. [Hempel's theory has a similar "problem", since if E Hempel-confirms H, then so does $E \,\&\, X$, for *any* X (M!).]
- OK, enough about deductive-logical approaches to confirmation. Let's look at some *probabilistic* accounts.

- Nicod's early intuition was that confirmation had to do with *probability-raising*. And, contemporary Bayesians have come back around to this view. What happened in between?
- In the first edition of LFP, Carnap [3] undertakes a precise probabilistic explication of the concept of confirmation.
- Carnap was interested not only in the qualitative confirmation relation. He also wanted explications of comparative and quantitative confirmation concepts.
  - **Qualitative**. E inductively supports H.
  - **Comparative**. E supports H *more strongly than* E' supports H'.
  - **Quantitative**. E inductively supports H *to degree* r.
- Carnap begins by clarifying the *explicandum* (the confirmation concept) in various ways, including:

Qualitative. ($\star$) E gives some (positive) evidence for H.

- Note two things. First, ($\star$) sounds *epistemic* (not *logical*). Second, ($\star$) sounds like it involves (positive) *relevance*.
- Strangely, Carnap proceeds (in LFP_1) to offer a *logical* account of confirmation that does *not* involve relevance…

- In the 1st ed. of LFP, Carnap characterizes "the degree to which E confirms H" as $\mathfrak{c}(H, E) = \Pr(H \mid E)$, which leads to:

  Quantitative. $\Pr(H \mid E) = r$.
  Comparative. $\Pr(H \mid E) > \Pr(H' \mid E')$.
  Qualitative. $\Pr(H \mid E) > t$ (for some "threshold value" t).
  - Doesn't sound like ($\star$). More on this dissonance below.
- Like Hempel, Carnap wanted a *logical* explication of confirmation (as a relation between sentences in FOLs).
- For Carnap, this meant that the probability functions used in confirmation theory must *themselves* be "logical".
- This leads naturally to the Carnapian project of providing a "logical explication" of conditional probability $\Pr(\cdot \mid \cdot)$ *itself*.
- Note: Here, Carnap (like Nicod) was influenced by Keynes [20], who believed that there were "partial entailments" out there in logical space. I'm skeptical (as are most Bayesians).
- *Even if* there are "logical probabilities", are they *required* for a *logical* conception of confirmation *based on* probability? I'll come back to that. Continuing with Carnapian $\mathfrak{c}$ …

- Later in LFP$_1$, Carnap gives counterexamples to Hempel's (SCC), which presupposes a more ($\star$)-like **qualitative** conception of confirmation. There, he presupposes:

  Qualitative. E confirms H iff $\Pr(H \mid E) > \Pr(H \mid \top)$.
- This *probabilistic relevance* conception *violates* (SCC), whereas the previous Pr-threshold conception *implies* (SCC).
- The second edition of LFP [4] includes a preface which acknowledges an "*ambiguity*" in LFP$_1$, and concedes that the (**qualitative**) relevance conception is "more interesting".
  - **Firmness**. The degree to which E confirms$_f$ H:
    $$\mathfrak{c}_f(H,E) = \Pr(H \mid E).$$
  - **Increase in Firmness**. The degree to which E confirms$_i$ H:
    $$\mathfrak{c}_i(H,E) = \mathfrak{f}[\Pr(H \mid E), \Pr(H \mid \top)]$$
    $\mathfrak{f}$ measures "the degree to which E *increases* the Pr of H."
- The 1st ed. of LFP was mainly about firmness, and the 2nd edition only adds the preface, which says very little about $\mathfrak{c}_i$. Specifically, no function $\mathfrak{f}$ is rigorously defended there.

- *Many* candidate functions $\mathfrak{f}$ satisfy the *relevance* constraint:

  ($\mathcal{R}$) $\mathfrak{f}[\Pr(H \mid E), \Pr(H \mid \top)] \gtreqless 0$ iff $\Pr(H \mid E) \gtreqless \Pr(H \mid \top)$
- The three historically most popular such functions $\mathfrak{f}$ are:
  - $d(H,E) = \Pr(H \mid E) - \Pr(H \mid \top)$
  - $r(H,E) = \log\left[\frac{\Pr(H \mid E)}{\Pr(H \mid \top)}\right]$
  - $l(H,E) = \log\left[\frac{\Pr(H \mid E)(1-\Pr(H \mid \top))}{(1-\Pr(H \mid E))\Pr(H \mid \top)}\right] = \log\left[\frac{\Pr(E \mid H)}{\Pr(E \mid \sim H)}\right]$
- Interestingly, these measures are *not comparatively equivalent*. They disagree on many comparative claims.
- The most radical (and interesting) disagreement between these measures occurs in the context of *favoring* claims [9] of the form $\mathfrak{c}(H,E) \geq \mathfrak{c}(H',E)$. For instance, only l satisfies:

  If $E \vdash H$ and $E \nvdash H'$, then $\mathfrak{c}(H,E) \geq \mathfrak{c}(H',E)$.
- Only r satisfies: $\Pr(E \mid H) > \Pr(E \mid H') \Rightarrow \mathfrak{c}(H,E) > \mathfrak{c}(H',E)$.
- I'll say more about disagreement between (these and other) relevance measures below (and next week). Back to Carnap.

- From an inductive-logical point of view, confirmation measures should *quantitatively generalize* entailment:

  ($\mathcal{D}$) Provided that both E and H are *contingent* claims[1] $\mathfrak{c}_i(H,E)$ should be *maximal* if $E \vdash H$, and *minimal* if $E \vdash \sim H$. [Note: $\Pr(H \mid E)$ satisfies *this*, but *not* $\mathcal{R}$.]
- Kemeny & Oppenheim [19] used this consideration (and others) to argue that the best explication of $\mathfrak{c}_i(H,E)$ is:
  $$F(H,E) = \frac{\Pr(E \mid H) - \Pr(E \mid \sim H)}{\Pr(E \mid H) + \Pr(E \mid \sim H)} \doteq l(H,E)$$
- F can be expressed as a function $\mathfrak{f}$ of $\Pr(H \mid E)$ and $\Pr(H \mid \top)$, and it satisfies $\mathcal{R}$, $\mathcal{D}$, and various other IL desiderata.
- One can use F to define **comparative** $[F(H,E) > F(H',E')]$ and **qualitative** $[F(H,E) > 0]$ confirmation$_i$ concepts.
- I think F (or any comparative equivalent, like l) has the proper *form* for an *inductive-logical* relevance measure of degree of confirmation. Whither (relevance) inductive logic?

[1] Here, I'm bracketing the "paradox of entailment" cases, which are tricky.

	Does Theory have property?					
Theory	EQC	EC	CC	M	SCC	CCC
Nicodian "Instantial"	No	No?[1]	Yes?[1]	Yes?[1]	No	No
Hempelian "Instantial"	Yes	Yes	Yes[2]	Yes	Yes	No
Hypothetico-Deductivism	Yes	No	No	No	No	Yes
Firmness	Yes	Yes[3]	No	No	Yes	No
Increase in Firmness	Yes	Yes[4]	No	No	No	No

The last row — three counterexamples for increase in firmness:

(CC) E = card is black, H = card is the A♠, H' = card is the J♣. E confirms both H and H', even though they are inconsistent.

(SCC) E = card is black, H = card is the A♠, and H' = card is an ace.

(CCC) E = card is the A♠, H = card is an ace, and H' = card is the A♦.

[1] Nicod's theory may not come down clearly on these (or it may *trivially*).
[2] Assuming that E is not self-contradictory.
[3] Assuming that $\Pr(E \mid K) \neq 0$.
[4] Assuming that $\Pr(H \mid K) \in (0,1)$, and $\Pr(E \mid K) \in (0,1)$.

- Another popular relevance measure from the literature is:

$$s(H,E) = \Pr(H \mid E) - \Pr(H \mid \sim E)$$

- Six properties of measures $\mathfrak{c}$ (for contingent H,E), see [7]:
  (1) If $E \vdash H$ and $E \nvdash H'$, then $\mathfrak{c}(H,E) \geq \mathfrak{c}(H',E)$.
  (2) If $\Pr(E \mid H) > \Pr(E \mid H')$, then $\mathfrak{c}(H,E) > \mathfrak{c}(H',E)$.
  (3) If $\Pr(H \mid E) > \Pr(H \mid E')$, then $\mathfrak{c}(H,E) > \mathfrak{c}(H,E')$.
  (4) $\mathfrak{c}(H,E) = \mathfrak{c}(E,H)$
  (5) $\mathfrak{c}(H,E) = -\mathfrak{c}(H,\sim E)$
  (6) $\mathfrak{c}(H,E) = -\mathfrak{c}(\sim H,E)$

	Does Measure have property?					
Relevance Measure	(1)	(2)	(3)	(4)	(5)	(6)
$d(H,E)$	No	No	Yes	No	No	Yes
$r(H,E)$	No	Yes	Yes	Yes	No	No
$l(H,E)$	Yes	No	Yes	No	No	Yes
$s(H,E)$	No	No	No	No	Yes	Yes

- Carnap (and Keynes) sought a more general theory of argument evaluation/goodness — "inductive soundness".
- Assessing the "goodness" (*soundness*) of a *deductive* argument from E to H requires the determination of:
  - Whether the argument is valid. [logical]
  - Whether E is true. [(generally) non-logical]
- How do we *generalize* this to include the *inductive* case?
- Of course, we *still* have to determine whether E is true.
- What *else*? Carnap would say we need to determine "the degree to which E confirms H." But, he would also say that this determination must be made in accordance with:

  The Requirement of Total Evidence. In the application of IL to a given knowledge situation, the total evidence available must be taken as a basis for determining the degree of confirmation.

- For Carnap, "take E as a basis" means "conditionalize Pr on E." Problem: let K_C express "everything we (assessor) know." Then, if we know E, $\Pr(H \mid E \,\&\, K_C) = \Pr(H \mid K_C)$!

- Therefore, if we know that E is true, we *cannot* — on a Carnapian approach — determine that E confirms$_i$ H.
- This is problematic. The non-logical component of our all things considered assessment of the argument's "goodness" has *interfered* with its (Carnapian) logical component!
- This is *not* a problem for *firmness*, since it doesn't prevent the logical probability $\Pr(H \mid E \& K_C) = \Pr(H \mid K_C)$ from being greater than *a threshold value*. This is *just* a problem for $\mathfrak{c}_i$.
- Bayesians face a similar problem: "old evidence" (on which more below). General problem: *no* Pr*-assignment such that* $\Pr(E) = 1$ *can reflect a correlation between* E *and* H.
- Thus, it seems, any Pr-relevance based approach to confirmation (logical or otherwise!) will have to abandon the principle of total evidence, *as Carnap understood it*.
- If RTE does *not* imply that we should *conditionalize* our evaluative probability assignment on our total evidence, then what *does* it imply? Good question! First, Bayesian $\mathfrak{c}$ . . .

- Most modern Bayesians don't believe there are "logical" probabilities. I'm inclined to agree, but I won't dwell on it.
- As a result, most modern Bayesians simply *give up on* the traditional project of confirmation theory *as a branch of IL*.
- Instead, they set their sights on explicating an explicitly *epistemic* (and subjective) notion of "inductive support":

  Qualitative. E confirms H *for agent* X *at time* t iff E and H are positively correlated under X's credence function at t.

- This is *formally* similar to the inductive-logical concept $\mathfrak{c}_i$. But, it is *subjective* and *epistemic*, *not* objective and logical.
- Like Carnap, Bayesians assume that *all confirmation relations supervene on* ***one kind*** *of probability*. They just disagree on **which kind** forms the supervenience base.
- There is controversy among Bayesians about **quantitative** and **comparative** $\mathfrak{c}_i$. I'll be talking about that next week (there's some interesting new psychological research here).
- Next: four views on the "logicality of Pr" (and the Carnapian/Bayesian supervenience assumption) . . .

- **Adams/Hailperin** [1, 14]. Individual probability assignments appearing in inductive logic are *never* logical. The *logical* properties (in IL) must hold for *all* probability assignments.
  - ⇒ Inductive logic does *not* undergird assessments of the strength of *particular* arguments. Rather, inductive logic characterizes "probabilistically valid" *forms* $\ulcorner \phi_1, \ldots, \phi_n \therefore \psi \urcorner$ such that ∀ **Pr**:
  $$\Pr(\phi_1) \in \alpha_1, \ldots, \Pr(\phi_n) \in \alpha_n \vDash \Pr(\psi) \in \beta$$
- **Carnap/Maher** [21]. Individual probability assignments that appear in confirmation functions ($\mathfrak{c}_f/\mathfrak{c}_i$) *are always* logical. *And*, inductive logic/confirmation theory *does* undergird assessments of the (*logical*) strength of *particular* arguments (*via logical* Pr's).
- **Subjective Bayesian**. Individual Pr assignments that appear in confirmation functions ($\mathfrak{c}_f/\mathfrak{c}_i$) are *never* logical. Confirmation theory *does* undergird assessments of the (*epistemic*/subjective) "strength"/"weight" of *particular* arguments (*via subjective* Pr's).
- **Alternative**. Individual Pr's appearing in $F_{\Pr}(H, E)$ are *not always* logical (*or* subjective). IL/CT *does* undergird assessments of the strength of *particular* arguments *in contexts* C. Which Pr(s) are appropriate for a given assessment (generally) *depends on* C.

- Probabilistic relevance approaches to confirmation theory have had various "successes", and problems of their own.
- On the "success" side, we have some interesting Pr-relevance "resolutions" of problems and paradoxes:
  - The Duhem–Quine problem. [6, 24, 11, 25, 12]
  - The irrelevant conjunction problem. [8, 15]
  - The ravens paradox [tomorrow!]. [10]
  - The value of varied/diverse evidence. [26]
  - The value of unification/coherence. [22, 2, 5]
  - Explanations of Kahneman & Tversky "fallacies" [next week!]
- I'd be happy to talk about any of these problems in detail (except the two problems I'll be discussing later this trip!).
- Next, I'll focus on the "old evidence" problem (for Bayesian confirmation), and how Bayesians have responded to it.
- Example: a highly reliable pregnancy test comes out + (for Mary). Call this evidence E. You learn E, and (in this context C) you assign $\Pr(E) = 1$. So, E cannot confirm$_i$ (H) that Mary is pregnant (in context C). There are 3 Bayesian responses.

- **Response 1**. The "look at another context C'" response [17]:
  - OK, in your *actual* context ***C*** where you *know* E, you can't apply confirmation$_i$. So, think about *another* (historical, counterfactual, *etc.*) context C' in which you *do not know* E.
  - Then, see if your "counterpart's" credence function $\Pr'$ (or a "logical" $\Pr'$ *conditioned on their total evidence in* C') reflects a correlation between E and H in C', and then *expropriate*.
  - This seems bizarre to me. Why should what one would or should believe in C' bear on what one should believe in C?
- **Response 2**. The "look at another evidence E'" response [13]:
  - OK, you can't assess whether evidence ***E*** supports H in C. So, think about *another* evidential proposition E' (*e.g.*, that "H predicts E in C"), and argue that E' supports H in C.
  - This one just *changes the subject*. It's ***E*** we're talking about.
- **Response 3**. The "use non-standard Pr theory" response [18]:
  - Move to a theory of probability that allows E and H to be "correlated" under Pr, *even if* E has probability 1 under Pr.
  - This avoids the two problems above. But, (a) it *disunifies* $\mathfrak{c}_i$-theory, (b) what if the pregnancy test *always* yields + results?, and (c) what if you *also* know H in C [$\Pr(H) = 1$]?

- I don't think any of these responses will work. My two take-away lessons from the "old evidence" problem (and the analogous problem for Carnapian increase in firmness) is:
  - The requirement of total evidence must not be interpreted as *requiring* that we (*always*) **conditionalize** evaluative (*i.e.*, confirmation-theoretic) probability assignments on everything we know (in the evaluative context).
  - Not all confirmation relations (in all contexts) supervene on credences (or logical probabilities, or any other kind of Pr).
- Note: In *some* contexts C, confirmation-theoretic probability assignments *should* assign $\Pr(E) = 1$. *E.g.*, if the pregnancy test in *known* to *always* yield positive results in C.
- To my mind, the RTE just means that when making an assessment of argument strength, we should do so on the basis of everything we know. Thus, the RTE is not a very "helpful" principle from a methodological point of view.
- But, it is naïve to hope for "helpful" principles in this sense: either for credences or confirmation-theoretic probabiltiies.

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