Favoring, Likelihoodism, and Bayesianism

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In Chapter 1 of *Evidence and Evolution*, Sober (2008) defends a Likelihodist account of *favoring*. The main tenet of Likelihoodism is the so-called *Law of Likelihood*. In this note, I explain why the Law of Likelihood fails to undergird an adequate explication of favoring.

1. Some Background on the Favoring Relation

This (brief) note is about the (evidential) "favoring" relation. Pre-theoretically, *favoring* is a three-place (epistemic) relation, between an evidential proposition E and two hypotheses H_1 and H_2. Favoring relations are expressed *via* locutions of the form:

E favors H_1 *over* H_2.

Strictly speaking, favoring should really be thought of as a *four*-place relation, between E, H_1, H_2, *and a corpus of background evidence K*. But, for present purposes (which won't address issues involving K), I will suppress the background corpus, so as to simplify our discussion. Moreover, the favoring relation is meant to be a *propositional* epistemic relation, as opposed to a *doxastic* epistemic relation. That is, the favoring relation is not meant to be restricted to bodies of evidence that are *possessed* (as evidence) by some actual agent(s), or to hypotheses that are (in fact) entertained by some actual agent(s). In this sense, favoring is analogous to the relation of *propositional* justification — as opposed to *doxastic* justification (Conee 1980).

In order to facilitate a comparison of Likelihoodist *vs* Bayesian explications of favoring, I will presuppose the following *bridge principle*, linking *favoring* and *evidential support*:

- *E favors* H_1 *over* H_2 iff *E supports* H_1 *more strongly than E supports* H_2.[1]

Finally, I will only be discussing instances of the favoring relation involving *contingent*, *empirical* claims. So, it is to be understood that "favoring" will not apply if any of E, H_1, or H_2 are *non*-contingent (and/or *non*-empirical). With this background in place, we're ready to begin.

2. A *Popperian* Sufficient Condition for Favoring

Here is an eminently plausible *sufficient condition* for favoring:

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[1] Likelihoodists may balk at this presupposition. But, as I explain in some detail elsewhere (Fitelson 2007), without this bridge principle, no meaningful comparison between Likelihoodism and Bayesianism seems possible.

(PP) If H_2 entails $\sim E$ but H_1 does *not* entail $\sim E$, then E *favors* H_1 *over* H_2.

This is a (weak) "Popperian Principle" concerning the *evidential asymmetry* between *refutation* and *non*-refutation. The Popperian slogan for (PP) would be:

- *Non-refuting evidence supports more strongly than refuting evidence.*

This slogan expresses the kernel of truth in Popperian Falsificationism. The so-called *Law of Likelihood* (Sober 2008, Royall 1997) is meant to *probabilistically generalize* (PP). To wit:

(LL) Suppose H_1 confers probability p_1 on E, and H_2 confers probability p_2 on E.
Then, E favors H_1 over H_2 iff $p_1 > p_2$.

In other words, (LL) reduces *favoring* to a comparison of the *likelihoods* of the H_1 and H_2, relative to evidence E [*viz.*, $p_1 = \Pr(E \mid H_1)$ and $p_2 = \Pr(E \mid H_2)$]. In the *limiting, deductive case* involved in (PP), $p_2 = 0$ and $p_1 > 0$. And, in such *special cases*, *every* (adequate) theory of favoring will endorse the conclusion implied by (LL) [*viz.*, (PP)]. As such, I accept (PP) as a *sufficient condition* for favoring, and I think (LL) is OK *in these special, "Popperian" cases*.

However, when we look at the consequences of (LL) for *other* cases, we can see that it *over*-generalizes (PP). A useful way to illustrate the nature of (LL)'s *over*-generalization of (PP) is to consider another, *non-Popperian (deductive) sufficient condition* for favoring.

3. A *Non*-Popperian Sufficient Condition for Favoring

To see why (LL) *over*-generalizes (PP), consider another (*deductive, limiting case*) *sufficient condition* for favoring that I think should be as uncontroversial as (PP):

(*) If E entails H_1 and E does not entail H_2, then E favors H_1 over H_2.

Principle (*) can be thought of as a "dual" of Principle (PP). Basically, (*) is meant to imply that if E *conclusively* supports H_1, but E *non*-conclusively supports H_2, then E favors H_1 over H_2. Consequently, the slogan for (*) would be:

- *Conclusive evidence supports more strongly than non-conclusive evidence.*

To my mind, this "dual" of (PP) seems just as plausible as (PP) itself.[2] But, while (PP) is (severally) compatible with each of (LL) and (*), it turns out that (LL) is *incompatible* with principle (*). Here is a concrete example illustrating the *incompatibility* of (LL) and (*).

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2 If one is a Popperian Falsificationist — in a *strong*, *Critical Rationalist* sense (Miller 1994) — then one will *deny* (*). But, *that* version of Falsificationism is *false*. And, I take it that contemporary defenders of (LL) [*e.g.*, Sober (2008) and Royall (1997)] do *not* want to embrace this stronger (and highly implausible) Popperian position. As such, contemporary Likelihoodists will need a *different* way to argue that (LL) does *not over*-generalize (PP).

Example. Suppose we have deck of 100 playing cards, and *we know nothing about how the cards in the deck are distributed*, *except* for the following two facts: (*i*) there are some clubs and some red cards in the deck, and (*ii*) at least one ace of spades is contained in the deck. We shuffle the cards well, and we sample a card (c) at random. Now, consider the following three claims regarding c:

(E) c is a spade.
(H_1) c is a black card.
(H_2) c is an ace of spades.

Because E entails H_1 and E does not entail H_2, ($\star$) implies that E favors H_1 over H_2 in this case (which clearly seems to be the correct verdict). However, because $\Pr(E \mid H_2) = 1 > \Pr(E \mid H_1) > 0$, (LL) implies that E favors H_2 over H_1, which *contradicts* ($\star$). This shows that, while (LL) can be seen as *generalizing one* sufficient condition for favoring [(PP)], it also *contradicts another* sufficient condition for favoring [($\star$)].[3]

4. Bayesian Diagnoses (and Explications)

From a Bayesian point of view, the debate about (LL) is really just a debate about the proper measure of *degree of confirmation*. Recall our bridge principle connecting favoring and support:

- *E favors H_1 over H_2* iff *E supports H_1 more strongly than E supports H_2*.

Bayesian confirmation theory provides various explications of "the degree to which *E supports H*." These come in the form of various *relevance measures* $\mathfrak{c}(H,E)$ of "the degree to *E confirms H*." For each of these precise Bayesian explications of evidential support, we get a precise *confirmation-theoretic* bridge principle, of the following kind:

($BP_{\mathfrak{c}}$) *E favors H_1 over H_2 — according to measure* $\mathfrak{c}$ — iff $\mathfrak{c}(H_1,E) > \mathfrak{c}(H_2,E)$.

Different choices of $\mathfrak{c}$ lead to different precise Bayesian bridge principles connecting favoring and confirmation. For instance, according to *the ratio measure* of degree of confirmation:

(r) The *degree* to which *E confirms H* = $r(H,E) = \Pr(H \mid E) / \Pr(H)$.

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[3] The entailment relations are *inessential* to the intuitive verdicts here. A simple modification of Example drives this point home. Suppose that a highly (but imperfectly) reliable witness is going to make three claims about c. The witness is going to report (1) either E or $\sim E$, and (2) either H_1 or $\sim H_1$, and (3) either H_2 or $\sim H_2$. Now, preface each of (E), (H_1), and (H_2) with the following: "The highly (but imperfectly) reliable witness testified that…". This modification does *not* undermine the intuitive verdict that E favors H_1 over H_2 in the Example. Moreover, (LL) will (still) give the (intuitively) *incorrect* verdict here. And, this is despite the fact that there are no entailment relations between the propositions in the revised testimonial rendition of the Example. So, the entailments are inessential to the Example. See (Fitelson 2007) and (Fitelson 2011) for more detailed diagnoses and discussions.

If we accept (r), then (LL) follows from the resulting bridge principle (BP_r). That is, if we plug $c(H,E) = r(H,E)$ into (BP_c), we get (LL). This allows us to see that (LL) *is just a consequence of one approach to Bayesian confirmation*.

Unfortunately, the ratio-measure approach to Bayesian confirmation (r) is flawed in various ways. Perhaps the most telling objection to (r) is that it entails *commutativity* of "degree of evidential support" (Eells & Fitelson 2002):

(C) For all E and H, $c(H,E) = c(E,H)$.

But, (C) is clearly incorrect, since (*e.g.*) E might *entail* H, while H does *not* entail E. And, in such cases, it is clear that commutativity of evidential support (hence, degree of confirmation) can *fail*. I think this flaw is one of the underlying reasons why (LL) gives counter-intuitive results, including those which contradict the intuitively compelling sufficient condition for favoring articulated by principle ($\star$).

There are various (Bayesian) *alternatives* to (LL)/(r) that are compatible with both (PP) and ($\star$), and which do *not* imply the commutativity of quantitative confirmation. One *naïve* Bayesian alternative to (LL) would involve a comparison of *posteriors* $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid E)$:

(NB) E favors H_1 over H_2 iff $\Pr(H_1 \mid E) > \Pr(H_2 \mid E)$.

But, this "*naïve* Bayes" approach to favoring (NB) is *also inadequate*. Popper (1954) showed that (NB) *violates* the following *sensitivity to evidential relevance requirement*:

(R) Suppose E is *positively* relevant to H_1 and E is *negatively* relevant to H_2.
Then, E does *not* favor H_2 over H_1.

Principle (R) makes sense because *favoring* is a relation of *comparative evidential support*. Moreover, (LL) *entails* (R), so (R) is something that Likelihoodists *must* (also) accept. In this sense, (R) is *common ground* between Likelihoodists and Bayesians. That is, anyone who accepts *any* version of (BP_c) must also accept (R). This covers *all* Bayesian explications of favoring — including the Likelihoodist [*viz*., (BP_r)/(LL)] approach.

To sum up: we seek an (probabilistic) explication of *favoring* that is compatible with (PP), ($\star$), *and* (R). As it happens, there are many such contenders within the Bayesian stable.

At the *quantitative* level, there are various measures of confirmation (c) that undergird — *via* (BP_c) — explications of favoring that are compatible with (PP), ($\star$), *and* (R). For instance,

- Likelihood-ratio-based measures (Good 1984, Fitelson 2007, Fitelson 2011).
- An alternative to the likelihood-ratio, which has recently been defended by some philosophers and cognitive scientists (Crupi *et. al.* 2007, Tentori *et. al.* 2007).

At the *qualitative* level, there are various sets of *probabilistic* sufficient conditions for favoring that can be seen as (proper) *generalizations* of (PP), ($\star$), *and* (R). For instance,

(WLL) Suppose $\Pr(E \mid H_1) > \Pr(E \mid H_2)$ *and* $\Pr(E \mid {\sim}H_1) \leq \Pr(E \mid {\sim}H_2)$.
Then, E favors H_1 over H_2.

Joyce (2008) calls this the "Weak Law of Likelihood" [*aptly*, since (LL) entails (WLL), *but not conversely*]. It's a principle that (almost all) Bayesian approaches to favoring [based on (BP_c)] will agree upon. Of course, (WLL) appeals to so-called "catch-all likelihoods" [*viz.*, $\Pr(E \mid {\sim}H_1)$ and $\Pr(E \mid {\sim}H_2)$], and so it(s antecendent) will be controversial for many non-Bayesian philosophers. I don't have the space here to delve into these more subtle aspects of the dialectic between Likelihoodism and its Bayesian rivals. For a detailed discussion of the Likelihoodism/Bayesianism debate about the favoring relation, see (Fitelson 2007). And, for a more recent discussion of "constrastivism", Likelihoodism, and Bayesianism — with applications to problems in philosophy of science and cognitive science — see (Fitelson 2011).

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