Wigner and his many friends: A new no-go result?

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1.- Introduction

In April 2016, Daniela Frauchiger and Renato Renner published an article online entitled “Single-world interpretations of quantum theory cannot be self-consistent” in which they introduce a *Gedankenexperiment* that allows them to conclude that, if “quantum theory is applied to model an experimenter who herself uses quantum theory”, then “no single-world interpretation can be logically consistent.” (Frauchiger and Renner 2016: 1). The argument intends to support the many-worlds interpretation of quantum mechanics, to the extent that it forces us “to give up the view that there is one single reality.” (Frauchiger and Renner 2016: 22). In a new version of the paper, now entitled “Quantum theory cannot consistently describe the use of itself” and published in *Nature Communications* in September 2018, the authors moderate their original claim. In this new version, the same *Gedankenexperiment* is proposed to “investigate the question whether quantum theory can, in principle, have universal validity”, and the conclusion is “that quantum theory cannot be extrapolated to complex systems, at least not in a straightforward manner.” (Frauchiger and Renner 2018: 1); on this basis, the authors consider how the different interpretations of standard quantum mechanics and the different quantum theories should face their result.

Since its first online publication, the Frauchiger and Renner (F-R) argument has caused quite a splash in the field of quantum foundations. In general, it has been considered as a *new* no-go result for quantum mechanics. For instance, in the website of the Perimeter Institute of Theoretical Physics one can find a video of the talk entitled “Frauchiger-Renner no-go theorem for single-world interpretations of quantum theory”, given by Lidia del Rio (2016) only two months after the original publication, in June 2016. But, in many cases, more extreme reactions can be found, based on conceiving the F-R argument as a kind of proof of the *inconsistence* of quantum mechanics. This idea, for instance, is suggested by a post of the Department of Physics of the ETH Zürich (the university to which Frauchiger and Renner belong), entitled “Searching for errors in the quantum world” (Würsten 2018), which asks “How is it possible for a theory to be inconsistent when it has repeatedly been so clearly confirmed by experiments?” (the post is reproduced in the website of *Science Daily*). In turn, with the title “Reimagining of Schrödinger’s cat breaks quantum mechanics — and stumps physicists", an article appeared in the section "News" of *Nature* (the article is reproduced in *Scientific American*). And if one does not restrict the attention to highly reputed journals and websites, it turns to be impossible to keep track of the huge number of comments to the supposedly new no-go result in other websites and personal blogs.

In spite of the differences between the two versions of Frauchiger and Renner's article, both are based on the same *Gedankenexperiment*, which, according to the authors, leads to an argument whose conclusion is a contradiction that supposedly expresses the new no-go result. The purpose of this short article is to clarify the core of the F-R argument, in order to show how the contradiction is obtained. On the basis of this clarification, we conclude that the result of the F-R argument has been overestimated and should be reconsidered from a more cautious perspective.

2.- The Frauchiger-Renner argument

The *Gedankenexperiment* proposed in Frauchiger and Renner's article is a sophisticated reformulation of Wigner's friend experiment (Wigner 1961). In that original thought experiment, Wigner considers the superposition state of a particle in a closed laboratory where his friend is confined. When Wigner's friend measures the particle, its state collapses to one of the components. However, from the outside of the laboratory, Wigner still assigns a superposition state to the whole composite system particle+friend+laboratory.

The F-R argument relies on duplicating Wigner's setup. Here we will follow Jeffrey Bub's presentation of the argument, since it extracts its structure in a simple and elegant way (Bub 2018). Let us consider Alice and Bob located in separate and isolated labs S_A and S_B. Alice measures the observable A_c of a biased "quantum coin" in the state $\left(1/\sqrt{3}\right)|h_c\rangle_A + \left(\sqrt{2/3}\right)|t_c\rangle_A$, where $|h_c\rangle_A$ and $|t_c\rangle_A$ are the eigenstates of A_c. She prepares a qubit in the state $|0_q\rangle_B$ if the outcome is h_c, or in the state $\left(1/\sqrt{2}\right)\left(|0_q\rangle_B + |1_q\rangle_B\right)$ if the outcome is t_c, and sends it to Bob. When Bob receives the qubit, he measures its observable B_q with eigenstates $|0_q\rangle_B$ and $|1_q\rangle_B$.

Now, let us consider the two labs S_A and S_B with all their content (the quantum coin/the qubit, Alice/Bob, the measuring apparatuses, etc.) as two composite many-body systems. The state of lab S_A is now described by $\left(1/\sqrt{3}\right)|h\rangle_A + \left(\sqrt{2/3}\right)|t\rangle_A$, where $|h\rangle_A$ and $|t\rangle_A$ are the eigenstates of the observable $A = A_c \otimes I_{A-c}$, where I_{A-c} is the identity observable corresponding to all the degrees of freedom of S_A different from those of the quantum coin. In other words, A is the observable A_c of the quantum coin but "viewed" from the perspective of the whole lab S_A. Analogously, the two alternative states of lab S_B are described by $|0\rangle_B$ and $\left(1/\sqrt{2}\right)\left(|0\rangle_B+|1\rangle_B\right)$, where $|0\rangle_B$ and $|1\rangle_B$ are the eigenstates of the observable $B=B_q\otimes I_{B-q}$, where I_{B-q} is the identity observable corresponding to all the degrees of freedom of S_B different from, those of the qubit. In other words, B is the observable B_q of the qubit but "viewed" from the perspective of the whole lab S_B. Therefore, state of the composite system S_A+S_B can be expressed as:

$$|\Psi\rangle=\frac{1}{\sqrt{3}}\left(|h\rangle_A|0\rangle_B+|t\rangle_A|0\rangle_B+|t\rangle_A|1\rangle_B\right) \quad (1)$$

As Bub clearly notes, eq. (1) does not presuppose "a suspension of unitary evolution in favor of an unexplained "collapse" of the quantum state." (Bub 2018: 2).

The *Gedankenexperiment* continues by considering two observers, Wigner and Friend, located outside the labs, who will describe the whole situation from the viewpoint of the observables X and Y of the systems S_A and S_B, respectively:

- X has eigenvectors $|\text{fail}\rangle_X$ and $|\text{ok}\rangle_X$, such that:

$$|\text{fail}\rangle_X=\frac{1}{\sqrt{2}}\left(|h\rangle_A+|t\rangle_A\right) \qquad |\text{ok}\rangle_X=\frac{1}{\sqrt{2}}\left(|h\rangle_A-|t\rangle_A\right) \quad (2)$$

- Y has eigenvectors $|\text{fail}\rangle_Y$ and $|\text{ok}\rangle_Y$, such that:

$$|\text{fail}\rangle_Y=\frac{1}{\sqrt{2}}\left(|0\rangle_B+|1\rangle_B\right) \qquad |\text{ok}\rangle_Y=\frac{1}{\sqrt{2}}\left(|0\rangle_B-|1\rangle_B\right) \quad (3)$$

So, in terms of these new eigenvectors, the state $|\Psi\rangle$ of S_A+S_B (see eq. (1)) can be alternatively expressed as:

$$|\Psi\rangle=\sqrt{\frac{2}{3}}|\text{fail}\rangle_X|0\rangle_B+\frac{1}{\sqrt{3}}|t\rangle_A|1\rangle_B \quad (4)$$

$$|\Psi\rangle=\frac{1}{\sqrt{3}}|h\rangle_A|0\rangle_B+\sqrt{\frac{2}{3}}|t\rangle_A|\text{fail}\rangle_Y \quad (5)$$

$$|\Psi\rangle=\frac{1}{\sqrt{12}}|\text{ok}\rangle_X|\text{ok}\rangle_Y-\frac{1}{\sqrt{12}}|\text{ok}\rangle_X|\text{fail}\rangle_Y+\frac{1}{\sqrt{12}}|\text{fail}\rangle_X|\text{ok}\rangle_Y+\sqrt{\frac{3}{4}}|\text{fail}\rangle_X|\text{fail}\rangle_Y \quad (6)$$

In order to simplify the presentation, let us use the expression 'O: o' to represent the proposition '*the observable O has the value o*'. Following Bub's presentation, the following results can be obtained. From eq. (4), the pair $(X:\text{ok},B:0)$ has zero probability, so $(X:\text{ok},B:1)$ is the only possibility when $X:\text{ok}$. From eq. (5), the pair $(Y:\text{ok},A:t)$ has zero probability, so $(Y:\text{ok},A:h)$ is the only possibility when $Y:\text{ok}$. In turn, eq. (6) shows that the pair $(X:\text{ok},Y:\text{ok})$ has a probability of $1/12$ in a joint measurement of X and Y by Wigner and Friend. But the combination of these three results "is inconsistent with any pair of outcomes for Alice's and Bob's measurements" (Bub 2018: 2), since the pair $(A:h, B:1)$ has probability zero in the state as expressed by eq. (1).

Frauchiger and Renner notice that the argument relies on three assumptions:

(QT) *Compliance with quantum theory*: quantum mechanics applies to systems of any complexity, including observers.

(SW) *Single-world*: measurements have a single outcome

(SC) *Self-consistency*: measurement outcomes for different observers are logically consistent.

In the 2016 version of their paper, Frauchiger and Renner implicitly accept (QT) and (SC): as a consequence, they claim that their argument shows that "no single-world interpretation can be logically consistent" (2016: 1) and, therefore, "we are forced to give up the view that there is one single reality" (2016: 22). In the 2018 version, they admit the possibility of different theoretical and interpretive viewpoints regarding their result, and include a table that shows which of the three assumptions each interpretation or quantum theory violates (2018: 9).

For our purpose it is essential to stress why the result obtained by Frauchiger and Renner has been so appealing for the quantum foundations community. The F-R argument is based exclusively on standard quantum mechanics: it is independent of any interpretation of the standard formalism. In particular, it does not appeal to the hypothesis of collapse or to any other assumption about measurement. In the original Wigner's friend argument, the paradox arises when comparing the collapsed state of the friend inside the lab and the superposition assigned by Wigner from the outside. The F-R argument, on the contrary, does not assume that the measurements made by the observers collapse the state of the measured system: eq. (1) is the complete uncollapsed quantum state of the composite system S_A+S_B, and the contradiction is obtained by considering exclusively the probabilities that this state allows us to infer. The only trick is to consider cases of probability equal to zero or to one. Besides its simplicity, the advantage of Bub's presentation of the argument is that it makes completely clear how the contradiction arises with no appeal to any interpretive assumption about measurement.

The reactions to the F-R argument have been multiple and varied. An interesting response emphasizes an implicit assumption of the argument: the non-relational view of quantum mechanics is an indispensable premise of the derivation. This is the view of Časlav Brukner (2018), who considers, from an operational perspective, that the self-consistent condition SC is too restrictive, since "the states referring to outcomes of different observers in a Wigner-friend type of experiment cannot be defined without referring to the specific experimental arrangements of the observers, in agreement with Bohr's idea of contextuality" (2018: 8). From a non-operational standpoint, Dennis Dieks (2019) advocates, in the line of Carlo Rovelli's relational view (1996), for a perspectivalist interpretation of quantum mechanics, according to which more than one state can be assigned to the same physical system: the state and physical properties of a system *A* are different in relation to different reference systems B_i; when the perspectival nature of quantum states is included as a premise, no contradiction can be inferred from the F-R argument. According to Richard Healey (2018), the F-R argument implicitly depends on an inconclusive additional assumption, intervention insensitivity, which guarantees that the truth-value of an outcome-counterfactual is insensitive to the occurrence of a physically isolated intervening event.

After supplying his clear and elegant reconstruction of the F-R argument, Jeffrey Bub (2018) claims that what he calls the "Frauchiger-Renner contradiction" shows that quantum mechanics should be understood probabilistically, as a new sort of non-Boolean probability theory, rather than representationally, as a theory about the elementary constituents of the physical world and how these elements evolve dynamically over time. In resonance with his information-theoretic interpretation of quantum mechanics, Bub conceives quantum mechanics formulated in Hilbert space is fundamentally a theory of probabilistic correlations that are structurally different from correlations that arise in Boolean theories. Analogously to special relativity, as a theory about the structure of space-time that provides an explanation for length contraction and time dilation through the geometry of Minkowski space-time with no dynamical considerations, "[q]uantum mechanics, as a theory about randomness and nonlocality, provides an explanation for probabilistic constraints on events through the geometry of Hilbert space, but that's as far as it goes." (Bub 2018: 3).

From a completely different perspective, the conclusion of the F-R argument was rejected on the basis of Bohmian mechanics, the paradigmatic one-world no-collapse quantum theory. For instance, Anthony Sudbery (2017) offers a Bell-Bohmian reconstruction of the argument, claiming that it supplies a counter-example to the conclusion obtained by Frauchiger and Renner. With a similar reasoning, Dustin Lazarovici and Mario Hubert (2018) assert that any Bohm-type theory provides a logically consistent description of F-R *Gedankenexperiment* if the state of the entire system and the effects of all measurements are taken into account.

In the following section we will reconstruct the F-R argument in detail only on the basis of the standard formalism of quantum mechanics: its consequences for other quantum theories will not be analyzed. Nevertheless, such a reconstruction will allow us to bring to light the logical structure of the argument in order to discuss its validity and scope.

3.- Reconstructing the argument

Frauchiger and Renner assume that the states involved in their argument always evolve unitarily. In fact, collapse is not included as one of the three assumptions on which the argument relies. Moreover, in the 2016 article they informally discuss the alternatives left by their result: either future experiments will show the need of replacing the original theory by adding, for example, an objective collapse, or we are forced to reject any single-world interpretation (Frauchiger and Renner 2016: 3). The assumption of unitarity also explains the fact that, in the 2018 article, collapse interpretations are included in the list of interpretations of quantum mechanics as those that violate the assumption QT (compliance with quantum theory) in order to circumvent the contradiction resulting from the argument (Frauchiger and Renner 2016: 9).

Following this idea suggested in the original papers, in the previous section we have pointed out that the F-R argument does not appeal to the hypothesis of collapse. However, not everybody agrees with this. For instance, Franck Laloë considers that the argument illustrates no inconsistency in quantum mechanics, but only the well-known fact that “the exact point at which the von Neumann reduction postulate should be applied is ill defined.” (Laloë 2018: 1). Mateus Araújo (2018), in turn, finds “the flaw in Frauchiger and Renner’s argument” in the fact that the predictions that Frauchiger and Renner claim to be followed from quantum mechanics can only be obtained when collapse is added. Independently of the soundness of these opinions, it seems quite clear that, if the conclusion of the F-R argument depended on collapse, it would lose much of its appealing since it would offer no much novelty when compared with the original Wigner’s friend argument. By contrast, what has shocked the physical community is that the argument seems to show an internal inconsistency of quantum mechanics, independently of any interpretive addition.

Nevertheless, Araújo’s and Laloë’s claims show that there is no consensus about which F-R argument’s premises are. For this reason, it is worth reconstructing the argument in order to show that the “Frauchiger-Renner contradiction” follows with no need of observers (or measurements) collapsing the quantum state. But the same reconstruction will allow us to bring to light a controversial step in the development of the argument.

Let us use the symbols '¬', '∧', '∨', and '→' for negation, conjunction, disjunction and conditional, respectively, as usual. Still following Bub's presentation, the contradiction is obtained in the following way:

- Eq. (4) shows that the probability of $X:\text{ok}$ and $B:0$ is zero. Therefore, it is certain that $X:\text{ok}$ and $B:0$ is not the case:

$$\Pr\left(X:\text{ok} \wedge B:0\right) = 0 \quad \Rightarrow \quad \neg\left(X:\text{ok} \wedge B:0\right) \tag{7}$$

  But this last proposition amounts to say that (recall the definition of the conditional in terms of conjunction: $p \to q \equiv \neg\left(p \wedge \neg q\right)$), if $X:\text{ok}$, then $B:0$ is not the case or, equivalently, if $X:\text{ok}$, then $B:1$:

$$\neg\left(X:\text{ok} \wedge B:0\right) \;\equiv\; X:\text{ok} \to \neg B:0 \;\equiv\; X:\text{ok} \to B:1 \tag{8}$$

- Eq. (5) shows that the probability of $Y:\text{ok}$ and $A:t$ is zero. Therefore, it is certain that $Y:\text{ok}$ and $A:t$ is not the case:

$$\Pr\left(Y:\text{ok} \wedge A:t\right) = 0 \quad \Rightarrow \quad \neg\left(Y:\text{ok} \wedge A:t\right) \tag{9}$$

  Analogously to the previous case, this last proposition amounts to say that, if $Y:\text{ok}$, then $A:t$ is not the case or, equivalently, if $Y:\text{ok}$, then $A:h$:

$$\neg\left(Y:\text{ok} \wedge A:t\right) \;\equiv\; Y:\text{ok} \to \neg A:t \;\equiv\; Y:\text{ok} \to A:h \tag{10}$$

- Eq. (6) shows that the probability of $X:\text{ok}$ and $Y:\text{ok}$ is not zero (in particular, it is $1/12$). Therefore, it may happen that

$$X:\text{ok} \wedge Y:\text{ok} \tag{11}$$

- From eqs. (11), (8) and (10), it can be concluded that, in the case that $X:\text{ok}$ and $Y:\text{ok}$, then $A:h$ and $B:1$:

$$A:h \wedge B:1 \tag{12}$$

- But from eq. (1), it is clear that the probability of $A:h$ and $B:1$ is zero. Therefore, it is certain that $A:h$ and $B:1$ is never the case:

$$\Pr\left(A:h \wedge B:1\right) = 0 \quad \Rightarrow \quad \neg\left(A:h \wedge B:1\right) \tag{13}$$

- Therefore, in the case that $X:\text{ok}$ and $Y:\text{ok}$, the following contradiction obtains

$$\left(A:h \wedge B:1\right) \wedge \neg\left(A:h \wedge B:1\right) \tag{14}$$

  This means that the conclusions obtained by Alice and Bob, who rely on the state vector as expressed in eq. (1), contradicts the conclusions obtained by Wigner and Friend, who have access to the state vector as expressed by eq. (6).

As this reconstruction shows, the contradiction between the observers' conclusions does not need that the quantum state collapses: they can agree on their disagreement just by looking at the uncollapsed quantum state. As stressed above, it is precisely this fact that makes the F-R argument novel and astonishing: it seemingly shows an internal contradiction at the level of probabilities, independently of any interpretation.

But, is this argument legitimate? We will show that, despite of its persuasiveness, one of its steps requires further scrutiny. First, it is necessary to identify the Hilbert space in which the whole argument unfolds. If Alice's lab S_A is represented by the Hilbert space $\mathcal{H}_A$, then the observables A and X are represented by operators acting on $\mathcal{H}_A$ and their respective eigenvectors $|h\rangle_A$ and $|t\rangle_A$, and $|\text{ok}\rangle_X$ and $|\text{fail}\rangle_X$ belong to $\mathcal{H}_A$. Analogously, if Bob's lab S_B is represented by the Hilbert space $\mathcal{H}_B$, then the observables B and Y are represented by operators acting on $\mathcal{H}_B$ and their respective eigenvectors $|0\rangle_B$ and $|1\rangle_B$, and $|\text{fail}\rangle_Y$ and $|\text{ok}\rangle_Y$ belong to $\mathcal{H}_B$. Therefore, the state $|\Psi\rangle$ of the composite system S_A+S_B is represented by a vector belonging to the Hilbert space $\mathcal{H}_{AB}=\mathcal{H}_A\otimes\mathcal{H}_B$.

Now let us consider the three propositions involved in the derivation of the contradiction:

- In order to conclude that $\Pr(X:\text{ok}\wedge B:0)=0$, eq. (4) should be expressed in the basis $X\text{-}B=\{|\text{fail}\rangle_X|0\rangle_B, |\text{fail}\rangle_X|1\rangle_B, |\text{ok}\rangle_X|0\rangle_B, |\text{ok}\rangle_X|1\rangle_B\}$ of $\mathcal{H}_{AB}$, that is, the basis defined by the observables X and B:

  $$|\Psi\rangle=\sqrt{\frac{2}{3}}|\text{fail}\rangle_X|0\rangle_B+\frac{1}{\sqrt{6}}|\text{fail}\rangle_X|1\rangle_B-\frac{1}{\sqrt{6}}|\text{ok}\rangle_X|1\rangle_B \tag{15}$$

  Therefore, the proposition $\neg(X:\text{ok}\wedge B:0)\equiv X:\text{ok}\rightarrow B:1$ is obtained in the X-B context.

- Analogously, in order to conclude that $\Pr(Y:\text{ok}\wedge A:t)$, eq. (5) should be expressed in the basis $Y\text{-}A=\{|\text{fail}\rangle_Y|h\rangle_A, |\text{fail}\rangle_Y|t\rangle_A, |\text{ok}\rangle_Y|h\rangle_A, |\text{ok}\rangle_Y|t\rangle_A\}$ of $\mathcal{H}_{AB}$, that is, the basis defined by the observables Y and A:

  $$|\Psi\rangle=\frac{1}{\sqrt{6}}|h\rangle_A|\text{fail}\rangle_Y+\frac{1}{\sqrt{6}}|h\rangle_A|\text{ok}\rangle_Y+\sqrt{\frac{2}{3}}|t\rangle_A|\text{fail}\rangle_Y \tag{16}$$

  Therefore, the proposition $\neg(Y:\text{ok}\wedge A:t)\equiv Y:\text{ok}\rightarrow A:h$ is obtained in the Y-A context.

- Finally, in order to conclude that $\Pr(X:\text{ok}\wedge Y:\text{ok})\neq 0$, eq. (6) is expressed in the basis $X\text{-}Y=\{|\text{fail}\rangle_X|\text{fail}\rangle_Y, |\text{fail}\rangle_X|\text{ok}\rangle_Y, |\text{ok}\rangle_X|\text{fail}\rangle_Y, |\text{ok}\rangle_X|\text{ok}\rangle_Y\}$ of $\mathcal{H}_{AB}$, that is, the basis defined by the observables X and Y. Therefore, the proposition $X:\text{ok}\wedge Y:\text{ok}$ can be meaningfully expressed only in the X-Y context (both when it is true and when it is false).

- Eq. (12) is obtained by combining eqs. (11), (8) and (10) in the following simple logical argument:

$$X:\text{ok}\wedge Y:\text{ok}\ ,\ \ X:\text{ok}\rightarrow B:1\ ,\ Y:\text{ok}\rightarrow A:h\ \Rightarrow\ A:h\wedge B:1 \qquad (17)$$

But, in which context can those three propositions be combined so as to obtain $A:h\wedge B:1$? In order to simultaneously assert a proposition referred to observables *X* and *Y*, a proposition referred to observables *X* and *B*, and a proposition referred to observables *Y* and *A* –the three premises of the reasoning of eq. (17)–, the context *X*-*Y*-*A*-*B* should be defined. The derivation expressed in eq. (17) would be valid if the bases *X*-*B*, *Y*-*A* and *X*-*Y* were really alternative expressions of a same basis of the complete Hilbert space $\mathcal{H}_{AB}$. However, this is not the case: they are three different bases, rotated with respect to each other. This is a consequence of the fact that:

(i) *X* does not commute with *A*: in the Hilbert space $\mathcal{H}_A$, the two eigenvectors $|\text{ok}\rangle_X$ and $|\text{fail}\rangle_X$ of *X* are rotated with respect to the two eigenvectors $|h\rangle_A$ and $|t\rangle_A$ of *A* (see in eq. (2) how they are interdefined), and

(ii) *Y* does not commute with *B*: in the Hilbert space $\mathcal{H}_B$, the two eigenvectors $|\text{ok}\rangle_Y$ and $|\text{fail}\rangle_Y$ of *Y* are rotated with respect to the two eigenvectors $|0\rangle_B$ and $|1\rangle_B$ of *B* (see in eq. (3) how they are interdefined).

The fact that the bases *X*-*B*, *Y*-*A* and *X*-*Y* are different bases of $\mathcal{H}_{AB}$ can also be demonstrated by defining three observables O_{XB}, O_{YA}, and O_{XY} acting on $\mathcal{H}_{AB}$, whose eigenvectors are the members of the bases *X*-*B*, *Y*-*A*, and *X*-*Y* respectively: it can be proved that those three observables do not commute with each other, $[O_{XB},O_{YA}]\neq 0$, $[O_{XB},O_{XY}]\neq 0$, and $[O_{YA},O_{XY}]\neq 0$ (see the detailed proof in the Appendix).

Summing up, the bases *X*-*B*, *Y*-*A*, and *X*-*Y*, in the context of which the propositions $X:\text{ok}\rightarrow B:1$, $Y:\text{ok}\rightarrow A:h$ and $X:\text{ok}\wedge Y:\text{ok}$ can be respectively asserted, are different bases of the same Hilbert space $\mathcal{H}_{AB}$. But we have learnt since the first courses on quantum mechanics that propositions corresponding to different bases (assigning precise values to non-commuting observables) cannot be simultaneously asserted. Therefore, if those old lessons are taken into account, the "Frauchiger-Renner contradiction" does not follow.

4.- What the argument shows

But, then, what does the F-R argument show? When the reasoning is carefully reconstructed, it is quite clear that the argument proves that, in quantum mechanics, if we combine propositions corresponding to different contexts by means of standard logic, consistency is not guaranteed. Or, in other words, the argument proves that the structure of the quantum propositions is a non-Boolean lattice (see, e.g., the already classical Bub 1997). But this is clearly not a new result.

Somebody might retort that, since the argument derives the contradiction only in terms of probabilities, the combination of different contexts is a legitimate strategy. But this is not the case if the probabilities equal to one are used to assert non-probabilistic propositions. In fact, given a spin-1/2 particle and two orthogonal directions z and x in physical space, it can be inferred that $\Pr\left(S_z :\uparrow \vee S_z :\downarrow\right) = 1$ and $\Pr\left(S_x :\rightarrow \vee S_x :\leftarrow\right) = 1$. Moreover, as usual, in the S_z context, the first probabilistic result can be used to assert the proposition $S_z :\uparrow \vee S_z :\downarrow$, and in the S_x context, the second probabilistic result can be used to assert the proposition $S_x :\rightarrow \vee S_x :\leftarrow$, with no need of any measurement. Nevertheless, the fact that the two propositions were inferred from probability-one results does not legitimize the assertion of the combined proposition $(S_z :\uparrow \vee S_z :\downarrow) \wedge (S_x :\rightarrow \vee S_x :\leftarrow)$: we cannot conclude that the spin in z is in one of its two possible values *and* the spin in x is in one of its two possible values.

Summing up, the F-R argument derives a contradiction without appealing to collapse but by using classical logic to connect propositions coming from different contexts. What are the possible reactions to this result? One of them, as advanced above, is to admit the contradiction as a *reductio-ad-absurdum* proof of the non-Boolean structure of the quantum propositions. In this case, the result can be considered correct but not novel at all. However, from a less benevolent perspective somebody might claim that the argument is plainly wrong, because *at present* everybody knows, at least, that the conjunction of propositions corresponding to the values of non-commuting observables is forbidden in the quantum domain. And the word ‘forbidden’ must be understood with a meaning rooted in the very praxis of science: if in a quantum mechanics exam a student concludes a conjunction of propositions corresponding to the values of non-commuting observables, the exam would certainly be disapproved. From this second position, physics is not reduced to a mere set of formalisms; it is a dynamical body of knowledge in continuous development. Therefore, a physical result must be assessed not only in the context of the formalism from which it is derived, but also in the broader framework of the physics’ community knowledge at the historical time when it is obtained: a result that is novel at one time, may be trivial at a later time, and may even be strictly wrong when, after many years, it is already well known that the assumptions on which its derivation was based are unacceptable.

5.- Conclusions

In this brief article we have analyzed the F-R argument by following the clear and illuminating presentation offered by Jeffrey Bub in a very recent work. On the basis of a detailed reconstruction of the argument, we have shown that the F-R argument is interpretively neutral; in particular, it does not require collapse to lead to its conclusion. The contradiction clearly pointed out by Bub arises by considering exclusively the quantum state of the whole situation, without appealing to effective measurements or to any interpretive addition to the formalism. This fact is what explains the strong repercussion of the Frauchiger and Renner's paper, and the alarmist consequences that have been drawn from it: that the argument "breaks quantum mechanics" or that the theory is inconsistent, or that there are "errors in the quantum world". In fact, the paper seems to show that the problem does not lie in any interpretive addition to, or reformulation of, the standard formalism, but in the core of quantum mechanics itself. The final aim of our reconstruction of the argument was to show that the derivation of the F-R argument's conclusion requires the conjunction between propositions corresponding to different contexts, that is, propositions that assign precise values to incompatible observables; this result should suffice to temper those worrying opinions about quantum mechanics.

In the last section of his article on the F-R argument, Bub asks "What are the options in the light of the Frauchiger-Renner result?" (Bub 2017: 4). Of course, one option is adopting a non- representationalist, informational view of quantum mechanics, as Bub himself does since several year ago (Clifton, Bub, and Halvorson 2003). Another option is advocating for a relationalist (Rovelli 1996) or perspectivalist (Dieks 2009) interpretation of the theory, as explained above. Certainly, the many-worlds interpretation (Everett 1957; for an updated version, see Wallace 2012), which Frauchiger and Renner presented as almost the only way out to their contradiction in the first version of their paper, is also an alternative. But one may also be a Qbist (Fuchs, Mermin, and Schack 2014), or may adhere to a modal interpretation (Lombardi and Castagnino 2008) or to a transactional interpretation (Kastner 2013). And one may furthermore prefer to admit modifications of the standard formalism and endorse dynamical collapse theories (Ghirardi, Rimini and Weber 1986; for an updated review, see Ghirardi 2018), or Bohmian mechanics (Bohm 1952; for an updated review, see Goldstein 2017), or the consistent histories approach to quantum mechanics (Griffiths 1984; for an updated review, see Griffiths 2017). And we apologize in advance for all the interpretations that we have not mentioned here. But the fact is that all these options were already open before the F-R argument was proposed. And, beyond organizing some of those interpretations according to the violation of some of the assumptions of the paper, the F-R argument closes none of them. Of course, F-R is a no-go argument, but one that was very well known since long ago.

Appendix

As explained in the body of the article, what is at stake is the conjunction of three propositions corresponding to three contexts of a single Hilbert space. The problem derives from the fact that the three contexts are strictly different, that is, they correspond to different bases of the Hilbert space. This can be proved in the following way.

The propositions $X:\mathrm{ok}\wedge Y:\mathrm{ok}$, $X:\mathrm{ok}\rightarrow B:1$, and $Y:\mathrm{ok}\rightarrow A:h$ can be respectively asserted in the following bases of $\mathcal{H}_{AB}$:

– The basis $X\text{-}Y=\left\{|\mathrm{fail}\rangle_X\,|\mathrm{fail}\rangle_Y,|\mathrm{fail}\rangle_X\,|\mathrm{ok}\rangle_Y,|\mathrm{ok}\rangle_X\,|\mathrm{fail}\rangle_Y,|\mathrm{ok}\rangle_X\,|\mathrm{ok}\rangle_Y\right\}$

– The basis $X\text{-}B=\left\{|\mathrm{fail}\rangle_X\,|0\rangle_B,|\mathrm{fail}\rangle_X\,|1\rangle_B,|\mathrm{ok}\rangle_X\,|0\rangle_B,|\mathrm{ok}\rangle_X\,|1\rangle_B\right\}$

– The basis $Y\text{-}A=\left\{|\mathrm{fail}\rangle_Y\,|h\rangle_A,|\mathrm{fail}\rangle_Y\,|t\rangle_A,|\mathrm{ok}\rangle_Y\,|h\rangle_A,|\mathrm{ok}\rangle_Y\,|t\rangle_A\right\}$

In order to simplify notation, let us express the two first bases as:

$$X\text{-}Y=\left\{|\mathrm{fail}_X,\mathrm{fail}_Y\rangle,|\mathrm{fail}_X,\mathrm{ok}_Y\rangle,|\mathrm{ok}_X,\mathrm{fail}_Y\rangle,|\mathrm{ok}_X,\mathrm{ok}_Y\rangle\right\} \tag{A-1}$$

$$X\text{-}B=\left\{|\mathrm{fail}_X,0_B\rangle,|\mathrm{fail}_X,1_B\rangle,|\mathrm{ok}_X,0_B\rangle,|\mathrm{ok}_X,1_B\rangle\right\} \tag{A-2}$$

Now let us define the observables O_{XY} and O_{XB}, associated with the bases *X*-*Y* and *X*-*B* respectively, as follows:

$$\begin{aligned}O_{XY}=&\alpha_1|\mathrm{fail}_X,\mathrm{fail}_Y\rangle\langle\mathrm{fail}_X,\mathrm{fail}_Y|+\alpha_2|\mathrm{fail}_X,\mathrm{ok}_Y\rangle\langle\mathrm{fail}_X,\mathrm{ok}_Y|+\\&+\alpha_3|\mathrm{ok}_X,\mathrm{fail}_Y\rangle\langle\mathrm{ok}_X,\mathrm{fail}_Y|+\alpha_4|\mathrm{ok}_X,\mathrm{ok}_Y\rangle\langle\mathrm{ok}_X,\mathrm{ok}_Y|\end{aligned} \tag{A-3}$$

$$\begin{aligned}O_{XB}=&\beta_1|\mathrm{fail}_X,0_B\rangle\langle\mathrm{fail}_X,0_B|+\beta_2|\mathrm{fail}_X,1_B\rangle\langle\mathrm{fail}_X,1_B|+\\&+\beta_3|\mathrm{ok}_X,0_B\rangle\langle\mathrm{ok}_X,0_B|+\beta_4|\mathrm{ok}_X,1_B\rangle\langle\mathrm{ok}_X,1_B|\end{aligned} \tag{A-4}$$

In order to know whether O_{XY} and O_{XB} commute, let us express O_{XB} (eq. (A-4)) in the basis *X*-*Y* (eq. A-1)):

$$\begin{aligned}O_{XB}=\frac{1}{2}\big[&(\beta_1+\beta_2)|\mathrm{fail}_X,\mathrm{fail}_Y\rangle\langle\mathrm{fail}_X,\mathrm{fail}_Y|+(\beta_1-\beta_2)|\mathrm{fail}_X,\mathrm{fail}_Y\rangle\langle\mathrm{fail}_X,\mathrm{ok}_Y|+\\&+(\beta_1-\beta_2)|\mathrm{fail}_X,\mathrm{ok}_Y\rangle\langle\mathrm{fail}_X,\mathrm{fail}_Y|+(\beta_1+\beta_2)|\mathrm{fail}_X,\mathrm{ok}_Y\rangle\langle\mathrm{fail}_X,\mathrm{ok}_Y|+\\&+(\beta_3+\beta_4)|\mathrm{ok}_X,\mathrm{fail}_Y\rangle\langle\mathrm{ok}_X,\mathrm{fail}_Y|+(\beta_3-\beta_4)|\mathrm{ok}_X,\mathrm{fail}_Y\rangle\langle\mathrm{ok}_X,\mathrm{ok}_Y|+\\&+(\beta_3-\beta_4)|\mathrm{ok}_X,\mathrm{ok}_Y\rangle\langle\mathrm{ok}_X,\mathrm{fail}_Y|+(\beta_3+\beta_4)|\mathrm{ok}_X,\mathrm{ok}_Y\rangle\langle\mathrm{ok}_X,\mathrm{ok}_Y|\end{aligned} \tag{A-5}$$

The commutator $[O_{XY}, O_{XB}]$ is zero if O_{XB} is also diagonal in the basis *X*-*Y*. As eq. (A-5) shows, this happens only when $\beta_1 = \beta_2$ and $\beta_3 = \beta_4$, that is, when O_{XB} is degenerate regarding its eigenvalues $\beta_1 = \beta_2$ and $\beta_3 = \beta_4$. This would imply that the propositions $X:\text{fail} \wedge B:0$ and $X:\text{fail} \wedge B:1$ would be indistinguishable since represented by the same number, and the same would happen with the $X:\text{ok} \wedge B:0$ and $X:\text{ok} \wedge B:1$. But this is contrary to the starting point of the F-R argument, which assumes that the observable *B* has two distinguishable values, represented by different eigenvalues of the corresponding operator. Therefore, since $\beta_1 \neq \beta_2$ and $\beta_3 \neq \beta_4$, then $[O_{XY}, O_{XB}] \neq 0$. Completely analogous arguments can be developed to prove that $[O_{XY}, O_{YA}] \neq 0$ and $[O_{XB}, O_{YA}] \neq 0$.

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