A Dichotomic Analysis of the Surprise Examination Paradox

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This paper proposes a new framework to solve the surprise examination paradox. I survey preliminary the main contributions to the literature related to the paradox. I introduce then a distinction between a monist and a dichotomic analysis of the paradox. With the help of a matrix notation, I also present a dichotomy that leads to distinguish two basically and structurally different notions of surprise, which are respectively based on a conjoint and a disjoint structure. I describe then how Quine's solution and Hall's reduction apply to the version of the paradox corresponding to the conjoint structure. Lastly, I expose a solution to the version of the paradox based on the disjoint structure.

I shall present in what follows a new conceptual framework to solve the *surprise examination paradox* (henceforth, SEP), in the sense that it reorganizes, by adapting them, several elements of solution described in the literature. The solution suggested here rests primarily on the following elements: (i) a distinction between a monist and a dichotomic analysis of the paradox; (ii) the introduction of a matrix definition, which is used as support with several variations of the paradox; (iii) the distinction between a conjoint and a disjoint definition of the cases of surprise and of non-surprise, leading to two structurally different notions of surprise.

In section 1, I proceed to describe the paradox and the main solutions found in the literature. I describe then in section 2, in a simplified way, the solution to the paradox which results from the present approach. I also introduce the distinction between a monist and a dichotomic analysis of the paradox. I present then a dichotomy which makes it possible to distinguish between two basically and structurally different versions of the paradox: on the one hand, a version based on a conjoint structure of the cases of non-surprise and of surprise; in the other hand, a version based on a disjoint structure. In section 3, I describe how Quine's solution and Hall's reduction apply to the version of SEP corresponding to the conjoint structure of the cases of non-surprise and of surprise. In section 4, I expose the solution to SEP corresponding to the disjoint structure. Lastly, I describe in section 5, within the framework of the present solution, what should have been the student's reasoning.

1. The paradox

The surprise examination paradox finds its origin in an actual fact. In 1943-1944, the Swedish authorities planned to carry out a civil defence exercise. They diffused then by the radio an announcement according to which a civil defence exercise would take place during the following week. However, in order to perform the latter exercise under optimal conditions, the announcement also specified that nobody could know in advance the date of the exercise. Mathematician Lennart Ekbom understood the subtle problem arising from this announcement of a civil defence exercise and exposed it to his students. A broad diffusion of the paradox throughout the world then ensued.

SEP first appeared in the literature with an article of D. O' Connor (1948). O' Connor presents the paradox under the form of the announcement of a military training exercise. Later on, SEP appeared in the literature under other forms, such as the announcement of the appearance of an ace in a set of cards (Scriven 1951) or else of a hanging (Quine 1953). However, the version of the paradox related to the professor's announcement of a surprise examination has remained the most current form. The traditional version of the paradox is as follows: a professor announces to his/her students that an examination will take place during the next week, but that they will not be able to know in advance the precise day where the examination will occur. The examination will thus occur surprisingly. The students reason as follows. The examination cannot take place on Saturday, they think, for otherwise they would know in advance that the examination would take place on Saturday and thus it could not occur surprisingly. Thus, Saturday is ruled out. Moreover, the examination cannot take place on Friday, for otherwise the students would know in advance that the examination would take place on Friday and thus it could not occur surprisingly. Thus, Friday is also ruled out. By a similar reasoning, the students eliminate successively Thursday, Wednesday, Tuesday and Monday. Finally, all days of the week are then ruled out. However, this does not prevent the examination from finally occurring surprisingly, say, on Wednesday. Thus, the students' reasoning proved to be fallacious. However, such a reasoning appears intuitively valid. The paradox lies here in the fact that the students' reasoning seems valid, whereas it finally proves to be in contradiction with the facts, namely that the examination can truly occur surprisingly, in accordance with the announcement made by the professor.

In the literature, several solutions to SEP have been proposed. There does not exist however, at present time, a consensual solution. I will briefly mention the principal solutions which were proposed, as well as the fundamental objections that they raised.

A first attempt at solution appeared with O' Connor (1948). This author pointed out that the paradox was due to the contradiction which resulted from the professor's announcement and the implementation of the latter. According to O' Connor, the professor's announcement according to which the examination was to occur by surprise was in contradiction with the fact that the details of the implementation of the examination were known. Thus, the statement of SEP was, according to O' Connor, self-refuting. However, such an analysis proved to be inadequate, because it finally appeared that the examination could truly take place under some conditions where it occurred surprisingly, for example on Wednesday. Thus, the examination could finally occur by surprise, confirming thus and not refuting, the professor's announcement. This last observation had the effect of making the paradox re-appear.

Quine (1953) also proposed a solution to SEP. Quine considers thus the student's final conclusion according to which the examination can occur surprisingly on no day of the week. According to Quine, the student's error lies in the fact of having not considered from the beginning the hypothesis that the examination could not take place on the last day. For the fact of considering precisely that the examination will not take place on the last day makes it finally possible for the examination to occur surprisingly, on the last day. If the student had also taken into account this possibility from the beginning, he would not concluded fallaciously that the examination cannot occur by surprise. However, Quine's solution has led to criticisms, emanating notably from commentators (Ayer 1973, Janaway 1989 and also Hall 1999) who stressed the fact that Quine's solution did not make it possible to handle several variations of the paradox. Ayer imagines thus a version of SEP where a given person is informed that the cards of a set will be turned over one by one, but where that person will not know in advance when the ace of Spades will be issued. Nevertheless, the person is authorized to check the presence of the ace of Spades before the set of cards is mixed. The purpose of the objection to Quine's solution based on such a variation is to highlight a situation where the paradox is quite present but where Quine's solution does not find to apply any more, because the student knows with certainty, given the initial data of the problem, that the examination will take place as well.

According to another approach, defended in particular by R. Shaw (1958), the structure of the paradox is inherently self-referential. According to Shaw, the fact that the examination must occur by surprise is tantamount to the fact that the date of the examination cannot be deduced in advance. But the fact that the students cannot know in advance, by deduction, the date of the examination constitutes precisely one of the premises. The paradox thus finds its origin, according to Shaw, in the fact that the structure of the professor's announcement is self-referential. According to the author, the self-reference which results from it constitutes thus the cause of the paradox. However, such an analysis did not prove to be convincing, for it did not make it possible to do justice to the fact that in spite of its self-referential structure, the professor's announcement was finally confirmed by the fact that the examination could finally occur surprisingly, say on Wednesday.

Another approach, put forth by Richard Montague and David Kaplan (1960) is based on the analysis of the structure of SEP which proves, according to these authors, to be that of the *paradox of the Knower*. The latter paradox constitutes a variation of the *Liar paradox*. What thus ultimately proposes Montague and Kaplan, is a reduction of SEP to the Liar paradox. However, this last approach did not prove to be convincing. Indeed, it was criticized because it did not take account, on the one hand, the fact that the professor's announcement can be finally confirmed and on the other hand, the fact that one can formulate the paradox in a non-self-referential way.

It is also worth mentioning the analysis developed by Robert Binkley (1968). In his article, Binkley exposes a reduction of SEP to Moore's paradox. The author makes the point that on the last day, SEP reduces to a variation of the proposition 'P and I don't know that P' which constitutes Moore's paradox. Binkley extends then his analysis concerning the last day to the other days of the week. However, this approach has led to strong objections, resulting in particular from the analysis of Wright and Sudbury (1977).

Another approach also deserves to be mentioned. It is the one developed by Paul Dietl (1973) and Joseph Smith (1984). According to these authors, the structure of SEP is that of the *sorites paradox*. What then propose Dietl and Smith, is a reduction of SEP to the sorites paradox. However, such an analysis met serious objections, raised in particular by Roy Sorensen (1988).

It is worth lastly mentioning the approach presented by Crispin Wright and Aidan Sudbury (1977). The analysis developed by these authors[1] results in distinguishing two cases: on the one hand, on the last day, the student is in a situation which is that which results from Moore's paradox; in addition, on the first day, the student is in a basically different situation where he can validly believe in the professor's announcement. Thus, the description of these two types of situations leads to the rejection of the *principle of temporal retention*. According to this last principle, what is known at a temporal position T_0 is also known at a later temporal position T_1 (with $T_0 < T_1$). However, the analysis of Wright and Sudbury appeared vulnerable to an argument developed by Sorensen (1982). The latter author presented indeed a version of SEP (the *Designated Student Paradox*) which did not rely on the principle of temporal retention, on which the approach of Wright and Sudbury rested. According to Sorensen's variation, the paradox was quite present, but without the conditions of its statement requiring to rely on the principle of temporal retention. Sorensen describes thus the following variation of the paradox. Five students, A, B, C, D and E are placed, in this order, one behind the other. The professor then shows to the students four silver stars and one gold star. Then he places a star on the back of each student. Lastly, he announces to them that the one of them who has a gold star in the back has been designated to pass an examination. But, the professor adds, this examination will constitute a surprise, because the students will only know that who was designated when they break their alignment. Under these conditions, it appears that the students can implement a similar reasoning to that which prevails in the original version of SEP. But this last version is diachronic, whereas the variation described by Sorensen appears, by contrast, synchronic. And as such, it is thus not based on whatever principle of temporal retention.

Given the above elements, it appears that the stake and the philosophical implications of SEP are of importance. They are located at several levels and thus relate[2] to the theory of knowledge, deduction, justification, the semantic paradoxes, self-reference, modal logic, and vague concepts.

2. Monist or dichotomic analysis of the paradox

Most analyses classically proposed to solve SEP are based on an overall solution which applies, in a general way, to the situation which is that of SEP. In this type of analysis, a single solution is presented, which is supposed to apply to all variations of SEP. Such type of solution has a unitary nature and appears based on what can be termed a *monist* theory of SEP. Most solutions to SEP proposed in the literature are monist analyses. Characteristic examples of this type of analysis of SEP are the solutions suggested by Quine (1953) or Binkley (1968). In a similar way, the solution envisaged by Dietl (1973) which is based on a reduction of SEP to the sorite paradox also constitutes a monist solution to SEP.

Conversely, a dichotomic analysis of SEP is based on a distinction between two different scenarios of SEP and on the formulation of an *independent* solution for each of the two scenarios. In the literature, the only analysis which has a dichotomic nature, as far as I know, is that of Wright and Sudbury mentioned above. In what follows, I will present a dichotomic solution to SEP. This solution is based on the distinction of two variations of SEP, associated with concepts of surprise that correspond to different structures of the cases of non-surprise and of surprise.

At this step, it proves to be useful to introduce the matrix notation. With the help of this latter, the various cases of non-surprise and of surprise be modelled with the following S[k, s] table, where k denotes the day where the examination takes place and S[k, s] denotes if the corresponding case of non-surprise (s = 0) or of surprise (s = 1) is made possible (S[k, s] = 1) or not (S[k, s] = 0) by the conditions of the announcement (with $1 \leq k \leq n$).[3] If one considers for example 7-SEP [4], S[7, 1] = 0 denotes the fact that the surprise is not possible on the 7th day, and conversely, S[7, 1] = 1 denotes the fact that the surprise is possible on the 7th day; in the same way, S[1, 0] = 0 denotes the fact that the non-surprise is not possible on the 1st day by the conditions of

[1] I simplify here considerably.

[2] Without pretending to being exhaustive.

[3] In what follows, n denotes the last day of the term corresponding to the professor's announcement.

[4] Let 1-SEP, 2-SEP,..., n-SEP be the problem for respectively 1 day, 2 days,..., n days.

the announcement, and conversely, S[1, 0] = 1 denotes the fact that the non-surprise is possible on the 1st day.

The dichotomy on which rests the present solution results directly from the analysis of the structure which makes it possible to describe the concept of surprise corresponding to the statement of SEP. Let us consider first the following matrix, which corresponds to a *maximal* definition, where all cases of non-surprise and of surprise are made possible by the professor's announcement (with ■ = 1 and □ = 0):

(D1)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	■
S[6,*s*]	■	■
S[5,*s*]	■	■
S[4,*s*]	■	■
S[3,*s*]	■	■
S[2,*s*]	■	■
S[1,*s*]	■	■

At the level of (D1), as we can see it, all values of the S[*k*, *s*] matrix are equal to 1, which corresponds to the fact that all the cases of non-surprise and of surprise are made possible by the corresponding version of SEP. The associated matrix can be thus defined as a *rectangular* matrix.

At this stage, it appears that one can conceive of some variations of SEP associated with more restrictive matrix structures, where certain cases of non-surprise and of surprise are not authorized by the announcement. In such cases, certain values of the matrix are equal to 0. It is now worth considering the structure of these more restrictive definitions. The latter are such that it exists at least one case of non-surprise or of surprise which is made impossible by the announcement, and where the corresponding value of the matrix S[*k*, *s*] is thus equal to 0. Such a condition leaves place [***room] with a certain number of variations, of which it is now worth studying the characteristics more thoroughly.

One can notice preliminarily that certain types of structures can be discarded from the beginning. It appears indeed that any definition associated with a restriction of (D1) is not adequate. Thus, there are minimal conditions for the emergence of SEP. In this sense, a first condition is that the *base step* be present. This base step is such that the non-surprise must be able to occur on the last day, that is to say S[*n*, 0] = 1. With the previously defined notation, it presents the general form *n***n** and corresponds to 7*7* for 7-SEP. In the lack of this base step, there is no paradoxical effect of SEP. Consequently, a structure of matrix such as S[*n*, 0] = 0 can be discarded from the beginning.

One second condition so that the statement leads to a genuine version of SEP is that the examination can finally occur surprisingly. This renders indeed possible the fact that the professor's announcement can be finally satisfied. Such a condition – let us call it the *vindication step* – is classically mentioned as a condition for the emergence of the paradox. Thus, a definition which would be such that all the cases of surprise are made impossible by the corresponding statement would also not be appropriate. Thus, the structure corresponding to the following matrix would not correspond either to a licit statement of SEP:

(D2)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	□
S[6,*s*]	■	□
S[5,*s*]	■	□
S[4,*s*]	■	□
S[3,*s*]	■	□
S[2,*s*]	■	□
S[1,*s*]	■	□

because the surprise is possible here on no day of the week (S[*k*, 1] = 0) and the validation step is thus lacking in the corresponding statement.

Taking into account what precedes, one is now in a position to describe accurately the minimal conditions which are those of SEP:

(C3) S[*n*, 0] = 1 (*base step*)
(C4) ∃*k* (1 ≤ *k* ≤ *n*) such that S[*k*, 1] = 1 (*validation step*)

At this step, it is worth considering the structure of the versions of SEP based on the definitions which satisfy the minimal conditions for the emergence of the paradox which have just been described, i.e. which contain at the same time the basic step and the validation step. It appears here that the structure associated with the cases of non-surprise and of surprise corresponding to a variation with SEP can present two forms of a basically different nature. A first form of SEP is associated with a structure where the possible cases of non-surprise and of surprise are such that it exists during the *n*-period at least one day where the non-surprise and the surprise are simultaneously possible. Such a definition can be called *conjoint*. The following matrix constitutes an example of this type of structure:

(D5)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	■
S[6,*s*]	■	■
S[5,*s*]	■	■
S[4,*s*]	■	■
S[3,*s*]	□	■
S[2,*s*]	□	■
S[1,*s*]	□	■

because the non-surprise and the surprise are simultaneously possible here on the 7th, 6th, 5th and 4th days. However, it proves that one can also encounter a second form of SEP the structure of which is basically different, in the sense that for each day of the *n*-period, it is impossible to have simultaneously the surprise and the non-surprise.[5] A definition of this nature can be called *disjoint*. The following matrix thus constitutes an example of this type of structure:

(D6)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	□
S[6,*s*]	■	□
S[5,*s*]	■	□
S[4,*s*]	□	■
S[3,*s*]	□	■
S[2,*s*]	□	■
S[1,*s*]	□	■

Consequently, it is worth distinguishing in what follows two structurally distinct versions of SEP: (a) a version based on a *conjoint* structure of the cases of non-surprise and of surprise made possible by the announcement; (b) a version based on a *disjoint* structure of these same cases. The need for making such a dichotomy finds its legitimacy in the fact that in the original version of SEP, the professor does not specify if one must take into account a concept of surprise corresponding to a *disjoint* or a *conjoint* structure of the cases of non-surprise and of surprise. With regard to this particular point, the professor's announcement of SEP appears ambiguous. Consequently, it is necessary to consider successively two different concepts of surprise, respectively based on a disjoint or conjoint structure of the cases of non-surprise and of surprise, as well as the reasoning which must be associated with them.

3. The surprise notion corresponding to the conjoint structure

Let us consider first the case where SEP is based on a concept of surprise corresponding to a conjoint structure of the cases of non-surprise and of surprise. Let SEP(I) be the version associated with such a concept of surprise. Intuitively, this version corresponds to a situation where there exists in the *n*-period at least one day where the non-surprise and the surprise can occur at the same time. Several types of definitions are likely to satisfy this criterion. It is worth considering them in turn.

4.1 The definition associated with the rectangular matrix and Quine's solution

To begin with, it is worth considering the structures which are such that all cases of non-surprise and of surprise are made possible by the statement. The corresponding matrix is a *rectangular* matrix. Let thus SEP(I□) be such a version. The definition associated with such a structure is maximal since all cases of non-surprise and of surprise are authorized. The following matrix corresponds thus to such a general structure:

[5] The cases where neither the non-surprise nor the surprise are made possible on the same day (i.e. such that S[*k*, 0] + S[*k*, 1] = 0) can be purely and simply ignored.

(D7)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	■
S[6,*s*]	■	■
S[5,*s*]	■	■
S[4,*s*]	■	■
S[3,*s*]	■	■
S[2,*s*]	■	■
S[1,*s*]	■	■

and the associated professor’s announcement is the following:

(S7) An examination will occur in the next week but the date of the examination will constitute a surprise.

At this step, it appears that we also get a version of SEP for *n* = 1 which satisfies this definition. The structure associated with 1-SEP(I□) is as follows:

(D8)

	S[1, 0]	S[1, 1]
S[1,*s*]	■	■

which corresponds to the following professor’s announcement:

(S8) An examination will occur on tomorrow but the date of the examination will constitute a surprise.

Thus, 1-SEP(I□) is the minimal version of SEP which satisfies not only the above condition, but also the base step (C3) according to which the non-surprise must possibly occur on the last day, as well as the validation step (C4) in virtue of which the examination can finally occur by surprise. Moreover, it is a variation which excludes, by its intrinsic structure, the emergence of the version of SEP based on a concept of surprise corresponding to a disjoint structure. For this reason, (D8) can be regarded as the canonical form of SEP(I□). Thus, it is the genuine core of SEP(I□) and in what follows, we will thus endeavour to reason on 1-SEP(I□).

At this stage, it is worth attempting to provide a solution to SEP(I□). For that purpose, let us recall first Quine’s solution. The solution to SEP proposed by Quine (1953) is well-known. Quine highlights the fact that the student eliminates successively the days *n*, *n* -1..., 1, by a reasoning based on *backward-induction* and concludes then that the examination will not take place during the week. The student reasons as follows. On day *n*, I will predict that the examination will take place on day *n*, and consequently the examination cannot take place on day *n*; on day *n* -1, I will predict that the examination will take place on day *n*-1, and consequently the examination cannot take place on day *n* -1; ...; on day 1, I will predict that the examination will take place on day 1, and consequently the examination cannot take place on day 1. Finally, the student concludes that the examination will take place on no day of the week. But this last conclusion finally makes it possible to the examination to occur surprisingly, including on day *n*. According to Quine, the error in the student’s reasoning lies precisely in the fact of not having taken into account this possibility since the beginning, which would then have prevented the fallacious reasoning.[6]

Quine, in addition, directly applies his analysis to the canonical form 1-SEP(I□), where the corresponding statement is that of (S8). In this case, the error of the student lies, according to Quine, in the fact of having considered only the single following assumption: (a) "the examination will take place tomorrow and I will predict that it will take place". In fact, the student should have also considered three other cases: (b) "the examination will not take place tomorrow and I will predict that it will take place"; (c) "the examination will not take place tomorrow and I will not predict that it will take place"; (d) "the examination will take place tomorrow and I will not predict that it will take place". And the fact of considering the assumption (a) but also the assumption (d) which is compatible with the professor’s announcement would have prevented the student from concluding that the examination would not finally take place.[7] Consequently, it is the fact of

[6] Cf. (1953, 65) : 'It is notable that *K* acquiesces in the conclusion (wrong, according to the fable of the Thursday hanging) that the decree will not be fulfilled. If this is a conclusion which he is prepared to accept (though wrongly) in the end as a certainty, it is an alternative which he should have been prepared to take into consideration from the beginning as a possibility.'

[7] Cf. (1953, 66) : 'If *K* had reasoned correctly, Sunday afternoon, he would have reasoned as follows : "We must distinguish four cases : first, that I shall be hanged tomorrow noon and I know it now (but I do not) ; second, that I shall be unhanged tomorrow noon and do not know it now (but I do not) ; third, that I shall be unhanged tomorrow noon and know it now ; and fourth, that I shall be hanged tomorrow noon and do not know it now. The latter two alternatives are

having taken into account only the hypothesis (a) which can be identified as the cause of the fallacious reasoning. Thus, the student did only take partially into account the whole set of hypotheses resulting from the professor's announcement. If he had apprehended the totality of the relevant hypotheses compatible with the professor's announcement, he would not have concluded fallaciously that the examination would not take place during the week.

At this stage, it proves to be useful to describe the student's reasoning in terms of reconstitution of a matrix. For one can consider that the student's reasoning classically based on backward-induction leads to reconstitute the matrix corresponding to the concept of surprise in the following way:

(D9)

	S[1, 0]	S[1, 1]
S[1,*s*]	■	□

In reality, he should have considered that the correct way to reconstitute this latter matrix is the following :

(D8)

	S[1, 0]	S[1, 1]
S[1,*s*]	■	■

4.2 The definition associated with the triangular matrix and Hall's reduction

As we have seen, Quine's solution applies directly to SEP(I□), i.e. to a version of SEP based on a conjoint definition of the surprise and a rectangular matrix. It is now worth being interested in some variations of SEP based on a conjoint definition where the structure of the corresponding matrix is not rectangular, but which satisfies however the conditions for the emergence of the paradox mentioned above, namely the presence of the base step (C3) and the validation step (C4). Such matrices have a structure that can be described as *triangular*. Let thus SEP(IΔ) be the corresponding version.

Let us consider first 7-SEP, where the structure of the possible cases of non-surprise and of surprise corresponds to the matrix below:

(D10)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	□
S[6,*s*]	■	■
S[5,*s*]	■	■
S[4,*s*]	■	■
S[3,*s*]	■	■
S[2,*s*]	■	■
S[1,*s*]	■	■

and to the following announcement of the professor

(S10) An examination will occur in the next week but the date of the examination will constitute a surprise. Moreover, the fact that the examination will take place constitutes an absolute certainty.

Such an announcement appears identical to the preceding statement to which the Quine's solution applies, with however an important difference: the student has from now on the certainty that the examination will occur. And this has the effect of preventing him/her from questioning the fact that the examination can take place, and of making thus impossible the surprise to occur on the last day. For this reason, we note S[7, 1] = 0 in the corresponding matrix. The general structure corresponding to this type of definition is:

(D11)

	S[*k*, 0]	S[*k*, 1]
S[*n*,*s*]	■	□
S[*n*-1,*s*]	■	■
....	....	....

And similarly, one can consider the following canonical structure (from where the denomination of *triangular* structure finds its justification), which is that of SEP(IΔ) and which corresponds thus to 2-SEP(IΔ):

the open possibilities, and the last of all would fulfill the decree. Rather than charging the judge with self-contradiction, let me suspend judgment and hope for the best."'

(D12)	S[k, 0]	S[k, 1]
S[2,s]	■	□
S[1,s]	■	■

Such a structure corresponds to the following announcement of the professor:

(S12) An examination will occur on the next two days, but the date of the examination will constitute a surprise. Moreover, the fact that the examination will take place constitutes an absolute certainty.

As we see it, the additional clause of the statement according to which it is absolutely certain that the examination will occur prevents here the surprise of occurring on the last day. Such a version corresponds in particular to the variation of SEP described by A. J. Ayer. The latter version corresponds to a player, who is authorized to check, before a set of playing cards is mixed, that it contains the ace, the 2, 3..., 7 of Spades. And it is announced that the player that he will not be able to envisage in advance justifiably, when the ace of Spades will be uncovered. Finally the cards, initially hidden, are uncovered one by one. The purpose of such a version is to render impossible, before the 7th card being uncovered, the belief according to which the ace of Spades will not be uncovered. And this has the effect of forbidding to Quine' solution to apply on the last day.

It is now worth presenting a solution to the versions of SEP associated with the structures corresponding to (D11). Such a solution is based on a reduction recently exposed by Ned Hall, of which it is worth beforehand highlighting the context. In the version of SEP under consideration by Quine (1953), it appears clearly that the fact that the student doubts that the examination will well take place during the week, at a certain stage of the reasoning, is authorized. Quine thus places himself deliberately in a situation where the student has the faculty of doubting that the examination will truly occur during the week. The versions described by Ayer (1973), Janaway (1989) but also Scriven (1951) reveal the intention to prevent this particular step in the student's reasoning. Such scenarios correspond, in spirit, to SEP(IΔ). One can also attach to it the variation of the *Designated Student Paradox* described by Sorensen (1982, 357)[8], where five stars – a gold star and four silver stars – are attributed to five students, given that it is indubitable that the gold star is placed on the back of the student who was designated.

However, Ned Hall (1999, 659-660) recently exposed a reduction, which tends to refute the objections classically raised against Quine's solution. The argumentation developed by Hall is as follows:

> We should pause, briefly, to dispense with a bad – though oft-cited – reason for rejecting Quine's diagnosis. (See for example Ayer 1973 and Janaway 1989). Begin with the perfectly sound observation that the story can be told in such a way that the student *is* justified in believing that, come Friday, he will justifiably believe that an exam is scheduled for the week. Just add a second *Iron Law of the School* : that there must be at least one exam each week. (...) Then the first step of the student's argument goes through just fine. So Quine's diagnosis is, evidently, inapplicable.
>
> Perhaps – but in letter only, not in spirit. With the second Iron Law in place, the last disjunct of the professor's announcement – that $E_5 \,\&\, \neg J(E_5)$ – is, from the student's perspective, a *contradiction*. So, from his perspective, the *content* of her announcement is given not by SE_5 but by SE_4 : $(E_1 \,\&\, \neg J_1(E_1)) \vee \ldots \vee (E_4 \,\&\, \neg J_4(E_4))$. And now Quine's diagnosis applies straightforwardly : he should simply insist that the student is not justified in believing the announcement and so, come Thursday morning, not justified in believing that crucial part of it which asserts that if the exam is on Friday then it will come as a surprise – which, from the student's perspective, is tantamount to asserting that the exam is scheduled for one of Monday through Thursday. That is, Quine should insist that the crucial premise that $J_4(E_1 \vee E_2 \vee E_3 \vee E_4)$ is *false* – which is exactly the diagnosis he gives to an ordinary 4-day surprise exam scenario. Oddly, it seems to have gone entirely unnoticed by those who press this variant of the story against Quine that its only real effect is to convert an n-day scenario into an n-1 day scenario.

Hall puts then in parallel two types of situations. The first corresponds to the situation in which Quine's analysis finds classically to apply. The second corresponds to the type of situation under consideration by the opponents to Quine's solution and in particular by Ayer (1973) and Janaway (1989). On this last hypothesis, a stronger version of SEP is taken into account, where one second *Iron Law of the School* is considered and it is given that the examination will necessarily take place during the week. The argumentation developed by Hall leads to the *reduction* of a version of n-SEP of the second type to a version of (n-1)-SEP of the quinean type. This equivalence has the effect of annihilating the objections of the opponents to Quine's solution.[9] For the effect of this reduction is to make it finally possible to Quine's solution to apply in the situations described by Ayer and Janaway. In spirit, the scenario under consideration by Ayer and Janaway corresponds

[8] 'The students are then shown four silver stars and one gold star. One star is put on the back of each student.'.

[9] Hall refutes otherwise, but on different grounds, the solution proposed by Quine.

thus to a situation where the surprise is not possible on day n (i.e. S[n, 1] = 0). This has indeed the effect of neutralizing Quine's solution based on n-SEP(I□). But Hall's reduction then makes it possible to Quine's solution to apply to (n-1)-SEP(I□). The effect of Hall's reduction is thus of reducing a scenario corresponding to (D11) to a situation based on (D8). Consequently, Hall's reduction makes it possible to reduce n-SEP(IΔ) to (n-1)-SEP(I□). It results from it that any version of SEP(IΔ) for one n-period reduces to a version of SEP(I□) for one (n-1)-period (formally n-SEP(IΔ) ≡ (n-1)-SEP(I□) for $n > 1$). Thus, Hall's reduction makes it finally possible to apply Quine's solution to SEP(IΔ).[10]

4. The surprise notion corresponding to the disjoint structure

It is worth considering, second, the case where the notion of surprise is based on a *disjoint* structure of the possible cases of non-surprise and of surprise. Let SEP(II) be the corresponding version. Intuitively, such a variation corresponds to a situation where for a given day of the n-period, it is not possible to have at the same time the non-surprise and the surprise. The structure of the associated matrix is such that one has exclusively on each day, either the non-surprise or the surprise.

At this step, it appears that a preliminary question can be raised: can't Quine's solution apply all the same to SEP(II)? However, the preceding analysis of SEP(I) shows that a necessary condition in order to Quine's solution to apply is that there exists during the n-period at least one day when the non-surprise and the surprise are at the same time possible. However such a property is that of a *conjoint* structure and corresponds to the situation which is that of SEP(I). But in the context of a *disjoint* structure, the associated matrix, in contrast, verifies $\forall k$ S[k, 0] + S[k, 1] = 1. Consequently, this forbids Quine's solution to apply to SEP(II).

In the same way, one could wonder whether Hall's reduction wouldn't also apply to SEP(II). Thus, isn't there a reduction of SEP(II) for a n-period to SEP(I) for a (n - 1)-period? It also appears that not. Indeed, as we did see it, Quine's solution cannot apply to SEP(II). However, the effect of Hall's reduction is to reduce a given scenario to a situation where Quine's solution finally finds to apply. But, since Quine's solution cannot apply in the context of SEP(II), Hall's reduction is also unable to produce its effect.

Given that Quine's solution does not apply to SEP(II), it is now worth attempting to provide an adequate solution to the version of SEP corresponding to a concept of surprise associated with a disjoint structure of the cases of non-surprise and of surprise. To this end, it proves to be necessary to describe a version of SEP corresponding to a disjoint structure, as well as the structure corresponding to the canonical version of SEP(II).

In a preliminary way, one can observe that the minimal version corresponding to a disjoint version of SEP is that which is associated with the following structure, i.e. 2-SEP(II):

(D13)

	S[1, 0]	S[1, 1]
S[2,s]	■	□
S[1,s]	□	■

However, for reasons that will become clearer later, the corresponding version of SEP(II) does not have a sufficient degree of realism and of plausibility to constitute a genuine version of SEP, i.e. such that it is susceptible of inducing in error our reasoning.

In order to highlight the canonical version of SEP(II) and the corresponding statement, it is first of all worth mentioning the remark, made by several authors[11], according to which the paradox emerges clearly, in the case of SEP(II), when n is large. An interesting characteristic of SEP(II) is indeed that the paradox emerges intuitively in a clearer way when great values of n are taken into account. A striking illustration of this phenomenon is thus provided to us by the variation of the paradox which corresponds to the following situation, described by Timothy Williamson (2000, 139):

> Advance knowledge that there will be a test, fire drill, or the like of which one will not know the time in advance is an everyday fact of social life, but one denied by a surprising proportion of early work on the Surprise

[10] Hall's reduction can be easily generalised. It is then associated with a version of n-SEP(IΔ) such that the surprise will not possibly occur on the m last days of the week. Such a version is associated with a matrix such that (a) $\exists m$ ($1 \leq m < n$) and S[n-m, 0] = S[n-m, 1] = 1 ; (b) $\forall p > n\text{-}m$ S[p, 0] = 1 and S[p, 1] = 0 ; (c) $\forall q < n\text{-}m$ S[q, 0] = S[q, 1] = 1. In this new situation, a *generalised Hall's reduction* applies to the corresponding version of SEP. In this case, the extended Hall's reduction leads to : n-SEP(IΔ) ≡ (n-m)-SEP(I□).

[11] Cf. notably Hall (1999, 661), Williamson (2000).

Examination. Who has not waited for the telephone to ring, knowing that it will do so within a week and that one will not know a second before it rings that it will ring a second later?

The variation suggested by Williamson corresponds to the announcement made to somebody that he will receive a phone call during the week, without being able however to determine in advance at which precise second the phone call will occur. This variation underlines how the surprise can appear, in a completely plausible way, when the value of *n* is high. The unit of time considered by Williamson is here the second, associated with a period which corresponds to one week. The corresponding value of *n* is here very high and equals 604800 (60 x 60 x 24 x 7) seconds. This illustrates how a great value of *n* makes it possible to the corresponding variation of SEP(II) to take place in both a plausible and realistic way. However, taking into account such large value of *n* is not indeed essential. In effect, a value of *n* which equals, for example, 365, seems appropriate as well. In this context, the professor's announcement which corresponds to a disjoint structure is then the following:

(S14) An examination will occur during this year but the date of the examination will constitute a surprise.

The corresponding definition presents then the following structure :

(D14)

	S[1, 0]	S[1, 1]
S[365,*s*]	■	□
....	....	....
S[1,*s*]	□	■

which is an instance of the following general form :

(D15)

	S[1, 0]	S[1, 1]
S[*n*,*s*]	■	□
....	....	....
S[1,*s*]	□	■

This last structure can be considered as corresponding to the canonical version of SEP(II), with *n* large. In the specific situation associated with this version of SEP, the student predicts each day – in a false way but justified by a reasoning based on backward-induction – that the examination will take place on no day of the week. But it appears that at least one case of surprise (for example if the examination occurs on the first day) makes it possible to validate, in a completely realistic way, the professor's announcement..

The form of SEP(II) which applies to the standard version of SEP is 7-SEP(II), which corresponds to the classical announcement:

(S7) An examination will occur on the next week but the date of the examination will constitute a surprise.

but with this difference with the standard version that the context is here exclusively that of a concept of surprised associated with a disjoint structure.

At this stage, we are in a position to determine the fallacious step in the student's reasoning. For that, it is useful to describe the student's reasoning in terms of matrix reconstitution. The student's reasoning indeed leads him/her to attribute a value for S[*k*, 0] and S[*k*, 1]. And when he is informed of the professor's announcement, the student's reasoning indeed leads him/her to rebuild the corresponding matrix such that all S[*k*, 0] = 1 and all S[*k*, 1] = 0, in the following way (for *n* = 7):

(D16)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	□
S[6,*s*]	■	□
S[5,*s*]	■	□
S[4,*s*]	■	□
S[3,*s*]	■	□
S[2,*s*]	■	□
S[1,*s*]	■	□

One can notice here that the order of reconstitution proves to be indifferent. At this stage, we are in a position to identify the flaw which is at the origin of the erroneous conclusion of the student. It appears indeed that the student did not take into account the fact that the surprise corresponds here to a disjoint structure. Indeed, he should have considered here that the last day corresponds to a proper instance of non-surprise and thus that S[*n*, 0] = 1. In the same way, he should have considered that the 1st day[12] corresponds to a proper instance of surprise and should have thus posed S[1, 1] = 1. The context being that of a disjoint structure, he could have legitimately added, in a second step, that S[*n*, 1] = 0 and S[1, 0] = 0. At this stage, the partially reconstituted matrix would then have been as follows:

(D17)	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	□
S[6,*s*]		
S[5,*s*]		
S[4,*s*]		
S[3,*s*]		
S[2,*s*]		
S[1,*s*]	□	■

The student should then have continued his reasoning as follows. The proper instances of non-surprise and of surprise which are disjoint here do not capture entirely the concept of surprise. In such context, the concept of surprise is not captured exhaustively by the extension and the anti-extension of the surprise. However, such a definition is in conformity with the definition of a vague predicate, which characterizes itself by an extension and an anti-extension which are mutually exclusive and non-exhaustive[13]. Thus, the surprise notion associated with a disjoint structure is a *vague* one.

What precedes now makes it possible to identify accurately the flaw in the student's reasoning, when the surprise notion is a vague notion associated with a disjoint structure. For the error which is at the origin of the student's fallacious reasoning lies in lack of taking into account the fact that the surprise corresponds in the case of a disjoint structure, to a vague concept, and thus comprises the presence of a penumbral zone corresponding to *borderline* cases between the non-surprise and the surprise. There is no need however to have here at our disposal a solution to the sorites paradox. Indeed, whether these borderline cases result from a succession of intermediate degrees, from a precise cut-off between the non-surprise and the surprise whose exact location is impossible for us to know, etc. is of little importance here. For in all cases, the mere fact of taking into account the fact that the concept of surprise is here a concept vague forbids to conclude that S[*k*, 1] = 0, for all values of *k*.

Several ways thus exist to reconstitute the matrix in accordance with what precedes. In fact, there exists as many ways of reconstituting the latter than there are conceptions of vagueness. One in these ways (based on a conception of vagueness based on fuzzy logic) consists in considering that there exists a continuous and gradual succession from the non-surprise to the surprise. The corresponding algorithm to reconstitute the matrix is then the one where the *step* is given by the formula 1/(*n*-*p*) when *p* corresponds to a proper instance of surprise. For *p* = 3, we have here 1/(7-3) = 0,25, with S[3, 1] = 1. And the corresponding matrix is thus the following one:

(D18)	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	1	0
S[6,*s*]	0,75	0,25
S[5,*s*]	0,5	0,5
S[4,*s*]	0,25	0,75
S[3,*s*]	0	1
S[2,*s*]	0	1
S[1,*s*]	0	1

where the sum of the values of the matrix associated with a day given is equal to 1. The intuition which governs SEP (II) is here that the non-surprise is total on day *n*, but that there exists intermediate degrees of surprise s_i ($0 < s_i < 1$), such as the more one approaches the last day, the higher the effect of non-surprise. Conversely, the effect of surprise is total on the first days, for example on days 1, 2 and 3.

[12] It is just an example. Alternatively, one could have chosen here the 2nd or the 3rd day.

[13] This definition of a vague predicate is borrowed from Soames. Considering the extension and the anti-extension of a vague predicate, Soames (1999, 210) points out thus: "These two classes are mutually exclusive, though not jointly exhaustive".

One can notice here that the definitions corresponding to SEP (II) which have just been described, are such that they present a property of *linearity* (formally, $\forall k$ (for $1 < k \leq n$), $S[k, 0] \geq S[k\text{-}1, 0]$). It appears indeed that a structure corresponding to the possible cases of non-surprise and of surprise which would not present such a property of linearity, would not capture the intuition corresponding to the concept of surprise. For this reason, it appears sufficient to limit the present study to the structures of definitions that satisfy this property of linearity.

An alternative way to reconstitute the corresponding matrix, based on the epistemological conception of vagueness, could also have been used. It consists of the case where the vague nature of the surprise is determined by the existence of a precise cut-off between the cases of non-surprise and of surprise, of which it is however not possible for us to know the exact location. In this case, the matrix could have been reconstituted, for example, as follows:

(D19)

	S[*k*, 0]	S[*k*, 1]
S[7,*s*]	■	□
S[6,*s*]	■	□
S[5,*s*]	■	□
S[4,*s*]	□	■
S[3,*s*]	□	■
S[2,*s*]	□	■
S[1,*s*]	□	■

At this stage, one can wonder whether the version of the paradox associated with SEP(II) cannot be assimilated with the *sorites paradox*. The reduction of SEP to the sorites paradox is indeed the solution which has been proposed by some authors, notably Dietl (1973) and Smith (1984). The latter solutions, based on the assimilation of SEP to the sorites paradox, constitute monist analyses, which do not lead, to the difference of the present solution, to two independent solutions based on two structurally different versions of SEP. In addition, with regard to the analyses suggested by Dietl and Smith, it does not clearly appear whether each step of SEP is fully comparable to the corresponding step of the sorites paradox, as underlined by Sorensen.[14] But in the context of a conception of surprise corresponding to a disjoint structure, the fact that the last day corresponds to a proper instance of non-surprise can be assimilated here to the base step of the sorites paradox.

Nevertheless, it appears that such a reduction of SEP to the sorites paradox, limited to the notion of surprise corresponding to a disjoint structure, does not prevail here. On the one hand, it does not appear clearly if the statement of SEP can be translated into a variation of the sorites paradox, in particular for what concerns 7-SEP(II). Because the corresponding variation of the sorites paradox would run too fast, as already noted by Sorensen (1988).[15] It is also noticeable, moreover, as pointed out by Scott Soames (1999), than certain vague predicates are not likely to give rise to a corresponding version of the sorites paradox. Such appears well to be the case for the concept of surprise associated with 7-SEP(II). Because as Soames[16] points out, the continuum which is semantically associated with the predicates giving rise to the sorites paradox, can be fragmented in units so small that if one of these units is intuitively F, then the following unit is also F. But such is not the case with the variation consisting in 7-SEP(II), where the corresponding units (1 day) are not fine enough with regard to the considered period (7 days).

Lastly and overall, as mentioned above, the preceding solution to SEP(II) applies, *whatever the nature of the solution which will be adopted for the sorites paradox*. For it is the ignorance of the semantic structure of the vague notion of surprise which is at the origin of the student's fallacious reasoning in the case of SEP(II).

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[14] Cf. Sorensen (1988, 292-293) : 'Indeed, no one has simply asserted that the following is just another instance of the sorites.

i. Base step : The audience can know that the exercise will not occur on the last day.
ii. Induction step : If the audience can know that the exercise will not occur on day *n*, then they can also know that the exercise will not occur on day $n - 1$
iii. The audience can know that there is no day on which the exercise will occur.

Why not blame the whole puzzle on the vagueness of 'can know'? (…) Despite its attractiveness, I have not found any clear examples of this strategy.'

[15] Cf. (1988, 324): 'One immediate qualm about assimilating the prediction paradox to the sorites is that the prediction paradox would be a very 'fast' sorites. (...) Yet standard sorites arguments involve a great many borderline cases.'

[16] Cf. Soames (1999, 218): 'A further fact about Sorites predicates is that the continuum semantically associated with such a predicate can be broken down into units fine enough so that once one has characterized one item as F (or not F), it is virtually irresistible to characterize the same item in the same way'.

And this fact is independent of the solution which should be provided, in a near or far future, to the sorites paradox – whether this approach be of epistemological inspiration, supervaluationnist, based on fuzzy logic..., or of a very different nature.

5. The solution to the paradox

The above developments make it possible now to formulate an accurate solution to the surprise examination paradox. The latter solution can be stated by considering what should have been the student's reasoning. Let us consider indeed, in the light of the present analysis, how the student should have reasoned, after having heard the professor's announcement:

– The student: Professor, I think that two semantically distinct conceptions of surprise, which are likely to influence the reasoning to hold, can be taken into account. I also observe that you did not specify, at the time of your announcement, to which of these two conceptions you referred. Isn't it?
– The professor: Yes, it is exact. Continue.
– The student: Since you refer indifferently to one or the other of these conceptions of surprise, it is necessary to consider each one of them successively, as well as the reasoning to be held in each case.
– The professor: Thus let us see that.
– The student: Let us consider, on the one hand, the case where the surprise corresponds to a *conjoint* definition of the cases of non-surprise and of surprise. Such a definition is such that the non-surprise and the surprise are possible at the same time, for example on the last day. Such a situation is likely to arise on the last day, in particular when a student concludes that the examination cannot take place on this same last day, since that would contradict the professor's announcement. However, this precisely causes to make it possible for the surprise to occur, because this same student then expects that the examination will not take place. And in a completely plausible way, as put forth by Quine, such a situation corresponds then to a case of surprise. In this case, the fact of taking into account the possibility that the examination can occur surprisingly on the last day, prohibits eliminating successively the days n, n-1, n-2, ..., 2, and 1. In addition, the concept of surprise associated with a conjoint structure is a concept of total surprise. For one faces on the last day either the non-surprise or the total surprise, without there existing in this case some intermediate situations.
– The professor: I see that. You did mention a second case of surprise...
– The student: Indeed. It is also necessary to consider the case where the surprise corresponds to a *disjoint* definition of the cases of non-surprise and of surprise. Such a definition corresponds to the case where the non-surprise and the surprise are not possible on the same day. The intuition on which such a conception of the surprise rests corresponds to the announcement made to students that they will undergo an examination in the year, while being moreover unaware of the precise day where it will be held. In such a case, it results well from our experience that the examination can truly occur surprisingly, on many days of the year, for example on whatever day of the first three months. It is an actual situation that can be experienced by any student. Of course, in the announcement that you have just made to us, the period is not as long as one year, but corresponds to one week. However, your announcement also leaves place to such a conception of surprise associated with a disjoint structure of the cases of non-surprise and of surprise. Indeed, the examination can indeed occur surprisingly, for example on the 1st day of the week. Thus, the 1st day constitutes a proper instance of surprise. In parallel, the last day constitutes a proper instance of non-surprise, since it results from the announcement that the examination cannot take place surprisingly on this day. At this stage, it also appears that the status of the other days of the corresponding period is not determined. Thus, such a disjoint structure of the cases of non-surprise and of surprise is at the same time disjoint and non-exhaustive. Consequently, the concept of corresponding surprise presents here the criteria of a *vague* notion. And this casts light on the fact that the concept of surprise associated with a conjoint structure is a vague one, and that there is thus a zone of penumbra between the proper instances of non-surprise and of surprise, which corresponds to the existence of borderline cases. And the mere existence of these borderline cases prohibits to eliminate successively, by a reasoning based on backward-induction, the days n, n-1, n-2, ..., 2, and then 1. And I finally notice, to the difference of the preceding concept of surprise, that the concept of surprise associated with a conjoint structure leads to the existence of intermediate cases between the non-surprise and the surprise.
– The professor: I see. Conclude now.
– The student: Finally, the fact of considering successively two different concepts of surprise being able to correspond to the announcement which you have just made, resulted in both cases in rejecting the classical reasoning which results in eliminating successively all days of the week. Here, the motivation to reject the

traditional reasoning appears different for each of these two concepts of surprise. But in both cases, a convergent conclusion ensues which leads to the rejection of the classical reasoning based on backward-induction.

6. Conclusion

I shall mention finally that the solution which has been just proposed also applies to the variations of SEP mentioned by Sorensen (1982). Indeed, the structure of the canonical forms of SEP(I□), SEP(IΔ) or SEP(II) indicates that whatever the version taken into account, the solution which applies does not require to make use of whatever principle of temporal retention. It is also independent of the order of elimination and can finally apply when the duration of the *n*-period is unknown at the time of the professor's announcement.

Lastly, it is worth mentioning that the strategy adopted in the present study appears structurally similar to the one used in Franceschi (1999): first, establish a dichotomy which makes it possible to divide the given problem into two distinct classes; second, show that each resulting version admits of a specific resolution.[17] In a similar way, in the present analysis of SEP, a dichotomy is made and the two resulting categories of problems lead then to an independent resolution. This suggests that the fact that two structurally independent versions are inextricably entangled in philosophical paradoxes could be a more widespread characteristic than one could think at first glance and could also partly explain their intrinsic difficulty.[18]

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[17] One characteristic example of this type of analysis is also exemplified by the solution to the *two-envelope paradox* described by David Chalmers (2002, 157) : 'The upshot is a disjunctive diagnosis of the two-envelope paradox. The expected value of the amount in the envelopes is either finite or infinite. If it is finite, then (1) and (2) are false (...). If it is infinite, then the step from (2) to (3) is invalid (...)'.

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