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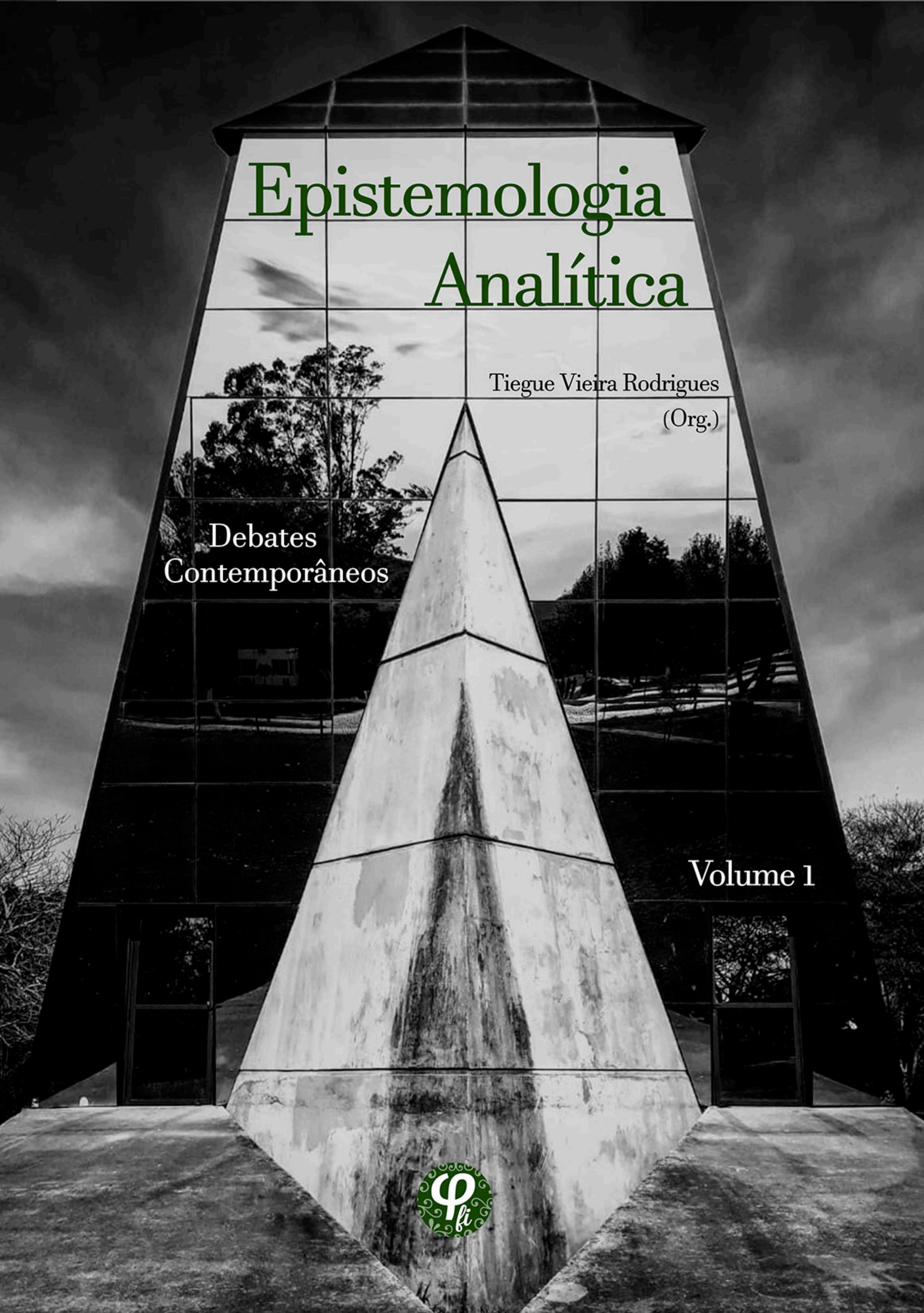

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Logical Omnipotence and Two notions of Implicit Belief

Danilo Fraga Dantas [1]

The most widespread models of rational reasoners are Hintikka's model (Hintikka, 1962) and the Bayesian model (de Finetti, 1937; Ramsey, 1926). Hintikka's model is based on modal epistemic logic. In this model, a reasoner is a set of possible worlds and an accessibility relation. The reasoner believes the propositions that are true in all accessible possible worlds[2]. The Bayesian model is based on probability theory. In this model, a reasoner is a probability function ranging over a set of propositions. The reasoner has a degree of belief x in a proposition ϕ iff the probability function returns x for ϕ.

Both Hintikka's and the Bayesian models exhibit the problem of logical omniscience. Reasoners are said to be logically omniscient when they believe all the logical consequences of their beliefs, believe all the logical tautologies[3], etc (see Jago, 2006, p. 327, for seven different characterizations of logical omniscience). The problem of logical omniscience is characterized as follows:

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[2] Hintikka's model may be interpreted in terms of knowledge or belief (Hintikka, 1962, develops both options). I want to sidestep the discussion about how exactly knowledge differs from belief. I will usually talk about belief, but most of my considerations will also apply to knowledge because (presumably) knowledge implies belief.

[3] In this paper, I will use “tautology” in the wide sense of logical (usually, classical) truth.

Definition 1 (Problem of logical omniscience). A model of rational reasoners exhibits the problem of logical omniscience iff all reasoners described in the model are logically omniscient.

All reasoners described in Hintikka's model are logically omniscient. These reasoners believe all propositions that are true in all accessible possible worlds. Possible worlds are usually construed as maximally specific and consistent truth assignments to all propositions (see Menzel, 2016). If all propositions in some set of propositions are true in all accessible possible worlds, then so are all their logical consequences. All reasoners described in the Bayesian model are also logically omniscient. These reasoners are described as probability functions. Probability theory requires probability functions to assign maximum probability to all tautologies (Kolmogorov, 1950). Then Bayesian reasoners have maximum degree of belief in all tautologies (see Garber, 1983, for a discussion on Bayesian logical omniscience). In the following, I will focus on Hintikka's model.

What makes the problem of logical omniscience relevant to epistemology is the fact that, at first, real reasoners cannot be logically omniscient (see fn. 9). The most common strategy for avoiding this problem is to interpret the models as describing the explicit beliefs of ideal reasoners, but only the implicit beliefs of real reasoners, for some notion of implicit belief. For example, sometimes real reasoners are said to implicitly believe the logical consequences of their explicit beliefs (e.g. Hintikka, 1962, p. 38)[4]. Ideal reasoners are often construed as logically omnipotent reasoners[5], i.e. as reasoners without cognitive limitations (e.g.

[4] "We study the logic of the closely related notions obtained by reading 'Kap' as follows: 'It follows from what a knows that p'".

[5] See (Kaneko and Suzuki, 2011, p. 14) for a similar use of 'logical omnipotence'.

Pollock, 1987, p. 504)[6]. Real reasoners (e.g. humans) are finite reasoners, i.e. as reasoners with cognitive limitations[7].

But why model ideal instead of real reasoners? The answer to this question usually involves normative considerations. Hintikka, for example[8], claims that the reasoners described in his model are ideal in the sense of being immune to the criticism of being irrational for not believing something that is a logical consequence of their beliefs[9]. But if not believing something that is a logical consequence of your beliefs may render criticisms of irrationality, then there must be some normative parameter of rationality requiring reasoners to believe the logical consequences of their beliefs. The (logically omniscient) reasoners described in the model would "instantiate" this parameter of rationality (they would be "maximally rational"), serving as a "model" of rationality for real reasoners. This move exploits an ambiguity in "model of reasoners": "model" as a schematic representation of reasoners (Frigg, 2006, p. 49) and "model" as a parameter of rationality (in this case, logical omniscience would be a "normative idealization", see Colyvan, 2013, p. 1339). In the following, I employ 'model' for the first use and "parameter of rationality" for the second.

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[6] "An ideal reasoner is unconstrained by a finite memory or processing capacity". Other examples are Chalmers (2010, p. 143) and Menzies (1998, p. 268-269).

[7] Informally, a finite reasoner has cognitive limitations such as finite perceptual input (perception transmits only a finite amount of information), finite memory (memory is able to store only a finite amount of information), and finite inferential power (reasoning is able to execute only finitely many inferential steps in a finite time interval), where an inferential step is the application of an inferential rule to a group of sentences (see Def. 4).

[8] The answer is also normative in the Bayesian case. Reasoners described in the Bayesian model would be ideal in the sense of not being vulnerable to Dutch books (see Ramsey, 1926, p. 182).

[9] This is the kind of criticism that Hintikka is talking about: "Suppose that someone says to you "I know that p but I don't know whether q" and suppose that p can be shown to entail logically q ... Then you can point out to him that what he says he does not know is already implicit in what he claims he knows. If your argument is valid, it is irrational for our man to persist in saying that he does not know whether q is the case" (Hintikka, 1962, p. 31).

It is not always reasonable to use reasoners without cognitive limitations as parameters of rationality for finite reasoners. Some patterns of inference that are optimal for reasoners without cognitive limitations are not minimally functional for finite reasoners. For example, if there is a procedure for checking guesses, a reasoner that is able to execute inferences instantaneously can solve any problem (instantaneously) simply by generating and checking successive random guesses. This pattern of inference is not remotely functional for finite reasoners because it trades (and depends) on the logical omnipotence of reasoners without cognitive limitations. This is the problem of logical omnipotence:

Definition 2 (Problem of logical omnipotence). A parameter of rationality exhibits the problem of logical omnipotence iff it requires logical omnipotence.

The notion of logical omniscience is relative to a logic and a notion of belief. Under some interpretations, logical omniscience entails logical omnipotence. If logical omniscience is interpreted in term of explicit beliefs, then all logically omniscient reasoners must be logically omnipotent[10]. Under other interpretations, however, logical omniscience does not entail logical omnipotence. For example, if logical omniscience is interpreted in terms of implicit beliefs and reasoners implicitly believe all the logical consequences of their explicit beliefs, then finite reasoners can be logically omniscient. Finally, logical omnipotence usually does not entail logical omniscience. If the set of explicit beliefs of a reasoner is closed under conjunction introduction (but not under other logical

[10] Supposing that the adoption of each explicit belief demands some amount of space in memory, finite reasoners cannot explicitly believe infinitely many tautologies because they have only a finite amount of space in memory (finite memory). Supposing that the adoption of each explicit belief demands the execution of some number of inferential steps, finite reasoners cannot explicitly believe infinitely many tautologies because they are able to execute only finitely many inferential steps in a finite time interval (finite inferential power).

rules), then the reasoner is logically omnipotent, but not omniscient (given some notions of logical omniscience).

Where logical omniscience and logical omnipotence diverge, logical omnipotence is the relevant problem. (Human) epistemology is especially concerned with human rationality and it seems to be an essential feature of human rationality that humans have finite amounts of cognitive resources; that humans are finite reasoners. For this reason, epistemology is (should be!) specially concerned with parameters of rationality that do not exhibit the problem of logical omnipotence. The avoidance of the problem of logical omniscience is not as central. In fact, parameters of rationality that exhibit logical omniscience, but not omnipotence seem to be especially relevant to epistemology. For example, it is a common contention that a parameter of rationality should exhibit some sort of logical closure of beliefs, given some notion of closure and belief (see Hintikka, 1962, p. 31). Then it seems to be a goal of epistemology to investigate notions of belief that generate logically omniscient, but not omnipotent parameters of rationality[11].

In this paper, I survey two developments of the most common strategy applied to the problem of logical omnipotence. The first strategy is to interpret the parameter of rationality in terms of accessible beliefs (notion inspired by Konolige, 1986, p. 19). The second strategy is to interpret the parameter of rationality in terms of stable beliefs (notion inspired by Pollock, 1995, p. 133). In section 1, I introduce a model of reasoners that is free from normative presuppositions. Then I define a finite reasoner and the general form of the parameter of rationality using this model. In section 2, I describe the two strategies using the model introduced in section 1 and show that both succeed in avoiding the problem of logical omnipotence in classical settings.

[11] The problem of logical omnipotence has interesting features of it own. For example, whereas there are several notions of logical omniscience (all of them relative to a logic and to a notion of belief), the notion of logical omnipotence is fixed, what suggests some sort of fundamentality.

1. The Model

The evaluation of whether parameters of rationality based on models such as Hintikka's exhibit the problem of logical omnipotence is impaired due to the normative import of those models: all reasoners described in Hitinkka's model, for example, trivially meet the corresponding parameter of rationality. In investigating the problem of logical omnipotence, it is important to keep aside descriptive and normative considerations about reasoners (the model of reasoners and the parameter of rationality, respectively). Syntactic models of reasoners (e.g. Konolige, 1986) are often criticized for not providing insights about the normative structure of knowledge (see Fagin et al., 2003, p. 339). For this reason, the model of reasoners presented in this paper is fully syntactic. The parameter of rationality, on the other hand, is described in semantic terms.

The model of reasoners is the following:

Definition 3 (Reasoner (R)). A reasoner R = <L; INPUT; KB; π> is a 4-tuple, where L is a formal language, INPUT and KB are sets of sentences in L, and π is a function $\pi: 2^L \times 2^L \times Z^+ \rightarrow 2^L$.

The first element (L) is a language that models the concepts available for the reasoner. The second element (INPUT) is a set of sentences in L that models the reasoner's perception. The third element (KB, from “knowledge base”) is another set of sentences in L that models the reasoner’s memory. The final element (π, from “pattern of inference”) is a function for updating KB from INPUT and KB that models the reasoner's ability for reasoning. A fact about patterns of inference is that reasoners can execute different inferences from the same premises[12]. This fact is expressed in the

[12] For example, given premises (p1) $\phi \rightarrow \psi$ and (p2) $\neg\psi$, a reasoner can infer that ϕ from p1 and p2 (modus tollens), infer that $\neg\phi$ (although not an interesting conclusion) from p2 alone (disjunction introduction), etc.

model using a function π that has a numeric input (positive integer) in addition to INPUT and KB: π(INPUT; KB; 1) models inference 1 from INPUT and KB, π(INPUT; KB; 2) models inference 2, etc.

In the model, the notion of a finite reasoner is the following:

Definition 4 (Finite reasoner). A reasoner R = <L; INPUT; KB; π> is finite iff L is recursively enumerable, INPUT and KB contain finitely many sentences, and π is a recursive function, where a function is recursive iff there exists an algorithm that can do the job of the function (Boolos et al., 2007, p. 63) and a set is recursively enumerable (r.e.) iff it is the range of a recursive function on the positive integers (Boolos et al., 2007, p. 96)[13].

That L is r.e. is a requirement for L being learnable by a finite reasoner[14]. If L is r.e., then L is finitary, i.e. all sentences in L have finite length. INPUT being finite models finite perceptual input. KB being finite models finite memory. Function π being recursive is related to finite inferential power. If π is recursive, then every belief of the reasoner is generated executing at most finitely many basic operations, what models the reasoner being able to execute only finitely many inferential steps in a finite time interval. In the following, I will usually investigate a priori reasoners, i.e. reasoners with R = <KB; π>, (initial) KB = ∅, and π: $2^L \times Z^+ \longrightarrow 2^L$[15].

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[13] A set is decidable if both the set and its complement are r.e. (Boolos et al., 2007, p. 96).

[14] Here, I am following Davidson's contention that a finite reasoner can only learn a language if it is constructive, in the sense of having compositional syntax and semantics (Davidson, 1965, p. 387). Davidson himself requires those languages to contain finitely many terms, sentences, etc but, (Haack, 1978) gives reasons to require only that they are r.e..

[15] A priori reasoners don't reason from perceptual information. This is modeled with INPUT = ∅, but, for simplicity, I will suppress INPUT from the model and make π : $2^L \times Z+ \longrightarrow 2^L$. Since the initial KB may store perceptual information, I will make KB = ∅. Finally, I will suppress L from the model. Since I always talk about reasoners in the context of a logic, I will presuppose that L is a r.e. arbitrary language for that logic.

This is a first approximation to the parameter of rationality: maximally rational reasoners believe all the logical consequences of their epistemic situation and have nontrivial sets of beliefs, where the epistemic situation of a reasoner is the information that the reasoner has available for reasoning (KB ∪ INPUT). In discussing the surprise test paradox, (Binkley, 1968) proposes a notion of an ideal reasoner with these requirements (also do Duc, 1995; E. Giunchiglia and F. Giunchiglia, 2001; Grim, 1988; Halpern and Moses, 1985; Stalnaker, 2006)[16]. This notion is not an adequate parameter of rationality because a reasoner with these features may still have all sorts of random beliefs[17]. This problem may be avoided implementing formally the following informal definition of the parameter of rationality:

Definition 5 (Maximum rationality – Informal). A reasoner R is maximally rational iff:

(r1) R believes all and only the logical consequences of the current epistemic situation;

(r2) R has a nontrivial set of beliefs.

The notion of maximum rationality is relative to a logic and to a notion of belief. Maximum rationality is relative to a logic because it is defined in terms of logical consequence and triviality, which are relative to a logic. The choice of a logic provides the

[16] In the literature, the second requirement is usually expressed as 'R has a consistent set of beliefs'. A set of beliefs is consistent iff it does not entail a contradiction. A set of beliefs is nontrivial iff it does not entail every sentence in the language. If the logic is explosive (see section 3), these requirements coincide. (Duc, 1995; E. Giunchiglia and F. Giunchiglia, 2001; Grim, 1988) drop the second requirement. However, this requirement is important for blocking a fully credulous reasoner, a reasoner that believes all sentences in the language, from being maximally rational.

[17] As long as those beliefs do not contradict the second requirement. The existence of this phenomenon depends on the notion of belief used, but it holds for accessible and stable beliefs (see sec. 2)

normative content of the parameter of rationality. For example, the discussion about whether it is always irrational to have inconsistent beliefs is equivalent to the question about whether the relevant logic is consistent or paraconsistent. In the following, I will always talk about maximum rationality given a logic. The notion of maximum rationality is also relative to a notion of belief because the term "belief" used in Def. 5 is ambiguous among explicit belief, implicit belief, etc. In the following, 'maximally rational^x_y' denotes the parameter of rationality given a logic x and a notion of belief y (belief$_y$).

Most results of logical omnipotence are generated using supraclassical logics and the notion of explicit belief (e.g. Hintikka, 1962). For example, the parameter of rationality in Def. 5 exhibits the problem of logical omnipotence when it is interpreted using classical logic and explicit beliefs (in this paper, 'classical logic' always refers to first-order logic). Informally, reasoners explicitly believe ϕ iff a representation with that content is inscribed in their "belief box" (see Schwitzgebel, 2015, sec. 2.2.1). Consider the notion of explicit belief (belief$_{ex}$) in the model:

Definition 6 (Belief$_{ex}$). A reasoner R = <KB; π> believes$_{ex}$ ϕ iff ϕ ∈ KB.

This is the parameter of rationality given (a logic x with consequence relation) $\vDash^x$, defined in terms of beliefs$_{ex}$:

Definition 7 (Maximum rationality$^x_{ex}$). A reasoner R = <KB; π> is maximally rational$^x_{ex}$ iff:
(r1$^x_{ex}$) KB $\vDash^x$ ϕ iff ϕ ∈ KB;
(r2$^x_{ex}$) (Σϕ) KB $\nvDash^x$ ϕ,

where Σ is the meta-linguistic "for some" (the meta-linguistic "for all" is often omitted, as in r1$^x_{ex}$). The parameter of rationality, when interpreted in terms of $\vDash^c$ (the consequence relation of classical logic) and $beliefs_{ex}$, exhibits the problem of logical omnipotence. This is the case because maximally $rational^{c}_{ex}$ reasoners $believe_{ex}$ all $tautologies^{c}$ ($r1^{c}_{ex}$). There are infinitely many $tautologies^{c}$. $Believing_{ex}$ infinitely many $tautologies^{c}$ requires having infinitely many sentences in KB (Def. 6), but a finite reasoner cannot have infinitely many sentences in KB (Def. 4, finite memory).

2. Two strategies

The problem of logical omnipotence may be avoided by interpreting the parameter of rationality using a notion of belief different from $belief_{ex}$, as, for example, implicit belief. In general, reasoners implicitly believe ϕ when they believe ϕ independently of having a representation with that content inscribed in their belief box (Schwitzgebel, 2015, sec. 2.2.1). There are several developments of the notion of implicit belief in the literature. Here, 'implicit belief' denotes the crudest of these developments: reasoners implicitly believe the logical consequences of their $beliefs_{ex}$ (Fagin et al., 2003, p. 363). Consider the notion of implicit belief (beliefim) in the model:

Denition 8 ($Belief_{im}$). A reasoner R = <KB; π> $believes_{im}$ ϕ iff KB $\vDash^{x}$ ϕ.

This is the parameter of rationality given $\vDash^{x}$, defined in terms of $beliefs_{im}$:

Denition 9 (Maximum $rationality^{x}_{im}$). A reasoner R = <KB; π> is maximally $rational^{x}_{im}$ iff:

($r1^{x}_{im}$) KB $\vDash^{x}$ ϕ iff KB $\vDash^{x}$ ϕ;

($r2^{x}_{im}$) ($\Sigma\phi$) KB $\nvDash^{x}$ ϕ.

The parameter of rationality, when interpreted in terms of $\vDash^{c}$ and beliefs$_{im}$, does not exhibit the problem of logical omnipotence. But avoiding this problem comes with the price of 'trivializing' the parameter of rationality: requirement $r1^{x}_{im}$ is obviously trivial. In this interpretation, all reasoners with nontrivial KBs meet the parameter of rationality (a reasoner with KB = Ø is maximally rational^{x} im given any $\vDash^{x}$). In fact, what a reasoner believes$_{im}$ depends mostly on the parameter of rationality (and not on the reasoner's epistemic situation). This feature of the notion of belief$_{im}$ defeats the purpose of keeping aside descriptive and normative considerations about reasoners.

I survey two strategies for avoiding the problem of logical omnipotence. The first strategy is to interpret the parameter of rationality in terms of accessible beliefs. Informally, the belief that ϕ is accessible to a reasoner when the reasoner is able to infer ϕ from the available information[18]. Consider the notion of accessible belief (belief$_{ac}$) in the model:

Definition 10 (Beliefs$_{ac}$). A reasoner R = <KB; π> believes$_{ac}$ ϕ iff $\phi \in \pi(KB)$.

The set of beliefs$_{ac}$, $\pi(KB)$, is the union of the outputs of function π for KB and every positive integer i. Formally, $\pi(KB) = \bigcup_{i} \pi(KB; i)$.

This is the parameter of rationality given $\vDash^{x}$, defined in terms of beliefs$_{ac}$:

Definition 11 (Maximum rationality$^{x}_{ac}$). A reasoner R = <KB; π> is maximally rational$^{x}_{ac}$ iff:

[18] This is the definition of "belief" in (Konolige, 1986, p. 19): "A formula ϕ is said to be believed by an agent i, which we write $B_i\phi$ i it is either in the agent's initial knowledge base or else is derivable from the knowledge base by applying the agent's deduction rules."

($r1^{x}_{ac}$) KB $\vDash^{x}$ ϕ iff $\phi \in \pi(KB)$;
($r2^{x}_{ac}$) ($\Sigma\phi$) $\pi(KB) \nvDash^{x} \phi$.

The parameter of rationality, when interpreted in terms of $\vDash^{c}$ and beliefs$_{ac}$, does not exhibit the problem of logical omnipotence. Consider a reasoner R^{c}_{ac} = <KB; π> and an axiomatization of classical logic (e.g. Smullyan, 1995, ch. 8). Let KB = Ø. For each axiom schema in the axiomatization, consider an inferential schema with no premises and with the axiom schema as conclusion. For each rule in the axiomatization, consider an inferential schema with the same premises and conclusion. Consider an ordering for the terms and sentences in L (L is r.e.). Then the execution of function π for KB and some positive integer i may be (roughly) defined as follows: for j = 1 to j = i, (i) execute all inferential schema without premises instantiated to all possible combinations of the terms and sentences with positions ≤ j and (ii) execute all inferential schema with premises instantiated for all possible combinations of sentences in KB before step ii and terms with positions ≤ j[19].

Function π generates all and only the theorems of classical logic for KB = Ø. The loop generates all theorems of classical logic because it executes all rules of classical logic for all possible combinations of sentences in L and the axiomatization is complete (Smullyan, 1995, ch. 8). The loop generates only the theorems of classical logic because the axiomatization is sound (Smullyan, 1995, ch. 8). Also, classical logic is consistent. Then R^{c}_{ac} is maximally rational$^{c}_{ac}$.

Since KB = Ø, KB is finite. Function π is recursive because the number of operations executed by π for KB and an arbitrary integer i is finite. For i = 1, the number of operations executed by π is a product of the number of inferential schema without premises, the number of inferential schema with premises, and the number

[19] The existence of such a function is a consequence of classical logic being r.e. (see Boolos et al., 2007, sec. 11).

of sentences in KB before step ii. Since the number of inferential schema is nite, this number is nite. For i = n + 1, the number of executed operations is the number of operations for i = n plus the number of operations for i = n + 1. Those two numbers are finite for the same reasons as for i = 1. Then R^{c}_{ac} is finite.

The second strategy is to interpret the parameter of rationality in terms of stable beliefs. Informally, a reasoner has the stable belief that ϕ iff the reasoner could reason indefinitely from the available information, then there would be a moment such that the reasoner would believe_{ex} ϕ at every moment after that. Consider the notion of stable belief (belief_{ω}) in the model:

Definition 12 (Belief_{ω}). A reasoner R = <KB; π> believes_{ω} ϕ iff $\phi \in KB_{\omega}$.

Function π defines a reasoning sequence KB_0, KB_1, ... ; KB_i, ..., where KB_0 is the initial KB of the reasoner and $KB_{i+1} = \pi(KB_i; i + 1)$[20]. In this context, KB_{ω}, the set of beliefs_{ω} of a reasoner with reasoning sequence KB_0, KB_1, ..., KB_i, ... is composed of the ϕ for which there is an i such that, for all j ≥ i, $\phi \in KB_j$. Formally, $KB_{\omega} = \bigcup_i \bigcap_{j \geq i} KB_j$[21].

This is the parameter of rationality given $\vDash^{x}$, defined in terms of beliefs_{ω}:

Definition 13 (Maximum $\text{rationality}^{x}_{\omega}$). A reasoner R = <KB; π> is maximally $\text{rational}^{x}_{\omega}$ iff:
($r1^{x}_{\omega}$) $KB \vDash^{x} \phi$ iff $\phi \in KB_{\omega}$;
($r2^{x}_{\omega}$) $(\Sigma\phi)\ KB_{\omega} \nvDash^{x} \phi$.

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[20] In this case, for all i, $\pi(KB_i; i + 1)$ must designate how the reasoner would reason from KB_i. This adds additional structure to the model, but I think that this addition pays off.

[21] The notion of belief_{W} is related to defeasible enumeration (Pollock, 1995, p. 143), identification in the limit (Kelly, 1990), limiting recursion (Gold, 1965), trial and error predicate (Putnam, 1965), and defeasible consequence (Antonelli, 2005, p. 87).

The parameter of rationality, when interpreted in terms of $\vDash^c$ and beliefs$_\omega$, does not exhibit the problem of logical omnipotence. R^c_{ac} is not only maximally rational$^c_{ac}$ but also maximally rational$^c_\omega$. This is the case because function π of R^c_{ac} is such that, for every i, π(KB; i) ⊆ π(KB; i + 1). Then π(KB) = KB_ω. Let R^c_ω = R^c_{ac}. Then R^c_ω is maximally rational$^c_\omega$ for the same reasons that R^c_{ac} is maximally rational$^c_{ac}$. All KB output by π are finite because KB_0 = Ø and each execution of function π has finitely many loops that add finitely many sentences to KB. The limiting knowledge base KB_ω has infinitely many sentences, but KB_ω is not an output of π. The function π of R^c_ω is recursive for the same reasons of that of R^c_{ac}. Then R^c_ω is finite.

3. Conclusions

Both strategies avoid the problem of logical omnipotence in classical settings, but they differ in an essential feature: whereas the notion of belief$_w$ is sensitive to the order of inferences, the notion of belief$_{ac}$ is not. As a consequence, the beliefs$_{ac}$ and beliefs$_\omega$ of a reasoner do not always coincide. This distinction doesn't show up in classical settings because maximally rational^c_y reasoners do not need rules for deleting sentences. However, the notion of belief$_{ac}$ and the notion of belief$_\omega$ yield different parameters of rationality in nonmonotonic settings, where maximally rational reasoners need rules for deleting sentences. In those settings, it is easy to construct cases in which a reasoner believes$_{ac}$ ϕ, but does not believe$_\omega$ ϕ. For example, suppose that R = <KB; π> is such that ϕ ∈ KB, π(KB; 1) deletes ϕ and adds to KB, and π(KB; i) does nothing for every i > 1. In this case, R believes$_{ac}$ ϕ because ϕ ∈ π(KB; 2) but R does not believe$_w$ ϕ because ϕ ∉ KB_i for every i > 1 (see Pollock, 1995, p. 132, for a similar case).

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