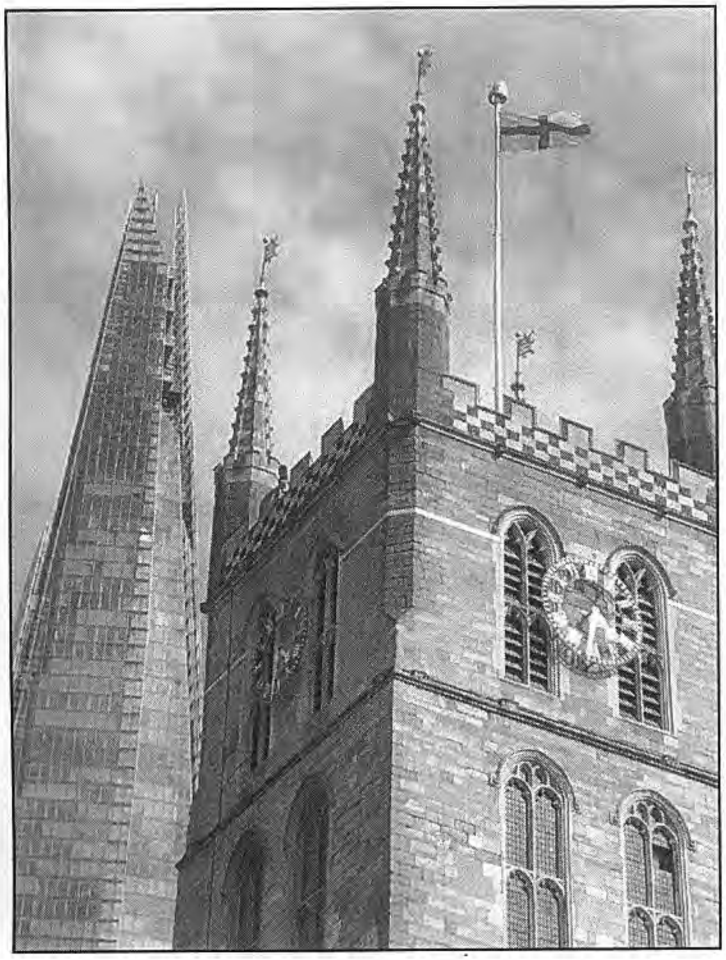

The Shard and Southwark Cathedral, London.

The cathedral is built on the site of 800 years of cathedrals from mid-19th century. The Shard by Renzo Piano built in 2009-2012.

Reclaiming Education

Renewing Schools and Universities in Contemporary Western Culture

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DISCIPLINES AND METHODS

Mathematics, Core of the Past and Hope of the Future

James Franklin

Mathematics has always been a core part of western education, from the medieval *quadrivium* to the large amount of arithmetic and algebra still compulsory in high schools. It is an essential part. Its commitment to exactitude and to rigid demonstration balances humanist subjects devoted to appreciation and rhetoric as well as giving the lie to postmodernist insinuations that all "truths" are subject to political negotiation.

In recent decades, the character of mathematics has changed—or rather broadened: it has become the enabling science behind the complexity of contemporary knowledge, from gene interpretation to bank risk. Mathematical understanding is all the more necessary for future jobs, as well as remaining, as ever, a prophylactic against the more corrosive philosophical views emanating from the humanities.

The Ancients and Deductive Proof

In the mid-fifth century BC, the Parthenon was rising over Athens, built according to the best geometrical principles. The construction lines are not visible on it, though on some other Greek temples they are.[1] Among the tragedians, artists, sophists and merchants creating Western civilization in the city below, a number of visitors from out of town brought an interest in what we would now call scientific questions. According to Parmenides of Elea, for example, thinking about the geometry of eclipses is enough for a convincing argument that the earth is round, even for someone fixed in one place on the earth's surface.[2]

A lesser-known genius who visited Athens was a certain Hippocrates of Chios.[3] He came to appreciate that geometry could be organised so that the complicated and less obvious propositions followed with strict logic from the simple and obvious ones. His project was perfected a century or more later in

[1] Lothar Haselberger, "The construction plans for the Temple of Apollo at Didyma," *Scientific American* 253, no. 6 (1985): 126-32.

[2] Otto Neugebauer, *A History of Ancient Mathematical Astronomy* (Berlin: Springer, 1975), 576.

[3] Confusingly, not the same person as Hippocrates of Cos, the medical writer after whom the Hippocratic Oath is called.

the *Elements* of Euclid, which was such a success that its predecessors have not survived. But one fragment of Hippocrates's work is left, enough to demonstrate his extraordinary brilliance. Being still at the "bare hands" phase of mathematical development, it can be appreciated by anyone.

The fragment concerns the "quadrature of lunes", that is, finding the area of a crescent-shaped figure as shown in the shaded portion at the top left of the figure:

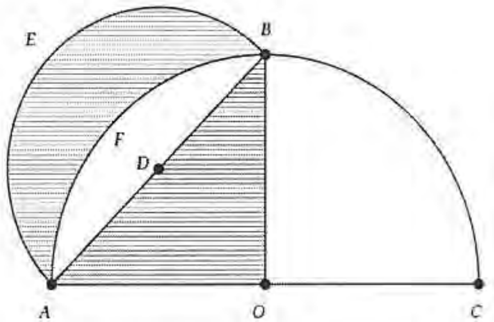

Fig. 1: The Lune of Hippocrates [4]

The lune is bounded by two arcs of circles: the upper one AEB which is half of a circle with diameter ADB, and the lower one AFB which is a quarter of a circle with diameter AOC. Hippocrates proves, amazingly, that the area of the lune is exactly the same as the area of the right-angled triangle AOB (also shaded).

Note that there is no π in the answer, as there is in the formula for the area of a circle: Hippocrates does not use anything about the area of a circle, but proves directly that the two areas, one curved and one straight-sided, are equal. If the radius AO is one unit of length, both shaded areas are exactly half a square unit. The original text of his proof is given in this footnote in case some readers wish to skip it.[5]

[4] Michael Hardy, "Lune of Hippocrates", 7 March 2009,. https://upload.wikimedia.org/wikipedia/commons/e/e0/Lune.svg

[5] "He started with, and laid down as the first of the theorems useful for the purpose, the proposition that similar segments of circles have the same ratio to one another as the squares on their bases. And this he proved by first showing that the squares on the diameters have the same ratio as the circles. After proving this, he proceeded to show in what way it was possible to square a lune the outer circumference of which is that of a semicircle. This he effected by circumscribing a semicircle about an isosceles right-angled triangle and a segment of a circle similar to those cut off by the sides. Then, since the segment about the base is equal to the sum of those about the sides, it follows that, when the part of the triangle above the segment about the base is added to both alike, the lune will be equal to the triangle." "Text of Simplicius", translated in Thomas L. Heath, *A Manual of Greek Mathe-*

The Greeks immediately realised there was something special about deductive proof as a way of acquiring knowledge. It is not like measuring a number of lunes and triangles and finding that in all cases the lune equals the triangle in area. Proof somehow gets to a deeper level of reality. It reveals not only what *is* so, but what *must* be so. And the proof allows us not only to know what must be so, but to understand *why* it must be so.

The results are not thereby cut off from physical reality, as if they are about a Platonic realm of ideal forms. If real lunes and triangles are drawn on paper, they are nearly equal in area, and the more perfectly they are drawn, the nearer the drawn shapes approximate the exact parts of real space to which the proof applies, and so the nearer they are equal in area.

There are plenty more exact and provable results where that came from, as laid out in the thirteen books of Euclid's *Elements* and many thousands of mathematical books and papers since. The method of deductive proof generates indefinitely many results. It is just a matter of human interest and energy keeping up with them.

A very simple example shows how the initial and most basic results of mathematics are known.

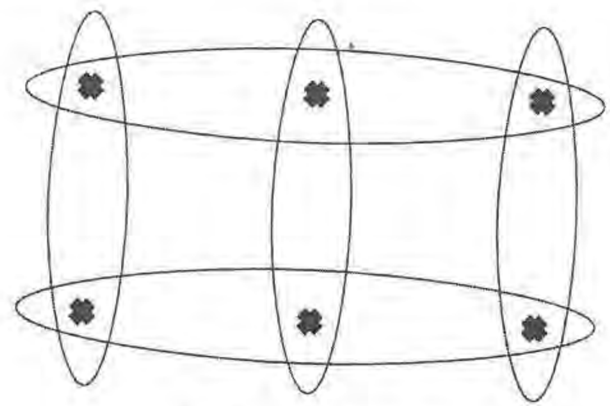

Fig. 2: Why $2 \times 3 = 3 \times 2$

In the figure, the six crosses are arranged both as two rows of 3 and as three columns of 2. Since they are the same six crosses, $2 \times 3 = 3 \times 2$. We can literally see not only *that* $2 \times 3 = 3 \times 2$, but that 2×3 *must be* 3×2.[6]

Further, we can easily see that the same reasoning applies if we add more rows and columns. If we have *m* columns and *n* rows, the same reasoning is

matics (Mineola NY: Dover, 2003), 121-132; a more accessible explanation in W.S. Anglin, *Mathematics: A Concise History and Philosophy* (New York: Springer, 1994), 52.

[6] Catherine Legg and James Franklin, "Perceiving necessity," *Pacific Philosophical Quarterly* 98, no. 3 (2017), 320-343.

valid. Therefore $m \times n = n \times m$, for any numbers m and n. So the insight produces an infinite number of truths, all of them understood to be true with certainty. Again, these truths apply directly to the real world of actual crosses written on paper, or of any other objects whatsoever. There is no need for getting out in the wet and observing, but the abstract truths predict what will be observed by anyone who does get out there.[7]

Excited by these possibilities, Aristotle proposed in his *Posterior Analytics* that *all* sciences should follow the model of mathematics, with immediately understood axioms supporting a superstructure of more complex theorems, and proofs explaining why the theorems were true.[8] That did not quite work out. Contrary to Aristotle's hope, in the natural and human sciences observation and measurement are still essential to establishing the truth of theories. Only in mathematics, and in some closely related fields like computer science and just possibly ethics, is proof in the full sense feasible.[9] Thus mathematics remains the ideal training ground for the human faculty of understanding and proof.

Mathematics at the Centre of Western Education

Western education has not lost sight of the point of mathematics and has always made it central to education, despite the fact that it is quite hard to learn and subject to a certain degree of customer resistance.

Medieval liberal tertiary education, the preparation for specialised studies such as theology, law and medicine, was divided into two levels, the *trivium* and the *quadrivium*. The *trivium* consisted of grammar, logic and rhetoric, that is, studies in words and how they work. Then came the *quadrivium*: arithmetic, geometry, music (theory) and astronomy, all of which are mathematics in one form or another.[10] Then the Italian merchant schools of the later Middle Ages made the remarkable discovery that calculation with indefinitely large numbers could be reduced to rules and taught to seven-year-olds.[11] Basic numeracy became widespread along with basic literacy. The new technologies of early modern times found a population able to understand and manipulate them.

[7] Briefly in James Franklin, "The mathematical world," *Aeon* 7 Apr 2014 https://aeon.co/essays/aristotle-was-right-about-mathematics-after-all; fully in James Franklin, *An Aristotelian Realist Philosophy of Mathematics: Mathematics as the science of quantity and structure* (Basingstoke: Macmillan, 2014).

[8] Richard D. McKirahan, *Principles and Proofs: Aristotle's theory of demonstrative science* (Princeton: Princeton University Press, 1992).

[9] See Svetlana V. Drachova *et al*, "Teaching mathematical reasoning principles for software correctness and its assessment," *ACM Transactions on Computing Education* 15, no. 3 (2015): article 15; and James Franklin, "On the parallel between mathematics and morals," *Philosophy* 79 (2004): 97-119.

[10] David L. Wagner, ed., *The Seven Liberal Arts in the Middle Ages* (Bloomington, Ind: Indiana University Press, 1983).

[11] Richard W. Hadden, *On the Shoulders of Merchants: Exchange and the mathematical conception of nature in early modern Europe* (Albany NY: SUNY Press, 1994).

There was some backsliding from the humanists of the Renaissance, who preferred words, but the Scientific Revolution of the seventeenth century was and was seen to be based on mathematics and data instead of wordy disputation. Galileo explains:

> If what we are discussing were a point of law or of the humanities, in which neither true nor false exists, one might trust in subtlety of mind and readiness of tongue and in the greater experience of the writers, and expect him who excelled in those things to make his reasoning more plausible, and one might judge it to be the best. But in natural sciences whose conclusions are true and necessary and have nothing to do with human will, one must take care not to place oneself in the defense of error; for here a thousand Demosthenes and a thousand Aristotles would be left in the lurch by every mediocre wit who happened to hit upon the truth for himself.[12]

When the Jesuit missionary Matteo Ricci reached China in 1582, he soon found that the Chinese scholars were particularly impressed with Western mathematical science. The first work translated into Chinese by him and his collaborator Xu Guangqi was the first six books of Euclid's *Elements*. Ricci says in his diary:

> Nothing pleased the Chinese as much as the volume on the Elements of Euclid. This perhaps was due to the fact that no people esteem mathematics as highly as the Chinese, despite their method of teaching, in which they propose all kinds of propositions but without demonstrations. The result of such a system is that anyone is free to exercise his imagination relative to mathematics without offering a definitive proof of anything.[13]

That is quite right about the difference between Western and other mathematics. Non-Western mathematics was in many ways impressive, especially in Babylon, India and China, but it looks more like modern computer science than modern mathematics: a series of recipes for calculating rather than an organised body of proofs of theorems.[14]

In Victorian England, the study of Euclid was presumed suitable for training boys of the upper classes in the intellectual tasks that awaited them

[12] Galileo, *Dialogue Concerning the Two Chief World Systems*, trans. S. Drake (2nd ed, Berkeley: University of California Press, 1967), 53-54.

[13] *China in the Sixteenth Century: The Journals of Matthew Ricci*, trans. L.J. Gallagher (New York: Random House, 1953), 476; details in Peter M. Engelfriet, *Euclid in China: The Genesis of First Chinese Translation of Euclid's* Elements *Books I-VI (Jihe yuanben; Beijing, 1607) and its Reception up to 1723* (Leiden: Brill, 1998); typical postmodernist complaints about the resulting "loss of indigenous mathematics" in Sara N. Hottinger, *Inventing the Mathematician: Gender, Race and Our Cultural Understanding of Mathematics* (Albany NY: SUNY Press, 2016), 121-2.

[14] George Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics* (3rd ed, Princeton: Princeton University Press, 2011).

such as governing India. Animated debate proceeded merely as to whether Euclid was best swallowed whole or whether modern re-hashings were easier but not oversimplified; Charles Lutwidge Dodgson (Lewis Carroll)'s book *Euclid and His Modern Rivals* defended teaching straight Euclid.[15]

The upshot of this long process was that mathematics—and mathematics at a substantially high level—has become a compulsory part of education across the board. That has applied not just in Western countries. The Soviet Union, for all its Marxist ideology in the humanities and Lysenkoist delusions in biology, left pure mathematics alone and maintained a very high standard of mathematics education in schools.[16] East Asian countries have surpassed Western ones in school mathematics education, having successfully grafted Western mathematics onto their cultural traditions.[17] Third World countries are doing their best to catch up. The culture of research mathematics is the same worldwide, with the same symbols used everywhere and virtually everything written in English. Mathematics is a universal culture—as international and standardised as air traffic control but impacting vastly more people.

Enemies of Mathematics

Naturally, the enemies of Western civilisation and of rationality have not taken the achievements of mathematics lying down.

The Greeks in Athens in the fifth century BC, multi-talented as they were, invented not only Western civilisation but how to complain about it. The Sophist Gorgias of Leontini, in the course of making a lot of money corrupting the youth of Athens around 420, defended the propositions:

> Nothing exists
> If anything existed, it could not be known
> If anything were known, it could not be communicated.[18]

That just about covers everything, and the postmodernists of the late twentieth century did not have much to add to it (except prolixity, obviously). They found plenty of life left in Gorgias's insights, and recycled them in such forms as:

[15] Charles Lutwidge Dodgson, *Euclid and His Modern Rivals* (London: Macmillan, 1879); Rafael Montoito and Antonio Vicente Marafioti Garnica, "Lewis Carroll, Education and the Teaching of Geometry in Victorian England," *História da Educação* 19, no. 45 (2015): 9-27.

[16] Alexander Karp and Bruce R. Vogeli, *Russian Mathematics Education: History and World Significance* (New Jersey: World Scientific, 2010).

[17] Frederick K.S. Leung, Klaus-D. Graf and Francis J. Lopez-Real, *Mathematics Education in Different Cultural Traditions: A Comparative Study of East Asia and the West: The 13th ICMI study* (New York: Springer, 2010).

[18] Bruce McComiskey, "Gorgias, "On Non-Existence": Sextus Empiricus, "Against the Logicians" 1.65-87, translated from the Greek text in Hermann Diels's "Die Fragmente der Vorsokratiker"," *Philosophy & Rhetoric* 30 (1997): 45-49.

1. Doubt whether there is any solid reality out there for science to know (or at least, label "naïve" the assumption that there is such a reality)
2. Maintain that even if there is some sort of reality out there, we cannot know what it is because we are trapped in our own evolutionarily-determined brains/cultural understandings/specific historicities/reactionary educations[19]
3. Allege that language is incapable of communicating any truth about objective reality, as it cannot refer directly to things and their properties

The evils of postmodernism are a well-studied field and need not be rehearsed again.[20] Here we confine ourselves to the case of mathematics. Naturally, the most shameless and foolhardy in the irrationalist camp have been keen to make a name for themselves by trying to knock over this last line of defence. Sokal and Bricmont's exposé of postmodernist absurdities, *Intellectual Impostures,* easily collected a whole chapter of garblings of mathematics from the best French theorists. Gilles Deleuze, one of the most worshipped gurus of "theory", had a special taste for pieces of mathematics (or apparent mathematics), such as:

> The respective independence of variables appears in mathematics when one of them is at a higher power than the first. That is why Hegel shows that variability in the function is not confined to values that can be changed ($^2/3$ and $^4/6$) or are left undetermined ($a = 2b$) but requires one of the variables to be at a higher power ($y^2/x = P$). For it is then that a relation can be directly determined as differential relation dy/dx, in which the only determination of the value of the variables is that of disappearing or being born, even though it is wrested from infinite speeds...[21]

The writings of the irrationalists about mathematics have had absolutely no impact on mathematicians. Mathematicians have not read one word of any complainers. It has been all water off a duck's back to them because they have remained unaware of and unaffected by the sell-out by the humanities. The production of theorems and the application of mathematics to climate modelling and airline scheduling have proceeded, totally untroubled by any

[19] James Franklin, "Stove's Discovery of the Worst Argument in the World," *Philosophy* 77 (2002), 615-24.

[20] One of the earliest and best refutations is Raymond Tallis, *Not Saussure: A Critique of Post-Saussurean Literary Theory* (Basingstoke: Macmillan, 1997). Needless to say it was as ineffective as it was brilliant.

[21] Gilles Deleuze and Félix Guattari, *What is Philosophy?* (New York: Columbia University Press, 1994), 122, in Alan Sokal and Jean Bricmont, *Intellectual Impostures* (London: Profile Books, 1998), 150-1; Australian imitations in James Franklin, *Corrupting the Youth: A History of Philosophy in Australia* (Sydney: Macleay Press, 2003), 368.

doubts as to the fallibility of proof (but subject to the politics of funding, of course).

That's the good news. The bad news is that it is otherwise in mathematics education. There, works like Paul Ernest's *Social Constructivism as a Philosophy of Mathematics* are taken very seriously indeed.[22] It may be that the paper 'Toward a feminist algebra' that was analysed in Gross and Levitt's *Higher Superstition* was somewhat beyond what is typical of the field,[23] but it is not hard to find text still being produced like this extract from *Educational Studies in Mathematics*, 2004:

> The supposed apolitical nature of mathematics is an institutional frame that functions to sustain specific power structures within schools. This paper disrupts the common assumption that mathematics (as a body of knowledge constructed in situated historical moments) is free from entrenched ideological motives. Using narrative inquiry, the paper examines the ways in which novice mathematics teachers negotiate the intersection of curriculum and institutional politics …[24]

Recent trends may be observed by following up in Google Scholar the 41 works that cite this article, such as 'Criticising with Foucault: towards a guiding framework for socio-political studies in mathematics education', 2016. The research field remains active. And activist.

Mathematics as it is Now

Those outside mathematics, especially those in such distant regions as the humanities, may have missed some important trends in the last century or so. People with a general school education know some very old pure mathematics such as arithmetic, geometry and algebra, have a rough idea of statistics, and have a notion that mathematics of some advanced sort is used in physics, computing and finance. That is not exactly wrong but is a very partial view. The truth is more complex and interesting.

In the mid-century, there appeared a number of disciplines at the edge of mathematics, variously called the "formal" or "mathematical" sciences or sciences of complexity—operations research, systems engineering, control theory, theoretical computer science and others.[25] To take just one example of the kind of thing they do, consider train timetabling. Train speeds, length of stops,

[22] Paul Ernest, *Social Constructivism as a Philosophy of Mathematics* (Albany NY: SUNY Press, 1998). (800+ citations on Google Scholar)
[23] Paul R. Gross and Norman Levitt, *Higher Superstition: The Academic Left and its Quarrels With Science* (Baltimore: Johns Hopkins University Press, 1994), 113-5.
[24] Elizabeth de Freitas, "Plotting Intersections Along the Political Axis: The Interior Voice of Dissenting Mathematics Teachers," *Educational Studies in Mathematics* 55 (2004): 259-274.
[25] James Franklin, "The Formal Sciences Discover the Philosophers' Stone," *Studies in History and Philosophy of Science* 25 (1994): 513-33.

and crew rosters create (quantitative) constraints that train journeys must satisfy, on top of which there are delays and breakdowns of various degrees of predictability. Those constraints must be described mathematically and software must be programmed to timetable the trains, ideally so as to minimise delays. Though developed originally for use in various such applications, these bodies of knowledge are in themselves purely mathematical disciplines (though often now housed in faculties of engineering).

More recently, the situation has changed through the ability of computers to deal with large amounts of data. Where once the strength of computers was in calculation, the focus has moved, as the rise of the phrase "information technology" suggests, to the processing of the flood of data. There are unstoppable streams of data arriving from satellites, from telescopes, from weather buoys, from medical imaging machines, from supermarket scanners, from wire services serving up tick data of all the world's financial transactions. Huge databases await mining and matching. As the science journalist Mitchell Waldrop says, "drink from the firehose of data".[26] Data interpretation tasks that the brain finds easy, like opening the eyes and seeing what's in front of them, remain very challenging for computers. Computer vision is still unreliable on the problem of listing the objects in a natural scene.

While there is plenty of activity in these fields, it is recognised that the capabilities of the hardware and software in storing and accessing the data have raced well ahead of the mathematical algorithms needed to make sense of the data. There is sophisticated technology for recording and displaying Pap smears, say, but for recognising whether they are cancerous, the method of choice is still to have a trained human looking at them—or at least, supervising any results generated automatically. That is the typical situation.

Data mining, interpretation of pathology results, target recognition and the like are all held up because the mathematics of pattern recognition, and statistical methods appropriate to large data sets in general, remain in a grossly primitive state. Surely one of the main directions for future mathematics is to sort that out. The problems to be solved include fraud detection and finding missing planes.[27]

The future of mathematics is hard to predict, but perhaps less so than the future of just about everything else. Mathematics has been going a very long time, and its changes have been expansions rather than revolutions. Mathe-

[26] M. Mitchell Waldrop, "Learning to Drink From A Firehose," *Science* 248 (1990): 674-5.
[27] Bart Baesens, Véronique Van Vlasselaer and Wouter Verbeke, *Fraud Analytics Using Descriptive, Predictive and Social Network Techniques: A guide to data science for fraud detection* (Hoboken: Wiley, 2015).; "How Statisticians Found Air France Flight 447 Two Years After It Crashed into the Atlantic," *MIT Technology Review*, May 27, 2014, https://www.technologyreview.com/s/527506/how-statisticians-found-air-france-flight-447-two-years-after-it-crashed-into-atlantic/; Sam Davey, Neil Gordon, Ian Holland, Mark Rutten and Jason Williams, *Bayesian Methods in the Search for MH370* (Singapore: Springer, 2016).

matics does not go back to square one, and there is some sense of what will come next from the pattern of the unsolved problems of today.

Undoubtedly many advances in mathematics in the near term will be simply cracking the problems that seem today to be in the course of yielding. Not long ago speech recognition by computer looked intractable, but the algorithms advanced and it is now, if not perfect, usable enough to answer phones for large organisations. Just now at the cutting edge is the recent sudden improvement in Google Translate arising from a powerful statistical learning algorithm applied to huge corpora of translated texts.[28]

Some problems are a little farther "out there". A big one that is not so far going very well but surely ought to be reasonably solvable is that of extracting causality from data. Among the many amazing things that human infants learn very fast is how to infer what causes what from looking at how things behave. They can manipulate a few close things, but most things can only be observed from a distance, and causality is not something that can be directly observed. Yet infants can work out what causes what and hence predict what will happen next. It is a difficult problem because "correlation does not imply causality".[29] If we could learn the algorithms behind that, we could trawl through medical data to learn the causes of disease. We would be able to determine automatically whether, for example, lower cholesterol is a cause or symptom of lower heart disease. The causes of global warming would be established beyond doubt and we would know what interventions will work.

A different kind of problem lies at, so to speak, the opposite end of the spectrum from big data. It concerns the estimation of "extreme risks"—risks of major disasters that are not exactly like anything that has happened yet, such as terrorist attacks or major quarantine incursions. It is "data-free statistics", in that there are no data, or hardly any, directly relevant to the event to be predicted. In that case one must combine the little data of some relevance (such as the occurrence of somewhat similar events) with expert opinion on possible scenarios. How to use data to keep opinion honest is a challenging problem.[30]

Plenty of problems remain, too, in mathematical finance, the area most popular as an employment destination for maths graduates. It would be good to understand how to apply mathematics to enhance instead of undermine

[28] Gideon Lewis-Kraus, "The Great A.I. Awakening," *New York Times Magazine*, Dec 14, 2016, https://www.nytimes.com/2016/12/14/magazine/the-great-ai-awakening.html

[29] Divorce rate in Maine correlates with per capita consumption of margarine, correlation = 99.26% (http://www.tylervigen.com/spurious-correlations)

[30] James Franklin, Scott A. Sisson, Mark A. Burgman and Jennifer K. Martin, "Evaluating Extreme Risks in Invasion Ecology: Learning from Banking Compliance," *Diversity and Distributions* 14 (2008): 581-91.

the stability of the global financial system. Medicine too presents a huge number of problems needing mathematics, in areas like gene expression.[31]

Directions for Mathematics Education

We are used to a stream of reports of falling standards in schools, falling enrolments in advanced maths, and the better performance of our Asian neighbours.[32] The first thing, plainly, is to make sure all students study an appropriate level of mathematics, and do not, for example, ditch mathematics because easier subjects gain higher HSC marks.[33] Efforts to explain the need for mathematics to students and their parents need to continue. It is true, however, that enrolments by the brighter students in university mathematics have held up well in recent years, especially in those courses related to jobs in finance.

Having said that, the mathematics taught needs a few adjustments to be suitable for an intelligent person in the present century.[34] Syllabuses at school and university are dominated by pure mathematics, about numbers and algebraic techniques that are promised to be useful in the "real world" in an indefinite future. It is indeed necessary to study those things, but an exclusive focus on them is both unmotivating and untrue to real mathematics.

The first thing that needs adding is some serious mathematical modelling. Modelling is the process of describing some real-world structure or problem in mathematical terms, so that mathematical techniques can be brought to bear on solving it. It is quite a different skill from pure mathematics. Take a problem like: is it feasible to tow an iceberg from Antarctica to provide fresh water for Adelaide? That does not require solving a set mathematical problem like "Simplify $(x-3)(x^3 + 4x - 7)$" and the skills involved are not the same. It needs a team to model what would need to happen to tow an iceberg and work out what quantitative information is needed—such as the size of icebergs, rates of melting, feasible speeds of towing, Adelaide's water demands—and finding that information. The outcome should be a clearly written report advising what the result is and setting out the reasons clearly.[35]

[31] E.g. Ahmet Ay and David N. Arnosti, "Mathematical Modelling of Gene Expression: A Guide for the Perplexed Biologist," *Critical Reviews in Biochemistry and Molecular Biology* 46, no. 2 (2011): 137-151.

[32] Most recently Samantha Hutchinson, "Australia looks to China for Maths, Science Lessons," *The Australian* 11/7/2017; figures in Frank Barrington and Michael Evans, "Participation in Year 12 Mathematics 2004-2014," Australian Mathematical Sciences Institute report, http://amsi.org.au/publications/participation-in-year-12-mathematics-2004-2014/ .

[33] Eryk Bagshaw, "HSC Maths: Students Studying Advanced Maths Stung with Lower Marks in ATAR," *Sydney Morning Herald*, 19/5/2015.

[34] James Franklin, "Mathematics for the Intelligent," *Gazette of the Australian Mathematical Society* 24 (1997): 2-3.

[35] Robert B. Banks, *Towing Icebergs, Falling Dominoes and Other Adventures in Applied Mathematics* (Princeton: Princeton University Press, 1998); UNSW's course 'Mathematical Modelling For Real

Statistics too needs a higher profile in mathematics education, as dealing with data and reaching conclusions from it are now the main uses of mathematics. In fact mathematics syllabuses have been moving in that direction, at both tertiary and secondary levels.[36] However syllabuses have been slower to adapt to the statistical needs of the "big data" that is flooding into data warehouses from the sophisticated hardware collecting it. Extracting meaning from huge datasets in real time is not quite the same task as the one for which classical statistical methods were developed, of extracting the most from small and expensive datasets such as in medicine.

The next thing that needs adding is a course on proof. Although mathematics advertises itself as "teaching you how to think"[37] and, as we have seen, proof is central to the Western mathematical tradition, mathematics degrees rarely include an explicit course on how to prove mathematical results. It is an easy enough skill to pick up but it is not inborn. Here is a simple example:

> Prove that the square of every even number is even.
>
> Proof: Let x be an even number (so that the result has generality by applying to *every* even number)
> So $x = 2 \times k$ for some whole number k (that is the meaning of "even")
> So $x^2 = (2 \times k)^2 = 2 \times 2 \times k^2$ (basic algebra)
> which is even (again from the meaning of "even": it is twice some whole number)
> Therefore the result is proved: for every even number, its square is even.

Understanding that does not require any stroke of genius: it is a perfectly mechanical application of the definition of "even" and simple algebraic manipulations. Such proof techniques can be and should be taught directly.[38]

Mathematical communication also needs to be taught. Communicating via graphs and diagrams is not the same as communicating via text. Mathematical professionals face the task of communicating their recommendations to people who cannot really understand them, and the challenges of communicating a simplified but sufficiently honest account of the mathematics are considerable. Those employers who habitually complain about the poor communication skills of graduates may want to consider the fact that they are

World Systems' is described at https://www.maths.unsw.edu.au/courses/math3041-mathematical-modelling-real-world-systems

[36] NSW Education Standards Authority, Mathematics K-10 syllabus, Statistics and Probability, https://syllabus.nesa.nsw.edu.au/mathematics/mathematics-k10/statistics-and-probability/

[37] Research on this tricky question in Matthew Inglis and Nina Attridge, *Does Mathematical Study Develop Logical Thinking? Testing the Theory of Formal Discipline* (New Jersey: World Scientific, 2017).

[38] James Franklin and Albert Daoud, *Proof in Mathematics: An Introduction* (Sydney: Kew Books, 2010).

acquiring their supply of graduates absolutely free and draw the obvious conclusion.

Mathematics has enormous effects in the "real world", as shown by such headlines as 'The Formula That Killed Wall Street'[39] and the fact that the military has always been an enthusiastic investor in mathematically-based technologies.[40] It follows that education in mathematics should include attention to ethical questions, in the same way as is normal in medical education. The issues for professional mathematicians are much the same as in other information management professions—duty of care, conflict of interest, honesty in drawing and communicating conclusions, confidentiality and whistleblowing, and the like—but the effects of malfeasance can be orders of magnitudes larger than in most professions.[41]

Conclusion

For those who wished to retain their sanity amid the stress of twentieth-century cultural chaos, where was there to escape to? In the humanities world, there was always the past, and many a cultural refugee from various Modernisms recuperated through communion with Monteverdi, or Vermeer, or Jane Austen. But for those who preferred their culture still living and breathing, the most extensive vandal-free space was science and mathematics.

Two regions of science stayed particularly free of any modern nervousness about themselves. One was engineering, for the obvious reason that bridge construction on cultural relativist principles is forbidden by the laws of nature as strictly as by those of man. The other was mathematics.

Mathematics has several advantages as a cultural counterweight to relativisms and scepticisms. Everyone knows something about it—in fact quite a lot about it—so it is not necessary to take the word of experts about everything in it, as it is for, say, quantum physics. Secondly, the truths in it are subject to proof, and what is proved does not become unproved (though it can be proved better).[42] For these reasons mathematics has always been an unfailing support for rationalist views, views which exalt the capacity of the human

[39] Felix Salmon, "Recipe For Disaster: The Formula That Killed Wall Street," *Wired* 23/2/2009, https://www.wired.com/2009/02/wp-quant/

[40] Examples in Nicholas Daras, ed, *Applications of Mathematics and Informatics in Military Science* (New York: Springer, 2012).

[41] James Franklin, "A 'Professional issues and ethics in mathematics' course," *Gazette of the Australian Mathematical Society* 32 (2005): 98-100; an Australian case study of the misuse of actuarial information in David Jackson QC, Report of the Special Commission of Inquiry into the Medical Research and Compensation Foundation, 2004, ch. 23 http://www.ir.jameshardie.com.au/jh/asbestos_compensation/special_commission_of_inquiry.jsp.

[42] It is sometimes claimed that non-Euclidean geometries are a counterexample, where Euclid's proofs were "overturned". That is not correct; the case is examined in James Franklin, *An Aristotelian Realist Philosophy of Mathematics: Mathematics as the science of quantity and structure* (Basingstoke: Macmillan, 2014), 160-2.

mind to find out the truth. Conversely, mathematics has been a perennial thorn in the side of opinions that abase human knowledge, and claim it is limited by sense experience, cultural horizons or one's personal education and perspective. Any culture or person that can count to 4 has discovered that 2 + 2 = 4, and should any fear arise of losing a grasp of that truth, resort to counting stones will quickly relieve any anxiety.

The truths of mathematics, unfortunately, cannot defend themselves, as they are not physical things with a causal action on the physical world. Neither ethical nor mathematical truths and ideals can fight tanks, or blizzards of allegations about history or politics (though again, neither can they be liquidated by those enemies). They depend on human minds to attune to them to act on their behalf—to implement those ideals and teach them to the next generation.

Bolstered by the continuing success of mathematics in applications, that makes an education in mathematics the surest way forward to a grasp of eternal verities.

Art Teaching as Part of a Liberal Education

Christopher Allen

Most art education curricula, whether in primary schools, secondary schools or vocational colleges, seem to have been written by people who have no real understanding of art, either as practitioners, historians or critics. Art classes, especially in primary school, are often treated as a kind of playtime, a break from serious academic subjects, and when art is taken a little more seriously, it is either thought of as a form of self-expression or as a way of articulating some kind of social critique.

But pupils are hardly ever taught any discipline of drawing, painting or any other art form that might make expression or articulation possible. Nor are they taught art history, which would allow them to reflect critically on their own practice; instead they are provided with a pre-digested set of ideological messages which they are encouraged to use both as inspiration for their own work and in the interpretation of any works by other artists they are shown.

This essay will begin by asking what role the teaching of art plays in a liberal education, and will then go on to suggest what approaches to the teaching of both art practice and art history, theory and criticism might best serve those ends. It will propose that art practice develops the intellect and the imagination in ways different from but complementary to other disciplines such as mathematics and languages, and that art history expands our understanding of human experience in a way comparable to the study of literature and music.

Art, in its many manifestations, means different things to different countries in the modern world: in Japan, for example, the aesthetic sensibility remains central to cultural life; in Italy, the art and architecture of past centuries is a source of pride even to uncultivated people, and opera remains a kind of popular music; the French take great pride in their literature and cinema, and in Iran such poets as Hafez and Sa'adi are loved by learned and unlearned alike.

In the British tradition, literature has always been richer and more highly regarded than either music or painting and sculpture, and perhaps that is why the English-speaking countries have comparatively little appreciation of the visual arts even today. We have biennales and art fairs, but these contrived