Proof in Mathematics
An Introduction

James Franklin and Albert Daoud

This book provides a short and straightforward introduction to the essential core of mathematics: proof. The book features:

- Brief discussions of the nature and necessity of proof
- Simple explanations of the basic proof techniques
- Immediate application to familiar mathematical material
- Numerous graded exercises
- Fully worked solutions to selected exercises
- A compelling, clear presentation

Introduction to Proofs in Mathematics (Prentice Hall of Australia, 1988), by the same authors, was warmly received around the world:

"Delightfully written... "
- *Mathematics Teacher* (USA) 82, (December 1989)

"The language is easy to read and stimulating... A definite step in the right direction."
- *The Mathematical Gazette* (UK), vol. 73 (Oct 1989)

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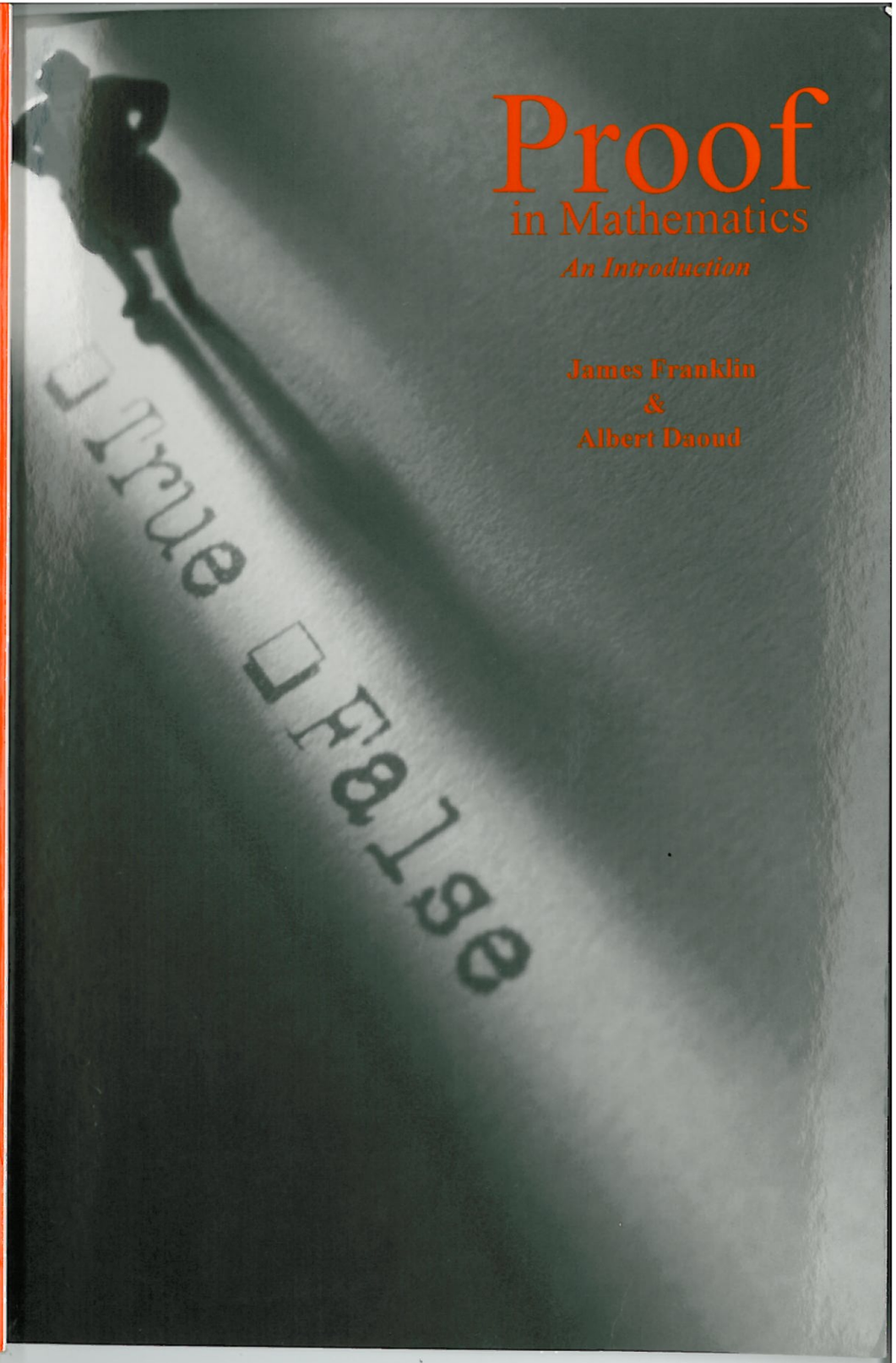

PROOF in MATHEMATICS

An Introduction

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Preface

University teachers of mathematics agree that their students' lack of knowledge is not uniformly distributed across topics, proofs being, however, usually at the head of the list of problems. Why do students take the instruction, "Prove ... " in examinations to mean, "Go to the next question"?

Students of mathematics and computing need to learn how to understand and construct proofs. Proofs are central to mathematics—and more so as the level of mathematics rises. In computer science, it has become clear with the increased importance of software that a computer program is a logical object, the understanding of which requires the same tools as mathematical proof. Proof, however, has not been well served by textbooks. While students do not have the background their teachers had in school of deductive geometry, nothing has replaced Euclid. Textbooks on mathematics for computer science are plentiful, but, as stated in *Carnegie-Mellon Curriculum for Undergraduate Computer Science* (ed. M. Shaw, Springer-Verlag, New York, 1985) on p. 81, their chief problem is "inadequate treatment of logic".

The present book aims to fill this gap. Some of its features are:

1. Techniques of proof are introduced in the context of mathematics, such as arithmetic, already familiar to the student.
2. The techniques are later applied to the usual topics of first-year university mathematics, such as linear algebra and calculus.
3. At all times, matters of context and motivation are kept in mind.
4. Issues of strategy and tactics in constructing proofs are emphasised.
5. The skills of setting out proofs to result in a convincing argument are insisted upon.
6. The continuity of proof with ordinary mathematical techniques, such as calculation, is respected.
7. There is no discussion of symbolic logic, as there are enough books on this subject already.
8. Since the propositional calculus has little importance in mathematical proof, it is barely mentioned. The quantifiers "all" and "some" are the central concern.

We owe a great debt to Jack Gray who read the manuscript in full and made many very valuable suggestions. Useful suggestions were also made by Rod James, John Loxton and David Hunt, as well as by Daniel Solow, whose book *How to Read and Do Proofs* (Wiley, New York, 1982) has broadly the same approach as this book. We would also like to thank Rose Gonzalvez for her fast and efficient typing of the manuscript.

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