# The Problem of the Relationships of Love, Hate and Indifference 

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#### Abstract

In Franceschi (2002), I presented a theory based on the matrices of concepts aiming at providing an alternative to the classification proposed by Greimas, in the field of paradigmatic analysis. The problem of specifying the relationships of the concepts of love, hate and indifference, arises in this construction. I attach myself to describe the problem of the love-hate-indifference relationships in detail, and several solutions that have been proposed to solve it. Finally I expose a solution to this problem, based on an extension of the theory of matrices of concepts.


I shall be concerned in this paper with presenting a problem related to the proper definition of the relationships of the following concepts: love, hate and indifference. I will describe first the problem in detail and some proposed solutions. Lastly, I will present my own solution to the problem.

## 1. The problem

The problem is that of the proper definition of the relationships of the concepts love, hate and indifference. Let us call it the LHI problem. What are then the accurate relationships existing between these three concepts? At first sight, the definition of the relation between love and hate is obvious. These concepts are contraries. The definition of such a relation should be consensual. Nevertheless, the problem arises when one considers the relationship of love and indifference, and of hate and indifference. In these latter cases, no obvious response emerges.
However, the issue needs clarifying. In this context, what should we expect of a solution to the LHI problem? In fact, a rigorous solution ought to define precisely the three relations $R, S, T$ such that love $R$ hate, love S indifference and hate T indifference. And the definitions of these relations should be as accurate as possible.
It is worth mentioning that several authors must be credited for having mentioned and investigated the LHI problem. In particular, it is worth stressing that the difficulties presented within propositional calculus by some assertions of the type $x$ loves $y$, $x$ hates $y$, or $x$ is indifferent to $y$ have been hinted at by Emile Benzaken (1990) ${ }^{1}$ :

Nevertheless, the difficulty can arise from pairs of words where the one expresses the contrary (negation) of the other; 'to hate' can be considered as the strong negation of 'to love', whereas 'to be indifferent' would be its weak negation.

The author exposes then the problem of the relationships of love/hatelindifference and proposes his own solution: hate is the strong negation of love, and indifferent is the weak negation of love.
However, it turns out that Benzaken's solution is unsatisfying for a logician, for the following reasons. On the one hand, this way of solving the problem defines the relations between love and hate (strong negation, according to the author) and between love and indifference (weak negation, on the author's view), but it fails to define accurately the relations existing between indifference and hate. There is a gap, a lack of response at

[^0]this step. And mentioned above, a satisfying solution should elucidate the nature of the relationships of the three concepts. On the other hand, the difference between weak negation and strong negation is not made fully explicit within the solution provided by Benzaken. For these reasons, Benzaken's solution to the LHI problem proves to be unsatisfying.
In a very different context, Rick Garlikov (1998) stresses some difficulties of essentially the same nature as those underlined by Benzaken:

In a seminar I attended one time, one of the men came in all excited because he had just come across a quotation he thought very insightful - that it was not hate that was the opposite of love, but that indifference was the opposite of love, because hate was at least still an emotion. I chuckled, and when he asked why I was laughing, I pointed out to him that both hate and indifference were opposites of love, just in different ways, that whether someone hated you or was indifferent toward you, in neither case did they love you.
Garlikov describes in effect the problem of the relationships of love/hate/indifference and implicitly proposes a solution of a similar nature as that provided by Benzaken. For this reason, Galikov's account suffers from the same defects as those presented by Benzaken's solution.

In what follows, my concern will be with settling first the relevant machinery, in order to prepare a few steps toward a solution to the LHI problem.

## 2. A framework

I will sketch here the formal apparatus described in more detail in Franceschi (2002). To begin with, consider a given duality. Let us denote it by $\mathrm{A} / \overline{\mathrm{A}}$. At this step, A and $\overline{\mathrm{A}}$ are dual concepts. Moreover, A and $\overline{\mathrm{A}}$ can be considered as concepts that are characterized by a contrary component $c \in\{-1,1\}$ within a duality $\mathrm{A} / \overline{\mathrm{A}}$, such that $c[\mathrm{~A}]=-1$ and $c[\overline{\mathrm{~A}}]=1$. Let us also consider that A and $\overline{\mathrm{A}}$ are neutral concepts that can be thus denoted by $\mathrm{A}^{0}$ and $\overline{\mathrm{A}}^{0}$.

At this point, we are in a position to define the class of the canonical poles. Consider then an extension of the previous class $\left\{\mathrm{A}^{0}, \overline{\mathrm{~A}}^{0}\right\}$, such that $\mathrm{A}^{0}$ and $\overline{\mathrm{A}}^{0}$ respectively admit of a positive and a negative correlative concept. Such concepts are intuitively appealing. Let us denote them respectively by $\left\{A^{+}, A^{-}\right\}$and $\left\{\bar{A}^{+}, \bar{A}^{-}\right\}$. At this step, for a given duality $A / \bar{A}$, we get then the following concepts: $\left\{\mathrm{A}^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{-}\right\}$. Let us call them canonical poles. It should be noted that one could use alternatively the notation $\alpha(\mathrm{A} / \overline{\mathrm{A}}, c, p)$ for a canonical pole. ${ }^{2}$ In all cases, the components of a canonical pole are a duality $\mathrm{A} / \overline{\mathrm{A}}$, a contrary component $c$ $\in\{-1,1\}$ and a canonical polarity $p \in\{-1,0,1\}$. This definition of the canonical poles leads to distinguish between the positive $\left(\mathrm{A}^{+}, \overline{\mathrm{A}}^{+}\right)$, neutral $\left(\mathrm{A}^{0}, \overline{\mathrm{~A}}^{0}\right)$ and negative $\left(\mathrm{A}^{-}, \overline{\mathrm{A}}^{-}\right)$canonical poles. Lastly, the class made up by the 6 canonical poles can be termed the canonical matrix: $\left\{\mathrm{A}^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{-}\right\}$.


Figure 1
Let us investigate now into the nature of the relations existing between the canonical poles of a given matrix. Among the combinations of relations existing between the 6 canonical poles $\left(\mathrm{A}^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{-}\right)$ of a same duality $\mathrm{A} / \overline{\mathrm{A}}$, it is worth emphasizing the following relations: duality, antinomy, complementarity,

[^1]corollarity, connexity, and anti-connexity. Thus, two canonical poles $\alpha_{1}\left(\mathrm{~A} / \overline{\mathrm{A}}, c_{1}, p_{1}\right)$ and $\alpha_{2}\left(\mathrm{~A} / \overline{\mathrm{A}}, c_{2}, p_{2}\right)$ of a same matrix are:
(i) dual if their contrary components are opposite and their polarities are neutral ${ }^{3}$
(ii) contrary (or antinomical) if their contrary components are opposite and their polarities are non-neutral and opposite ${ }^{4}$
(iii) complementary if their contrary components are opposite and their polarities are non-neutral and equal ${ }^{5}$
(iv) corollary if their contrary components are equal and their polarities are non-neutral and opposite ${ }^{6}$
(v) connex if their contrary components are equal and the absolute value of the difference of their polarities equals $1^{7}$
(vi) anti-connex if their contrary components are opposite and the absolute value of the difference of their polarities equals $1^{8}$
To sum up: $\left\{\mathrm{A}^{0}, \overline{\mathrm{~A}}^{0}\right\}$ are dual, $\left\{\mathrm{A}^{+}, \overline{\mathrm{A}}^{-}\right\}$and $\left\{\mathrm{A}^{-}, \overline{\mathrm{A}}^{+}\right\}$are contraries, $\left\{\mathrm{A}^{+}, \overline{\mathrm{A}}^{+}\right\}$and $\left\{\mathrm{A}^{-}, \overline{\mathrm{A}}^{-}\right\}$are complementary, $\left\{\mathrm{A}^{+}, \mathrm{A}^{-}\right\}$and $\left\{\overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{-}\right\}$are corollary, $\left\{\mathrm{A}^{0}, \mathrm{~A}^{+}\right\},\left\{\mathrm{A}^{0}, \mathrm{~A}^{-}\right\},\left\{\overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{+}\right\}$and $\left\{\overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{-}\right\}$are connex, $\left\{\mathrm{A}^{0}, \overline{\mathrm{~A}}^{+}\right\},\left\{\mathrm{A}^{0}, \overline{\mathrm{~A}}^{-}\right\},\left\{\overline{\mathrm{A}}^{0}, \mathrm{~A}^{+}\right\}$and $\left\{\overline{\mathrm{A}}^{0}, \mathrm{~A}^{-}\right\}$are anti-connex.

I shall focus now on the types of relations existing, under certain circumstances between the canonical poles of different dualities. Let us define preliminarily the includer relation. Let a concept $\alpha$ be an includer for two other concepts $\beta$ and $\chi$ if and only if $\alpha=\beta \vee \chi$. Such a definition captures the intuition that $\alpha$ is the minimal concept whose semantic content includes that of $\beta$ and $\chi$. To give an example concerning truthvalue, determinate is an includer for $\{$ true, false $\}$.
Let now $A$ and $E$ be two matrices whose canonical poles are respectively $\left\{\mathrm{A}^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{-}\right\}$and $\left\{\mathrm{E}^{+}\right.$, $\left.\mathrm{E}^{0}, \mathrm{E}^{-}, \overline{\mathrm{E}}^{+}, \overline{\mathrm{E}}^{0}, \overline{\mathrm{E}}^{-}\right\}$. These matrices are such that $\mathrm{E}^{+}, \mathrm{E}^{0}, \mathrm{E}^{-}$are the respective includers for $\left\{\mathrm{A}^{+}, \overline{\mathrm{A}}^{+}\right\},\left\{\mathrm{A}^{0}, \overline{\mathrm{~A}}^{0}\right\}$, $\left\{\mathrm{A}^{-}, \overline{\mathrm{A}}^{-}\right\}$i.e. the two matrices are such that $\mathrm{E}^{+}=\mathrm{A}^{+} \vee \overline{\mathrm{A}}^{+}, \mathrm{E}^{0}=\mathrm{A}^{0} \vee \overline{\mathrm{~A}}^{0}$ and $\mathrm{E}^{-}=\mathrm{A}^{-} \vee \overline{\mathrm{A}}^{-} .9$


Figure 2
Let us denote this relation by $\mathrm{A}<\mathrm{E}$. One is now in a position to extend the relations previously defined between the canonical poles of a same matrix, to the relations of a same nature between two matrices presenting the properties of A and E , i.e. such that $\mathrm{A}<\mathrm{E}$. The relations of 2-duality, 2-antinomy, 2-

[^2]complementarity, 2-anti-connexity ${ }^{10}$ ensue then straightforwardly. Thus, two canonical poles $\alpha_{1}\left(\mathrm{~A} / \overline{\mathrm{A}}, c_{1}, p_{1}\right)$ and $\alpha_{2}\left(\mathrm{E} / \overline{\mathrm{E}}, c_{2}, p_{2}\right)$ of two different matrices are:
(i') 2-dual (or trichotomic dual) if their polarities are neutral and if the dual of $\alpha_{2}$ is an includer for $\alpha_{1}$
(ii') 2-contrary ${ }^{11}$ (or trichotomic contrary) if their polarities are non-neutral and opposite and if the contrary of $\alpha_{2}$ is an includer for $\alpha_{1}$
(iii') 2-complementary (or trichotomic complementary) if their polarities are non-neutral and equal and if the complementary of $\alpha_{2}$ is an includer for $\alpha_{1}$
(vi') 2-anti-connex (or trichotomic anti-connex) if the absolute value of the difference of their polarities is equal to 1 and if the anti-connex of $\alpha_{2}$ is an includer for $\alpha_{1}$

To sum up now: $\left\{\mathrm{A}^{0}, \overline{\mathrm{E}}^{0}\right\}$ and $\left\{\overline{\mathrm{A}}^{0}, \overline{\mathrm{E}}^{0}\right\}$ are 2-dual, $\left\{\mathrm{A}^{+}, \overline{\mathrm{E}}\right\},\left\{\mathrm{A}^{-}, \overline{\mathrm{E}}^{+}\right\},\left\{\overline{\mathrm{A}}^{+}, \overline{\mathrm{E}}\right\}$ and $\left\{\overline{\mathrm{A}}, \overline{\mathrm{E}}^{+}\right\}$are 2-contrary, $\left\{\mathrm{A}^{+}, \overline{\mathrm{E}}^{+}\right\},\left\{\mathrm{A}^{-}, \overline{\mathrm{E}}\right\},\left\{\overline{\mathrm{A}}^{+}, \overline{\mathrm{E}}^{+}\right\}$and $\{\overline{\mathrm{A}}, \overline{\mathrm{E}}\}$, 2 are 2 complementary, $\left\{\mathrm{A}^{0}, \overline{\mathrm{E}}^{+}\right\},\left\{\mathrm{A}^{0}, \overline{\mathrm{E}}^{-}\right\},\left\{\overline{\mathrm{A}}^{0}, \overline{\mathrm{E}}^{+}\right\}$and $\left\{\overline{\mathrm{A}}^{0}, \overline{\mathrm{E}}\right\}$ are 2-anti-connex.
Lastly, the notion of a complement of a canonical pole also deserves mention. Let $\alpha$ be a canonical pole. Let us denote by $\sim \alpha$ its complement, semantically corresponding to non- $\alpha$. In the present context, the notion of a complement entails the definition of a universe of reference. I shall focus then on the notion of a complement of a canonical pole defined with regard to the corresponding matrix. In this case, the universe of reference is equal to $\left\{\mathrm{A}^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}\right\}$ and then $\sim \alpha=\left\{\mathrm{A}^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}\right\}-\alpha$. On has thus for example $\sim \mathrm{A}^{+}=\left\{\mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}\right\}$ and a similar definition for the complements of the other canonical poles of the matrix. Consider now two matrices such that $\mathrm{A}<\mathrm{E}$. Under these circumstances, the universe of reference ${ }^{12}$ is equal to $\left\{\mathrm{A}^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{-}, \overline{\mathrm{E}}^{+}, \overline{\mathrm{E}}^{0}, \overline{\mathrm{E}}\right\}$. Call it the 2-matrix of $\alpha$. It ensues that $\sim \alpha=\left\{\mathrm{A}^{+}\right.$, $\left.\mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}, \overline{\mathrm{~A}}^{-}, \overline{\mathrm{E}}^{+}, \overline{\mathrm{E}}^{0}, \overline{\mathrm{E}}^{-}\right\}-\alpha$. We have then the notion of a 2 -complement of a canonical pole $\alpha$, defined with regard to a universe of reference consisting of the 2 -matrix of $\alpha$. More generally, one has the notion of a $n$-complement ( $n>0$ ) of a canonical pole with regard to the corresponding $n$-matrix.

## 3. A solution

With the relevant machinery in place, we are now in a position to present a solution to the LHI problem. Let us now analyze the problem in the light of the above framework. To begin with, let us analyze the relevant concepts in more detail. The concept love has a positive connotation. It is a meliorative concept that can be denoted by love ${ }^{+}$. Conversely, the concept hate has a negative connotation. It is a pejorative concept that can be rendered by hate ${ }^{-}$. Similarly, the concept indifference also has a negative connotation. It can be considered a pejorative notion that can be denoted by indifference ${ }^{-}$.
At this step, a difficulty emerges. In effect, it should be stressed that the three concepts are either meliorative or pejorative at a certain degree. And such a degree might be different from one concept to another. For example hate- might be pejorative at a 0.95 degree, while indifference- might be pejorative at a lesser degree of 0.7 . Moreover, it could be said that such a degree might vary from culture to culture, from a given language to another. In sum, the meliorative or pejorative degree of the three concepts, so the objection goes, could be culture-relative.
Nevertheless, such difficulties can be avoided in the present context, since our reasoning will not bear upon the concepts inherent to a specific culture or language, but rather on the canonical concepts described above. Accordingly, we shall replace our usual concepts by the corresponding canonical concepts. There is room for variation in degrees, from culture to culture in the usual concepts of love, hate and indifference. But this point does not affect the current line of reasoning, since it only focuses on canonical concepts. The passage from the non-canonical concepts to the canonical ones goes straightforwardly as follows. Let $d[\alpha]$ be the pejorative or meliorative degree of a concept $\alpha$. Hence if $d[\alpha] \in] 0.5 ; 1]$ then $p[\alpha]=1$ else if $d[\alpha] \in[-1$; $-0.5[$ then $p[\alpha]=-1$. At this point, one can pose legitimately that $p[$ Love $]=1, p[$ Hate $]=-1$ and $p[$ Indifference $]=-1^{13}$. As a result, the three concepts can be denoted by Love ${ }^{+}$, Hate ${ }^{-}$, Indifference ${ }^{-}$.

[^3]As noted from the beginning, the relationship of love/hate is unproblematic and identifies itself with the relation of contrary. This applies straightforwardly to the relationship of the canonical concepts Love $^{+} /$Hate $^{-}$. Hence, the corresponding matrix has the following structure: $\left\{\right.$Love $^{+}, \mathrm{A}^{0}, \mathrm{~A}^{-}, \overline{\mathrm{A}}^{+}, \overline{\mathrm{A}}^{0}$, Hate $\}$. Now the next step is the reconstitution of the complete matrix. This task can be accomplished with the help of the definition of the relations of the canonical poles, namely: $\mathrm{A}^{-}$is corollary to Love ${ }^{+}, \overline{\mathrm{A}}^{+}$is corollary to Hate, $\mathrm{A}^{0}$ is connex to Love ${ }^{+}$and anti-connex to Hate ${ }^{-} \overline{\mathrm{A}}^{0}$ is connex to Hate and anti-connex to Love ${ }^{+}$. Given these elements, we are now in a position to reconstitute the corresponding canonical matrix: \{Love ${ }^{+}$, Attraction ${ }^{0}, \mathrm{~A}^{-}$, Defiance $^{+}$, Repulsion ${ }^{0}$, Hate $\left.{ }^{-}\right\} . .^{14}$


Figure 3
Let us examine now the case of the concept Indifference ${ }^{-}$. Such a concept inserts itself into a matrix the structure of which is: $\left\{\mathrm{E}^{+}, \mathrm{E}^{0}, \mathrm{E}^{-}, \overline{\mathrm{E}}^{+}, \overline{\mathrm{E}}^{0}\right.$, Indifference $\}$. Just as before, it is now necessary to reconstitute the complete matrix. This can be done with the help of the corresponding definitions: $\overline{\mathrm{E}}^{+}$is corollary to Indifference, $\mathrm{E}^{-}$is complementary to Indifference, $\mathrm{E}^{+}$is contrary to Indifference-, $\overline{\mathrm{E}}^{0}$ is connex to Indifference and to the corollary of Indifference, $\mathrm{E}^{0}$ is anti-connex to Indifference- and to the corollary of Indifference- ${ }^{-}$The associated matrix is then: $\left\{\mathrm{E}^{+}\right.$, Interest $^{0}, \mathrm{E}^{-}$, Phlegm $^{+}$, Detachment ${ }^{0}$, Indifference $\left.{ }^{-}\right\} .{ }^{15}$


Figure 4
It should be observed now that Interest $t^{0}=$ Attraction $^{0} \vee$ Repulsion ${ }^{0}$ i.e. that Interest $^{0}$ is an includer for Attraction ${ }^{0}$ and Repulsion ${ }^{0}$. At this step, given that $\left\{\right.$ Love $^{+}$, Attraction ${ }^{0}$, A ${ }^{-}$, Repulsion ${ }^{+}$, Repulsion ${ }^{0}$, Hate $\left.{ }^{-}\right\}<$ $\left\{\mathrm{E}^{+}\right.$, Interest ${ }^{0} \mathrm{E}^{-}$, Phlegm ${ }^{+}$, Detachment ${ }^{0}$, Indifference $\left.{ }^{-}\right\}$, the relationship of Love ${ }^{+} /$Indifference ${ }^{-}$and Hate-/Indifference- now apply straightforwardly. In effect, it ensues from the above definitions that, on the

[^4]one hand, Love ${ }^{+}$and Indifference ${ }^{-}$are trichotomic contraries and on the other hand, Hate ${ }^{-}$and Indifference ${ }^{-}$ are trichotomic complementaries. At this point, one is finally in a position to formulate a solution to the LHI problem:
(i) love is contrary to hate
(ii) love is 2-contrary to indifference
(iii) hate is 2 -complementary to indifference

Hence, $R, S, T$ identify respectively themselves with contrary, trichotomic contrary, trichotomic complementarity.

## 4. Concluding remarks

At this point, it is tempting not to consider the above analysis as a solution to the LHI problem per se. In effect, the concepts love, hate and indifference seem to be instances of a wider class of concepts whose relationships are of the same nature. This suggests that the same type of solution should be provided to the general problem of the definition of the relations of three given concepts $\alpha, \beta, \chi$. At first sight, certain concepts such as true, false and indeterminate, fall under the scope of the current analysis. Nevertheless, such a claim should be envisaged with caution. To what extent does the present analysis apply to other concepts? This is another problem that needs to be addressed, but whose resolution goes beyond the scope of the present account. ${ }^{16}$

## References

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Garlikov, Rick (1998). "Understanding, Shallow Thinking, and School". At
http://www.garlikov.com/writings.htm

[^5]
[^0]:    ${ }^{1}$ My translation. The original text is as follows: 'La difficulté cependant peut provenir de paires de mots dont l'un exprime le contraire (négation) de l'autre; "haïr" peut être pris comme la négation forte de "aimer" tandis que "être indifférent" en serait la négation faible'. (p. 63).

[^1]:    ${ }^{2}$ With the latter notation, the matrix of the canonical poles is rendered as follows: $\{\alpha(\mathrm{A} / \overline{\mathrm{A}},-1,1), \alpha(\mathrm{A} / \overline{\mathrm{A}},-1,0), \alpha(\mathrm{A} / \overline{\mathrm{A}}$, $-1,-1), \alpha(\mathrm{A} / \overline{\mathrm{A}}, 1,1), \alpha(\mathrm{A} / \overline{\mathrm{A}}, 1,0), \alpha(\mathrm{A} / \overline{\mathrm{A}}, 1,-1)\}$.

[^2]:    ${ }^{3}$ Formally $\alpha_{1}$ and $\alpha_{2}$ are dual if and only if $c\left[\alpha_{1}\right]=-c\left[\alpha_{2}\right]$ and $p\left[\alpha_{1}\right]=p\left[\alpha_{2}\right]=0$.
    ${ }^{4}$ Formally $\alpha_{1}$ and $\alpha_{2}$ are antinomical if and only if $c\left[\alpha_{1}\right]=-c\left[\alpha_{2}\right]$ and $p\left[\alpha_{1}\right]=-p\left[\alpha_{2}\right]$ with $p\left[\alpha_{1}\right], p\left[\alpha_{2}\right] \neq 0$.
    ${ }^{5}$ Formally $\alpha_{1}$ and $\alpha_{2}$ are complementary if and only if $c\left[\alpha_{1}\right]=-c\left[\alpha_{2}\right]$ and $p\left[\alpha_{1}\right]=p\left[\alpha_{2}\right]$ with $p\left[\alpha_{1}\right], p\left[\alpha_{2}\right] \neq 0$.
    ${ }^{6}$ Formally $\alpha_{1}$ and $\alpha_{2}$ are corollary if and only if $c\left[\alpha_{1}\right]=c\left[\alpha_{2}\right]$ and $p\left[\alpha_{1}\right]=-p\left[\alpha_{2}\right]$ with $p\left[\alpha_{1}\right], p\left[\alpha_{2}\right] \neq 0$.
    ${ }^{7}$ Formally $\alpha_{1}$ and $\alpha_{2}$ are connex if and only if $c\left[\alpha_{1}\right]=c\left[\alpha_{2}\right]$ and $\left|p\left[\alpha_{1}\right]-p\left[\alpha_{2}\right]\right|=1$.
    ${ }^{8}$ Formally $\alpha_{1}$ and $\alpha_{2}$ are anti-connex if and only if $c\left[\alpha_{1}\right]=-c\left[\alpha_{2}\right]$ and $\left|p\left[\alpha_{1}\right]-p\left[\alpha_{2}\right]\right|=1$.
    ${ }^{9}$ It should be observed that one of the three conditions is sufficient. In effect, $\mathrm{E}^{+}=\mathrm{A}^{+} \vee \overline{\mathrm{A}}^{+}$entails $\mathrm{E}^{0}=\mathrm{A}^{0} \vee \overline{\mathrm{~A}}^{0}$ and $\mathrm{E}^{-}=$ $\mathrm{A}^{-} \vee \overline{\mathrm{A}}^{-} ; \mathrm{E}^{0}=\mathrm{A}^{0} \vee \overline{\mathrm{~A}}^{0}$ implies $\mathrm{E}^{+}=\mathrm{A}^{+} \vee \overline{\mathrm{A}}^{+}$and $\mathrm{E}^{-}=\mathrm{A}^{-} \vee \overline{\mathrm{A}}^{-} ; \mathrm{E}^{-}=\mathrm{A}^{-} \vee \overline{\mathrm{A}}^{-}$entails $\mathrm{E}^{0}=\mathrm{A}^{0} \vee \overline{\mathrm{~A}}^{0}$ and $\mathrm{E}^{+}=\mathrm{A}^{+} \vee \overline{\mathrm{A}}^{+}$.

[^3]:    ${ }^{10}$ The generalisation to $n$ matrices $(n>1)$ of the present construction ensues, with the relations of $n$-duality, $n$-antinomy, $n$-complementarity, $n$-anti-connexity.
    ${ }^{11}$ Or 2-antinomical.
    ${ }^{12}$ In this context, $\mathrm{E}^{+}, \mathrm{E}^{0}$ and $\mathrm{E}^{-}$can be omitted without loss of content, given their nature of includers.
    ${ }^{13}$ The fact of considering alternatively $p$ [indifference] $>-0.5$ and thus $p$ [Indifference] $=0$ also leads to a solution in the present framework. In this last case, the relations $S$ and $T$ both identify themselves with trichotomic anti-connexity.

[^4]:    ${ }^{14}$ In the process of reconstitution of the complete matrix, some concepts may be missing. The reason is that they are not lexicalized in the corresponding language. This is notably the case for $\mathrm{A}^{\text {: }}$. This last concept semantically corresponds to inappropriate, excessive attraction.
    ${ }^{15}$ As far as I can see, the concepts associated with $\mathrm{E}^{+}$and $\mathrm{E}^{-}$are not lexicalized. They respectively correspond to appropriate interest and inappropriate, excessive interest.

[^5]:    ${ }^{16}$ I thank Professor Claude Panaccio and Rick Garlikov for useful comments on an earlier draft.

