Metaphysical Underdetermination as a Motivational Device*

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Abstract

The view that quantum particles cannot be regarded as individuals was articulated in the early days of the 'quantum revolution' and became so well-entrenched that French and Krause (2006) called it 'the Received View'. However it was subsequently shown that quantum statistics is in fact compatible with a metaphysics of particle individuality, subject to certain caveats. As a consequent it has been claim that there exists a kind of underdetermination of the metaphysics by the physics which in turn has been used to motivate a form of 'notice' structural realism (Ladyman 1998; French 2014). In this essay I will review this purported underdetermination and the motivation for structural realism that it purportedly provides in the context of recent developments in both the philosophy of physics (specifically the work of Saunders) and metaphysics (specifically the work of Dasgupta). I aim to conclude that such developments reinforce the underdetermination and allow one to respond to certain critical concerns regarding its motivational power.

Introduction

As noted in (French and Krause 2006) the history of quantum statistics is the history of quantum mechanics itself, beginning with Planck's classic 1901 paper in which he extended Boltzmann's 'Combinatorial' approach from the statistical mechanics of gas atoms to the oscillators of a black body. However, in considering the distribution of energy quanta over oscillators[1], Planck used a different form of counting than Boltzmann, something that puzzled the latter's student, Ehrenfest, among others. Clarification was only achieved years later with the emergence of Bose-Einstein and Fermi-Dirac statistics and the associated formal treatment of the counting involved (again see French and Krause 2006, Ch. 3). Just in case there happens to be anyone reading this who is not aware of that difference between classical and quantum statistics, here is a simple example: consider the distribution of two particles over two energy states. In classical, or Maxwell-Boltzmann statistics, four arrangements are then counted: both particles in one state, both particles in the other state and one particle in each state which is given a double weight because it was deemed that a new arrangement could be formed by permuting the particles between the states. The probability of finding both particles in a particular state, then, is ¼. In the case of quantum statistics, crucially, a permutation of the particles is not counted as yielding a new arrangement. That means for Bose-Einstein statistics, the probability of finding both particles in a particular state rises to 1/3; with

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[1] As Kuhn (1978) notes it was Einstein who extended this to the electromagnetic radiation itself in the 1905 paper that won him the Nobel prize.

Fermi-Dirac statistics, of course, for which particles are excluded from being in the same state, the probability is 0.

The 'Received View': Quantum Particles as Non-Individuals

The reaction from the 'quantum revolutionaries' was immediate: Born and Heisenberg declared that the particles could no longer be considered to be distinct individuals (again, see French and Krause 2006, p. 106) and Schrödinger subsequently wrote that ". . . we have . . . been compelled to dismiss the idea that . . . a particle is an individual entity which retains its 'sameness' forever. Quite the contrary, we are now obliged to assert that the ultimate constituents of matter have no 'sameness' at all" (Schrödinger 1996, p.121)[2]

How did they arrive at this conclusion? One can think of their reasoning as a form of modus tollens: Boltzmann explicitly took as one of the premises of his approach the claim that the particles – gas atoms in his case – were individuals (French and Krause 2006, Ch. 2) and on that basis derived the classical Maxwell-Boltzmann statistics as sketched above; as also noted above, these incorporated the counting of permutations of particles between states; in quantum statistics, whether Bose-Einstein or Fermi-Dirac, these permutations are not counted; hence the particles cannot be regarded as individuals. One can also think in terms of a metaphysical explanation: just as the explanation for counting permutations in the classical case is that the particles are individuals, so that for not counting them in quantum statistics is that the particles are not individuals.

This became the established or 'received' view (once again, see French and Krause 2006, Ch. 3), whether due to the status of its advocates or the apparently straightforward nature of the above reasoning. However, the nature of this non-individuality remained unclear. At best, certain metaphors would be used: Post, in his radio broadcast that was one of the very few discussions of the topic at the time (the transcript of which was published as Post 1963), invited the listener to think of two umbrellas that, however apparently indiscernible they might be, could still be discerned by some nick or scratch and then insisted that quantum particles were indiscernible in an even stronger sense. Hesse offered the example of money in a bank account (Hesse 1963), for which we can say there are £*n*, say, without being able to point to which specific £ in the bank's accounts ours correspond to (Teller used the same metaphor many years later; Teller 1995).[3]

Such metaphors are interesting, not least because they suggest that by virtue of not being regarded as individuals, quantum particles should also not be regarded as *objects*, at least not in the sense that we think of umbrellas and £ coins as objects. This indeed was the conclusion also drawn by Cassirer and Eddington, who, also noting the work of Born and Heisenberg (Cassirer 1936, p. 184; Eddington 1936), used it to motivate their respective forms of structuralism (neo-Kantian in the former case, 'subjective' in the latter). Again reconstructing and thereby reshaping the core of their arguments, we can characterise it simply

[2] There is, of course, an issue of transtemporal identity here as well but I shall not be considering that.

[3] There is an issue here regarding the delineation of the category 'object'. Broadly we take it to involve a certain form of repeatability and a spatio-temporal profile, broadly understood. (Thanks to Claudio Calosi for this; also see Quinton 1973).

as follows: to be an object, something must be an individual; quantum particles are not individuals; hence they cannot be objects. Of course, that still leaves open the question of what they are, then and Cassirer's response is famously captured in the claim that *if* we insist on talking, in everyday language, about electrons as objects – perhaps because, at that time, we seemed to lack the logico-linguistic resources to do otherwise – then we can do so 'only indirectly', '... not insofar as they themselves, as individuals, are given, but so far as they are describable as "points of intersection" of certain relations' (Cassirer *op. cit.*).

This reconceptualisation of electrons as nodes in a network of relations acts as an entrance into the broader framework of Cassirer's structuralism, according to which 'higher-order' principles, such as those pertaining to causality and symmetry and so forth, laws and specific measurement outcomes are intertwined into a kind of Parmenidean whole. Together with a renewed appreciation of Cassirer's place in the history of twentieth century philosophy has come a reappraisal of both his and Eddington's contributions to the history of structuralist thought in particular (French 2014 Ch. 4). Standard accounts of that history, with their focus on the retention of structural elements of theories, as represented by the relevant laws for example, typically begin with Poincaré, and perhaps Duhem, mention Russell and the infamous Newman problem, before passing through Maxwell and then ending with Worrall. And also typically, throughout such historical studies, little if any mention is made of quantum physics – Worrall just mentions it at the end of his classic paper as, basically, work for the future and although Russell does go into more detail, in the context of his 'upward path' (Psillos 2001) to structuralism, his book was written just before the development of quantum statistics and hence before what Cassirer and Eddington took to be their implication for objects became clear. However, as Eddington insisted, in his famous debate with Braithwaite where he reflected on Russell's account (French 2014 Ch. 4), once those features are taken on board, the shape of the structuralist project becomes entirely different.

Having briefly outlined the relevant history, I should emphasise that, despite claims that have been made to the contrary, such features, in themselves, do *not* provide the motivation for the form of structuralism that I advocate, namely, ontic structural realism. And this is for two reasons.

First of all, it is not the case that it follows from the claim that quantum particles are not individuals that they cannot be considered to be objects, in the usual sense. This is clearly demonstrated by the development of quasi-set theory by Krause and his collaborators (see for example Krause 2010 and Arenhart 2017). Its application to quantum particles hinges on the idea that we can understand non-individuality in terms of the claim that self-identity is not well defined for such particles. Following the quote cited above, Schrödinger continued,

"I beg to emphasize this and I beg you to believe it: It is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of 'sameness', of identity, really and truly has no meaning". (Schrödinger 1996, pp. 121-122).

da Costa used this to motivate the development of a form of non-reflexive logic, that is, a logic where the principle of identity—<for all>*x (x = x)*—is not generally valid. Quasi-set theory provides a semantics for such a logic by extending conventional set theory to cover statements of identity that are not well formed for all set theoretic axioms. Within the theory one can derive quantum statistics and it can be extended further to cover quantum field theory (French and Krause Ch. 9). The point is, quasi-set theory demonstrates that we can formally characterize non-individual quantum particles as objects and so that particular implication of quantum statistics does not compel us to make the move to structuralism, as urged by Cassirer and Eddington.

Secondly, it is also not the case that it follows from quantum statistics that quantum particles must be regarded as non-individuals. One can, in fact, continue to regard them as individuals, a conclusion that was reached independently by a number of people (French 1989a; Huggett 1999; van Fraassen 1991).

Quantum Particles as Individuals

One way of seeing this is to dig a little deeper into the simple two-particles-and-two-states example given above. The crucial factor, as emphasized there, is the role of permutations in determining the arrangements to be counted. This can be captured mathematically by the permutation group and the non-counting of permuted arrangements is then accommodated by what is now recognized as the permutation symmetry of quantum mechanics (see, for example, French and Rickles 2003). If we examine more closely the action of the permutation group we can interpret it as imposing accessibility restrictions on the relevant Hilbert space, such that the space comes to be divided into Bose-Einstein and Fermi-Dirac sectors (there are also others, corresponding to so-called paraparticle statistics that do not appear to be manifested in nature). Particles that occupy the Bose-Einstein sector – namely bosons – cannot transition into the Fermi-Dirac sector; that is, they cannot become fermions, nor vice versa. In other words, the Fermi-Dirac sector is inaccessible to bosons and likewise, the Bose-Einstein sector of Hilbert space is inaccessible to fermions and it is this restriction on accessibility that explains the change in weight assigned to the relevant arrangements between the classical and quantum situations (French 1989a). This offers an entirely different explanation for the difference between classical and quantum statistics and there is no need to invoke any form of non-individuality.[4]

Of course, that does not mean that the treatment of quantum particles as individual objects is entirely straightforward. Consider the question: what is the ground (to use a fashionable term) of this individuality? One option would be to go for something like 'haecceity', understood as that which makes something the thing that it is, or primitive thisness, which as the name suggests, posits this

[4] One can then push the explanans back a step: what is that explains the accessibility constraints? One option would be to argue that they are to be explained in terms of the nature of bosons and fermions respectively and given Dorr's claim that identity claims provide the bedrock of explanations, this would be an appropriate explanatory stopping point (Dorr 2016; I am grateful to Claudio Calosi for suggesting this). However, leaving aside the essentialist character of such a suggestion, this effectively yields different explanations for each kind of particle. A more unificatory explanation is provided through the appeal to permutation symmetry (French and Rickles 2003), understood as a feature of the structure of the world (see later).

ground as something primitive.[5] From a naturalistic perspective that seeks to reduce the number of such primitive metaphysical items where we can, the former might not look like such an attractive option. The latter is naturalistically more acceptable insofar as nothing further is added, metaphysically, over and above the existence of the individual concerned. Still, a less inflationary alternative would be to seek the basis for individuality in the properties of the object, perhaps in the context of the so-called 'bundle' view of the latter which takes objects to be nothing but bundles of their properties. On this view, instead of a metaphysics of objects that are individuals, primitively or otherwise, over and above their properties, there are only the bundles of properties. However, given the multiple instantiability of properties, Leibniz's Principle of Identity of Indiscernibles is typically then appealed to in order to guarantee that no two such objects can share the same set of properties.

However, as is now well-known, Leibniz's Principle does not fare so well in the quantum context (French and Redhead 1988; Massimi 2001). Of course, if you've already decided to opt for the non-individuals package, then the issue of the status of the Principle is moot, a point that unfortunately has also sometimes been missed in subsequent discussions (French and Redhead *op. cit.*). It is only when the individuality option has been taken that the question arises whether the Principle can be sustained in the context of modern physics and unfortunately the answer is that it cannot, *if* we adopt the 'standard' form of both quantum mechanics and the principle itself (French and Redhead *ibid.*; I will return to this caveat shortly). This result then blocks the reductive move of seeking the ground of individuality in the properties of the object and leaves the alternative, naturalistically distasteful options of having to appeal to haecceities, or something like Lockean substance or some other metaphysical device in order to make metaphysical sense of this package.

Let us pause and review: quantum statistics supports two, metaphysically quite different, packages – according to one, quantum particles can be regarded as non-individual objects where this non-individuality can be formally understood in terms of quasi-set theory; according to the other, quantum particles can still be regarded as individuals, where that individuality is grounded in some form of primitive thisness or other metaphysical device going beyond the properties of the particles. Thus we have a form of 'metaphysical underdetermination', according to which the physics supports two different fundamental metaphysical options.

Van Fraassen's Challenge

Should we be bothered by this underdetermination? Van Fraassen thinks so. The concluding section of his book, setting out the above landscape as he sees it (1991), is entitled 'Goodbye to Metaphysics', by which he really means 'Goodbye to Metaphysically Informed Realism'. The undermining of realism can be unpacked as follows: the realist affirms a belief in the existence of electrons, say. Implicit in this belief is the view of electrons as objects (elsewhere I have referred to this as the 'object-oriented stance'; French 2014). That view only makes sense if the electrons, as objects, have a determinate 'individuality profile'

[5] These may be distinguished in that 'primitive' individuality is, as the term suggests, ungrounded, whereas the haecceity view grounds it in this further metaphysical concept (see Scarpati 2019; again I am grateful to Claudio Calosi for highlighting this distinction).

(Brading and Skiles 2012). However, the above considerations demonstrate that they do not have such a determinate profile; or at least, not one that is naturalistically grounded.

How might the realist respond? One option is to cut the argument off before it really gets going by insisting that belief in the existence of electrons does not involve belief in them *as objects*, or indeed, any other metaphysical category of entity. This eschewal of a metaphysical informing of her position would leave the realist at the 'shallow' end of the metaphysical range (Magnus 2012) and indeed, there have been moves recently to reconfigure realism in almost entirely epistemic terms (Saatsi 2017a). Thus one might insist that all we should say is that electrons are whatever science says they are – particles, fields, end-points of strings or whatever – and that the job of the realist is just to situate our 'best' (however that is determined) theories within an appropriate philosophical framework involving relevant notions of truth, reference, progress and so on.

However, one might feel that there is more to the job than this! So, one might follow Chakravartty in demanding that the realist provide a 'clear picture' of how the world is (Chakravartty 2007, p. 26) and, further, that the clarity of such a picture will be dependent on the extent to which it is metaphysically informed. This would be to slide down the spectrum towards 'deeper' forms of realism but how deep one goes will, again, depend on how much metaphysics one feels one has to appeal to in order to achieve a given level of clarity (see French 2018). So, van Fraasssen, at least, would presumably insist that if the realist is going to present a picture of electrons, say, as objects, she needs to make that picture clear by specifying whether *qua* objects they are individuals or not. But that she cannot do, at least not naturalistically in the sense of appealing to the relevant physics. Thus the underdetermination impacts any realist who seeks to go beyond the very 'shallowest' forms of the position towards some 'clear picture' informed by what might be taken to be the basic metaphysics of objecthood.[6]

Such considerations also help blunt the effect of an alternative response to the underdetermination, which is to throw up one's hands, as it were, and declare that forms of metaphysical underdetermination are ubiquitous, so the realist shouldn't worry about this one in particular.

Thus, for example, consider the metaphysics of properties. These may be regarded in terms of universals that are instantiated in particulars, or they may be understood as particulars themselves – as tropes say. The latter view also trifurcates, with some holding that tropes are abstract, others that they are concrete and still others maintaining that they are both abstract and concrete (for a useful overview see Maurin 2018) Now the realist believes that the electron 'has' (again in some metaphysical sense) properties such as (rest) mass, charge and spin. How metaphysically informed should her realism be in order to provide a clear picture of such properties? Should she be required to take a stance in the debate over whether tropes are abstract or concrete or both? That seems excessive and unnecessary in order to present a clear picture. On the

[6] Of course, one might cleave to a notion of object that goes beyond this 'basic metaphysics' but if such a notion is tied to a particular 'individuality profile' – by virtue of having a spatio-temporal location, say – it will be subject to this underdetermination. Again I am grateful to Claudio Calosi for his comments here.

other hand, it doesn't seem so extreme to press the realist on whether she takes properties to be tropes, in general, or universals. Perhaps some will insist that here too we have a clear enough picture without such metaphysical detail.

However, it can be argued that when it comes to electrons as *objects*, we surely have a less than clear picture if we cannot say whether they should be regarded as individuals or non-individuals. Here we seem to have moved back up the spectrum towards the shallow end and if we're not to end up holding a metaphysically attenuated form of realism, we should be able to say something on this issue. Supporting this argument is the point that the provision of such a picture, when it comes to objects at least, will involve not just providing an appropriate metaphysical framework but also a logical or more generally formal one. And in this case the latter frameworks are entirely different: when it comes to electrons regarded as individuals we can avail ourselves of standard set theory and classical logic (issues related to quantum mechanics notwithstanding) but if we take the electrons as non-individuals option, we should shift to quasi-set theory and Schrödinger logic. Certainly, as indicated above, prior to these formal developments, that option could only be articulated in terms of a metaphorical picture but now a degree of clarity has been achieved. The problem is, of course, that we can't take that particular clear picture to be how the world is, given the underdetermination.

There is more to say here of course, not least because this idea of providing a 'clear picture' may not itself be clear. How does informing one's realism with metaphysics provide a measure of clarity, for example? One possible answer is to suggest that it does so by relating the 'picture' we are concerned with, in quantum physics in this case, to what might be called more 'everyday' pictures. So, for example, by regarding electrons as objects and as individuals and taking that individuality as grounded in something like primitive thisness, we can relate that picture to our understanding (such as it is) of all kinds of objects as individuals more generally (Chakravartty refers to the 'umbrella' like nature of metaphysics in this regard). Of course, some would argue that it is through such attempts to metaphysically relate quantum entities to their 'everyday' counterparts that we have been led astray in our endeavours and that a properly naturalistic approach to modern physics should eschew such attempts entirely (Ladyman and Ross 2007, Ch. 1). However, this is a hard line to take and would culminate in a kind of 'bottom up' approach to metaphysics, effectively demanding that an appropriate metaphysical framework for quantum physics be constructed *ab initio*, something that is fraught with problems. Instead we might adopt a more moderate approach and treat metaphysics as a kind of toolbox from which we can appropriate various devices and techniques for our own purposes (French and McKenzie 2012, 2015; see also Bryant 2017), including that of providing a clear picture as Chakravartty suggests. Obviously that takes us away from a 'purely' naturalistic position but as just mentioned, that's problematic anyway and if we are going to leaven our realism with some metaphysics, it seems appropriate to acknowledge that we can do so along some kind of spectrum.

Of course, that doesn't in itself help us with the above metaphysical underdetermination, but within such a moderate approach an alternative set of responses can be obtained by appealing to some further factor to 'break' the underdetermination.

So, let us first consider what devices might be extracted from the metaphysical 'toolbox' and used to support the non-individuals package.

Breaking the Underdetermination1: Particles-as-Non-Individuals
Apparently independently of van Fraassen's work along similar lines, or indeed of the above context regarding individuality in physics in general[7], Dasgupta (2009) has argued for a 'revisionary metaphysics' that he calls 'generalism', that eschews 'individualistic' facts in favour of those that are only qualitative (for a comparison of generalism with ontic structural realism, see Glick 2018; French forthcoming)[8]. His argument is based on the claim that what he calls 'primitive individuals' are both empirically undetectable and physically redundant and hence can be dismissed as metaphysical 'danglers' or 'idlers' (Turner 2014). The argument turns on a central analogy that is drawn with absolute velocity in classical mechanics (Dasgupta op. cit., p. 37). Since according to the laws of Newtonian mechanics no measuring device could be constructed that would detect absolute velocity, he concludes that it is empirically undetectable. Furthermore, differences in absolute velocity at one time do not give rise to any other differences at later times and hence he maintains, it is physically redundant as well.

By analogy Dasgupta concludes that primitive individuals are also danglers, since, again according to the laws of Newtonian mechanics, two particles with the same mass, charge and so on, launched from the same spot with the same initial velocity and subject to the same forces will follow the same trajectory. Hence, differences in individualistic facts at a certain time do not give rise to any differences at later times and this is because the laws of Newtonian mechanics do not pertain to the individuality *per se* of the particles but only the relevant properties. Not only is the particles' individuality physically redundant, it is also empirically undetectable, since if, unbeknownst to us, a certain individual were permuted with another with exactly the same properties we could not tell the difference. Furthermore given the laws of physics it is impossible to build any device that could distinguish between these two situations, precisely because primitive individuals are physically redundant (*ibid*., pp. 42-43). Hence 'primitive individuality' should be eliminated.

My point in presenting Dasgupta's proposal here is to illustrate how a particular metaphysical device taken from the 'toolbox' may help render a certain naturalistically inclined package more acceptable: in this case we can appreciate how the above conclusions mesh nicely with the view of quantum particles as non-individuals. But of course, there are differences and also concerns that may be raised.

The most obvious difference is that the non-individuals package has been articulated in terms of quasi-set theory, whereas Dasgupta deploys an algebraic framework in which certain functors express the relevant features of the domain of properties without individuals: some express the conjunction and negation of properties, others express the permutation of properties, another captures the

[7] Turner (2014) in his response to Dasgupta does at least point to this context but associates it with 'quantum probabilities' without going into detail.
[8] Glick concludes that generalism does not provide an appropriate metaphysical framework for ontic structural realism because it cannot accommodate the motivations for the latter.

idea of partially 'filling' a property as in instantiating its first position and so on.[9] This affords a framework for his generalist metaphysics in which the fundamental facts of the world have the form P^0 obtains, where P^0 is a 0-place property that may be formed from more basic terms through the application of the above functors (2009, p. 53). Within this framework, statements that apparently refer to individuals, such as those we typically make about entities in the 'everyday' world can be accommodated through a form of error theory, according to which such statements are strictly false, or fictionalism, according to which they are true of the fiction that there are individuals, or reductionism, according to which they actually refer to more fundamental facts, such as those expressed in the generalist picture, and so on (*ibid*., p. 54). Of course, these might be seen as costs involved in using such a framework but Dasgupta insists that the ontological parsimony that results from eliminating primitive individuals more than compensates for such costs (*ibid*., p. 57).[10]

There are also obvious concerns having to do with Dasgupta's core argument and hence with its extension to the non-individuals case.[11] One might worry, for example, that the analogy with absolute velocity has led him astray. In this case, of course we're talking about a property not a primitive individual and as such it was only removed from physical theory after a great deal of theoretical and empirical effort, culminating in Einstein's theory of Special Relativity. You might think that individuality was likewise removed after considerable effort culminating in quantum mechanics but as we have seen, that's not quite the case – unlike absolute velocity and Special Relativity, individuality is compatible with quantum physics.[12] Furthermore, by focusing only on Newtonian mechanics, Dasgupta fails to take note of classical statistical mechanics, where, as noted above, individuality was assumed right from the start. In this case, Dasgupta's permutation argument does not go through: even though the objects concerned – gas atoms – are indiscernible such that we could not *tell* the difference, the permutation *makes* a difference insofar as it underpins the counting that lies at the heart of Maxwell-Boltzmann statistics. Dasgupta's argument is too quick in this respect.

Having said that, he could appeal to Saunders' argument that the so-called Gibbs Paradox involving the mixing of gases shows that individuality should be abandoned even in the context of classical statistical mechanics (Saunders 2006a).[13] Saunders' conclusion could then be recast in terms of Dasgupta's claim

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[9] Berto's, Bueno's and Jantzen's general concerns might equally apply to this framework, of course.

[10] There are further costs associated with the algebraic language that Dasgupta proposes (Turner 2014).

[11] Thus Glick (2018) argues that 'it's hard to see how the generalist can adopt the received view that takes only quantum particles to be 'non-individuals'' because the latter relies on 'distinctive features of indistinguishable quantum particles that are independent of the status of primitive individuals in general. ' (p. 15).

[12] Also, the redundancy of absolute velocity follows from the fact that Newton's Second Law involves second order derivatives whereas the argument from quantum statistics relies on no such features. One might be concerned that the former is less prone to interpretational worries than the latter but permutation symmetry holds whatever interpretation of quantum mechanics is chosen. Again, thanks to Claudio Calosi for encouraging me emphasise this point.

[13] Interestingly, Saunders also advocates a form of generalism; see Saunders, "The world is the totality of quantities, not of things", Presidential Address at the 2018 Meeting of the British Society for the Philosophy of Science.

that individuality is a 'dangler' even in classical mechanics. Still, that's not conclusive. One could, for example, take the paradox to be a kind of 'footprint' (in Post's sense; Post 1971), or retrospective intrusion within classical statistical mechanics of the forthcoming 'new' quantum statistics (French and Krause 2006, pp. 83-84). Here we bump up against issues of theory identity and specifically, where classical mechanics ends and quantum theory begins. Nevertheless, the point remains: there is a broader metaphysical framework available in terms of which we can make further sense of the non-individuals package.

However, one can also take metaphysical devices, moves and arguments out of the toolbox and deploy them against this package. Bueno, for example, (Bueno 2014) has argued that the notion of identity is fundamental and that it should not be given up without further reflection. In the case of the non-individuality package, in particular, he argues that concerns arise with regard to quantification. Certainly, without identity one cannot assign an ordinal to a collection of quantum particles but perhaps one can still say the collection has a kind of cardinality, in the sense that although one cannot specify them one by one as it were, one can still claim there are a certain number in the collection. However, one can ask how this cardinality might be measured. One suggestion (Domenech and Holik 2007) is to take such a collection, in the form of a helium atom say, and repeatedly ionize it. Counting the electron tracks produced then allows us to measure the cardinality of the collection of elections in the atom.[14] Unfortunately, Bueno argues, this assumes the identity of the electrons, lest we lose all possibility of taking the tracks to be those of distinct electrons to begin with. Hence identity cannot be dispensed with.[15]

This is an example of quite a general manoeuvre that may be (rather bluntly!) summarized as follows: take a metaphysical claim made in the context of quantum physics and raise concerns about it using an example explicated in a (broadly) classical context (a similar concern to Bueno's was raised by Chakravartty 2003). So, in the example above, once we have ionized the helium atom and are counting electron tracks we are no longer in the quantum 'domain' and by virtue of being allocated trajectories in space-time, the electrons can be taken to possess a form of 'spatio-temporal individuality' (French and Krause 2006, pp. 66-82). But of course, recalling the famous admonitions of Heisenberg et. al., we cannot uncontentiously take the electrons to be actually following such trajectories (unless we have already adopted the Bohm interpretation of

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[14] This of course is grossly simplified as stated; nevertheless, cardinality measurements are by necessity indirect: one might attribute n protons to a given nucleus (and hence n electrons to the atom), for example, by measuring the mass of the nucleus using a Penning trap, say (see http://aruna.physics.fsu.edu/ebss_lectures/F_Lecture3.pdf) and then knowing the mass of the proton and the effect of the strong nuclear force, infer the number of protons.

[15] Following Aristotle (and also as diverse a couple of philosophers as Auyang and Heidegger), Berto argues that 'identity amounts to unity' and shares the primitive nature of the latter (Berto 2017, p. 78). Permutation symmetry is then to be understood as an epistemological condition, with no metaphysical implications and indiscernible quantum objects understood as referred to only arbitrarily. All of this is simply to acknowledge the various aspects of the alternative metaphysical 'package' that takes such objects to be individuals, as discussed below. Berto insists that taking identity as primitive because it is one with 'being one' does not pre-judge the sort of comparison between such packages as I am engaged in here (*ibid*., p. 80). However, despite this insistence, it is hard to see how its identification of the concept with 'unity' does not beg the question against the non-individuals account. Thanks to Claudio Calosi for pointing me to Berto's paper.

quantum mechanics) and so this must be regarded as a form of 'mock' individuality, to use Toraldo di Francia's felicitous phrase (French and Krause 2006 pp. 230-231). To use this as the basis for attributing, within the quantum context, identity to the electrons is then illegitimate (Arenhart 2017).[16]

Of course, one might dispute the presumed distinction between quantum and classical 'domains', insisting perhaps that the world is quantum in its entirety, as it were. However, the clue to a response lies in the term 'mock' individuality – it is not being claimed that the electrons are individuals with identity in one 'domain' that is somehow lost in another; rather the idea is that there is only the appearance of individuality in the measurement context (both Born and Schrödinger were concerned about accounting for this appearance and attempted to account for it by drawing on Gestalt psychology; see French and Krause 2006 p. 120). What one should not do is take that appearance, base a certain metaphysics upon it (in this case, a metaphysics of identity as standardly understood) and then attempt to export that metaphysics beyond that specific measurement context in an attempt to apply it to the particles generally (for a more detailed response to Bueno's concern, see Arenhart and Krause 2019).

Granted the above, and the general principle of not begging the question against the non-individuals package by assuming, implicitly or otherwise, that for something to be an object it must be an individual object, one can also deploy certain devices from the metaphysical toolbox to support the particles-as-individuals alternative.

Breaking the Underdetermination2: Particles-as-Individuals

We recall that this package seems to require metaphysical grounding via some account of individuality. Of course, one could simply deny this requirement and take individuality to be primitive (see, for example, Morganti 2009). This, it has been claimed,

'…allows for a great methodological gain in terms of simplicity, clarity and conservativeness with respect to entrenched metaphysical beliefs and schemes, while being at least equally satisfactory in terms of defining an ontological interpretation that meets the criteria and constraints set by a naturalistic methodology.' Dorato and Morganti 2013, pp. 605-606).

One might immediately balk at the assertion that accepting individuality into our metaphysical pantheon as a primitive offers any gain in simplicity. Of course, the demand for reducing the number of such primitives in any system is a staple of metaphysicians' debates. However, evaluations of simplicity are notoriously problematic, even in science (from where such evaluative criteria are typically imported and of course further questions then arise as to the appropriateness of

[16] Similarly, Jantzen argues for the strong claim that '… identity and cardinality are tied together as a matter of meaning irrespective of metaphysics' (Jantzen 2019, p. 288). Consequently, he maintains, talk of entities without identity is either meaningless or, in fact, talk about something else altogether (he suggests mass terms). Perhaps the only response to such blunt claims as those made by Berto and Jantzen is to insist that *quasi*-cardinality is the appropriate formal counterpart to 'countability' when it comes to quantum entities, for which the 'conceptual link' between cardinality and identity must be taken as broken.

their use in a different domain; see Saatsi 2017b) and one may pay for such simplicity via reduction in terms of further complexity elsewhere (this is a core issue in the debate between Dasgupta and Turner for example and is adduced by Dorato and Morganti as a further reason for preferring their approach (op. cit.)).

One can engage in a similar 'to and fro' when it comes to the gain in 'clarity and conservativeness' afforded by primitive individuality but I shall set that to one side. More crucially for the current discussion is the assertion that acceptance of this notion does not necessarily conflict with a naturalistic stance in general. The basis for this is the claim that the formal language of quantum theory manifests a certain notion of 'countability' in the sense that the theory assumes that we are dealing with a system of *n* entities to begin with and, further, this notion can be regarded as metaphysically and physically significant in itself (Dorato and Morganti 2013, p. 606). The manifestation of *n* entities is thus taken to have an ontological counterpart such that we can consider them to be *n* individuals, independently of any consideration of any other factor (such as those to be considered below). Thus, 'Primitivism … provides us with the most straightforward and uncomplicated ontological interpretation of the theory.' (*ibid*.).

Now, this should not be taken to suggest that it is 'countability' that is serving as the ground for the attribution of individuality, in which case one might wonder in what sense the latter can be understood as primitive.[17] Rather, the idea seems to be that the formal language of the theory yields a determinate number of entities; hence the corresponding items within the theory – labels or whatever – are countable; this formal countability has an ontological counterpart and on the basis of *that* we can take the entities to be individuals, where this individuality is not be understood as reducible to anything else. A more serious concern has to do with some ambiguity in what we are supposed to take as the 'formal language' of the theory. Thus it is stated (*ibid*., p. 606, fn 14) that a similar idea is expressed in (Ladyman and Bigaj 2010) where it is assumed that anything that is the value of a first order variable can be taken to be an individual. The same Quinean line (see Dorato and Morganti op cit., p. 593 fn1) is taken by Saunders in his approach, to be considered below, but has been criticized for begging the question against the non-individuals package that, as we've seen, adopts a different formal framework. Elsewhere (Dorato and Morganti *op. cit*., p. 607) it is left unspecified whether we are supposed to consider such a formal reconstruction of 'the' theory (whatever that is taken to be) or rather the language of the theory as it appears 'au naturel' as it were, in papers, presentations, textbooks and so forth.[18] In the latter case it is not at all clear, at the very least, whether there is always the above manifestation of countability (as an example of the difference between the Quinean and 'au naturel' presentations of a theory, take a look at Landau and Lifshitz 1977). But even in cases where it is, the advocate of the non-individuals package could always insist that there is also a notion of 'countability' expressed within their framework, only now it is captured formally via 'quasi-cardinality' rather than the usual cardinality plus ordinality combination (and here we might recall what

[17] Again we recall Berto's claim: if individuality is 'one with being one' then if the latter is primitive, the former has to be as well, given their identification.

[18] I am sceptical as to whether one can even assert that there is *a* theory that is subject to different reconstructions or presentations (French 2020).

we've said above). Thus having a notion of countability on hand does not necessarily provide the basis for claiming that the entities concerned are individuals, understood in primitive terms or not.

Having said all that, appealing to the device of primitive individuality does not undermine my overall project here. On the contrary, given the non-naturalistic flavour of haecceity, as one option, allowing primitive individuality as a naturalistically more acceptable metaphysical alternative could be seen as further enhancing the particles-as-individuals package and thereby strengthening the metaphysical underdetermination. Nevertheless, whether for reasons of simplicity or clarity or even better naturalistic fit in general (despite what is argued in Dorato and Morganti op. cit.) one might prefer to ground individuality in the relevant properties that are represented within the theory, whether formally reconstructed or not. However, as noted above, that requires the Principle of Identity of Indiscernibles which has been ruled out in the quantum context. Fortunately, Saunders has revived a form of the Principle by himself delving into a kind of 'toolbox' – Quine's – and retrieving a notion of 'weak discernibility' that can be deployed in the case of quantum theory (Saunders 2006b).

The core idea is that two individuals can be said to be weakly discernible if they enter into irreflexive relations of the form '… has different P from …', where P is some predicate. Since two fermions in a singlet state, say, must possess different spins (one will have spin 'up', the other spin 'down') they can be said to enter into such a relation and hence are weakly discernible and can be regarded as individuals in this sense. We can understand this approach as offering a further means of articulating the quantum particles-as-individuals package but insofar as the individuality is grounded not in some metaphysical notion of primitive thisness or whatever but in certain relations holding between the particles in a certain state, it might be seen as more naturalistically acceptable than the alternatives. Furthermore, not only does this provide another nice example of the deployment of a metaphysical device, it also illustrates how such a device, having been initially discarded, may see its fortunes restored, as it were – Leibniz's Principle was rejected as false in quantum mechanics, only for Saunders to re-tool it using this Quinean notion of 'weak discernibility' (for further discussion of such moves, see French and Mckenzie 2015).

Needless to say, however, matters are still not entirely straightforward. As the above brief sketch indicates, weak discernibility as originally presented only applies to fermions that can exist in such singlet states. Bosons, on the other hand, do not enter into such irreflexive relations. One possible option is to draw a distinction between fermions and bosons that accommodates this difference in discernibility. Thus Saunders has suggested that whereas the former are 'material' particles, the latter are force 'carriers' and hence, as mere field excitations, might not be expected to be discernible. The obvious worry is that this seems to be a bit of an *ad hoc* move. Subsequently, Saunders' original argument was generalized, to conclude that fermions are always weakly discernible and bosons sometimes are, depending on the state they happen to be in (Muller and Saunders 2008). However this generalization has itself been criticized on the grounds that the operators appealed to in the bosonic case are unphysical, and those in the case of fermions are multiples of the identity, hence do not correspond to any physical relation (Huggett and Norton 2014). Fortunately, again, physically appropriate sets of observables can be constructed in terms of which it can be shown that both bosons and fermions can be categorically weakly discerned in certain states and probabilistically so in all (where the former pertains to being in an eigenstate of the relevant observable and the latter to possession of a given expectation value of an observable; *ibid*.).

Nevertheless, one might still have concerns about the use of this particular tool of 'weak discernibility'. Let us go back to the core idea, exemplified in the case of fermions by the irreflexive relation '… has different spin from …' conferring weak discernibility upon them. A well-known worry about the use of relations to ground individuality is that insofar as a relation is taken to hold between two relata, this move presupposes the identity of the latter. The issue here is one of ontological priority. One response is to shift that priority to the relations, so that instead of talking of relations holding 'between' two relata, already ontologically 'in place', as it were, one conceives of the latter as being constituted, in some sense, out of the former. Of course, effecting such a shift is precisely what has been taken to characterize structuralism in general (for further discussion, see French and Krause 2006, pp. 175-177). And of course it involves the (partial) abandonment of the object oriented stance, which is precisely the response to the underdetermination to be discussed below. Whether one sees this as a further cost or a positive feature of the 'weak discernibility' approach obviously depends on one's sympathies towards the structuralist tendency.

There is, however, an even deeper issue here that has to do with the justification for attributing properties to quantum systems in general. Standardly, this attribution is provided by the so-called 'Eigenvalue-Eigenstate Link' which, simply put, states that a given property can only be attributed to a system if and only if that system is in the corresponding eigenstate of the observable concerned. Typically this is used to attribute monadic properties, such as spin, to systems but as Muller and Saunders note, a crucial step in their argument for the claim that fermions are weakly discernible is to extend the Eigenvalue-Eigenstate Link to relations (Muller and Saunders 2008, p. 515). Norton, however, has raised concerns about this step; in particular, with regard to the attribution of the irreflexive relation 'has opposite spin to' to the specific formal expression used in Muller and Saunders' argument (Norton 2015). He asks three questions: why should we take this formal expression to represent any physical property of the system at all? If we do so take it, why should we take it to represent a dyadic property and not some property of different arity? And even if we accept *that*, why should we take it to represent a symmetric and irreflexive dyadic property?

As he goes on to note, what needs to be provided here is an extension of the Eigenvalue-Eigenstate Link that would enable us to move from the particular formal expression to the relevant n-adic property (*ibid*., p. 1194). Unfortunately, Muller and Saunders do not provide such an extension, nor is one part of the standard formulation of quantum mechanics. Furthermore, even if we were to accept that such a move could be made, some further justification would need to be given to interpret that property as 'has opposite spin to'. Here things are unclear, to say the least, since the relevant formalism as it stands does not rule out other interpretations of that formal expression (*ibid*., pp. 1195-1196).

Interestingly, part of the justification for the interpretation in terms of 'opposite spin to' refers back to an earlier argument of Saunders (2003) but as Norton notes (2015 p. 1196 fn 9) this in turn draws on Mermin's 'relationalist' interpretation of quantum mechanics that eschews relata to begin with (as noted in French and Krause 2006, p. 175) and so, in that case, there would be no objects to be weakly discernible and the status of PII would, again, be obviated!

More significantly, perhaps, Norton claims that what does the 'heavy lifting' in the identification of the relevant property as 'has opposite spin to' is simply the assertion that each of the two particles in the relevant state happen to have opposite values of spin (*ibid*., p. 1196). However, as he goes on to argue, a possible circularity arises here and to avoid it, we need to verify that assertion. Unfortunately, given that the particles are in an entangled state, neither one can be isolated in order to determine its spin. And granted that the relevant observable does pick out opposite eigenvalues in the case of the singlet state, one still has to provide grounds for identifying a given part of the relevant expression as representing which particle. Here it is easy to be misled by the way we write the relevant expression:

$\psi = 1/\sqrt{2}(\uparrow\downarrow - \downarrow\uparrow)$

Tempted as we might be to read ↑↓ as 'particle one with spin up and particle two with spin down', treating the terms in this way as separately giving information about the state is akin the regarding the above expression as describing a statistical mixture, rather than an entangled state (*ibid*., p. 1197). It is the state as a whole that has physical content, not what appear in the formalism as separate components. Thus, the above piece of formalism '…may tempt us into thinking that its pattern of arrows is revealing hidden relations between the single particles.' (ibid., p. 1198). However, absent further justification, such temptations should be resisted and hence, Norton concludes, '…there is nothing in the singlet state's description that warrants attributing opposite spin to its particles.' (*ibid*.; see also Calosi and Morganti forthcoming)

In fact, a similar concern has been raised previously, both in the context of an early consideration of Saunders' argument (French and Krause 2006, pp. 174-177) and with regard to the debate over the status of the 'standard' form of the Principle of Identity of Indiscernibles; that is, without weak discernibility (Massimi 2001; see also French and Krause 2006, pp. 166-168). It hinges on the question, why should we take the relevant expression – such as the example above – whether presented in a formal reconstruction of the theory or in the form of the latter as given in textbooks and the like, as representing two objects in the relevant state to begin with (cf. Norton 2015, p. 1198)?

Of course it is not enough to say, well that is simply how the expression is written; that is, in such a way that it invites interpretation in terms of two objects being in the relevant state. As Norton has rehearsed and as sketched above, such an interpretation is not straightforward even if the theory is taken 'au naturel'. And of course, more generally, when we write down the appropriate expression for a collective of particles, we might begin by assigning 'particle 1' say to 'state 1', 'particle 2' to 'state 2' and so on but then, following Schrödinger's prescription, we apply the particle permutation operator to obtain all possible permuted combinations and then select out of those the symmetric and antisymmetric forms (leaving to one side those corresponding to parastatistics, of course). In effect we begin by assuming the particles are objects that can be labeled and hence putatively attributed with individuality but then we permute these labels in such a way that we cannot 'tell' which particle is which so that the individuality is either 'lost' or, at best, 'masked' (Post 1963). But what is the basis for this assumption to begin with? We could appeal, again, to the observation of particle 'traces' in various experimental contexts but at best that allows us to attribute Toraldo di Francia's 'mock' individuality and as already noted, we cannot presume that this can straightforwardly be exported out of those contexts (again see French and Krause 2006, p. 166).

Alternatively, we can acknowledge, with Norton, that weak discernibility cannot be attributed to the particles and thus that there is simply no basis for contemplating the status of even this modified form of Leibniz's Principle in the quantum domain. Of course, that still doesn't necessarily rule out the particles-as-individuals package – one could ground their individuality in haecceities and metaphysically accommodate the entanglement in terms of some other device, such as relations of 'non-supervenient' relations (Teller 1986; see also French 1989b) or 'symmetric dependence' (Calosi and Morganti forthcoming).[19] As noted already, the extent to which one sees such options as running counter to an appropriate form of naturalism depends on one's attitude towards the use of such metaphysical devices.

Let us pause and review.

As we have seen, both 'packages' can be elaborated in terms of a variety of formal and metaphysical 'devices'. One could of course argue that the deployment of somesuch device or other confers a significant advantage of one package over the other and thereby 'breaks' the underdetermination. However, as we've also seen, whatever device one picks, its deployment is going to be subject to an array of caveats and counter-arguments of not only different strength – insofar as that can be measured – but of very different kinds, with the result that comparing them is not just a matter of 'apples vs. oranges' but of a whole basket of tropical fruit! So, for example, for all that the notion of 'weak discernibility' can be subjected to the above criticisms, it at least has the virtue of being presented within a Quinean and hence classical logical and set-theoretical framework; whereas if you feel that those criticisms, together with the unalloyed metaphysical nature of haecceities, compel you to adopt the non-individuals alternative, you have to accept the costs associated with quasi-set theory. And of course, none of these metaphysical moves and manoeuvres will impress van Fraassen in the slightest!

Back to the Challenge

Returning to his challenge, then, we can now better appreciate, perhaps, the motivational force of the underdetermination in pushing us to avoid all the above philosophical 'to and fro' and to drop the commitment to objects to begin with. Of course, shifting to a structuralist ontology does not obviate the need for some metaphysical explication and the above 'spectrum' running from 'shallow' to 'deeper' forms of realism can still be invoked. Indeed, on an autobiographical

[19] Norton suggests that we might interpret entangled states as representing 'emergent unities' (2015 p. 1198) but it is not clear what this could mean (cf. Howard 1989).

note, back in the early days of the ontic structural realist project, when asked 'what is this thing called structure?' I would point to some expression of the relevant piece of physics – such as the commutation relation between the particle permutation operator and the Hamiltonian for a system, expressing the role of permutation symmetry – and bluntly state 'It's that!'. Justifiably, perhaps, this was felt to be an unsatisfactory response, not least because it generated the impression that the structure to which commitment was being invited was mathematical structure and that ontic structural realists were Platonists (for a response see French and Ladyman 2003).

Subsequently, the above 'rummaging in the toolbox' methodology has been deployed, twice over, as it were: first to history, drawing on the structuralist developments of Cassirer and Eddington that were 'effaced' in the standard expositions of structural realism. Thus, appropriating Cassirer's framework, the structure of the world is envisaged as a kind of 'Parmenidean sphere' consisting of symmetry principles, laws and definite measurement outcomes, standing – metaphysically at least – in a non-hierarchical relationship (French 2014). One can then fill in that template in a number of ways.[20] So, for example, one might adopt a Humean form of structuralism, according to which those laws that feature in our best theories are nothing but expressions of the fundamental regularities that effectively, but only partly, constitute the world. I say 'only partly' because the Humean faces certain difficulties in accommodating symmetries (Mckenzie 2014b) although steps have been taken to overcome these (Duguid 2017).

Alternatively, one could acknowledge the role of modality in the world and incorporate 'powers' or dispositions into this picture, but with the 'seat' of such powers being not objects per se, but the structure itself. Again the nature of symmetry principles presents an obstacle to the adoption of the standard characterisation of such dispositions in terms of the 'stimulus and manifestation' condition, forcing the appropriation of some other device from the metaphysicians' toolbox, such as 'potentiality' (see French 2020b).

Finally, one could adopt the view that the laws are primitively modal (Maudlin 2007) and extend this to the structure of the world as a whole (French 2014). Thus Permutation Symmetry may be considered to be inherently modal in the sense that it encodes a whole range of possibilities, including those corresponding to 'parastatistics' as well as the more commonly discussed bosonic and fermionic forms discussed here (French 1987). In one sense, of course, this sits close to the dispositionalists' account, insofar as it accepts modality 'into' the world; but in another, it is not so distant from Humeanism in that it too denies any 'governing' role for laws and symmetries and effectively identifies them with features of (the structure of) the world.[21]

[20] This template can be seen as one way of understanding the 'Big Tent' structuralism pursued in French 2014, with the different 'fillings in' as different denizens living within the tent!

[21] One might worry that one underdetermination has been avoided only to introduce another here! However, the two forms are different insofar as these alternative 'fillings in' of the structuralist template are not equivalent to the two kinds of 'individuality profile' for objects. The former are independent of the notion of structure itself and hence the eliminativist argument does not get a toe-hold. Once again, thanks to Claudio Calosi for encouraging me to be clear on this.

Of course, this world is actual, not possible and so the structure of this world is not wholly modal – the possibilities represented by parastatistics do not appear to be actualised, for example. In this regard, Bose-Einstein and Fermi-Dirac statistics can be thought of as 'existential witnesses' (Wilson 2012) to the actuality of this world. This brings us to our second use of the methodology of 'rummaging in the toolbox', but this time the box is full of metaphysical devices[22]: we can capture the relationship between Permutation Symmetry, say, and the standard forms of quantum statistics in terms of that which holds between determinables and determinates (French 2014; see also French 2019). Generalising this to the other fundamental symmetries we can capture the relationship between these and the fundamental properties of particles, offering a 'top down' approach to the structuralist reconceptualisation of quantum particles (French 2014).

That accommodates the various particle kinds but still leaves the relationship between symmetries, laws and definite measurement outcomes to be accounted for. The approach taken to this will in part depend on one's chosen solution to the infamous measurement problem and again in the spirit of a 'Big Tent' approach to structuralism, one could delineate different forms depending on which interpretation of quantum mechanics is adopted.[23] The Everettian interpretation, for example, is perhaps most obviously compatible with structural realism (Wallace 2003) but it is not so hard to perceive how the 'probability field' of (some understandings of) the GRW approach could also be given a structuralist gloss; and despite appearances, even the Bohmian line might be amenable to similar treatment (French 2001). In effect this is to acknowledge, that even if one accepts the move to structure as a way of resolving the metaphysical underdetermination, there is a further underdetermination to deal with as represented by these (and other) different interpretations of the theory. Resolving *that* requires consideration of a further array of factors that I can't consider here (see the papers in French and Saatsi 2020).

Finally, there is still something else that should be accommodated by this filling in of the structuralist template that hasn't yet been considered: entanglement, regarded by Schrödinger, famously, as '…*the* characteristic trait of quantum mechanics ' (1935, p. 555). This is typically characterised in terms of the non-factorisability of the relevant wave function representing the state of two (or more) systems that have interacted (*ibid*., p. 556). However, as has been noted (see, for example, Ladyman, Linnebo and Bigaj 2013) some care must be taken here: anti-symmetrised states such as the fermionic singlet state discussed above are also non-factorisable[24] but do not exhibit the problematic empirical consequences typically associated with entanglement such as measurement

[22] Glick suggests that ontic structural realism should be understood as a 'meta-metaphysical' stance in the sense of acknowledging that metaphysics is simply not capable of providing us with an appropriate notion of fundamental structure and that this can only be achieved through mathematical representation. This does allow a role for metaphysics but only as 'pragmatically' illuminating the structure of the world for the practical purposes of communication, say. Here I am suggesting that metaphysics can play a more profound role in offering some understanding of this structure.

[23] This is why the criticism that the ontic structural realist has not given her own solution to the measurement problem is perhaps a little unfair (Esfeld 2015)!

[24] Certain symmetrised states are also non-factorisable but others, corresponding to the arrangements of two bosons in the same state, do not manifest entanglement.

correlations that violate the famous Bell inequalities. Thus the ontic structural realist cannot accommodate entanglement by simply folding it into the role of Permutation Symmetry, say.

One option would be to extend the relationalist approach hinted at above from the fermionic singlet state to entangled states in the latter sense. However this immediately runs afoul of the afore-mentioned concerns as to whether the relevant mathematical expression can even be taken to represent a relation to begin with (for consideration of these concerns in the specific context of entanglement, see Calosi and Morganti forthcoming). And of course, if such relations were taken to underpin 'weak discernibility', understood as grounding the individuality of the particles, then even if it could be pulled off, such a move would only work for so-called 'moderate' forms of ontic structural realism that still allow some role for objects (even a much attenuated one; for criticisms see French 2014 pp. 178-180).

Furthermore, even if the above concerns are put aside, and the relevant expressions are understood as representing relations, and it is acknowledged that this approach would only work for certain forms of structural realism, there remains the issue of accommodating the relationship between the relevant relation and the particles, where the former yields the weak discernibility of the latter. One might dive into the metaphysicans' toolbox yet again and appeal to some form of dependence here (French 2010). However, insofar as the relevant states can be expressed in terms of different observables, such as spin or position, say, so that different relations are in play, the form of dependence will be only 'generic' in the sense that it does not pick out which is the ontologically fundamental relation (McKenzie 2014a; Calosi and Morganti forthcoming). Here perhaps the issue of which is one's favoured interpretation intrudes again: the structuralist Bohmian (if there is such a beast!) would certainly argue for position relations being fundamental (on the usual grounds that all measurements are ultimately position measurements) and construct all other observables, including spin, from those. Alternatively, one could agree that position is fundamental in this sense but decline to engage in the Bohmian construction, noting instead that we can understand spin as a determinate of Poincaré symmetry. There is, as critics have noted, further work to be done (Calosi and Morganti ibid.).

Another device might be that of non-supervenience, already mentioned above and originally introduced precisely to offer a new metaphysical understanding of entanglement within a broadly relationalist framework (Teller 1986). Indeed, this was cited in Ladyman's classic work on ontic structural realism (Ladyman 1998) as a further motivation for shifting to a structuralist stance. Again, however, this seems best suited to the 'moderate' forms insofar as 'non-supervenience' is cashed out, negatively, in terms of the relevant relations not supervening (surprise surprise!) on the intrinsic or other properties of the particles; indeed, as also already noted, it has been taken to mesh nicely with the particles-as-individuals package (French 1989b).

Presumably non-supervenience in this case would also be dismissed as 'generic' but one could make the same sort of move as above and obtain the different kinds of non-supervenient relations by appealing to, for example, the way certain properties 'drop out' of the relevant symmetry. This move perhaps offers one way of meeting the challenge that '… we need to be told what exactly the physical content of the relevant relations is, which observables the correlated quantum objects depend on.' (Calosi and Morganti forthcoming). Nevertheless, if one is uncomfortable with such metaphysical devices, one might simply take the structure of the world to be represented, in part, by Hilbert space, as again alluded to in Ladyman's original piece (1998).[25] So, we recall that the standard way of representing the states of systems is in terms of the tensor product Hilbert space. The action of Permutation Symmetry then divides that space into different 'symmetrised' sectors – with two particles, only two sectors are possible, namely the antisymmetric and symmetric, corresponding to fermions and bosons respectively of course but for three or more particles other sectors become available, corresponding to paraparticles which, as already mentioned, do not appear to exist in this world. The fermionic singlet state underpinning the attribution of weak discernibility has the characteristics that it does by virtue of falling within the anti-symmetric sector.

This still does not accommodate the empirically more interesting form of entanglement but again, what the structuralist says on this may depend on certain other interpretational moves. So, for example, for Hilbert spaces of dimension greater than three, the Kochen-Specker theorem holds, which means that, putting things negatively, the structure of the world is such that it is impossible to assign values to all physical quantities whilst, at the same time, preserving the functional relations between them (Isham and Butterfield 1998). In effect, the theorem requires any no-hidden-variables interpretation of quantum theory to be contextual. Likewise, one can understand Bell's Theorem as ruling out any *local* hidden variables theory and given that this has been described as 'the most profound discovery in science' (Stapp 1975, p. 271), the structuralist might be justified in taking it to represent a fundamental feature of the structure of the world. Characterising that feature more positively however bumps up against the afore-mentioned interpretational attitudes, as an Everettian will have a very different view of what the theorem tells us than the Bohmian, say. All of this is to acknowledge that there is indeed more work to be done (again see Calosi and Morganti) but in fairness, insofar as it involves such attitudes, it is work that the non-structuralist has to do as well!

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[25] Again, issues of interpretation intrude as a structuralistically inclined wave function realist will insist on substituting configuration space for Hilbert space and the fundamental ontological arena (French 2013).

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