From Arbuthnot to Boltzmann:
The Past-hypothesis, Special Sciences and the Best System

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1. Introduction

In recent work on the foundations of statistical mechanics and the arrow of time, Barry Loewer and David Albert have developed a view that defends both a Humean account of laws and a physicalist fundamentalism. A proper understanding of the foundations of statistical physics, they argue, provides the missing piece for a successful completion of David Lewis's project of a unified account of laws, causes, counterfactuals, and probabilities that is at once Humean and foundationalist. In this paper I will critically examine their account and argue that there exists a tension between their account of laws and their reductivism or foundationalism. Albert and Loewer's version of a Best System account of laws departs from Lewis's own account by further emphasizing the pragmatic dimension that arguably was already implicit in Lewis's account. Here I find myself in broad agreement with them. But, as I will argue here, this pragmatism is in tension with the foundationalism to which they also subscribe.

I will proceed as follows. In section 2 I briefly summarize Albert and Loewer's account of the foundations of statistical physics and its place in a Lewisian conception of the world. In sections 3 and 4 I discuss worries that arise from the fact that their account, unlike Lewis's original account, allows for initial conditions to be laws and for terms referring to non-fundamental properties to be part of the language in which the laws are formulated. In section 5 I argue that pragmatic considerations similar to the ones which Albert and Loewer themselves invoke suggest that the laws of the special sciences should be added as additional axioms to the Best System. In section 6 I critically examine an argument by Albert aimed at defending his fundamentalism against an objection due to Philip Kitcher. I side with Kitcher and argue that the special sciences can offer explanations of the phenomena in their domain that are explanatorily independent from putative derivations of these phenomena from fundamental physics.

2. From Boltzmann to Lewis

According to the Boltzmannian account defended in (Albert 2000), the thermodynamic asymmetry that the entropy of a closed macroscopic system never decreases can be explained by appealing to a time-symmetric micro-dynamics and an asymmetric constraint on initial conditions. If we assume an equi-probability distribution of micro-states compatible with a given macro state of non-maximal entropy, then it can be made plausible that, intuitively, 'most' micro states will evolve into states corresponding to macro states of higher entropy. However, if the micro-dynamics governing the system is time-symmetric, then the same kind of considerations also appear to show that, with overwhelming probability, the system evolved *from* a state of higher entropy. This undesirable retrodiction, which is at the core of the *reversibility objection*, can be blocked, if we conditionalize the distribution of micro-states not only on the present macro-state but also on a low-entropy initial state of the system. Since the reversibility objection can be raised for any time in the past as well, Albert and others argue that we are ultimately led to postulate an extremely low-entropy state at or near the beginning of the universe. Albert calls this postulate "the past hypothesis" (*PH*). Albert and Loewer maintain that the statistical physical account (*SM*) consisting of the past hypothesis, a probability postulate *PROB*, which postulates a uniform probability distribution over the physically possible initial micro-states of the universe, and the dynamical micro-laws (which in this context are assumed to be deterministic) provide us with a complete physics—our fundamental theory of everything.

According to Albert and Loewer, both *PH* and *PROB* should be thought of as nomic constraints, even though they are statements about the initial conditions of the universe. For then, Loewer argues, the Boltzmannian account provides the missing piece in a broadly Lewisian account of counterfactuals and causation. Lewis himself attempted to derive the temporal asymmetry of counterfactuals and causation from a thesis of and asymmetry overdetermination. But this thesis is false and Lewis's account of the asymmetry fails (see Elga 2001, Frisch 2005, ch. 7). In its stead, Albert and Loewer appeal to the *SM* account as source of the asymmetry. Roughly, according to Albert and Loewer's amended Lewisian account, we evaluate counterfactuals by calculating the probability of the consequent, conditional on the laws of the actual world, the counterfactual antecedent event *c* at some time *t*, and the actual macro-state at *t* outside of the region where *c* occurs. The asymmetry is, then, supposedly a consequence of the fact that the laws include a time-asymmetric constraint, *PH*. And the reason for why *PH* is a nomic constraint rather than merely a de facto initial condition is that it is an axiom of the Lewisian Best System—the theory of the world "couched in a vocabulary that includes a conditional probability function, mathematical notions, and terms that represent fundamental properties" that "best combines simplicity, informativeness and fit." (Loewer 2007, 305)

3 Initial Conditions and Simple Systems

Loewer argues that *PH* and *PROB* are part of the best system, since adding the two conditions to the micro-dynamical laws "results in a system that is only a little less simple but is vastly more informative than is the system consisting only of the dynamical laws" (Loewer 2007, 305). That is, on Loewer's view, *all* axioms of the best deductive system qualify as laws, and not just the axioms which state regularities. In this respect Loewer's account departs from Lewis's original proposal, according to which only genuine regularities but not initial conditions could qualify as laws. On Lewis's account, the strength of different deductive systems is evaluated by including a statement of the relevant initial conditions, even though the initial conditions themselves are not candidates for being laws.

Allowing initial conditions to be laws is problematic, however. Consider a relatively simple 'world' that consists of nothing but a simple gravitational system, such as the Solar system, that satisfies Newton's gravitational theory. If we compare a deductive system for this world consisting of Newton's laws alone with a system that also includes a statement of the world's instantaneous state at some time, then the ranking of the two systems seems obvious: the initial conditions, specifying the instantaneous positions and momenta of the planets are not overly complex, while the gain in informativeness over a system without initial conditions is quite dramatic: Newton's laws on their own do not determine the positions or momenta of any planet, but adding the initial conditions determines the entire history of the world. Thus, according to Loewer's account, the statement of the initial conditions comes out as a law. But this has the consequence that the *entire* evolution of the world is nomologically necessary, since the state of the world at any time is derived from nomic constraints alone. Thus, the intuitive distinction between laws and nomologically contingent initial conditions seems to be obliterated for such a world.

This strikes me as a serious problem for any account that allows for all axioms of the best deductive system to count as laws. One condition of adequacy of any account of the notion of scientific laws is that it allow us to recover core features of the usage of the term 'law' by scientists—that it match scientific practice—and one such feature clearly is the distinction between laws and contingent boundary and initial conditions applied even to relatively simple systems. In examining the properties of a theory and even in applying the theory to real-world systems physicists routinely investigate very simple models or possible worlds allowed by a theory's dynamical laws, such as a simple gravitational system. Loewer's account has the seemingly absurd conclusion that *for each different initial state* of the system, the system could not have possibly been in any other state, since it is a law of the system that it started out in that state.

4. Comparing Different Systems

Lewis's Best System account requires that the axioms of the best system relate only fundamental properties. Albert and Loewer's account, by contrast, allows for *PROB* and *PH*, which contain terms referring to non-fundamental macro-properties, to be laws as well. Yet once we allow axioms to be formulated in different vocabularies and do not restrict our language to that of fundamental physics, we are faced with the problem of how to compare different deductive systems formulated in terms of different languages. As (Cohen and Callender 2009) argue, both simplicity and strength are immanent notions that are relative to an inventory of basic kinds or predicates. *PH* is a very simple hypothesis from the perspective of a language that includes terms referring to macro-properties, but is extremely complicated from the perspective of the language of micro-physics. As Cohen and Callender (2009) say, "if we translate the claim that entropy was low into a more fundamental microlanguage we end up with a long gruesome mess. […] The translated claim would still be strong, but hardly simple. For this reason, written in the language of the (*ex hypothesi*) fundamental kinds, the Past Hypothesis cannot be a MRL law."

Allowing initial conditions to be laws poses additional problems for the comparison of different deductive system. In relatively simple worlds, as we have seen, adding a statement concerning initial conditions leads on balance to a better system. Since the dynamical laws on their own do not imply any particular matter of fact, adding relatively simple initial conditions vastly increases the strength of the system with only a minimal loss in simplicity. But how do we balance simplicity and informativeness if we are trying to adjudicate between the system that contains a statement of the actual dynamical micro laws of our universe and the system that contains in addition a statement of the precise microscopic initial state of the universe? In this case it is no longer obvious how to rank the two systems. On the one hand, a statement of the complete initial conditions of the universe at one time would be extraordinarily complicated, on the other hand the gain in informativeness is enormous: The system consisting only of the dynamical laws implies not a single particular fact about the instantiation of any properties at any spacetime point, while the system that includes a statement of the initial conditions implies the complete history of the universe. It is unclear, at least to me, how one might possibly adjudicate between the two systems and how to balance the dramatic increase in complexity against the (equally as?) dramatic increase in informativeness. (Note that there is no such problem for Lewis's account: for Lewis the two systems, with and without initial conditions respectively, have exactly the same laws.)

One might reply that there need not be a complete ordering of all candidates for Best System. It might be enough if there was a partial ordering with a clear 'winner'—a winner, which Albert and Loewer would insist, is the system consisting of the true micro-dynamics in conjunction with *PH* and *PROB*. Yet if we compare this system with the system that contains a statement of the precise initial state of the universe, it is still not clear which system maximizes simplicity and strength and how we should balance the dramatic increase in informativeness in the system with complete microscopic initial conditions against the vast increase in complexity.

Now, Albert and Loewer's account has the resources to answer both problems of comparison. First, Loewer proposes that the simplicity of a system be measured not only in terms of the number or syntactic complexity of the its axioms, but also in terms of the lengths of the proofs required for deriving different theorems. Second, Albert and Loewer introduce a strongly pragmatic element into the metric for comparing different systems, which is brought out particularly vividly in their imaginary tale of how the best system is revealed to us during an audience with God. Here is how Albert describes the scenario. Imagine that you have an audience with God who provides you with as much information about the particular facts of the worlds as you could possible want to have. One way to provide this information is to recite long lists of particular facts concerning which properties are instantiated at which spatio-temporal locations. Yet as God begins to recite particular matter of fact,

> it begins to look as if all this is likely to drag on for a while. And you explain to God that you're actually a bit pressed for time, that this is not all you have to do today, that you are not going to be in a position to hear out the whole story. And you ask if maybe there's something meaty and pithy and helpful and informative and short that He might be able to tell you about the world which (you understand) would not amount to everything, or nearly everything, but would nonetheless still somehow amount to a lot. Something that will serve you well, or reasonably well, or as well as possible, in making your way about in the world. (Albert, unpublished manuscript)

The meaty and pithy information with which God provides you, Albert claims, will consist of the micro-dynamical laws together with *PH* and *PROB*.

Once we emphasize that it is *our* standard of simplicity and usefulness that is the ultimate criterion by which we adjudicate between systems, both Cohen and Callender's worry and the problem of comparing systems with or without initial conditions can be answered. The yardstick for simplicity and informativeness is how practically useful a system is for us—*how well it allows us to make our way about in the world*. For beings like us the deductive system that includes the *PH* is clearly simpler—even though from the perspective of the language of the fundamental micro-theory, stating the *PH* would be a 'gruesome mess.' And arguably both the system consisting only of the dynamical laws and that consisting of the laws and the exact initial conditions fail dramatically as far as their usefulness for us in making our way about in the world is concerned: The former system does not allow us to derive any particular goings-on in the world, not even in principle, while the latter is much too complicated to be of any practical use. Indeed, specifying the exact initial conditions of the universe, Albert says, would "violate" the stipulation of providing a simple summary. Thus, practical usefulness, for him, provides not only a criterion of *relative* goodness for a system but also a *necessary* condition for being minimally acceptable: a system that includes axioms that are too complex is practically useless and hence could not be the Best System, no matter how informative it might be in principle or how much more informative it might be than any of its competitors.[1]

There is another argument for including *PH* and *PROB* in our system. The evolution towards higher entropy states and eventually toward equilibrium is an evolution from a macroscopically atypical state—from a state associated with a tiny volume of phase space—to a macroscopically typical state. This macroscopic feature of the evolution of the universe cannot be captured in the purely microscopic theory. The micro-theory allows us to derive the universe's micro trajectory in phase space, but it is not apparent from the micro-theory alone in which sense the forward evolution of the universe is relatively insensitive to changes in the precise micro state (since most micro-states will evolve toward equilibrium macro-states) while the evolution backward in time is highly sensitive to slight changes (since most micro-states did not evolve from an extremely low entropy initial state). To be sure, if we knew the precise initial micro-state and the micro-dynamics, then we could derive the entire history of the universe. But this derivation would miss important macroscopic structural features of that evolution: it would miss the sense in which the precise details of the micro state are irrelevant in accounting for features of the future evolution of the universe *that are important to us*, while the precise details are not similarly irrelevant in ensuring an evolution backward in time with the same macro-properties. Thus the addition of the macro-constraints *PH* and *PROB* plays an important explanatory role.

The lesson to draw from the problems of comparison for Albert and Loewer's account is that the strong pragmatic element in their view plays an important role: without

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[1] Note that Cohen and Callender similarly construe simplicity not merely as offering a criterion of relative goodness but also as a necessary condition of minimal adequacy: they maintain that since the *PH* would be too complex if written in the language of fundamental physics it cannot be a law (no matter how informative it might be).

construing simplicity in terms of what is useful for beings like us, their account cannot answer these problems.

5. Why stop here?

Allowing predicates referring to thermodynamic macro-properties as part of the Best System constitutes an expansion of Lewis's original proposal, which only permits axioms containing predicates referring to fundamental properties and kinds. And as we have seen this expansion can be both motivated and defended by appealing to a radically pragmatic construal of what it is for a system to be the best: the best system is the one that summarizes as much information about the world as possible in a way that is useful for us. Now, Albert and Loewer want to stop there. They insist that once we have expanded the system consisting of the micro-dynamical laws to include *PROB* and *PH* we have found *the* Best System and have arrived at a complete, unified, and fundamental theory of the universe. Yet the very same considerations to which Albert and Loewer themselves appeal suggest that we ought to expand our system further and include the laws of the special sciences as well.

According to Albert and Loewer, adding *PH* and *PROB* to the micro-dynamical laws results in a better system than adding a statement of the exact microscopic initial state, since the latter statement would be absurdly complex—too complex to be even minimally acceptable as part of any contender as overall best system. But consider the derivation of any 'law' or regularity of the special sciences, such as Fisher's principle, which explains why the sex-ratio of most species is 1:1. Any derivation of this phenomenon from the fundamental micro-dynamics, *PH* and *PROB* would be absurdly complicated—in fact too complicated to be of any help to us in making our way about in the world. Thus it seems that the *SM* account as well violates the minimal condition of adequacy proposed by Albert of providing "something that will serve you well, or reasonably well, or as well as possible, in making your way about in the world." By contrast, a system that contains in addition to the laws of physics also the relevant laws of evolutionary biology would only be minimally more complex—it would contain only a few more simple axioms—yet the gain in deductive simplicity would be enormous.

Now, Albert and Loewer entertain the hypothesis that all the laws of the special sciences follow from the axioms of the *SM* account as theorems, presumably once we also introduce an appropriate translation manual. This hypothesis is open to challenge, but even if we, for the present purposes, grant this claim, we can insist that the laws of the special sciences be added as axioms to the deductive system. The expanded deductive system will not be deductively stronger, and it will be less simple along one of the dimensions of simplicity identified by Loewer: the system will have more axioms. But, first, the same holds for the *SM* account *vis-à-vis* the system consisting of micro-dynamics, initial micro-state, and translation manual: the former is not deductively stronger than the latter. And, second and more importantly, this loss of simplicity along one dimension will be more than made up for by a gain in simplicity along another dimension that Loewer has identified—the length of the proofs required to derive different theorems. Thus, contrary to their own intentions, Albert and Loewer's pragmatism supports a view similar to the "relativized" and anti-foundationalist Best System account defended in (Cohen and Callender).[2] Once we take the best system to be the one that best allows us to make our way about in the world, not only the postulates of thermodynamics but the regularities of the special sciences end up as part of our overall system as well.

6. Against explanatory fundamentalism

Explanatory considerations also count in favor of including higher-level laws in the Best System. (Albert unpublished) defends the explanatory universality and completeness of physics against an objection by Philip Kitcher. Kitcher contrasts R. A. Fisher's evolutionary explanation of a regularity discovered by Arbuthnot—that in each of the 82 years from 1623 onward the preponderance of children born in London were boys—with a microphysical derivation of the sex of every child and maintains that even if *per impossibile* we possessed a micro-physical derivation of the sex of each child, starting with the microscopic initial state of the world at each copulation, this derivation would not advance our understanding of the sex-ratio and would not show that Arbuthnot's regularity is

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[2] Cohen and Callender speak of many different best systems, each relativized to a set of kinds. What I take to be 'the best system' is the conjunction of all of these different systems.

anything but a gigantic coincidence. Albert argues against Kitcher that if we assume the statistical mechanical account consisting of *PROB*, *PH* and the micro-dynamics—that is, if we start with an equi-probability distribution over all initial micro-states of the universe compatible with *PH*, evolve that distribution forward until the beginning of Arbuthnot's observation period in accord with the (assumed to be) deterministic micro-laws, and conditionalize this on the existence of the solar system and on everything else taken for granted in the discussion of relative birth rates in London—then the resulting conditionalized probability distribution should be exactly in accord with the predictions based on Fisher's evolutionary explanation. If Fisher's explanation is correct, then, so Albert, the probabilities to which this explanation appeals must follow from the true and complete physics. Thus, Fisher's explanation provides us with some information about the "true and complete and universal physical theory", but it could not possibly add to that theory.

I agree with Kitcher that Fisher's explanation adds to the putative foundationalist derivation of the relative birth ratios sketched by Albert. To begin with, note that even Albert's explanation-sketch arguably does not propose a *purely* foundationalist account but already implicitly appeals to Fisher's explanation. Let us grant that Albert is right and that distributions for conditional macro-probabilities of male births could in principle be recovered from a micro-statistical account. But there are myriad such distributions, each conditionalizing on different facts characterizing the macro-state of the Earth and its vicinity. Moreover, Fisher's account, like any macro-theory presumably involves idealizations and abstractions and what is taken for granted in the account is left to some extent vague. Thus, arguably none of the conditional probability distributions recoverable from the *SM* account will match the probability distribution derived by Fisher exactly. Finally, in order to derive the sex-ratios of different species at different times the putative *SM* account would have to conditionalize on certain features of the macro-state at different times. These different macro-states will be similar in countless ways and will differ from one another in countless others. All this raises the worry that even if we were to grant Albert that the *SM* account allowed in principle derivations of probability distributions for sex ratios, the simple account given by Fisher relating birth ratios to sex ratios at maturity (and unifying the probability distributions for different times and species) would be deeply buried within the *SM* account. Fisher's account does not merely give us an observation about a particular probability distribution that follows from the *SM* account, as Albert suggests, but gives us an argument why it is *this* conditional probability distribution, rather than one of the many others implied by the *SM* account that is useful and interesting for us. Only from the perspective of evolutionary biology and its way of carving up the world, it seems, can we detect those conditional probability distributions that are relatively simple and informative about sex ratios at birth.

More specifically, Kitcher claims that a putative micro-account could not explain why Arbuthnot's regularity is more than a gigantic coincidence. I think this claim can be defended, too, against Albert's criticism. Consider a gerrymandered set *S* of events all of which have probability *P*=0.53—the probability of having a boy, according to Fisher's account—conditional on some relatively localized macro-state of the world, such as the state of London in 1623. One would expect there to be a very large number of such events, including, for example the probability that a certain not entirely symmetrical coin comes up heads, say. Now compare the *SM* account of the events in *S* with an *SM* account of the events in the set *B* of male births. The events in both *S* and *B* have negligible probability given just *PH* and *PROB* but all have the same probability *P* conditional on the respective later macro-states. Indeed from the perspective of the micro-physical account the two accounts are equivalent: both proceed by evolving the initial probability distribution forward in time and then conditionalizing on a later macro-states and showing that all events in the two sets have conditional probability *P*. Nevertheless, it seems intuitively that the fact that all the events in the gerrymandered set *S* have probability *P* is a mere coincidence, while the fact that all male births have probability *P* is not: in some intuitive sense, the probabilities of the events in *S*, unlike those of the events in *B*, have nothing to do with one another.

I want to suggest that this intuition can be cashed out in terms of the existence of unifying theories: the events in *B* can be unified *at the macro-level* in terms of the evolutionary account while the events in *S* cannot be similarly unified. The evolutionary account provides an explanation of the probabilities for the events in *B* that is both simple and unified. By contrast, by assumption no such simple unified explanation of the events in *S* is possible. Of course, the *SM* account unifies the phenomena as well—if successful it provides the ultimate unified account of the world—but it only provides us with conditional probabilities of events on the complete micro-physical goings-on entire 'slices' of the world, rather than singling out certain higher-level structural features that may account for the phenomena we are interested in. And it is not evident from the perspective of the micro-level, where for both sets of events we simply have the to-ing and fro-ing of the microscopic particles, that the events in *B* can be given a more highly unified explanation than the events in *S*.[3] By contrast, Fisher's explanation relies solely on certain structural features of evolutionary systems relevant for a derivation of the sex ratio and abstracts from any other details irrelevant for that phenomenon.

One might object that once we grant that the *SM* account implies all the correct probabilities, it seems to follow that the evolutionary explanation cannot possibly add anything, since it similarly is implied by the *SM* account. But it contributes to the goodness of an explanation, if it does not add irrelevant details and if it explicitly singles out the explanatorily relevant factors. To take an example purely from within physics, it constituted a genuine advance in our understanding, when physicists realized that the relativistic mass dependence could be derived solely from the Lorentz-invariance of the underlying dynamics rather than from specific assumptions about the structure of the electron (even when those assumptions implied Lorentz-invariance). Relativistic systems behave the way they do *no matter what* the microdynamics governing them is, as long as it is Lorentz-invariant. Similarly, the evolutionary account of Arbuthnot's regularity holds no matter what the microphysical details as long as they allow for the existence of evolutionary systems. The point here is not merely that these details would be absurdly complex and hence would violate Albert's minimal condition of adequacy, but that it is a virtue of an explanation if it accounts for its explanandum only by appealing to genuinely relevant features of the system at issue.

One might nevertheless wish to insist that the micro-physical account is in some sense deeper since any higher-order regularities are ultimately 'due to' the fundamental laws. The fundamental laws, one might want to say, are what are ultimately responsible for

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[3] In particular, the difference between *S* and *B* cannot be cashed out in terms of a difference of the spatiotemporal regions involved on the micro-level. The events in *S* may be highly localized, while *B* may be the set of human births throughout long periods of time.

the existence of the higher level laws. But this reply is not open to the Humean—especially not to Humeans who allow for laws that are not formulated in terms of a set of fundamental predicates. All there is ultimately, for a Humean, is the mosaic of particular matters of fact. The role of the laws, at all levels, is to provide us with particularly useful ways of summarizing features of that mosaic.

7 Conclusion

I find Albert and Loewer's emphasis of the pragmatic dimension in a Lewisian Best System account very attractive. But it seems to me that this pragmatism sits ill with their insistence on a physical foundationalism. If the role of laws is indeed to provide us with a meaty and pithy and useful summary about what happens in the world, in a manner "that will serve [us] well, or reasonably well, or as well as possible, in making [our] way about in the world", then, as I have argued here, the laws of the special sciences ought to be part of the Best System no less than the past hypothesis or the probability postulate of statistical mechanics.

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